Describing and Matching 2D Shapes by Their Points of Mutual Symmetry^{*}

Arjan Kuijper¹ and Ole Fogh Olsen²

 RICAM, Linz, Austria arjan.kuijper@oeaw.ac.at
 IT-University of Copenhagen, Denmark fogh@itu.dk

Abstract. A novel shape descriptor is introduced. It groups pairs of points that share a geometrical property that is based on their mutual symmetry. The descriptor is visualized as a diagonally symmetric diagram with binary valued regions. This diagram is a fingerprint of global symmetry between pairs of points along the shape. The descriptive power of the method is tested on a well-known shape data base containing several classes of shapes and partially occluded shapes. First tests with simple, elementary matching algorithms show good results.

1 Introduction

One method to describe 2D objects is by their outlines, or shapes. The complicated task of comparing objects then changes to comparing shapes. With a suitable representation, this task can be simplified. Several representations of shapes have been investigated in order to be able to perform this comparison efficiently and effectively. One of the earliest representations is Blum's biologically motivated skeleton [1]. As Kimia points out [2], there is evidence that humans use this type of representation.

Research on skeleton-based methods has been carried out in enormous extent ever since, see e.g. [3, 4]. The Shock Graph approach [5] has lead to a shape descriptor that can perform the comparison task very well [4, 6, 7]. This method depends on results obtained from the so-called Symmetry Set [8, 9], a super set of the Medial Axis. In these cases, the shape is probed with circles tangent to it at at least two places. The Symmetry Set is obtained as the centres of all these circles, while the Medial Axis is the sub set containing only maximal circles.

From the field of robotics, probing shapes is also of interest. Blake et al. [10, 11] describe a grasping method by the set of points that are pair wise parallel. At such a pair a parallel jaw gripper can grasp the object. These points form

^{*} This work was supported by the European Union project DSSCV (IST-2001-35443). A.K. acknowledges for funding the Johann Radon Institute (RICAM) of the ÖAW, the Austrian Science Funding Agencies FWF, through the Wittgenstein Award (2000) of Peter Markowich, and FFG, through the Research & Development Project 'Analyse Digitaler Bilder mit Methoden der Differenzialgleichungen'.

A. Leonardis, H. Bischof, and A. Pinz (Eds.): ECCV 2006, Part III, LNCS 3953, pp. 213–225, 2006. © Springer-Verlag Berlin Heidelberg 2006

the union of the Symmetry Set and a set they called anti-Symmetry Set, as it is closely related to the symmetry set [12].

In this work, we combine the ideas of these two fields of shape analysis by investigating the set of pairs of points at which a circle is tangent to the shape. We do not consider the centre of the circle, but the combination of the two points. A geometric method is given to derive the pairs of points, based on a zero crossing argument. Therefore, to each pair of points a signed value can be assigned, yielding a matrix of values (-1, 0, 1).

This matrix is then used as a shape descriptor. Its properties and allowed changes follow directly from the Symmetry Set, just as in the Shock Graph method. Next, a simple comparison algorithm is introduced to perform the task of object comparison. For this purpose, the two matrices for each pair of objects are set to equal dimensions and the normalised inner product is taken as equivalence measure. This procedure is tested on two data bases containing objects in different classes, where some objects are occluded or noisy. Given the simplicity of the algorithm, results are promising and main erroneous results are due to the algorithm, showing the potential power of the representation.

2 Problem Framework and Definitions

The Medial Axis can be defined as the closure of the loci of the maximal circles tangent to a shape (see e.g. [9]). This somewhat abstract formulation can be made clear by investigation of Figure 1a. A circle with radius r is tangent to a shape at two points. The unit length normal vectors $(N_1 \text{ and } N_2)$ of the circle and the shape coincide. The centre of the circle is a Medial Axis point, that is found by multiplying each normal vector with -r and taking the tangency point as tail of the vector $-rN_i$. As there are for each point several combinations satisfying this tangency argument¹, the set is taken for with -r is maximal, i.e. the set with the smallest radius.

