Liveness with $(0, 1, \infty)$-Counter Abstraction*

Amir Pnueli[1], Jessie Xu[2], and Lenore Zuck[2]

[1] Weizmann Institute of Science, Rehovot, Israel
amir@wisdom.weizmann.ac.il
[2] New York University, New York
{jessie,zuck}@cs.nyu.edu

Abstract. We introduce the $(0, 1, \infty)$-*counter abstraction* method by which a parameterized system of unbounded size is abstracted into a finite-state system. Assuming that each process in the parameterized system is finite-state, the abstract variables are limited counters which count, for each local state s of a process, the number of processes which currently are in local state s. The counters are saturated at 2, which means that $\kappa(s) = 2$ whenever 2 *or more* processes are at state s. The emphasis of the paper is on the derivation of an adequate and sound set of fairness requirements (both weak and strong) that enable proofs of liveness properties of the abstract system, from which we can safely conclude a corresponding liveness property of the original parameterized system. We illustrate the method on few parameterized systems, including Szymanski's Algorithm for mutual exclusion. The method is also extended to deal with parameterized systems whose processes may have infinitely many local states, such as the Bakery Algorithm, by choosing few "interesting" state assertions and $(0, 1, \infty)$-counting the number of processes satisfying them.

1 Introduction

Uniform verification of parameterized systems is one of the most challenging problems in verification today. Given a parameterized system $S(N) : P[1] \| \cdots \| P[N]$ and a property p, uniform verification attempts to verify $S(N) \models p$ for every $N > 1$. One of the promising approaches to the verification of infinite-state systems (and, due to the unbounded value of the parameter N, parameterized systems are essentially infinite-state) is the method of *finitary abstraction* by which we abstract an infinite-state system into a finite-state one, that can then be model checked. A general theory of finitary abstraction which abstracts a system *together* with the property to be proven is presented in [14], and can be applied to the verification of arbitrary LTL properties, including liveness properties.

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In this paper we present *counter abstraction*, a special finitary abstraction method for parameterized systems. Assume that the processes $P[i]$ in the parameterized system have similar programs, where the program has locations $0, \ldots, L$. Assume first that processes do not have local variables (except for the program counter). In this case, the abstract state space is defined by having abstract variables $\kappa_0, \ldots, \kappa_L$. Abstract variable κ_ℓ is intended to count the number of concrete processes that are at location ℓ. To guarantee that the abstract system is finite-state, we bound the range of κ_ℓ to $[0..2]$. For $k < 2$, $\kappa_\ell = k$ states that precisely k processes are at location ℓ, while $\kappa_\ell = 2$ states that 2 *or more* processes are at location ℓ. Certainly, such an abstraction is adequate for specifying properties of mutual exclusion which requires that the number of processes at critical locations never exceeds 1. The case that processes do possess additional (finite-state) local variables can also be treated by counter abstraction, only in this case we allocate a counter κ_s to each *local state* s of a process. The counter κ_s counts (up to 2) the number of processes whose local state is s.

The idea of this simple abstraction is certainly not new [17] and usually serves as one of the first examples of abstraction and abstract interpretation [6]. The main contribution of this paper is to present a systematic method for enriching the counter abstraction with an adequate and automatically derivable set of fairness conditions, and illustrate its application to verify effectively and efficiently various liveness properties of parameterized systems.

The method can be applied with no extensions to parameterized systems in which every process has only finitely many local states and the global variables range over finite domains. It can also be extended to deal with systems in which individual processes may have infinitely many local states. The necessary extension is that we identify some state assertions and then allocate (bounded) counters that count the number of processes whose local states satisfy these assertions. This extension is illustrated in Section 5 where we verify the correctness of Szymanski's algorithm and the Bakery Algorithm.

Related Work. The work which obtains results closest to ours is [3] and previous articles describing the PAX system. They also address the problem of verification of parameterized systems, with emphasis on liveness properties and manage to verify individual accessibility of Szymanski's protocol. One of the differences between the two approaches is that [3] is based on the method of *predicate abstraction* [4] by which the abstract space is determined by the assertions appearing in the transition system and in the temporal property to be verified. As a result, the structure of the abstraction varies from case to case and the computation of the added fairness requirements depends on a marking algorithm which has to be re-applied for each case separately, searching for appropriate well-founded ranking. In comparison, we propose a uniform abstraction approach based on the notion of counter abstraction, which allows us to provides a standard recipe for the additional compassion (strong fairness) requirements, as well as additional justice (weak fairness) requirements, which [3] claims cannot be automatically lifted from the concrete to the abstract systems (unless it is for a distinguished process which is not abstracted). Thus, due to our focus on a

single uniform abstraction, we are able to derive a richer set of abstract fairness conditions, including weak as well as strong fairness. Like [3], we also use an implementation of WS1S in order to compute the abstracted system. Another relative advantage of our method is its extended ability to deal with parameterized systems whose individual processes are not necessarily finite-state.

The theory of linear abstraction, as presented in [14], is aimed at verifying liveness (and general LTL) properties. However, the general recipe of abstraction is often too weak to provide a working finitary abstraction. In order to obtain a complete method, it is suggested to augment the system under consideration by an auxiliary monitoring module and abstract the composed system. While this may be theoretically satisfactory, no effective method is provided for designing the augmenting monitor.

The problem of uniform verification of parameterized systems is, in general, undecidable [1]. One approach to remedy this situation, pursued, e.g., in [7], is to look for restricted families of parameterized systems for which the problem becomes decidable. Many of these approaches fail when applied to asynchronous systems where processes communicate by shared variables.

