A NEW APPROACH TO MODELLING IN PLANNING OF ELECTRIC POWER SYSTEMS

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ABSTRACT

The Paper shows the structure of a planning concept for electricity supply systems which, although developed for the Austrian system, is applicable on a general basis to other, more extensive, environmental systems.

1. INTRODUCTION

A system to be studied can be characterised by its elements and their relationships. The operation of such a system consists in using its elements to meet a (public) demand. A growth in demand entails an extension of the system and this in turn involves aims that conflict with those of other systems (other systems serving to meet the demand or other environmental systems). This requires consideration of global variables and constraints, which leads to the consideration of more extensive systems.

2. THE PLANNING PROBLEM

The elements of the system under study (e.g. power stations) are successively added to the system so as to keep pace with the growing requirements of the system (meeting the demand). The planning problem is the timing of the system extensions. Due to the mutual influences between several extensions we must consider a relatively long planning period. The aim of planning is to make the system fulfil certain tasks at minimum cost, observing the given constraints.

3. TREATMENT OF THE PROBLEM

Let the configuration of the elements of a system be denoted by "state". As the state changes with time, it can be expressed as a function of time:

$$z(t) \, , \, t \in [t_o, t_e] \ ,$$

where $z(t)$ is a vector whose component $z_i(t)$ indicates the number of realisations (e.g. power stations placed in operation) of the ith element at time t ; if z_i denotes a particular project, either $z_i(t) = 0$ or $z_i(t) = 1$; if it is a standard unit (a power station type that can be realised any number of times), then $z_i(t) = k$ with k being a non-negative integer.

e.g. $z(t)$ for some t :

$$\left| \begin{array}{c} 0\ 1\ 0\ 1 \\ \text{projects} \end{array} \right| \begin{array}{c} 0\ 2\ 3\ 0\ 4\ 1 \\ \text{standard units} \end{array} \left| \right.$$

We are trying to find $z^*(t)$ with $\mathcal{K}(z^*) = \min\limits_{z \in \mathfrak{D}} (\mathcal{K}(z))$.

Let $\Delta z(t) := \lim\limits_{\varepsilon \to +0} (z(t) - z(t-\varepsilon))$.

$$\mathcal{K}(z) = \mathcal{K}^{op}(z) + \mathcal{K}^{inv}(\Delta z)$$

The dependence of $\mathcal{K}$ on $\Delta z(t)$ is primarily given by the investment costs of unit additions, or more exactly, the total costs which arise from the decision to extend the system.

The dependence of $\mathcal{K}$ on $z(t)$ is expressed by the operating costs. If $z(t)$ cannot meet the demand subject to the constraints this is expressed by $\mathcal{K}(z) = \infty$.

For a finite number of points t_{ij} : $\Delta z(t_{ij}) > 0$.

Hence we can write:

$$\mathcal{K}^{inv}(\Delta z) = \sum_i \sum_j K_i^{inv}(\Delta z(t_{ij}), t_{ij}) .$$

Let $y_z(t) = (y_1(t), \dots, y_i(t), \dots, y_n(t))$ denote the vector of output of components ("operation variables").

We have to include in our consideration the optimal operating costs:

$$\mathcal{K}^{op*}(z) = \mathcal{K}^{op}(z, y_z^*) = \min\limits_{y_z \in Y} \mathcal{K}^{op}(z, y_z) .$$

For the purpose of studying the operation of the system we subdivide the study period into intervals Δt whose length is based on the longest operating cycle in the system (e.g. one year in the Austrian hydro system). With respect to system operation, these time intervals can be regarded as being independent and we can study the relationships within each interval T_j separately.

$$T_j = [t_o + (j-1) * \Delta t , t_o + j * \Delta t]$$

then:

$$\mathcal{K}^{op*}(z) = \sum_j \mathcal{K}^{op*}_{T_j}(z) = \sum_j \left(\min_{y_z \in Y_j} \mathcal{K}^{op}_{T_j}(z, y_z) \right)$$

with $Y_j \subset Y$.

