Rough Surface Estimation Using the Kirchhoff Model

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Abstract. In this paper, we exploit the theory of light scattering from rough surfaces to estimate surface characteristics through reflectance measurements. Here, we analyse the Beckmann formulation of the Kirchhoff theory. We then suggest two classes of surfaces for which the appropriate techniques can be used for estimating the surface roughness, the correlation length and the surface slope. Finally, we show how the Beckmann model can be fitted to reflectance data for materials with very-rough surfaces. Since the Kirchhoff theory is inadequate for large angles of incidence, we make use of a modification to the Beckmann model. The proposed techniques have significant potential in computer vision for texture model acquisition and realistic reflectance modelling.

1 Introduction

Reflectance modelling is a task of pivotal importance in the analysis of image data. For instance, in computer graphics it is necessary for generating realistic images of synthetic scenes. In computer vision, on the other hand, reflectance models form the basis of shape analysis techniques such as shape-from-shading and photometric stereo, and may also be used for surface analysis tasks which may be used to estimate the physical properties of materials from passively sensed image data [8]. In this paper we are interested in using reflectance models to estimate surface roughness. This is a topic of current importance since recent theories of surface texture have moved away from the naive idea that texture is painted onto the surface. Instead, attempts are being made to understand the formation of texture in terms of surface relief distributions [5–7]. As first observed by Lambert, under diffuse reflection, surfaces without macroscopic roughness appear equally bright from all viewing directions. For a rough surface, however, the surface appears brighter as the viewing direction approaches the illumination direction.

The Torrance-Sparrow model [11] is among the most popular models which aims to incorporate the effect of roughness into the specular reflectance component. The calculation of reflectance is based on geometrical optics, and is hence applicable when the surface irregularities are much larger than the wavelength of incident radiation. Nayar et al. [9] showed that under these conditions the Torrance-Sparrow model approximates the physical optics model developed by Beckmann [1]. Unfortunately these models ignore the effect of roughness on the

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diffuse component. However, this effect has been incorporated into the model developed by Oren and Nayar [13]. Van Ginneken et al. have recently developed a model that can be used to predict reflectance from isotropic rough surfaces that have both specular and diffuse components [8]. As mentioned earlier, the majority of the reflectance models used in the computer vision literature are either very simple and hence ignore surface roughness (e.g. Lambertian model), or are empirical or semi-empirical (e.g. [13]) in nature. There are reports of using either semi-empirical models or improved semi-empirical models (e.g. [8]) in computer vision. However, there has been relatively little effort expended at developing purely physics-based models that account for directly measurable surface characteristics. To address this omission, in this paper, we provide an analysis of the physics-based model of Beckmann-Kirchhoff (B-K) for the scattering of light from rough surfaces [1] and investigate how this model can be utilized for surface roughness measurement from digital images. Our second contribution is to develop techniques for fitting rough reflectance models to data.

Previous methods for determining surface characteristics, and in particular roughness, work at long wavelengths and use special purpose instruments to make reflectance measurements. However, we perform our experiments in the visible region of the spectrum and make use of an ordinary digital camera. To avoid possible noise, saturation or digitization artifacts, we compute the mean-intensity value by averaging the measured reflectance values over a neighborhood of points in the image. This technique agrees with the definition of mean optical intensity from the Beckmann scattering theory [1]. Furthermore, we need to know neither the strength of the light-source nor the surface albedo. The reason for this is that the techniques discussed in this paper use the ratio of two intensities, and not the absolute intensity values. For slightly-rough dielectrics, we use the Beckmann model for the specular direction to estimate the surface roughness σ. For very-rough dielectrics, our main contribution is as follows. We suggest a method for estimating a measure of the surface slope σ/T, where T is the correlation length. We do this using intensity values measured at two different angles of illumination incidence. In practice, we capture the first image with the planar surface perpendicular to the viewing direction. The second image is captured with the camera and the object fixed, but the light-source direction is moved by a predetermined amount. We then compute the mean-intensity ratio, which we define as the ratio of the mean-intensity values from the two images. From the mean-intensity ratio, we can compute σ/T using the B-K model. Similar slope quantities have been used previously in the literature [1, 4].