Another approach is to look for sound but incomplete methods. Representative works of this approach include methods based on: explicit induction ([8]), network invariants that can be viewed as implicit induction ([15]), abstraction and approximation of network invariants ([5]), and other methods based on abstraction ([9]). Other methods include those relying on "regular model-checking" (e.g., [12]) that require special *acceleration* procedures and thus involve user ingenuity and intervention, methods based on symmetry reduction (e.g., [10]), or compositional methods (e.g., ([20]) that combine automatic abstraction with finite-instantiation due to symmetry. These works, from which we have mentioned only few representatives, require the user to provide auxiliary constructs and thus do not provide for fully automatic verification of parameterized systems.

2 Parameterized Systems

The systems we consider here consist of a parallel composition of N processes, all executing the same program, that may refer to the id (identity) of the executing process. In addition, the process programs may access a set of global shared variables and each other's local variables. Accesses to the local variables of other processes can be done under existential or universal quantification. A simple example of such a system is MUX-SEM in Fig. 1. There, each of the processes is executing exactly the same code (that does not refer to the id of the executing process), and there are only references to the single shared variable y.
The semaphore instructions "**request** y" and "**release** y" appearing in the program stand, respectively, for $\langle$**when** $y = 1$ **do** $y := 0\rangle$ and $y := 1$. As seen in Fig. 1, we use the simple programming language SPL [19,18] for presenting programs.

2.1 The Computational Model

For a computational model we use *fair discrete systems*, which is a slight variation on the *fair transition systems* (FTS) model of [19]. An FDS $\mathcal{D} : \langle V, \Theta, \rho, \mathcal{J}, \mathcal{C} \rangle$ consists of the following components.

- $V = \{u_1, ..., u_n\}$: A finite set of typed *system variables*, containing data and control variables. The set of *states* (interpretation) over V is denoted by Σ.
- Θ : The *initial condition* – an *assertion* (first-order state formula) characterizing the initial states.
- ρ : A *transition relation* – an assertion $\rho(V, V')$, relating the values V of the variables in state $s \in \Sigma$ to the values V' in a $\mathcal{D}$-successor state $s' \in \Sigma$.
- $\mathcal{J} : \{J_1, \dots, J_k\}$: A (possibly parameterized) set of *justice* (*weak fairness*) *requirements*. The justice requirement $J \in \mathcal{J}$ is an assertion, intended to guarantee that every computation contains infinitely many J-state (states satisfying J).
- $\mathcal{C} : \{\langle p_1, q_1 \rangle, \dots \langle p_n, q_n \rangle\}$: A (possibly parameterized) set of *compassion* (*strong fairness*) *requirements*. The compassion requirement $\langle p, q \rangle \in \mathcal{C}$ is a pair of assertions, intended to guarantee that every computation containing infinitely many p-states also contains infinitely many q-states.

We require that every state $s \in \Sigma$ has at least one $\mathcal{D}$-successor. This is often ensured by including in ρ the *idling* disjunct $V = V'$ (also called the *stuttering* step). In such cases, every state s is its own $\mathcal{D}$-successor.

The FDS corresponding to program MUX-SEM is presented in Fig. 2.

A *computation* of an FDS $\mathcal{D} : \langle V, \Theta, \rho, \mathcal{J}, \mathcal{C} \rangle$ is an infinite sequence of states $\sigma : s_0, s_1, s_2, ...$, satisfying the following requirements:

Initiality: s_0 is initial, i.e., $s_0 \models \Theta$.
Consecution: For each $j = 0, 1, ...$, the state s_{j+1} is a $\mathcal{D}$-successor of the state s_j.
Justice: For each $J \in \mathcal{J}$, σ contains infinitely many J-positions.
Compassion: For each $\langle p, q \rangle \in \mathcal{C}$, if σ contains infinitely many p-positions, then it also contain infinitely many q-positions.

For an FDS $\mathcal{D}$, we denote by $\mathit{Comp}(\mathcal{D})$ the set of all computations of $\mathcal{D}$.

$$
\begin{array}{l}
\textbf{in} \quad N : \textbf{natural where } N > 1 \\
\textbf{local } y \;\; : \textbf{boolean where } y = 1 \\
\overset{N}{\underset{i=1}{\Big\|}} P[i] :: \left[\begin{array}{l} \textbf{loop forever do} \\ \quad \left[\begin{array}{l} 0 : \textbf{Non-Critical} \\ 1 : \textbf{request } y \\ 2 : \textbf{Critical; release } y \end{array} \right] \end{array} \right]
\end{array}
$$

Fig. 1. Program MUX-SEM

$$V : \begin{cases} N : \textbf{natural} \\ y : \ \textbf{boolean} \\ \pi : \ \textbf{array}\ [1..N]\ \textbf{of}\ \{0,1,2\} \end{cases} \qquad \Theta : y \ \wedge \ N > 1 \ \wedge \ \forall i : [1..N] : \pi[i] = 0$$

$$\rho : \ \exists i : [1..N] \left(\begin{array}{l} \quad \pi'[i] = \pi[i] \ \wedge \ y' = y \\ \vee\ \pi[i] = 0 \ \wedge \ \pi'[i] = 1 \ \wedge \ y' = y \\ \vee\ \pi[i] = 1 \ \wedge \ y = 1 \ \wedge \ \pi'[i] = 2 \ \wedge \ y' = 0 \\ \vee\ \pi[i] = 2 \ \wedge \ \pi'[i] = 0 \ \wedge \ y' = 1 \end{array} \right) \ \wedge$$
$$\forall j \neq i : \pi'[j] = \pi[j] \ \wedge \ N = N'$$

$$\mathcal{J} : \left\{ \pi[i] \neq 2 \mid i \in [1..N] \right\} \qquad \mathcal{C} : \ \left\{ \langle \pi[i] = 1 \ \wedge \ y \ , \ \pi[i] = 2 \rangle \mid i \in [1..N] \right\}$$

Fig. 2. MUX-SEM–The FDS corresponding to program MUX-SEM

2.2 Verification of Parameterized Systems

When verifying properties of a parameterized system *Sys*, one wants to show that the property of interest is valid for all values of N. We focus on properties that are expressible by a formula φ of the type $\forall i_1, \ldots, i_k : f(i_1, \ldots, i_k)$ where k is a constant independent of N, $i_1, \ldots, i_k$ are indices of processes and f is a temporal formula over the free variables $i_1, \ldots, i_k$. Thus, to show that φ is valid over a parameterized system *Sys*, it is necessary to show that every computation $\sigma \in \mathcal{C}omp(Sys)$, $\sigma \ \models \ N \geq k \rightarrow \forall i_1, \ldots, i_k : [1..N] : f(i_1, \ldots, i_k)$.

