THE USE OF MIXED INTEGER PROGRAMMING FOR THE EVALUATION OF SOME ALTERNATE AIR POLLUTION ABATEMENT POLICIES

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INTRODUCTION

As a part of the preliminary work performed in the Nervion River Valley Bilbao Air Pollution Study, the following method has been developed for the evaluation and selection of emission control policies and standards.

Escudero and Jimenez, 1975 describes a methodology used to estimate the probability distribution of pollutant concentrations in each receptor grid square of a studied area for a seasonal time period, given the predicted pollutant emissions due to the point and area sources with significant influence on the pollutant concentration. This probability distribution is estimated over the total range of different meteorological conditions that affect the concentration significantly.

The probability distribution, which is estimated on the basis of a stochastic diffusion model, gives the probability for each meteorological condition and set of contributing emissions that the concentration in each grid square will exceed the maximum limit of concentration.

A polluted grid square is considered to be one in which this probability is greater than the maximum probability allowed, called the relative limit. A contributing areas is the set of point sources and area sources which affect the concentration in the receptor grid square. A polluted area is defined as the set of contributing areas and polluted grid squares which have at least one grid square or emitter source in common.

GENERAL OBJECTIVE AND CONDITIONS FOR THE MASC-AP MODEL

The objective of this model is to evaluate the alternatives for reducing the emission at the point and area sources so that the problem area will no longer be polluted area.

The reduction alternatives may be programmed for a single seasonal period or for a set of these periods adapting the reduction alternative for each emitter source to fixed and constant abatement levels for the entire period programmed (Escudero, 1973). This paper treats a single seasonal period, and in contrast to other types of models (Gorr and Kortanek, 1970; Kortanek and Gorr, 1971; Teller, 1968), the probabilistic limits of the real concentration are used, as it is one of the principal characteris-

tics of the reduction model.

The probabilistic limits of the concentration include the new average theoretical concentrations in the polluted grid square when the emissions are reduced (Escudero and Jimenez, 1975). These concentrations correspond to a given set of probabilities that the real concentration exceed the absolute limit, so that once the new theoretical concentration has been estimated it is assumed that it is the upper limit of the interval which corresponds to it in the limit concentrations relative to the given set of probabilities. In this way an estimate is made of the probability of excess real concentration for each theoretical concentration, and therefore for each emission reduction alternative.

The emission reduction alternatives are estimated such that the emission reduction to be imposed on each influence source be both the minimum possible and be in proportion to its influence on the polluted area.

THE ELEMENTS OF THE EMISSIONS REDUCTION MODEL

The elements needed for using this mixed integer programming model for each problem area are the following:

The receptor grid square r$\forall r \epsilon R$ which makes up the polluted area where the concentration is to be reduced.

The influence emitter grid square e$\forall e \epsilon E$ within the problem area.

The meteorological condition m$\forall m \epsilon M$ in grid square r which, given the set of emissions, causes the real concentration C_{rm} to exceed the absolute limit AL in the seasonal period under consideration.

The type of probability p$\forall p \epsilon P$ which corresponds to the probability PAL_{rmp} of the limit concentration UC_{rmp}.

The predicted pollutant emission AQA_e ($\mu g/s$) for the seasonal period under consideration coming from the area sources located in emitter grid square e. Idem ($\mu g/s$) for point sources (AQP_e).

The theoretical unit influence KA_{rem} of the area emissions from emitter grid square e upon grid square r under meteorological condition m, according to the stochastic diffusion model. Idem for the point source emissions (KP_{rem}).

The average theoretical pollutant concentration TC_{rm} ($\mu g/m^3$) which, in the stochastic diffusion model, is found in grid square r, under meteorological condition m, given the actual set of emissions AQA_e and AQP_e. The pollutant concentration TC_{rm} is represented by:

$$TC_{rm} = a_{rm} + \sum_{e=1}^{E} (KA_{rem} AQA_e + KP_{rem} AQP_e) \qquad \forall m \epsilon M,\ e \epsilon R \qquad (1)$$

where a_{rm} is an independant term considering the background concentration and others.

The frequency MP_m of meteorological condition m during the seasonal period under consideration. Only those meteorological conditions are taken into account whose

probability MP_m is significant.

The meximum allowable probability (relative limit) RL that the real concentration C_r in any grid square exceed the absolute limit AL.

The theoretical concentration UC_{rmp} in grid square r under meteorological condition m corresponding to probability PAL_{rmp}.

The given set of probabilities PAL_{rmp} that the real concentration in each situation rm exceed the absolute limit AL. Using the stochastic diffusion model, the theoretical concentration is obtained for each possible set of emissions. It is assumed that the probability that the real concentration exceed the limit AL is the probability estimated by the UC_{rmp} value inmediately above the corresponding theoretical concentration.