As in most of the other methods of treating this problem (see e.g. (1),(3),(4)), we make this problem a discrete one with respect to these intervals and we apply a discrete Dynamic Programming algorithm. This means that we allow changes in state only at the times $t_o, t_o + \Delta t,$ $t_o + 2 \cdot \Delta t, \ldots$ and calculate the operating costs for each interval $[t_o + (j-1) \cdot \Delta t, t_o + j \cdot \Delta t]$ separately.
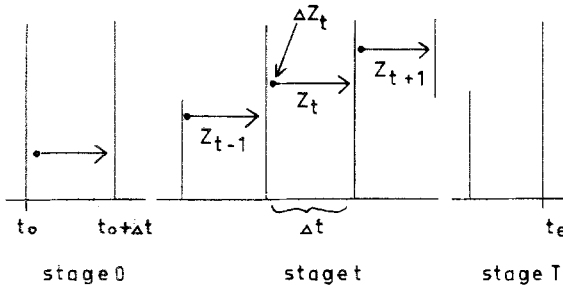
Let $z_t \in D_t$ and $z_t = z_o + \sum_{\tau=1}^{t} \Delta z_\tau$

The problem can be written:

$$\min_{\Delta z_\tau, \tau = 1, \ldots, T} \left(\sum_{\tau=0}^{T} K^{op*}_\tau(z_\tau) + \sum_{\tau=1}^{T} K^{inv}_\tau(\Delta z_\tau) \right) .$$

Now define:

$$K_t(z_t) := \min_{\Delta z_\tau, \tau = 1, \ldots t} \left(\sum_{\tau=0}^{t} K^{op*}_\tau(z_\tau) + \sum_{\tau=1}^{t} K^{inv}_\tau(\Delta z_\tau) \right)$$

then:

$$K_t(z_t) = \min_{\Delta z_t} \left(K^{inv}_t(\Delta z_t) + K_{t-1}(z_{t-1}) \right) + K^{op*}_t(z_t)$$

Since hydro power accounts for a large proportion of the Austrian system, the operation model is very complex. To deal both with the questions of long-term planning and with questions of detail, two programs have been developed:

(a) an extension planning program with special routines for defining the states, with a simplified nonoptimising operatio simulation model checked with the help of program (b).

(b) an operation scheduling program which can solve large-scale
Nonlinear Programming problems and is used to solve questions
of detail (especially for the hydro system), and to assist
in planning by means of model (a).

4. STRUCTURE OF THE EXTENSION PLANNING MODEL (Fig. 1)

Let $z_t = (z_{1t}, \ldots z_{it}, \ldots z_{nt})$ denote a state at t with $1 \le t \le T$.

We now must generate all the $z_t \in D_t$:

This is done by using a GENER model, which "sieves out" feasible states,
i.e. those which satisfy the given constraints associated with stage t .

Other models are now applied to the sets of states so generated, yield-
ing results which we associate with each state. Thus e.g.: The SECUR
model determines a security level of a state; or the SIMUL model deter-
mines the operating cost over a year calculated on the basis of a suit-
able but non-optimising operation strategy.

Running the problem with different security levels, we study only
the respective associated values and temporarily delete some states if
these do not meet the requirements. (We superimpose a mask $\bar{D}_t \subset D_t$
on the sets of states). In the case of running the problem with differ-
ent parameters, some of the models need be run only once. In each case,
however, the DP algorithm must be run.

This is done in the STEP model:
It serves to associate in each DP step from stage $t-1$ to stage t
the total cost value $K_t(z_t)$ with each z_t . These values are the optimal
costs in relation to state z_t if state $z_t \in \bar{D}_t$ has been reached by
an optimal path through the stages 1 to t . The optimal extension alter-
native results from the optimal path to z^*_T of the last stage with

$$K_T(z^*_T) = \min_{z_T \in \bar{D}_T} K_T(z_T) \; .$$

5. OPERATION SCHEDULING OPTIMISATION MODEL

As stated earlier, large-scale systems must be studied. The relation-
ships between their elements can usually be subdivided, in a multistage
manner, into "global" and "local" ones, that is to say, we can subdivide
the total system into subsystems (as e.g. overall generating system and
hydro power stations on a certain river), which in turn contain local
relationships. These can again be subdivided into global and local ones.
Thus we obtain a multi-level structured system model.

Now consider the operation scheduling model to treat questions of detail.

The dimension of the operation model results from the great number of intervals of which the optimisation period consists, and from the fact that we express the period (year) by some characteristic daily load curves. We can derive the following NLP problem:

The thermal subsystem provides a cost basis for evaluating the hydro system. This results in the following objective function:

$$\max \sum_{t \in T_y} v_t (PH_t)$$

where v_t is a nonlinear evaluation function for the hydro capacity PH_t and $PH_t = \sum_{i \in \mathcal{U}} PH_{it}$, where PH_{it} denotes the capacity of power plant

$i \in \mathcal{U}$ at time interval t.