For example, the safety property of MUX-SEM, requiring that no two processes are ever in the critical section at the same time, is given by $\forall i \neq j : \Box\neg(\pi[i] = 2 \wedge \pi[j] = 2)$. Thus, to prove safety one has to show that for every $N \geq 2$, the property $\forall i \neq j : \Box\neg(\pi[i] = 2 \wedge \pi[j] = 2)$ holds for every computation of $\mathcal{C}omp$(MUX-SEM).

As stated in the introduction, automatic verification of parameterized systems is notoriously difficult. Many methods, including [2], have been proposed for proving safety properties for a large class of such systems. Our goal here is to automatically prove liveness properties of such systems. It should be noted that the methodology reported here allows for automatic verification of safety properties, at times more efficiently than by other methods.

Our approach is based on the method of *finitary abstraction* introduced in [14]. The idea behind finitary abstraction (as well as other abstraction methods, all inspired by [6]) is to reduce the problem of verifying that a given *concrete* system $\mathcal{D}$ satisfies its (concrete) specifications ψ, into a problem of verifying that some (carefully crafted, finite-state) *abstract* system $\mathcal{D}^\alpha$ satisfies its (finite-state) abstract specifications ψ^α.

We sketch here the basic elements of the method, and refer the reader to [14] for additional details. Consider an FDS $\mathcal{D} = \langle V, \Theta, \rho, \mathcal{J}, \mathcal{C}\rangle$. Assume a set of *abstract variables* V_A and a set of expressions $\mathcal{E}^\alpha$, such that $V_A = \mathcal{E}^\alpha(V)$ expresses the values of the abstract variables in terms of concrete variables. For an assertion p, $\alpha^+(p)$ is the set of abstract states that have *some* p-states abstracted into them, and $\alpha^-(p)$ is the set of abstract states such that *all*

states abstracted into them are p-states. Both α^- and α^+ can be generalized to temporal formulae ([14]). The temporal formula ψ^α is obtained by taking $\alpha^-(\psi)$. The abstract system $\mathcal{D}^\alpha$ that corresponds to $\mathcal{D}$ under α is the FDS $(V^\alpha, \Theta^\alpha, \rho^\alpha, J^\alpha, \mathcal{C}^\alpha)$ where $V^\alpha = V_A$, $\Theta^\alpha = \alpha^+(\Theta)$, $\rho^\alpha = \alpha^{++}(\rho)$, $J^\alpha = \bigcup_{J \in \mathcal{J}} \alpha^+(J)$, and $\mathcal{C}^\alpha = \bigcup_{\langle p,q\rangle \in \mathcal{C}} \langle \alpha^-(p), \alpha^+(q)\rangle$ where $\alpha^{++}(\rho) = \exists V, V' : V_A = \mathcal{E}^\alpha(V) \ \wedge \ V'_A = \mathcal{E}^\alpha(V') \ \wedge \ \rho(V, V')$.

It is proven in [14] that this abstraction method is sound. That is, if $\mathcal{D}^\alpha \models \psi^\alpha$ then $\mathcal{D} \models \psi$. Since formula ψ is an arbitrary temporal formula, this provides a sound abstraction method for verifying liveness as well as safety properties. In the next section we show the procedure above is not sufficient for deriving abstract fairness requirements.

3 Counter Abstraction

We present *counter abstraction*, a simple data abstraction method which is a special case of the finitary abstraction approach. Counter abstraction was studied in several other works (e.g., in [22,17]), but not in the context of fully automatic verification of liveness properties. The standard methodology for abstracting fairness requirements is inadequate when applied to counter abstraction. We show how to derive stronger fairness requirements for abstracted systems in general and counter abstracted systems in particular. These stronger fairness requirements support fully automatic verification of liveness properties.

3.1 Counter Abstracting States and Transitions

Consider a parameterized system *Sys* described by an SPL program as described in Section 2. Assume that the control variable of processes ($\pi[i]$) ranges over $0, \ldots, L$, and that the shared variables are $y_1, \ldots, y_b$. For simplicity, we assume first that the processes do not have any local variables. Thus, the concrete state variables are N, $\pi[1..N]$, and $y_1, \ldots, y_b$. We define a *counter abstraction* α, where the abstract variables are $\kappa_0, \ldots, \kappa_L : \{0, 1, 2\}$; $Y_1, \ldots, Y_b$ and the abstraction mapping $\mathcal{E}^a$ is given by

$$\begin{array}{ll} \kappa_\ell = \left\{ \begin{array}{l} 0 \text{ if } \forall i : [1..N] : \pi[i] \neq \ell \\ 1 \text{ if } \exists i_1 : [1..N] : \pi[i_1] = \ell \ \wedge \ \forall i_2 \neq i_1 : \pi[i_2] \neq \ell \\ 2 \text{ otherwise} \end{array} \right\} & \text{for } \ell \in [0..L] \\ Y_j = y_j & \text{for } j = 1, \ldots, b \end{array}$$

That is, κ_ℓ is 0 when there are no processes at location ℓ, it is 1 if there is exactly one process at location ℓ, and it is 2 if there are two or more processes at location ℓ.[1]

Since the systems we are dealing with are symmetric, all the processes have initially the same control value. Without loss of generality, assume it is location 0.

