A SYSTEM OF MODELS OF OUTPUT RENEWAL

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Regular renewal of output produced is a law in contemporary industrial production. The processes of output renewal are very diverse; peculiar to them are intricate hierarchical interrelationships between their components and uncertainty of parameters which characterize both the components and the processes as a whole. For these, inherent are certain specific features which, taken together, make possible their representation and analysis with discrete means, and, in particular, the effectiveness of applying models based on different types of graphs.

The diversity of output renewal processes and their specificity necessitate the use of a system of problem-directed models instead of a single universal model. Of the initial stages in output designing it is typical to have numerous alternative situations representing a set of various technical solutions. Of special interest are those competing variants whose preference is dependent either on random parameters, or on conditions needing additional research. According to the laws of combinatorics, the total number of variants on an item can be large, but the main point is that combinations of individual variants generate situations which are far from being obvious.

A model applied to the analysis of this kind of processes is based on a representation of a process as a multi-variant alternative stochastic directed graph $G(X, U)$ where X is a set of its vertices, U - a set of arcs. In the graph $G(X,U)$ the vertices correspond to different stages in the process of designing and are, for this reason, heterogeneous. In this model provided are 9 types of vertices representing all possible situations found in implementing complex projects. To represent different kinds of alternatives, at inputs and outputs of vertices logical conditions $\wedge$, V and $\overline{V}$ (symbol $\overline{V}$ denoting the excluding V) are realized. To represent statistical and dynamic rules of preference and choice of alternatives, to the arcs (i,j) going from the vertices i with logical conditions V or $\overline{V}$ at the output, the probabilities P_{ij} are confronted which may, in their turn, be connected by complicated Bayesian relations with the probabilities of the realization of inputs into the given vertex i :

$$\forall_j \in \Gamma i, \quad P_{ij} = f_j \ (P_{\kappa i} / \kappa \in \Gamma i^{-1}).$$

The development of these models is a multi-stage process in which the joint experience and intellect of all the participants in program developing is used. At the first stage, a structural scheme of technical complex under study is made, with a certain degree of detailing, as a graph $G_S(X_S, U_S)$. On the basis of expertise vertices $a \in A \subset X_S$ are discerned for which alternative solutions are permissible. An essential element of this stage is determining a type and logical conditions at input and output for $\forall a \in A$. For all the vertices $a \in A$ a set of permissible alternatives is determined, and each of them is represented by an arc (a,e), and $e \overline{\in} X_S$. For each alternative (a,e) a subgraph $G_e(X_e, U_e)$ of its realization is built. It is allowed in the model that any subgraph $G_e(X_e,U_e)$ in its turn can be stochastic alternative graph, and , hence, for its construction one needs analogous procedures.

The stochastic alternative graph $G(X, U)$ representing the process as a whole is obtained through adjoining, on the basis of the graph G_S , all the graphs of type G_e and through successively replacing the arcs $(a,j) \in U_S$ by a set of subgraphs G_e representing the alternatives provided for the vertex a .

The closing stage in building the stochastic alternative graph $G(X,U)$ is determining the parameters of all of its arcs. The parameters of arcs representing jobs are determined according to the normative approach of /1/, and for the probabilities P_{ae} of the alternatives (a,e) special procedures of group judges' evaluations treatment are used. Depending on the type of the event α there can be either vector $\{P_{\alpha e}\}, e\in\Gamma\alpha, \{\alpha\}=\Gamma\alpha$, or matrix $\{P_{\alpha e}^{\kappa}\}$ $e\in\Gamma\alpha, \kappa\in\Gamma\bar{\alpha}^{1}, \{\alpha\}=\Gamma\alpha, \{\kappa\}=\Gamma\bar{\alpha}^{1}$ in case of Bayesian relations. Apart from this, for conditions $\overline{V}$ at the vertex output there must be provided a rating $\sum_{e} P_{\alpha e}=1$ or $\forall\kappa: \sum_{e} P_{\alpha e}^{\kappa}=1$. In certain instances parameters P representing the probabilities of the choice of alternatives are, in their turn, random variables and, according to this, the expertise results will not be the numbers $P_{\alpha e}$ and $P_{\alpha e}^{\kappa}$ but the distribution functions of these variables set in one form or another.