Let Q_{it} be the flow rate at power station i at time interval t. Then we can write $PH_{it} = f_i (Q_{it})$.

For Q_{it}, we have to consider lower and upper bounds

$$Q_{i_{min}} \leq Q_{it} \leq Q_{i_{max}} \qquad \forall i \in \mathcal{U}, \forall t \in T_y.$$

Let us define a storage - unit - incidence matrix as shown in Fig. 2.

Let V_{kt} be the volume of storage node k at the beginning of the time

interval t, then $V_{kt} = V_{k0} + \sum_{\tau=1}^{t-1} \Delta V_{k\tau}$ with $\Delta V_{kt} = \sum_{i \in \mathcal{U}} (-ic_{ki}) * Q_{it}$.

The V_{kt} are constrained by $V_{k_{min}} \leq V_{kt} \leq V_{k_{max}}$.

This results in a NLP problem in the variables Q_{it} with the linear constraints resulting from those of the V_{kt}.

Let us now consider these constraints: If we are faced with a structured problem, this is expressed by the ic -matrix, which might e.g. take the following form:
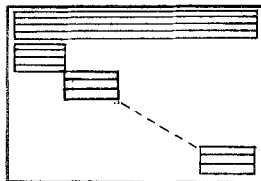
That means that we can subdivide the nodes $k \in K$ into

(1) coupling storage nodes $k \in K_c$ and

(2) storage nodes in subsystem S_i : $k \in K_{S_i}$.

Since in hydro power systems individual storages are scheduled on different cycles (daily, weekly and yearly cycles), we can also define "subsystems" with respect to time, because the constraints for the V_{kt} do not extend beyond the operation period of the power stations.

That is: $\quad K = K_y + K_w + K_d$.

$$\underset{\text{yearly}}{|} \quad \underset{\text{weekly}}{|} \quad \underset{\text{daily cycle}}{|}$$
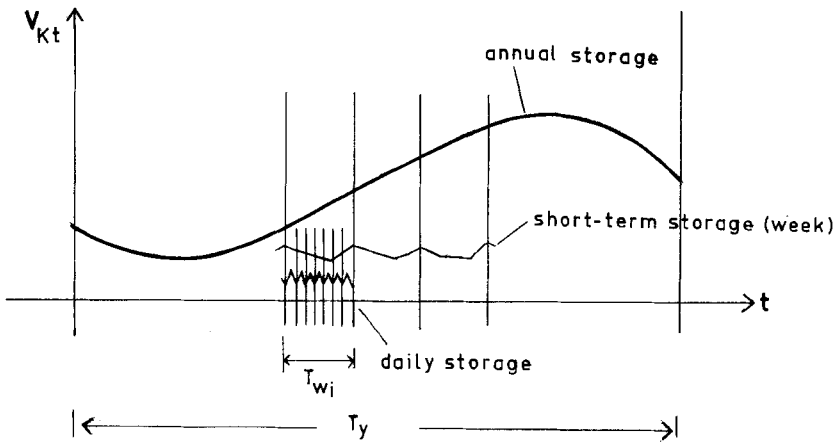
Let: $\quad K_{dc} := K_d \cap K_c \;,\; K_{dS_i} := K_d \cap K_{S_i}$.

Thus we obtain a multi-level structure with coupling constraints as shown in Fig. 3.

6. APPLICATION OF A MULTI-LEVEL ALGORITHM

The above NLP problem has been handled by a method taking advantage of the structure of the constraint matrix. The projection p of the gradient g is used as a feasible direction. Instead of calculating the projection vector by the projection matrix $P = I - Q(Q^T Q)^{-1} Q^T$ and $p = Pg$ where $Q = (q_1, \ldots q_m)$ is the matrix of active constraints (as e.g. in Rosen's Gradient Projection Method), we use orthogonal vectors $\tilde{q}_1, \ldots, \tilde{q}_m$, which form a basis for the linear space $L(q_1, \ldots, q_m)$.