[1] The upper bound of 2 can be substituted with other positive integer. In particular 1 can be used for most locations.

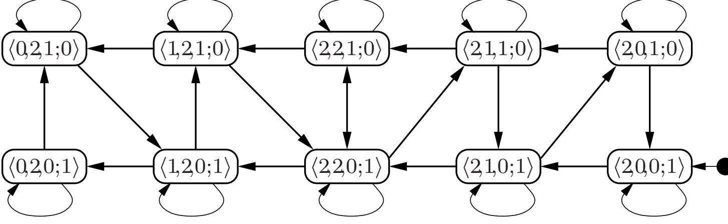


Fig. 3. Counter abstraction of MUX-SEM ($N \geq 5$)

Thus we have that Θ^α is $\kappa_0 = 2 \;\wedge\; \kappa_1 = \cdots = \kappa_L = 0 \;\wedge\; Y_1 = y_1^0 \;\wedge\; \cdots \;\wedge\; Y_b = y_b^0$, where $y_1^0, \ldots y_b^0$ denote the initial values of the concrete shared variables $y_1, \ldots, y_b$.

Next, we obtain ρ^α according to the "recipe" of the previous section. This can be computed automatically using a system supporting the logic WS1S, such as TLV[P] [13] or MONA [11]. Alternately, we can follow the more efficient procedure sketched below. The details are easy to flesh out and omitted here for brevity. For every location ℓ in the SPL program that describes a process, assume the instruction at location ℓ is of the form:

$$\ell: \ \textbf{if } c \textbf{ then } [\ \boldsymbol{y} := f_1(\boldsymbol{y});\ \textbf{goto } \ell_1\ ] \textbf{ else } [\ \boldsymbol{y} := f_2(\boldsymbol{y});\ \textbf{goto } \ell_2\ ]$$

Suppose c^α is defined and that $\ell \neq \ell_1, \ell_2$. For $i = 1, 2$, let τ_i be the formula

$$\kappa_\ell > 0 \;\wedge\; \kappa'_\ell = \kappa_\ell \ominus 1 \;\wedge\; \kappa'_{\ell_i} = \kappa_{\ell_i} \oplus 1 \;\wedge\; \forall j \neq \ell, \ell_i : \kappa_j = \kappa'_j \;\wedge\; y' = f_i(y)$$

where $x' = x \oplus 1$ is an abbreviation for: $x' = \min\{x + 1, 2\}$ and $x' = x \ominus 1$ is an abbreviation for: $x' = \max\{0, x - 1\}$. Then, we include in ρ^α the disjunction $c^\alpha \wedge \tau_1 \;\vee\; \neg c^\alpha \wedge \tau_2$.

Example 1 Fig. 3 presents graphically the FDS obtained by counter abstracting MUX-SEM, where each state displays the values assigned to the abstract variables $\kappa_0, \kappa_1, \kappa_2; Y$. To make the figure more compact, we removed states and transitions that occur only for small values of N (namely, for $N < 5$.) For example, all states where every κ_i is less than two (implying $N < 4$) is removed; so is the transition that leads from $\langle 2, 1, 1; 0\rangle$ into $\langle 1, 2, 1; 0\rangle$ (since it implies that $N = 4$.) The initial state is identified by an entry edge. Suppose c^α is defined and that $\ell \neq \ell_1, \ell_2$. For $i = 1, 2$, let τ_i be the formula

$$\kappa_\ell > 0 \;\wedge\; \kappa'_\ell = \kappa_\ell \ominus 1 \;\wedge\; \kappa'_{\ell_i} = \kappa_{\ell_i} \oplus 1 \;\wedge\; \forall j \neq \ell, \ell_i : \kappa_j = \kappa'_j \;\wedge\; y' = f_i(y)$$

where $x' = x \oplus 1$ is an abbreviation for: $x' = \min\{x + 1, 2\}$ and $x' = x \ominus 1$ is an abbreviation for: $x' = \max\{0, x - 1\}$. Then, we include in ρ^α the disjunction $c^\alpha \wedge \tau_1 \;\vee\; \neg c^\alpha \wedge \tau_2$.

We note that there are self loops on each of the states, which is to be expected considering we assume there is always an idle transition, and when there are self loops in the concrete systems, they also appear in the abstract system.

Establishing the safety property is now trivial, since the counter abstraction of the safety property $\varphi : \ \forall i \neq j : \Box\neg(\pi[i] = 2 \;\wedge\; \pi[j] = 2)$ is $\varphi^\alpha \;=\; \Box(\kappa_2 \leq 1)$, which trivially holds for MUX-SEM$^\alpha$.

3.2 Deriving Abstract Justice Requirements

Consider a single justice requirement of MUX-SEM $J: (\pi[i] \neq 2)$. The recipe in the previous section defines $J^\alpha = \alpha^+(J)$, which should encompass all abstract states S that have at least one α-source satisfying $\pi[i] \neq 2$. It is not difficult to see that this yields the abstract assertion $J^\alpha = \kappa_0 + \kappa_1 > 0$. Since, as can be seen from Fig. 3, all reachable states satisfy this assertion, J^α does not introduce any meaningful constraint on computations, and the counter abstraction of a system with the requirement J will be equivalent to the system without this requirement. It follows that any liveness property (such as accessibility) that is not valid for system MUX-SEM without the justice requirement J cannot be proven by an abstraction which only abstracts J into $\alpha^+(J)$.

We now develop a method by which stronger abstract justice requirements can be derived. Let φ be an abstract assertion, i.e., an assertion over the abstract state variables V_A. We say that φ *suppresses the concrete justice requirement* J if, for every two concrete states s_1 and s_2 such that s_2 is a ρ-successor of s_1, and both $\alpha(s_1)$ and $\alpha(s_2)$ satisfy φ, $s_1 \models \neg J$ implies $s_2 \models \neg J$.