The two points can be found using geometrical arguments [8], see Fig. 1b. Take an arbitrary origin point and let p_1 and p_2 be vectors pointing to the two locations of tangency. Then $p_1 - p_2$ is a vector pointing from one tangency point to the other. From the construction of the circle as described before, the vector $-rN_1 + rN_2$ (and when normal vectors are pointing inward and outward $-rN_1 - rN_2$) is parallel to $p_1 - p_2$. Consequently $(p_1 - p_2).(N_1 \pm N_2) = 0$ for these two points. Let a shape be continuously parameterised then for each point p several points q_i can be found for which

$$(p - q_j).(N(p) \pm N(q_j)) = 0$$
(1)

where N(.) denotes the normal vector. Note that if the normal vectors are parallel, the inner product is zero as well. Such points are the anti-Symmetry Set points described by Blake et al. [10, 11] for the parallel jaw gripper. If the shape

¹ It can be shown that for each point there are at least two other points [8]. Constellations with tangency normal vectors pointing inside and outside can occur [9].



Fig. 1. a) A pair of tangency points that gives rise to a Medial Axis point. b) The constellation of position and normal vectors is special at such points.

is parameterised by N points (p_1, p_2, \ldots, p_N) , then the tangency pairs are found as the zero crossings of Eq. 1. To find these zero crossings, it suffices to look at the square sign of inner product diagram P(i, j) of the signed values of Eq.1:

$$P(i,j) = \operatorname{sign}\left[(p_i - p_j).(N_i \pm N_j)\right]$$
(2)

In Figure 2 a fish shape is shown, together with its sign of inner product diagram. When actual zero crossings are computed, i.e. when the boundaries of the regions in such a diagram are taken, one obtains a so-called pre-Symmetry Set that is used to derive the distinct branches of the Symmetry Set [8, 13]. The possible changes of these boundaries when the shape changes, are known [14] and relate to the possible changes of the Medial Axis [9].

Changes in the shape lead to movement of the boundaries and therefore to changes of areas. Topological changes fall apart into two classes: Firstly, boundaries can meet and establish a different connection when a white (or black) region



Fig. 2. a) A fish shape. b) Sign of inner product diagram for the fish shape.

is locally split into two parts. Secondly, regions can be annihilated or created, either on the diagonal or pair-wise off-diagonal. Other possible changes of the Symmetry Set do not lead to topological changes.

As may be clear from Eq. 2, the diagram is symmetric in the diagonal. It can be identified with the shape, just as (by definition) the axes of the diagram. The values on the diagonal equal zero, as these points cannot be evaluated in Eq. 1. Second, on all other point combinations it is non-generic to encounter exactly a zero-crossing, so either a positive or a negative sign is obtained.

3 Sign of Inner Product Diagram Based Matching

The task of comparing objects has now become the task of comparing diagrams. If the parameterisations of two shapes consist of the same amount of points n, the corresponding sign of inner product diagrams can be multiplied element wise with each other. If the shapes are identical and the parameterisations are equal, this inner product equals n(n-1), since the diagonal consists of n points.

If the parameterisations are taken at a different starting position, so that $p_i = q_{i+\alpha}$, rotated version of the sign of inner product diagram should be taken into account. This rotation takes place in horizontal and vertical directions simultaneously, as $P(i, j) = Q(i + \alpha, j + \alpha)$, values taken modulo n. So to validate each possible starting position, n instances need to be compared.

Finally, the number of points for both shapes need not be equal. If the difference is m rows (and columns), a method must be chosen that removes mrows and columns. One choice is to remove them equally spread over the largest sign of inner product diagram. This relates to removing a set of equidistant points along shape with the largest number of points. It can be regarded as a re-parameterisation of the shape with the largest number of points.