The algorithm for the analysis of the stochastic alternative graph $G(X,U)$ is based on Monte Carlo ideas. To simulate the choice of alternatives, the procedures given in /5/ are used. The results of the algorithm are characteristics of graph $G(X,U): F(T), \Psi(S)$, i.e. distribution functions of designing duration T and of its cost S; $E(R_t)$, $D(R_t)$ mathematical expectations and variances of demands for resources allocated by time intervals t where $0\leqslant t\leqslant T^{0}$ and T^{0} is a given variable; for $\forall_e$, $e\in X$, in particular, for $\forall_\beta$, $\Gamma_\beta=\emptyset$, P_e is determined, i.e. the probability of reaching the vertex e in the project implementation, and some other characteristics. There are some procedures for discerning the most probable structure of the graph $G(X,U)$.

In developing the software for this model a crucial requirement was its openness. This stemmed from the diversity of processes and problems studied with the help of this model; therefore, each user should have a possibility to extend the software of the model to new problems. The modular modification of the model software procedure realizes this principle.

The analysis and the obtaining of integral characteristics of a single alternative stochastic graph does not resolve the problem of the process implementation in that case where the plant performs a number of projects comparable in priority and complexity. One of the approaches to resources allocation for supplying several projects is based on the application of linear-programming models. The characteristics of each from L graphs L subject to implementation $G^{\nu}(X^{\nu},U^{\nu}), \nu=1(1)L: E(S^{\nu}), E(R_t^{\nu})$ obtained by the above described model are incorporated as information into a linear-programming model. The obtained solution gives, in the first approximation, information about the location of resources in accordance with a certain optimization criterion. Another approach in which the linear-programming problem serves as a procedure in a general scheme of simulation of implementing L projects is also used. The obtained characteristics S^{ν} and R_t^{ν} of the graph G^{ν}, $\nu=1(1)L$ in a single Θ step of modelling enable one to solve the linear programming problem and to estimate the value of the objective function K_{Θ}. These experiments generating $K_1, K_2, \dots, K_{\Theta}, \dots, K_N$ of the objective function K make it possible to obtain the function of the distribution of the values of the objective function K and to estimate on the whole the effectiveness of the implementation of the plant productive program under optimal allocation of resources at random demand.

At a stage in output renewal process where the main alternatives of the project have been basically resolved, a special stochastic graph of the project implementation is built up. In technology of producing complicated items an advanced system of stage control, matching other kinds of coordination is provided. The results of these stages which will be conventionally called control stages, are, generally speaking, fortuitous and, along with the planned ones and those allowing the continuation of developing, there may also occur results requiring backtracks to the stages in development already passed. The popular network model permits one to represent only the ideal course of a process. According to this, for adequate representation of a real process, the network graph $G_g(X_g, U_g)$ is first built which is then transformed into a stochastic graph with backtracks $G_e(X_c, U_c)$. The sets of vertices X_g and X_c may be the same, but the sets of arcs U_g and U_c are essentially different: $U_c = U_g \cup U_b$, here U_b is a set of backtrack arcs of the type (e_b, j), e_b the vertex generating a backtrack to the vertex j. With the help of multiple interview technique and processing judges' evaluations, the parameters of the arcs of the graph G_C are determined. Of interest are the coefficients of changes in parameters of the arc (i,j): $\mu_t(i,j)$, $\mu_S(i,j)$, $\mu_K(i,j)$ which determine respectively time, cost, resources needed in repeated executions of work (i,j) and parameters set on the backtrack arcs (e_b, j) : $P_{e_b,j}$, the probability of the backtrack arc, $\mu_p(e_b,j)$, the coefficient of the change in probability of events e_b generating the backtracks. These parameters are recognized in the model and allow one to obtain the clarified characteristics of the process.

The algorithm for the analysis of a graph with fortuitous backtracks based on Monte Carlo technique and using the specific features of the built model, makes it possible to get on computers various information about the process, in particular, the characteristics of a number of parameters, among them: the density function $\Phi(T)$, $\Psi(S)$ of time and cost of the project execution, mathematical expectation and the variance of demands for resources of inaccumulative type of power $E(R_t)$, $D(R_t)$. These demands can be defined for each interval t of the planning period $[0,T]$. This information and structure of the graph $G(X,U)$ is starting for scheduling. Scheduling programs have been worked out on algorithms /7/ and conducted with the observance of modularity principle. Scheduling plans-graphs are given on computers as working documents, the form of which takes into consideration the requirements of the backfeed principle in the control over the processes described.

Building up the modular complexes of programs makes it possible to initiate the effective functioning of the system in different regimes, including the regime "man - computer". The use of a set of programs in a regime of a business game enables one to reveal on models extreme conditions of the process and to foresee necessary measures and their effectiveness beforehand, before the actual implementation of a certain process of output renewal under productive conditions.

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