2 Background

Roughness is a measure of the statistical variation in the topographic relief of a surface [3]. The quantity can be obtained directly from surface-profile measurements, or it can be calculated from a scattering measurement using a theoretical model. Bennett [3] has stressed that there is no unique RMS roughness value for a surface. However, we limit our study to homogeneous and isotropic surfaces where assuming a single approximate value for the RMS roughness is feasible at all locations. Much of the literature on rough surfaces assumes that the height distribution is Gaussian, i.e. $W(z) = \exp(-z^2/2\sigma^2)$ where σ is the measure of roughness. However, evidence for the validity of the assumption that rough surfaces posses Gaussian distributions is conflicting and depends strongly on the nature of the surface being considered [10]. Unfortunately, the specification of a height distribution and the RMS roughness is insufficient to discriminate between surfaces with different length scales. Such surfaces may, however, be distinguished on the basis of their correlation function. The theory of wave scattering from rough surfaces [1] often assumes that surface correlation functions are also Gaussian, i.e. $C(\tau) = \exp(-\tau^2/T^2)$ where T is the correlation length. When the height data have been measured relative to the mean surface level, the RMS slope m can be defined as the root-mean-square of the slopes. Each slope is the difference between the heights of the adjacent points divided by the data sampling interval. The RMS slope is even more dependent on the measuring instrument than is the RMS roughness [3].

The scalar theory treatment of scattering from rough surfaces is based on the Helmholtz-Kirchhoff diffraction integral. To overcome some problems involved in solving this integral, an approximation known as the Kirchhoff boundary condition is made. This approximation limits the validity of the scalar theory to the case of scattering close to the specular direction. Vernold and Harvey [12] have recently modified the B-K theory to overcome this limitation and have extended the theory to large angles of incidence and scatter. Here, we discuss the Beckmann formulation of the scalar Kirchhoff theory which is most widely used in the study of wave scattering from rough surfaces.

3 Beckmann-Kirchhoff Scatter Theory

By requiring that a surface has both a Gaussian height distribution and a Gaussian correlation function, the Beckmann-Kirchhoff (B-K) theory leads to two interesting cases. For a surface that is smooth or of intermediate roughness, the following infinite series solution may be used to model the diffuse reflectance:

$$I(\theta_i, \theta_r, \phi_r) = (\pi T^2 F^2 e^{-g}/A) \sum_{m=1}^{\infty} [(g^m/m!m) \exp(-v_{xy}^2 T^2/4m)] \qquad (1)$$

In this model, the incident beam has the zenith angle θ_i and a fixed azimuth angle ($\phi_i = \pi$), whereas the reflected beam has the zenith angle θ_r and the azimuth angle ϕ_r (on local tangent planes). The expressions for v_x, v_{xy}, v_z and g are given by $v_x = k(\sin\theta_i - \sin\theta_r \cos\phi_r)$, $v_y = -k(\sin\theta_r \sin\phi_r)$, $v_{xy}^2 = v_x^2 + v_y^2$, $v_z = -k(\cos\theta_i + \cos\theta_r)$ and $g = \sigma^2 v_z^2$, where $k = 2\pi/\lambda$ (λ is the wavelength). The geometrical factor F is given by

$$F(\theta_i, \theta_r, \phi_r) = (1 + \cos\theta_i \cos\theta_r - \sin\theta_i \sin\theta_r \cos\phi_r)/[\cos\theta_i(\cos\theta_i + \cos\theta_r)]$$

The parameter A is the area of a plane sheet on which the scattering coefficient is defined [1]. For slightly-rough surfaces ($g \ll 1$) the series in Eq. (1) converges rapidly. In practice, only the first term needs to be considered. As a result the diffuse intensity for slightly-rough surfaces becomes

$$I(\theta_i, \theta_r, \phi_r) \approx (\pi g T^2 F^2 / A) \exp[-(g + v_{xy}^2 T^2/4)] \quad (2)$$

When the surface is very-rough ($g \gg 1$), when compared to the test wavelength, the expression for the diffuse reflectance is

$$I(\theta_i, \theta_r, \phi_r) \approx (\pi T^2 F^2 / A v_z^2 \sigma^2) \exp(-v_{xy}^2 T^2 / 4 v_z^2 \sigma^2) \quad (3)$$