Now let two shapes S_1, S_2 be parameterised with n_1 and n_2 points. Assume without loss of generality $n_1 \leq n_2$. The sign of inner product diagram of S_2 is denoted by P_1 . Let $n = n_1$ and $m = n_2 - n_1$. Build P_2 by removing each $(\frac{m}{n_2})^{th}$ row and column of the sign of inner product diagram of S_2 . Let P_1^r denote the sign of inner product diagram P_1 considered with as starting position point r on the shape, i.e. P_1 with its first r - 1 columns and rows transferred to positions $n + 1, \ldots, n + r - 1$:

$$P_1^r(i,j) = P_1(i-r+1,j-r+1),$$

where values are taken modulo n. This matches the shapes regardless of begin position of the parameterisations. Then the matching $D(P_1, P_2)$ between S_1, S_2 is set as

$$D(S_1, S_2) = \max_r (D(S_1^r, S_2))$$
(3)

with

$$D(S_1^r, S_2) = \frac{\sum_{i=1}^n \sum_{j=1}^n P_1^r(i, j) P_2(i, j)}{n(n-1)} - \frac{m}{2n_2}$$
(4)

The first term in Eq. 4 denotes the weighted equality of the two sign of inner product diagram P_1^r , P_2 . Perfect match is given by 1, while a complete mismatch equals -1 and a random match 0. The second term penalises the difference in number of points in a parameterisation, as this difference is ignored in the first term by construction. Adding this penalty is motivated by the way the shapes are obtained, viz. as the outlines of standardised binary images. Therefore, the number of points relates to the complexity of the shape.

4 Data Base Matching

As first test set 41 shapes from an online data base are taken². They form three classes: fishes, planes, and tools. Some fishes and planes are artificially drawn, and form inter class instances. The results of matching all shapes with each



Fig. 3. Matching of fishes, tools, and planes

² http://www.lems.brown.edu/vision/researchAreas/SIID/

other can be seen in Figure 3. For each shape, the best eight matches are shown: The first column has score zero, as each shape matches to itself without difference. The second column gives the second best match, etc.

The matching is consistent with [15], where this database is introduced. One can see, for instance, that tools match to tools, and that the wrenches and double wrenches match to the correct set. The erroneous matches – the appearances of shapes of a different class – occur at a match D = .5 or less. These errors can

class	score
1	11, 11, 11, 11, 11, 11, 11, 11, 8, 6, 7, 1
2	$11,\!11,\!9,\!10,\!8,\!6,\!8,\!5,\!4,\!5,\!2$
3	$11,\!10,\!10,\!10,\!10,\!10,\!9,\!8,\!9,\!7,\!2$
4	11, 11, 11, 10, 10, 8, 9, 10, 7, 6, 3
5	$11,\!10,\!9,\!9,\!7,\!8,\!1,\!2,\!0,\!3,\!6$
6	11, 11, 11, 11, 11, 11, 11, 11, 11, 11,
7	11, 11, 11, 10, 10, 8, 6, 8, 2, 3, 3
8	$11,\!10,\!10,\!11,\!9,\!9,\!9,\!8,\!7,\!3,\!2$
9	11, 11, 11, 11, 11, 11, 11, 11, 11, 11,

 Table 1. Score of inter-class matches

0	315	435	453	483	491	524	545	566	567	586
▶,	*		X	Ν,	Ň	N	1	*	-	×
0	395	419	503	514	545	613	645	649	649	653
1		N	X	X	▶,	*	-	- 1	*.	-
0	395	501	539	562	587	593	626	637	641	642
*	N 4	N .	Ň	N		×	*	-	•	-
0	399	449	467	491	501	503	523	539	551	554
\mathbf{N}	N		*	Х,	N,	1	м,	×.,	-	- 1
0	309	376	377	395	435	449	559	580	587	617
	N	*	X	1	▶,	Ň	м,	-	*.	- 1
0	309	399	419	503	507	524	562	609	613	614
		Ň	1	*	X	х,	*	*	N .	~
0	374	377	453	507	514	591	612	612	615	617
X	*		х,	N	1	₩.	-	1	•	1
0	466	483	501	501	518	556	613	636	654	656
N .	N 4	▶,	Ň	*	×	*	N		X	
0	442	518	568	586	593	609	669	691	692	700
$\mathbf{x}_{\mathbf{r}}$	N 4	Ν,	*	Х,	*	Ň	X		N	-
0	315	374	376	467	503	527	547	556	568	578
*	►,	X		Ň	N	*	-	Ν,	×	-
0	395	442	466	523	527	559	566	591	609	611
Χ.	*	×.	×.		*		*	X		