The projection vector p can be expressed as follows:

$$p = g - \sum_{i=1}^{m} \tilde{\lambda}_i \cdot \tilde{q}_i \quad , \quad \tilde{\lambda}_i = (g, \hat{q}_i) / |\hat{q}_i|^2 \quad ,$$
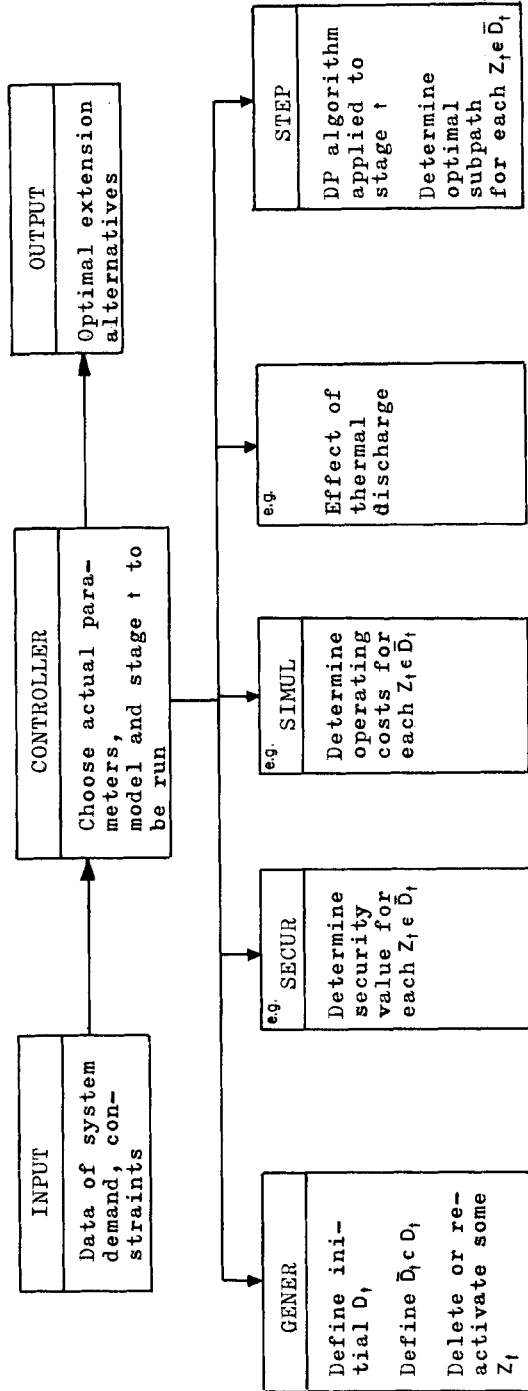
where (a, b) denotes the inner product of vectors a, b.

The advantage of the given structure is that constraints from parallel subsystems (i.e. from subsystems whose variable vectors have disjoint index sets) are a priori orthogonal.

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Fig. 1: Structure of the Program

INPUT	CONTROLLER	OUTPUT
Data of system demand, constraints	Choose actual parameters, model and stage t to be run	Optimal extension alternatives

GENER	SECUR	SIMUL		STEP
Define initial D_t Define $\bar{D}_t \subset D_t$ Delete or re-activate some Z_t	e.g. Determine security value for each $Z_t \in \bar{D}_t$	e.g. Determine operating costs for each $Z_t \in \bar{D}_t$	e.g. Effect of thermal discharge	DP algorithm applied to stage t Determine optimal subpath for each $Z_t \in \bar{D}_t$

Generation of states   Associating values with states   Optimisation

Fig. 2: Example of storage system with its incidence matrix
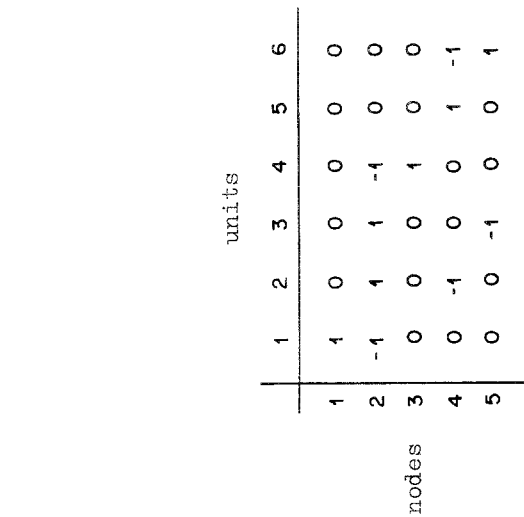
	units					
nodes	1	2	3	4	5	6
1	1	0	0	0	0	0
2	-1	1	1	-1	0	0
3	0	0	0	1	0	0
4	0	-1	0	0	1	-1
5	0	0	-1	0	0	1

Fig. 3: Example of multilevel constraint matrix and adjoint subsystem tree