For example, the abstract assertion $\varphi : \kappa_2 = 1$ suppresses the concrete justice requirement $J : \pi[i] \neq 2$. Indeed, assume two states s_1 and s_2 such that s_2 is a successor of s_1 and both are counter-abstracted into abstract states satisfying $\kappa_2 = 1$. This implies that both s_1 and s_2 have precisely one process executing at location 2. If s_1 satisfies $\neg J = (\pi[i] = 2)$ then the single process executing at location 2 within s_1 must be $P[i]$. Since s_2 is a successor of s_1 and also has a single process executing at location 2, it must also be the same process $P[i]$, because it is impossible for $P[i]$ to exit location 2 and another process to enter the same location all within a single transition.

The abstract assertion φ is defined to be *justice suppressing* if, for every concrete state s such that $\alpha(s) \models \varphi$, there exists a concrete justice requirement J such that $s \models \neg J$ and φ suppresses J.

For example, the assertion $\kappa_2 = 1$ is justice suppressing, because every concrete state s whose counter-abstraction satisfies $\kappa_2 = 1$ must have a single process, say $P[i]$, executing at location 2. In that case, s violates the justice requirement $J : \pi[i] \neq 2$ which is suppressed by φ.

Theorem 1. *Let $\mathcal{D}$ be a concrete system and α be an abstraction applied to $\mathcal{D}$. Assume that $\varphi_1, \ldots, \varphi_k$ is a list of justice suppressing abstract assertions. Let $\mathcal{D}^\alpha$ be the abstract system obtained by following the abstraction recipe described in Section 2 and then adding $\{\neg\varphi_1, \ldots, \neg\varphi_k\}$ to the set of abstract justice requirements. If $\mathcal{D}^\alpha \models \psi^\alpha$ then $\mathcal{D} \models \psi$.*

Thus, we can safely add $\{\neg\varphi_1, \ldots, \neg\varphi_k\}$ to the set of justice requirement, while preserving the soundness of the method.

The proof of the theorem is based on the observation that every abstraction of a concrete computation must contain infinitely many $\neg\varphi$-states for every justice suppressing assertion φ. Therefore, the abstract computations removed from the abstract system by the additional justice requirements can never correspond to abstractions of concrete computations, and it is safe to remove them.

Theorem 1 is very general (not even restricted to counter abstraction) but does not provide us with guidelines for the choice of the justice suppressing assertions. For the case of counter-abstraction, we can provide some practical guidelines, as follows:

G1. If the concrete system contains the justice requirements $\neg(\pi[i] = \ell)$, then the assertion $\kappa_\ell = 1$ is justice suppressing.
G2. If the concrete system contains the justice requirements $\neg(\pi[i] = \ell \ \wedge \ c)$, where c is a condition on the shared variables (that are kept intact by the counter-abstraction), then the assertion $\kappa_\ell = 1 \ \wedge \ c$ is justice suppressing.
G3. If the concrete system contains the justice requirements $\neg(\pi[i] = \ell)$ and the only possible move from location ℓ is to location $\ell + 1$, then the two assertions $\kappa_\ell > 0 \wedge \kappa_{\ell+1} = 0$ and $\kappa_\ell > 0 \wedge \kappa_{\ell+1} = 1$ are justice suppressing.
G4. If the concrete system contains the justice requirements $\neg(\pi[i] = \ell \ \wedge \ c)$, where c is a condition on the shared variables, and the only possible move from location ℓ is to location $\ell{+}1$, then the assertions $\kappa_\ell > 0 \wedge \kappa_{\ell+1} = 0 \wedge c$ and $\kappa_\ell > 0 \ \wedge \ \kappa_{\ell+1} = 1 \ \wedge \ c$ are justice suppressing.

Example 2 We state justice properties of MUX-SEM$^\alpha$ according to the above guidelines. Since for MUX-SEM we have the justice $\neg(\pi[i] = 2)$, and since every move from location 2 leads to location 0, then the assertions $\kappa_2 = 1$, $\kappa_2 > 0 \ \wedge \ \kappa_0 = 0$, and $\kappa_2 > 0 \ \wedge \ \kappa_0 = 1$ are all justice suppressing and their negation can be added to $\mathcal{J}^\alpha$. Similarly, the concrete compassion requirement $\langle \pi[i] = 1 \ \wedge \ y, \pi[i] = 2 \rangle$ implies that the concrete assertion $\neg(\pi[i] = 1 \ \wedge \ y)$ is a justice requirement for system MUX-SEM. We can therefore add $\neg(\kappa_1 = 1 \ \wedge \ y)$ to the abstract justice requirement. Since every move from location 1 leads to location 2, we can also add $\neg(\kappa_1 > 0 \ \wedge \ \kappa_2 = 0 \ \wedge \ y)$ to the abstract justice requirements.

Note: All justice requirements of MUX-SEM$^\alpha$ have been generated automatically by a general procedure implementing all the rules specified by the guidelines above.

3.3 Proving Liveness

The liveness property one usually associates with parameterized systems is *individual accessibility* of the form $\forall i : \Box(\pi[i] = \ell_1 \rightarrow \Diamond(\pi[i] = \ell_2))$. Unfortunately, counter-abstraction does not allow us to observe the behavior of an individual process. Therefore, the property of individual accessibility cannot be expressed (and, consequently, verified) in a counter-abstracted system. In Section 3.4 we show how to extend counter-abstraction to handle individual accessibility properties.