Fig. 4. Class 1

0 0 7 0 7 0 7 0 7 0 7 0 7 0 7	68 71 322 72 73 322 74 322 75 543 77 576 71 523 71 523	576 596 532 532 548 532 548 532 548	616 501 638 717 548 548 7657 760	700 ► 686 ► 650 ► 741 ★ 709 ► 708 € 708 €	737 743 743 686 752 743 743 721	754 752 700 754 783 724	770 ★ 758 ★ 726 ► 760 ★ 760 ★ 787 ►	781 ★ 772 ★ 762 ★ 764 ★ 795 ★	795 ₩ 799 ₩ 790 ★ 766 ★ 796	805 * 809 * 792 * 779 * 797
	 7 68 7 322 7 322 7 322 7 743 7 743 7 7<	 ▶ > >	548 657 710 717 717 717 717 717 717 71	 ▶ 686 ▶ 650 ▶ 741 ₹ 709 ▶ 708 ₹ 709 	743 686 752 743 9 721	752 700 754 783 724	★ 758 ★ 726 ★ 760 ★ 787 #	 ★ 772 ★ 762 ★ 764 ★ 795 ★ 	₩ 799 ₩ 790 ★ 766 ★ 796	 809 792 779 779 797
	68 322 322 543 467 576 523 11 523	596 467 532 548 548 532 601 737 3	601 638 717 755 638 548 7657 77 76	686 500 741 709 708 708	743 686 752 743 721	752 700 754 783 724	758 ★ 726 ★ 760 ★ 787 ▶	772 ★ 762 ★ 764 ★ 795 ★	799 ₩ 790 ★ 766 ★ 796	809 792 779 779 779
یر ہے۔ 1 ہے تو میر ہے ۔ 1 ہے تھ	322 322 543 467 576 523	467 532 548 532 532 601 737	638 717 737 638 548 748 748 757 757 757 757 757 757 757 75	650 741 * 709 * 708 * 709	686 752 743 721	700 754 783 724	726 h 760 x 787 b	∧ 762 ↑ 764 ★ 795	₩ 790 ★ 766 ★ 796	792 * 779 * 797
	322 322 543 1 467 576 523 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3	532 548 532 532 601 737	538 717 717 638 548 548 657 7	550 741 ★ 709 ★ 708 € 709	752 743 721	754 754 783 724	760 * 787	762 764 X 795	790 ★ 766 ★ 796	779 * 779 * 797
	322 543 1 467 1 576 1 523 1 523	532 548 532 601 737	717 538 548 657 1	741 * 709 * 708 * 709	752 743 721	754 783 724	760 * 787	764 ★ 795	766 ★ 796	779 10 797
0 1 0 1 0 1 0 1 0	543 1 467 1 576 1 523 1 523	548 532 601 737	548 548 657	 ★ 709 ▶ 708 ₹ 709 	743 * 721	783 724	★ 787 ⊯	★ 795 ┣	★ 796	* 797
0 1 0 1 0 1 1 0	543 1 467 1 576 1 523 1 523	548 532 501 737	638 548 657 7	709 * 708 * 709	743 * 721	783	787 بە	795	796	797
0 7 0 7	467 576 523	532 601 737	548 1 657 1	► 708 € 709	721	7 24	× .			
0 7 0 7	467 576 523 23 523	532 601 737	548 1 657 1 70	708 * 709	121	124	707	742	750	75
0 T	576 523 H 523	601	657 1	709	1.0		137	743	/52	/58
ħ	523 H 523	► 737	`T	-	719	721	726	761	774	- 806
	523 77 523	737	760	1.0	T	-	*	۲		7.
0	H 523	3	769	798	809	848	848	854	876	876
Ж	523	-7	7531	*	۳	*	•	1	<u>, </u>	*
0		782	814 w	815	836	839	840	840	849	855
<i>/</i> 77	70) 5/13	97 657	752	л 762	779	780	۳۹ 705	705	815	837
1		1	102	702		₩	₩	*	Ħ	4
0	596	616	650	717	763	766	769	784	785	785
7.7	1		*	. در	۳	1	Ħ	×	₩	×
0	260	270	272	222	442	626	617	650	660	721
*	209	270	212	555 A	442 ★	030 ★	•	030 A	009	121
Х	X	X	X	Х	X	X	X	X	X	•
0	259	260	333	515	578	644	708	720	727	750
X	\mathbf{x}	$\boldsymbol{\chi}$	X	X	X	٠	1	\mathbf{x}	۲	স
0	94	260	270	399	499	672	695	698	702	704
X	×	★	★	×	×	×	キ	٠	×	×
0	94	259	272	384	488	664	691	699	701	709
X	X	★	*	×	×	×	キ	X	×	×
0	230	269	384	399	487	498	510	515	523	713
★	×	★	X	X	×	×	ス	★	X	×
0	230	346	383	442	459	481	488	499	578	700
×	*	*	×	★	*	X	X	X	★	*
0	95	383	449	476	510	669	691	695	734	750
×	オ	×	×	×	★	★	X	X	۴	★
0	95	346	452	481	487	647	664	672	720	721
大	*	×	×	X	*	★	X	X	★	۴
0	620	682	687	688	691	706	713	714	726	728
≁	T	-	Ŧ	1	-	—	-	-	-	-
0	93	476	481	481	523	650	696	699	705	722
X	*	×	×	×	★	★	Ŧ	X	X	×
0	93	449	452	459	498	636	701	702	732	734
×	t	ス	*	×	*	★	X	X	*	ł