The B-K model depends on both the incidence (θ_i) and the reflectance (θ_r, ϕ_r) angles. However, in computer vision it is mainly the incidence angle behavior of the reflectance models that is of interest. For instance, shape recovery using shape-from-shading schemes is only tractable when the reflectance model is only dependent on the incidence angle. Hence, here we derive a formulation for the specific case when the angle between the light-source and the viewing directions is small, and so $\theta_i = \theta_r = \theta$ and $\phi_r = \pi$. In this case, Eq. (3) can be simplified by replacing the quantities v_z, v_{xy} and F with the corresponding functions of θ. Under the conditions assumed here, $v_{xy} = v_x = 2k \sin\theta$, $v_z = -2k\cos\theta$ and $F = 1/\cos^2\theta$. Hence, in this case the B-K model reduces to

$$I(\theta) \approx (T^2 \lambda^2 / 16\pi A \sigma^2 \cos^6\theta) \exp(-T^2 \tan^2\theta / 4\sigma^2) \quad (4)$$

3.1 Modified Beckmann-Kirchhoff Model

The B-K model fails for large incidence angles and large scattering angles. To overcome this problem, Vernold and Harvey [12] have recently developed a modification of the B-K model that gives excellent agreement with experimental scattering data from rough surfaces at both large angles of incidence and at large scatter angles. The failure of the B-K theory to handle wide-angle scattering and large angles of incidence has been highlighted by other authors [4] too. In the Vernold and Harvey modification [12], the geometrical factor (F^2) used in B-K model is replaced by the cosine of the incidence angle ($\cos\theta_i$) which comes from Lambert's cosine reflectance law. Hence, by replacing the term F^2 with $\cos\theta$ (where $\theta = \theta_i$) in Eq. (3), the Vernold-Harvey modification to the B-K model (for identical light-source and viewing directions) is

$$I(\theta) \approx (T^2 \lambda^2 / 16\pi A \sigma^2 \cos\theta) \exp(-T^2 \tan^2\theta / 4\sigma^2) \quad (5)$$

3.2 Validity of Kirchhoff Theory

Kirchhoff theory is based on a single scatter model and hence does not account for multiple scattering, which occurs at large values of the slope σ/T. When multiple scattering occurs, then a proportion of the incident intensity is absorbed due to subsurface light scattering. Hence, Kirchhoff theory underestimates the scattered intensity. Recently, Caron et al. [4] have attempted to model this effect using the energy conservation rate. This is the ratio of the scattered energy to the incident energy i.e. $\xi = E_{sc}/E_{inc}$. At normal incidence and for $g \gg 1$, they have found an expression for ξ in terms of σ and T. In the physical case, where $0 \le \theta_r \le \pi/2$, then $\xi_{phys} = 1 - \exp(-T^2/4\sigma^2)$. This model also accounts for missing energy due to subsurface scattering. This interpretation results from the fact that when the scattering process is integrated over the range $0 \leq \theta_r \leq \pi$ then $\xi_{non-phys} = 1$. Caron et al. also identify a second limitation of the Kirchhoff theory. Kirchhoff theory assumes that all locations on the surface receive light. However, for rough surfaces under oblique incidence there are locations that are not illuminated due to shadows cast by rough protrusions. To address this problem, they have introduced a limit $\theta_0 = \pi/2 - \tan^{-1}(\sigma\sqrt{2}/T)$ for angles of incidence under which the predictions by Kirchhoff model are valid. Specifically, for angles of incidence greater than the angle of RMS slope θ_0, the energy scattered is significantly overestimated and the scattered intensity cannot be calculated acceptably without shadowing functions [4].

4 Surface Characteristic Estimation

In this section, we discuss some techniques for estimating rough surface characteristics. Specifically, we estimate the RMS roughness σ and the correlation length T for slightly-rough surfaces ($g \ll 1$), and, the ratio σ/T for very-rough surfaces ($g \gg 1$).

4.1 Estimating RMS roughness for slightly-rough surfaces

When the surface is smooth enough to produce a well defined specular beam, the surface roughness can be determined from the relative intensity of the specular beam [2]. According to the B-K model [1], this relative intensity is given by

$$i_{spec} = \exp\left[-(4\pi\sigma\cos\theta/\lambda)^2\right] \tag{6}$$

where θ is the incidence angle. Here, we determine an experimental value for the relative intensity i_{spec} by measuring both the total peak intensity $\hat{I}_{tot}$ at the center of the specular highlight and the average diffuse intensity $\hat{I}_{dif}$ in the neighborhood of the specular lobe. The relative intensity can then be approximated using the formula $i_{spec} = 1 - (\hat{I}_{dif}/\hat{I}_{tot})$. The wavelength λ of the light-source and the incident angle θ are known. Hence, the RMS roughness is given by