The probability that for the predicted set of emissions AQA_e and $AQAP_e$ the real concentration in grid square r under meteorological condition m exceed the maximun limit AL, taking into consideration the probability of the meteorological condition, so that $PC_{rm} = MP_m\ PAL_{rm}$.

The probability PAL_{rm} that, for the predicted set of emissions, the real concentration exceeds the absolute limit will be equal to PAL_{rmP}. Therefore, $UC_{rmP} = TC_{rm}$.

The influence WA_e of the area sources in emitter grid square e during the seasonal period considered upon the pollutant concentration in all the grid squares which make up the polluted area under all the meteorological conditions considered (Escudero and Jimenez, 1975). Idem for the point sources (WP_e). The influences WA_e and WP_e are represented by:

$$WA_e = \sum_{r=1}^{R} \sum_{m=1}^{M} KA_{rem}\ AQA_e\ PC_{rm} \qquad (2a)$$

$$WP_e = \sum_{r=1}^{R} \sum_{m=1}^{M} KP_{rem}\ AQP_e\ PC_{rm} \qquad (2b)$$

The maximum percent MRA_e of the reduction allowed of the predicted emission AQA_e in emitter grid square e, based upon socioeconomic considerations. Idem for the point sources $AQP_e(MRP_e)$.

The variables used in the model are: The pollutant emission XQA_e to be reduced in the area source of emitter grid square e during the entire seasonal period being considered. Idem for the point sources (XQP_e). The new average theoretical concentration XC_{rm} in situation rm corresponding to the new emission from each emitter grid square. The binary variable Y_{rmp} whose value is 1 if the theoretical concentration UC_{rmp} is the upper limit of the concentration XC_{rm}. If not its value is 0.

THE FORMULATION OF THE MASC-AP MODEL

Using the elements described above, the model for the reduction of emissions that will eliminate the polluted area is the following:

Minimize in a weighted form the emission reduction:

$$\text{Min. } QR = \sum_{e=1}^{E} \left(\frac{1}{WA_e} XQA_e + \frac{1}{WP_e} XQP_e\right) \tag{3}$$

such that, with the conditions being the same, priority is given to the emission in the grid square that pollutes most.

Minimization of the Equation (3) is subject to the following conditions:

1) The estimation of the new theoretical average concentration XC_{rm} corresponding, according to the stochastic diffusion model, to the new set of emissions.

$$XC_{rm} = TC_{rm} - \sum_{e=1}^{E} (KA_{rem}\ XQA_e + KP_{rem}\ XQP_e) \qquad \forall m \epsilon M,\ r \epsilon R \tag{4}$$

2) The necessity that only variable Y_{rmp} take on the value 1 if UC_{rmp} is the limit inmediately above the new average theoretical concentration XC_{rm} $\forall m \epsilon M,\ r \epsilon R$.

$$XC_{rm} \leq \sum_{p=1}^{P} UC_{rmp}\ Y_{rmp} \qquad \forall m \epsilon M,\ r \epsilon R \tag{5}$$

$$1 = \sum_{p=1}^{P} Y_{rmp} \qquad \forall m \epsilon M,\ r \epsilon R \tag{6}$$

3) The equation (8) is the principal condition of the model requiring the probability

$$\sum_{m=1}^{M} MP_m\ PAL_{rmp}\ Y_{rmp} \tag{7}$$

that the real concentration C_r exceed the absolute limit AL not be greater than the relative limit RL in any of the polluted grid square.

$$\sum_{m=1}^{M} \sum_{p=1}^{P} MP_m\ PAL_{rmp}\ Y_{rmp} \leq RL \qquad \forall r \epsilon R \tag{8}$$

In this regard it is important to note that Equation (6) demands that each case have only one Equation (7) different from zero.

4) The variables for the amount of emission reduction are represented by:

$$XQA_e \leq MRA_e\ AQA_e \tag{9a}$$

$$XQP_e \leq MRP_e\ AQP_e \tag{9b}$$

The principal results of the reduction model are:

a) The reduction values, expressed in percentages, for the point and area sources of each grid square for the time period considered.

b) The values of the corresponding new emissions.

c) The new probabilistic distribution of concentration for each receptor grid square which makes up the polluted area.

d) The probability that the new concentration exceed the limit established.

THE BRANCH AND BOUND POSSIBILITIES IN THE MASC-AP MODEL

The model for the reduction of the pollutant emissions needs the use of mixed integer programming techniques, since the Y variables are binary being able to take on only the values 0 and 1. Among the many algorithms existing for its solution (Geoffrion and Marsten, 1972) the MASC-AP reduction model is based on the IBM MPSX/MIP system using the following possibilities.