There are, however, liveness properties that *are* expressible and verifiable by counter abstraction. Such are *livelock freedom* (sometime called *communal accessibility*) properties of the form $\psi : \ \Box(\exists i : \pi[i] = \ell_1 \rightarrow \Diamond(\exists i : \pi[i] = \ell_2))$, stating that if *some* process is at location ℓ_1, then eventually *some* process (not

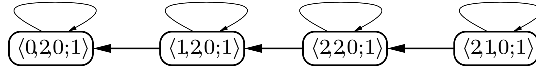


Fig. 4. Reachability graph for χ

necessarily the same) will enter ℓ_2. The counter-abstraction of such a property is $\psi^\alpha : \Box(\kappa_{\ell_1} > 0 \rightarrow \Diamond(\kappa_{\ell_2} > 0))$. Model checking that ψ^α holds over Sys^α can be accomplished by standard model checking techniques of response properties. E.g., the procedure in [16] suggests extracting from the state-transition graph of the subgraph of *pending states* and showing that it contains no infinite fair path. A pending state for a property $p \rightarrow \Diamond q$ is any state which is reachable from a p-state by a q-free path.

Example 3 Consider the system MUX-SEM$^\alpha$ of Fig. 3 and the abstract live-lock freedom property $\chi : \Box(\kappa_1 > 0 \rightarrow \Diamond(\kappa_2 > 0))$. In Fig. 4, we present the subgraph of pending states for the property χ over the system MUX-SEM$^\alpha$.

The way we show that this graph contains no infinite fair path is to decompose the graph into maximal strongly connected components and show that each of them is *unjust*. A strongly connected subgraph (SCC) S is unjust if there exists a justice requirement which is violated by all states within the subgraph. In the case of the graph in Fig. 4 there are four maximal SCC's. Each of these subgraphs is unjust towards the abstract justice requirement $\neg(\kappa_1 > 0 \;\wedge\; \kappa_2 = 0 \;\wedge\; y)$ derived in Example 2.

We conclude that the abstract property $\Box(\kappa_1 > 0 \rightarrow \Diamond(\kappa_2 > 0))$ is valid over system MUX-SEM$^\alpha$ and, therefore, the property $\Box(\exists i : \pi[i] = 1 \rightarrow \Diamond(\exists i : \pi[i] = 2))$ is valid over MUX-SEM.

3.4 Proving Individual Accessibility

As indicated before, individual accessibility cannot be directly verified by standard counter abstraction which cannot observe individual processes. To prove individual accessibility for the generic process $P[t]$, we abstract the system by counter abstracting all the processes except for $P[t]$, whom we leave intact. We then prove that the abstracted system satisfies the liveness property (the abstraction of which leaves it unchanged, since it refers to $\pi[t]$ that is kept intact by the abstraction), from which we derive that the concrete system satisfies it as well.

The new abstraction, called "counter abstraction save one", is denoted by γ. As before, we assume for simplicity that the processes possess no local variables except for their program counter. The abstract variables for γ are given by $\kappa_0, \ldots, \kappa_L : [0..2]$, $\Pi : [0..L]$; $Y_1, \ldots, Y_b$ and for the abstraction mapping $\mathcal{E}^\gamma$ we have $\Pi = \pi[t]$, $Y_k = y_k$ for $k = 1, \ldots, b$, and

$$\kappa_\ell = \left\{ \begin{array}{l} 0 \text{ if } \forall r \neq t : \pi[r] \neq \ell \\ 1 \text{ if } \exists r \neq t : \pi[r] = \ell \;\wedge\; \forall j \notin \{r,t\} : \pi[t] \neq \ell \\ 2 \text{ otherwise} \end{array} \right\} \text{ for } \ell \in [0..L]$$

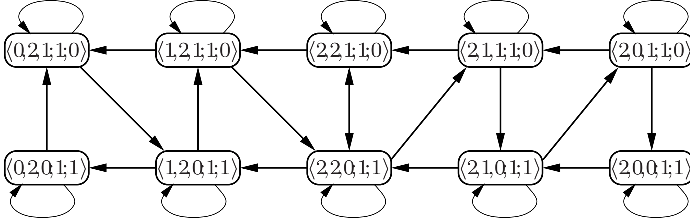


Fig. 5. The states of MUX-SEM$^\gamma$ which are pending with respect to φ

We obtain ρ^γ in the obvious way. For $\mathcal{J}^\gamma$, we include all the justice requirements obtained by the recipe of [14] and the guidelines of Subsection 3.2, with all requirements in $\mathcal{J}$ that relate to $P[t]$. For $\mathcal{C}^\gamma$ we take all the requirements in $\mathcal{C}$ that relate only to $P[t]$. To prove $\varphi : \square(\Pi = 1 \rightarrow \diamond(\Pi = 2))$, consider the subgraph of S^γ that consists of all the abstract states that are reachable from a $(\Pi = 1)$-state by a $(\Pi = 2)$-free paths, and show, as before, that this subgraph contains no infinite fair path.

Example 4 Consider the system MUX-SEM$^\gamma$ and the liveness property φ^γ given by $\square((\Pi = 1) \rightarrow \diamond(\Pi = 2))$. The subgraph of pending states is presented in Fig. 5. Each state in this graph is labeled by a tuple which specifies the values assigned by the state variables $\langle \kappa_0, \kappa_1, \kappa_2; \Pi; Y \rangle$.

Unlike the previous case, this system has the compassion requirement $\langle \Pi = 1 \wedge Y, \Pi = 2 \rangle$ associated with $P[t]$. After removing from the graph all states satisfying $\Pi = 1 \wedge Y$, it is easy to see that no infinite fair paths are left. We can thus conclude that the abstract property $\square((\Pi = 1) \rightarrow \diamond(\Pi = 2))$ is valid over MUX-SEM$^\gamma$ and, therefore, $\square((\pi[t] = 1) \rightarrow \diamond(\pi[t] = 2))$ is valid over MUX-SEM.

4 Compassion

In Section 3.2 we showed how to derive additional justice requirements for a counter-abstracted system. We now turn to abstract compassion requirements, which do not always correspond to concrete compassion requirements. Here we derive abstract compassion requirements which reflect well-founded properties of some of the concrete data domains. Consider program TERMINATE presented in Fig. 6(a).