$$\sigma = (\lambda/4\pi\cos\theta)[\ln(1/i_{spec})]^{1/2} \tag{7}$$

4.2 Estimating correlation length for slightly-rough surfaces

Here it is assumed that an estimate of the RMS roughness σ is available. First, we measure two values of reflectance I_1, I_2 at a single point (in practice the mean-intensity of the neighboring pixels) for two different angles of incidence θ_1 and θ_2, so that the planar surface is perpendicular to the viewing direction ($\theta_r = 0$). Here we use only one wavelength λ. Under such illumination conditions and from Section 3 we can write $F(\theta_i) = 1/\cos(\theta_i)$, $v_{xy}(\theta_i) = (2\pi/\lambda)\sin(\theta_i)$, $v_z(\theta_i) = (2\pi/\lambda)[1 + \cos(\theta_i)]$ and $g(\theta_i) = \sigma^2 v_z^2(\theta_i)$.

Using Eq. (2), we find the equation for the ratio I_1/I_2. Since all of the parameters appearing in this equation except T are known, we estimate the correlation length T using the equation

$$T = 2\{[g(\theta_1) - g(\theta_2) + \ln K]/[v_{xy}^2(\theta_2) - v_{xy}^2(\theta_1)]\}^{1/2} \quad (8)$$

where $K = [g(\theta_2)F^2(\theta_2)I_1]/[(g(\theta_1)F^2(\theta_1)I_2]$.

4.3 Estimating surface slope for very-rough surfaces

For very-rough surfaces, we estimate the ratio σ/T using the B-K reflectance model. Using these estimates, we can fit the B-K model to the reflectance data. Such a model is potentially useful for several applications in computer vision such as texture model acquisition and realistic reflectance modelling. The technique is as follows. First, we measure two values of reflectance (I_1, I_2) for two incidence angles (θ_1, θ_2) as described in Section 4.2. From Eq. (3), we first obtain the equation for the ratio I_1/I_2. Using this equation, the ratio σ/T is

$$\sigma/T = (1/4\ln K)^{1/2}\{[v_{xy}^2(\theta_2)/v_z^2(\theta_2)] - [v_{xy}^2(\theta_1)/v_z^2(\theta_1)]\}^{1/2} \quad (9)$$

where $K = [F^2(\theta_2)v_z^2(\theta_1)I_1]/[F^2(\theta_1)v_z^2(\theta_2)I_2]$.

5 Experiments

The study reported on this paper is one motivated by computer vision. Hence, we intend to experiment using visible light and a digital camera. The images used in our experiments have been captured using an Olympus 10E camera. Each surface has been imaged under controlled lighting conditions in a darkroom. The objects have been illuminated using a single collimated tungsten light-source whose wavelength is approximately $700nm$. The surfaces used in our experiments are either slightly rough or very rough.

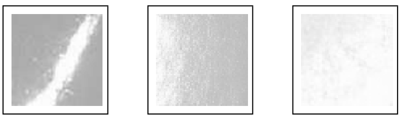

Fig. 1. Images of porcelain plate (left), glossy paper (centre) and plastic plate (right).

The slightly-rough surfaces are a porcelain plate, a white plastic plate and a glossy paper. For each of these surfaces we capture one image under off-normal illumination ($\theta_i = 14°$). These are shown in Fig. 1. Here, we use the B-K model for the specular direction to find the surface roughness σ (Section 4.1). We also estimate the correlation length T using the technique outlined in Section 4.2. We present the resulting estimates in Table 1. Our estimates are in good agreement with the ground-truth values found in the literature [10, 13] for similar surfaces.

Table 1. Results for the slightly-rough surfaces shown in Fig. 1.

Surface type	i_{spec}	RMS roughness (μm)	I_1/I_2	Correlation length (μm)
Porcelain plate	0.29874	0.06310	1.04716	0.30083
Glossy paper	0.23802	0.06878	1.04271	0.31057
Plastic plate	0.06491	0.09494	1.03179	0.36507

The very-rough surfaces used in our study are samples of stone tile, textured wall paper and sandpaper. For each of these surfaces, we capture two images under two different incidence angles ($30°$ and $45°$). These image pairs are shown in Fig. 2. For these surfaces we use the method explained in Section 4.3 to find the ratio σ/T. Since here the incident angles are greater than $20°$, we use the Vernold-Harvey modification. Using the estimates of σ/T, we also compute the angle of RMS slope θ_0 and the energy conservation rate for the physical case ξ_{phys} (Section 3.2). The results are presented in Table 2. The estimates are very close to the actual values measured using the well known stylus method [2, 10].