SOS conditions

A Special Order Set is a set (Beale and Tomlin, 1969) of binary variables of which one and only one has the value 1. In the emissions reduction model the SOS conditions are included in Equation (6) so that if one variable has the value 1 the others must be null. Accompanying each SOS row there must be another condition or some weighting that is responsible for the important attributed to it. For the SOS condition the corresponding weighting is the probability PC_{rm} that the concentration C_r exceed the maximum limit AL.

Accompanying each SOS row there is another weighting row that represents the importance attributed to each SOS variable, in this case to each Y_{rmp} variable. For the SOS row (Equation 6) the corresponding weighting row is the Equation (5).

The SOS rows possibility is used in the branch and bound phase, once the continuous optimum solution is obtained, when at a certain node the branching integer variable (in this case, SOS row) is selected. To branch the SOS row, the value of its weighting row (Equation 5) is analyzed, so that (Escudero, 1975b) if it were the case, for example that

$$1 = Y_1 + Y_2 + Y_3 + Y_4 + Y_5 \qquad \text{(SOS row)} \qquad (10)$$

$$W = 750Y_1 + 500Y_2 + 250Y_3 + 100Y_4 + 50Y_5 \qquad \text{(weighting row)} \qquad (11)$$

and the SOS row did not have any binary variable with a value of 1, the value of W is noted, and if this is, for example, 300, then, since the only other alternatives would be 750, 500, 250, 100 and 50, either $W > 300$ (in which case W is 750 or 500) or $W < 300$ (in which case W is 250 or 100 or 50). To establish this dichotomy it is necessary that in the first case $Y_3 = Y_4 = Y_5 = 0$ and in the second case that $Y_1 = Y_2 = 0$, so that in the corresponding branch 3 and 2 SOS variables respectively have been fixed, when in the normal procedure (Benichou et al., 1971) only one of the variables would have been acted upon.

Quasi-integer variables

In the optimization models with integer variables, it is necessary, in the optimum, that these variables take on integer values (0 or 1 in this case), but often in the branching formation the candidate nodes have some variables with integer values and others with quasi-integer values (for example, 0.001; 0.995) which means having many successor nodes in order to make them integers (primarily if the number of integer variables is high and their coefficients in the objective function are not very different).

The need in our case for binary variables only is motivated by the requirement in Equation (8) that the probability that the concentration exceed the absolute limit not be greater than the relative limit. Thus, even though the binary variables had only quasi-integer values, the objective would also be achieved, since the quasi-integrality of the binary variables will really bring about the probability of exceeding the absolute limit, even though it were not the relative limit, but were a value very close to it. Given the probabilistic form of the model this would not ruin the plan adopted for the reduction of the emissions.

Pseudo-cost of the integer variables

There exist in the literature many controversies over the strategy to be used in the branch and bound phase, mainly in regard to the choice of the next branching node and the choice of the branching variable. Although in the choice of the branching node the criterium may be used of the best functional value (Roy et al., 1970) or a mixed criterium, the best functional value until the first integer solution and then the best estimated value (Benichou et al., 1971), the MASC-AP model uses the criterium of the best estimated value. In regard to the branching variable, although the penalties criterium may be adopted (Roy et al., 1970), the MASC-AP model uses the criterium of the pseudo-costs (Benichou et al., 1971) since it has been observed experimentally that it better incorporates the influence of each variable upon the objective function.

Therefore, using the SOS rows as the "branching variables" (Escudero, 1975b), the elements of this strategy are the following:

$\bar{W}_c$. The real value of the weighting row W_c of the "branching SOS row" in the optimum solution of the subproblem (or node) k.

$f_c^{(r)}$. The sum of SOS variables ($\sum_{j \le r} Y_j$) that make up the SOS row c, being fixed to zero in the first branch of the dichotomy r. In the case of Equations (10) and (11), r=2 then $f_c^{(2)} = Y_1 + Y_2$. Therefore, for the second branch of the dichotomy r we have $1 - f_c^{(2)} = Y_3 + Y_4 + Y_5$.

F^k. The optimum functional value in subproblem k.

F^{n+1}. The optimum functional value in the subproblem (n+1).

F^{n+2}. The optimum functional value in the subproblem (n+2). The subproblems (n+1) and (n+2) have been generated by means of branching on the SOS row with the dichotomy r.