The liveness property we would like to establish for this program is given by the formula $\varphi : \diamond(\forall i : \pi[i] = 1)$, stating that eventually, all processes reach location 1. The counter abstraction for TERMINATE, (assuming $N > 2$), is TERMINATE$^\alpha$, presented in Fig. 6(b), where each state s is labeled by the values state s assigns to the abstract variables κ_0 and κ_1. The abstracted property φ^α is given by $\diamond(\kappa_0 = 0)$. However, this property is not valid over TERMINATE$^\alpha$, even when we take into account all the justice requirements derived according to Subsection 3.2. These justice requirements force the computation to eventually

exit states $\langle 2,0\rangle$, $\langle 2,1\rangle$, and $\langle 1,2\rangle$, but they do not prevent the computation from staying forever in state $\langle 2,2\rangle$.

To obtain a fairness requirement which will force the computation to eventually exit state $\langle 2,2\rangle$ we augment the system with two additional abstract variables and a corresponding compassion requirement that governs their behavior.

Let $Sys : \langle V, \Theta, \rho, \mathcal{J}, \mathcal{C}\rangle$ be an FDS representing a concrete parameterized system, where the locations of each process are $[0..L]$. We define an *augmented system* $\mathcal{D}^* : \langle V^*, \Theta^*, \rho^*, \mathcal{J}^*, \mathcal{C}^*\rangle$ as follows:

$$
\begin{array}{ll}
V^* & : V \ \cup\ \{\mathit{from}, \mathit{to} : [-1..L]\} \\
\Theta^* & : \Theta \ \wedge\ (\mathit{from} = -1)\ \wedge\ (\mathit{to} = -1) \\
\rho^* & : \rho \ \wedge \left(\begin{array}{l} \exists i : [1..N], \ell_1 \neq \ell_2 : [0..L] : \pi[i] = \ell_1 \ \wedge\ \pi'[i] = \ell_2 \ \wedge \\ \qquad\qquad\qquad\qquad (\mathit{from}' = \ell_1)\ \wedge\ (\mathit{to}' = \ell_2) \\ \vee\ \forall i : [1..N] : \pi'[i] = \pi[i] \ \wedge\ \ (\mathit{from}' = -1)\ \wedge\ (\mathit{to}' = -1) \end{array}\right) \\
\mathcal{J}^* & : \mathcal{J} \\
\mathcal{C}^* & : \mathcal{C} \ \cup\ \{\langle \mathit{from} = \ell,\ \mathit{to} = \ell\rangle \ \mid\ \ell \in [0..L]\}
\end{array}
$$

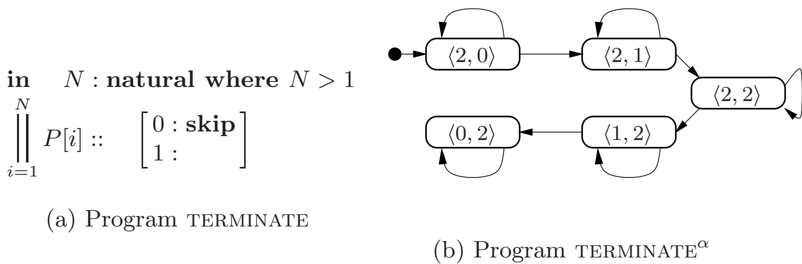
Thus, system *Sys* is augmented with two auxiliary variables, *from* and *to*. Whenever a transition causes some process to move from location ℓ_1 to location $\ell_2 \neq \ell_1$, the same transition sets *from* to ℓ_1 and *to* to ℓ_2. Transitions that cause no process to change its location set both *from* and *to* to -1. For every $\ell \in [0..L]$, we add to Sys^* the compassion requirement $\langle \mathit{from} = \ell,\ \mathit{to} = \ell\rangle$. This compassion requirement represents the obvious fact that, since the overall number of processes is bounded (by N), processes cannot leave location ℓ infinitely many times without processes entering location ℓ infinitely many times.

Comparing the observable behavior of *Sys* and Sys^*, we can prove the following:

Claim. Let σ be an infinite sequence of V-states. Then σ is a computation of *Sys* iff it is a V-projection of a computation of system Sys^*.

Thus, the augmentation of *Sys* does not change its observable behavior.

Consequently, instead of counter-abstracting the system *Sys*, we can counter-abstract Sys^*. We denote by $Sys^\dagger = (Sys^*)^\alpha$ the counter abstraction of the augmented system Sys^*. In the presence of the auxiliary variable *from*, we can derive



(a) Program TERMINATE

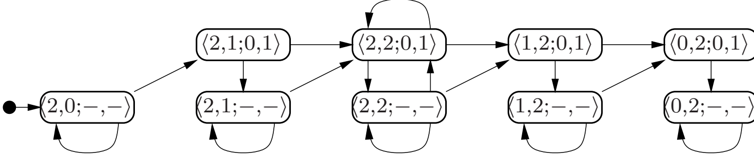
(b) Program TERMINATE^α

Fig. 6. Program TERMINATE and its counter abstraction

an even sharper justice requirements and replace the guidelines of Subsection 3.2 by the following single rule:

G5. If the concrete system contains the justice requirements $\neg(\pi[i] = \ell \ \wedge \ c)$, where c is a condition on the shared variables and process indices other than i, then we may add to $Sys^\dagger$ the justice requirement $\mathit{from} = \ell \vee \neg(\kappa_\ell > 0 \wedge c^\alpha)$, where c^α is the counter abstraction of c.

Reconsider program TERMINATE. Applying the augmented abstraction, we obtain the following abstraction TERMINATE†, where each state s is labeled by the values s assigns to the variables $\langle \kappa_0, \kappa_1, \mathit{from}, \mathit{to}\rangle$ ($-$ stands for -1):



This system is augmented by the additional justice requirement $(\mathit{from} = 0) \ \vee \ (\kappa_0 = 0)$ (generated according to G5) and the compassion requirement $\langle \mathit{from} = 0,\ \mathit{to} = 0\rangle$.