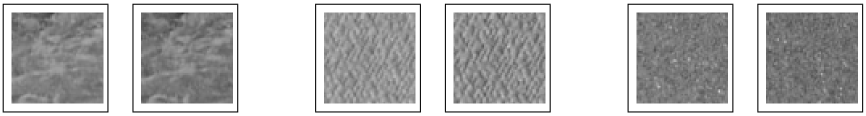

Fig. 2. Image pairs of very-rough surfaces: stone tile (left), textured wall paper (centre) and sandpaper (right) for $\theta_i = 30°$ and $\theta_i = 45°$ respectively ($\theta_r = 0$).

With estimates of σ/T available, we can fit the B-K model to the real-world data. To do this, we construct a cylindrical surface using the sample of sandpaper previously used (Table 2). We illuminate the cylinder in the viewing direction.

Table 2. Results for the very-rough surfaces shown in Fig. 2.

Surface type	I_1/I_2	σ/T	θ_0 (degs.)	$\xi_{phys}(\%)$
Stone tile	1.0562	0.9126	37.77	25.93
Wall paper	1.0696	0.7654	42.73	34.74
Sandpaper	1.0877	0.6483	47.48	44.83

We use the estimated value of σ/T for the sample of sandpaper to fit the model to the data in the following way. Across a horizontal line, perpendicular to the axis of the cylinder, we compute the mean-intensity at each point (on the right-hand half-cylinder) by averaging the intensity values of its vertical neighbors. Fig. 3 shows mean-intensity as a function of incidence angle as a solid curve.

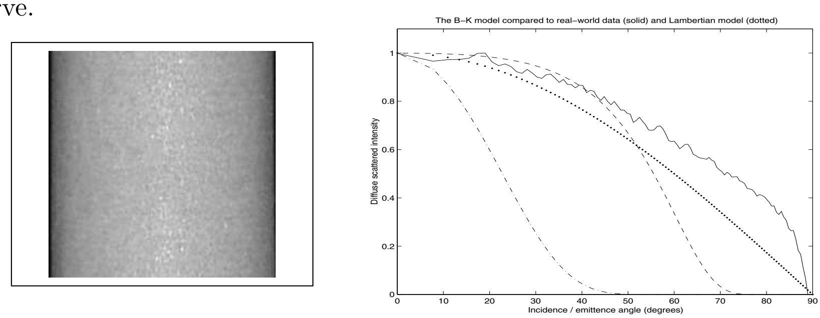

Fig. 3. Intensity-mean (solid) across a cylindrical sandpaper against incidence angle; the original (dash-dot) and modified (dashed) B-K model; Lambertian model (dotted).

Here, we show the Lambertian model (dotted curve), the original B-K model (Eq. 4, dash-dot curve), and the modified B-K model (Eq. 5, dashed curve). In this plot, for incidence angles less than almost $50°$, the data fits well to the modified B-K model while it deviates significantly from the Lambertian model. The original B-K model only fits well to the data for small incidence angles. There are a number of different reasons why the B-K model does not match the data at all locations on the cylinder. The main reason is the multiple scattering phenomena [10, 4] which is not included in the Kirchhoff's single scattering theory. For the sample of sandpaper used here, $\xi = 44\%$ suggests the possibility that the scattered energy may have been misestimated by a number as 44% when the Kirchhoff theory is used. This prediction is close to the experimental results. Also, $\theta_0 = 47°$ is very close to the incidence angles above which the Kirchhoff theory is expected to fail.

6 Conclusions

In this paper, we have analysed Beckmann formulation of the Kirchhoff's theory. The aim here has been to develop simple methods for estimating surface characteristics using a light-source and a digital camera. We estimate the RMS roughness and the correlation length for slightly-rough surfaces, and the surface slope for very-rough surfaces. This estimated slope allows the B-K model to be fitted to real-world data. While the modified B-K model fits to the data, the original B-K model only fits well to the data for small incidence angles. The results indicate that physics-based reflectance models can be used in conjunction with simple experimental instruments to make surface roughness measurements. Hence, we believe that the proposed techniques have significant potential in computer vision for texture model acquisition and realistic reflectance modelling.

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