The pseudo-costs lower and upper of the SOS row in the dichotomy r (if they have not been obtained previously) are estimated as follows:

$$PCL_c^{(r)} = \frac{|F^k - F^{n+1}|}{f_c^{(r)}} \tag{12}$$

$$PCU_c^{(r)} = \frac{|F^k - F^{n+2}|}{1 - f_c^{(r)}} \tag{13}$$

So that, if for each dichotomy r (in our case r=1, 2, 3 and 4) of each SOS row i we have obtained the value $\delta_i^{(r)}$:

$$\delta_i^{(r)} = \min \{PCL_i^{(r)} \; f_i^{(r)}, \; PCU_i^{(r)} \; (1-f_i^{(r)})\} \tag{14}$$

the estimations of each node are obtained in the following way. Using the pseudo-costs of the I SOS rows in which one SOS variable has not taken the value 1 in the node (k), the calculation is made on the basis of the formula:

$$E^k = F^k + \sum_{i=1}^{I} \delta_i^{(r)} \tag{15}$$

where r is the corresponding dichotomy of each of the I SOS rows considered. Equation (15) represents the funcional value E^k of the best integer solution which is estimated may be obtained with the nodes generated from the node k.

The pseudo-costs (Equations 12 and 13) represent the deteriorization of the optimum value (F^k) of the function for every unit of change in the corresponding SOS row. These values depend on the node in which they have been obtained. However, on the basis of the preliminary experiments which have been carried out, it may be assumed that although they do not remain constant, they have generally the same order of magnitude.

THE MASC-AP MODEL STRATEGY

The strategy of the MASC-AP model, since it has a large percentage of binary variables and can thus be considered "quasi-pure", is the following.

The selection of the branching variable and the branching node.

The SOS row in Equation (6) to bifurcate will be that which has not yet reached an integer or quasi-integer value, whose associated value PC_{rm} is greater, since this is the most difficult SOS condition to fulfill, and therefore the condition which causes a greater deteriorization in the objective function. Once the first integer solution is obtained, the SOS row is chosen whose value $\delta_i^{(r)}$ (Equation 14) is the one which offers a greater deteriorization in the objective function.

These deteriorizations are classified in dynamic order such that the "list" of the different deteriorizations is composed by the actualized pseudo-costs. When the SOS row to bifurcate has been selected, the criterium for creating the two successor nodes is based on Equation (5).

Except in the node (0), before selecting the branching SOS row it is necessary to select the branching node from among all the candidate nodes. The criterium adopted by the MASC-AP model is to choose the node with the best estimated value so that, since the pseudo-costs are not calculated for the SOS rows which are not yet branching SOS rows, the estimated value in the first branches, and practically until some integer solution is reached, differs very little from the functional value.

Dropped nodes and selection of the candidate nodes

Before obtaining the optimum continuous solution, the MASC-AP model obtains a feasible integer solution such that in the branch and bound phase those nodes are dropped whose functional value is worse than that of the previously obtained integer solution.

Once the integer solution has been obtained, the branch and bound phase drops the nodes whose functional value is greater say by 10% than that of the integer solution. Also, this phase "postpones" nodes whose functional value even if it is greater than the value of the integer solution does not have a difference greater than the previous value (e.g. 10%), so that once the optimality of the best integer solution has been proved it is observed whether among the postponed succesor nodes there is some integer solution which differs from the optimum solution by no more than 10%. In this way, different alternatives for the reduction of emissions are produced.

Now, given that the alternatives for the emission are estimated, since the effects of each emitter source on the pollution of the problem area are estimated for only the polluted area, a node is not admitted as candidate node if its functional value is not better, say, as a minimum by 10%, than the value of the best integer solution. In this way much of the CPU time is saved, and at the same time little of the accuracy of the quasi-optimum integer solution is lost.

CONCLUSION

The model presented in this paper must be considered to be an effective tool for establishing bases for corrective alternatives for an abatement problem of air pollution. It should also be considered a very useful instrument for qualifying, within the development policies for a given area, the standards that are more and more indispensible for protecting our air environment.

It should be noted that the basic statistical parameter considered in the formulation of the model is the maximum probability allowed that the concentration in a given grid square exceed the maximum limit allowed, in contrast to models which use averages as their standards of quality. This methods avoids the danger of large concentrations being masked with smaller concentrations. This probability depends conjointly on the probabilistic matrix of the typology by which the different meteorological factors have been stratified and the probability that for a theoretical concentration estimated on the basis of a predicted set of emissions the real concentration might exceed the maximum limit permited.

It is of interest to point out that in order to estimate the concentration in each grid square, stochastic diffusion models have been used for each meteorological stratum, depending on the emissions, so that the tabular form of the model is in function with the emitter grid squares.

The criterium which minimize the model is the weighted reduction of the emission levels for each contributing grid square in accord with the effect it has on the pollutant concentration in the sum of the grid squares which make up the polluted area.

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