Taking into account the new justice requirement $(\mathit{from} = 0) \ \vee \ (\kappa_0 = 0)$, and the new compassion requirement $\langle \mathit{from} = 0,\ \mathit{to} = 0\rangle$, it is easy to see that the program admits no infinite fair paths. It follows that TERMINATE† satisfies $\diamondsuit(\kappa_0 = 0)$ and, therefore, that the concrete system TERMINATE satisfies $\forall i : \diamondsuit(\pi[i] \neq 0)$.

5 Examples

We briefly describe two of our test cases: Szymanski's mutual exclusion algorithm [23], described in Fig. 7(a), and the Bakery algorithm, described in Fig. 7(b). The complete FDSs and codes for the test cases are omitted from here for space reasons and can be found in *http://cs.nyu.edu/˜zuck/counter*.

Szymanski's Algorithm. Strictly speaking, the algorithm is not symmetric since the instruction in ℓ_6 depends on the id of the executing process. To prove mutual exclusion with a counter abstraction, it was necessary to add an additional abstract variable *lmin* to the abstraction. This variable maintains the location of the process with the minimal index among those in 3..7; *lmin* is 0 when there are no processes in locations 3..7. Once the abstraction mapping of this variable is defined, the abstract transition relation can be automatically computed. Using this abstraction we were able to verify the property of mutual exclusion of the system.

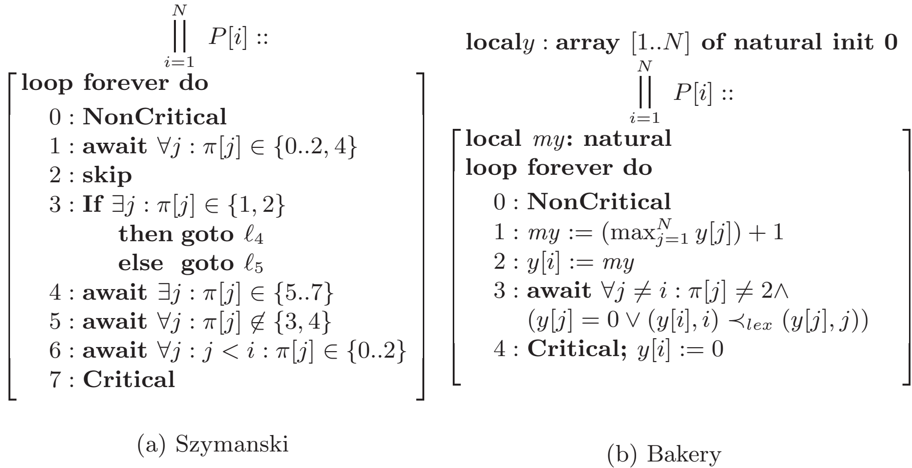
To facilitate the proof of the livelock freedom property $\chi : (\exists i : \pi[i] = 1) \rightarrow \diamondsuit(\exists i : \pi[i] = 7)$, the abstract system $\mathsf{szyman}^\dagger$ contains the abstract justice requirements $\mathit{from} = \ell \vee \neg En(\tau)$ for each abstract transition τ departing

from location $\ell \neq 0$, where $En(\tau)$ is the enabling condition for τ. Consider, for example, the transition corresponding to a process moving from location 6 to location 7. The enabling condition for the corresponding abstract transition is $\kappa_6 > 0 \ \wedge \ lmin = 6$. Consequently, we include in the abstract FDS the justice requirement $from = 6 \ \vee \ \neg(\kappa_6 > 0 \ \wedge \ lmin = 6)$. In addition, we include in the abstract system the compassion requirements $\langle from = \ell, \ to = \ell \rangle$, for each $\ell = 0, \ldots, 7$. We were able to verify χ. Note that the auxiliary variable *lmin* is not required for establishing livelock freedom.

To prove individual accessibility, we applied γ abstraction to szyman*. Some bookkeeping was needed to maintain the value of *lmin*, that was used, as before, to store the location of the non-t process whose index is minimal among those in locations 3..7 (including t.)

The Bakery Algorithm. An interesting feature of the algorithm is the infinite data domain–processes choose "tickets" whose values are unboundedly large. To counter abstract BAKERY, we used a variable *lmin*, with a role similar to the one used in *Szymanski*. Here *lmin* is the location of the process j whose $(y[j], j)$ is (lexicographically) minimal among those in locations 2..4. The mutual exclusion property of the protocol, $\Box(\kappa_4 < 2)$, as well as the livelock freedom property, $\Box(\kappa_1 > 0 \rightarrow \Diamond(\kappa_4 > 0))$ were easily established in TLV (the Weizmann Institute programmable model checker [21]). To establish the individual accessibility property, $\pi[t] = 1 \rightarrow \Diamond(\pi[t] = 4)$, we applied γ abstraction to BAKERY*. As in the case of *Szymanski*, some bookkeeping was needed to maintain the value of *lmin*, that was used, as before, to store the location of the non-t process whose index is minimal among those in locations 2..4 (including t.)

Run Time Results In Fig. 8 we give the user run-time (in seconds) results of our various experiments. All verifications were carried in TLV.



(a) Szymanski

(b) Bakery

Fig. 7. Szymanski's and the Bakery Mutual Exclusion Algorithms

	MUX-SEM (sec)	szyman (sec)	BAKERY (sec)
Mutual Exclusion and Livelock Freedom	0.02	0.69	0.12
Mutual Exclusion and Individual Accessibility (γ abstraction)	0.03	95.87	1.06

Fig. 8. Run time results

6 Conclusions

We have presented the abstraction method of *counter abstraction*. The main novelty and contribution is the derivation of additional fairness conditions (both weak and strong) which enable us to perform automatic verification of interesting liveness properties of non-trivial parameterized systems.

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