Fig. 5. Classes 2 and 3

visually be explained: A coarse plane "looks" more like a fish with two big fins than a very detailed plane.

Next, this approach is used on the data base used by Sebastian et al. [16]. This data base contains 9 classes with 11 shapes each. Some of the shapes are

occluded or deformed versions of another shape in the class. Just as in [16], a score $D^*(S_1, S_2)$ is set to be a non-negative number, ranging towards 1000. This is achieved by taking (recall Eq. 3)

$$D^*(S_1, S_2) = 1000(1 - D(S_1, S_2))$$
(5)

Now 0 denotes a perfect match and values towards 1000 a random match. The results per class are shown in Figs. 4-8. We have chosen to show all results, as this better reveals the potential of matching methodology.

In each of the figures, the first column resembles the shape matched with itself, resulting in a score of 0. The next 10 columns give the second to eleventh best match. Ideally, this would be shapes from the same class. The score of each shape is taken as an eleven dimensional vector with each value being zero or one. A one at position i denotes a shape at the i^{th} position that belongs to the same class, while a zero denotes a shape of a different class. The total class score is then given as the sum of the eleven vectors in the class, ideally being a vector containing 11 elevens. Table 1 gives these results.

5 Discussion of Results

Table 1 shows that some classes (6 and 9) yield a perfect score. Other classes contain matchings to objects of other classes. For some this occurs at higher positions, but in three cases already the second best match is wrong.

All these cases are caused by the choice of the matching algorithm, the removal of equidistant points. This is strongest visible in the third class, bottom of Fig. 5. The 9^th row introduces a shape that has a large occlusion. This relates to removing a set of neighbouring points along the shape instead of the taken approach. An indication that "something is wrong" is given by the high cost for the second best match (620), compared to the other second best matches in this class (≤ 269). Is introduces a complete row of wrong matches.

A similar effect, albeit in the opposite way, occurs in the fifth class, bottom of Fig. 6. The third row shows an occluded hand, which relates to a local addition of a set of neighbouring points along the shape. Again a high cost for the second best match is obtained. Assuming only equidistant removal of points, however, the second best match is visually correct. The fingers correspond to the four legs of the cow, while the blown-up thumb relates to the cow's head and body. The same thing can be said about the occluded rabbit in class 8, top of Fig. 8.

Obviously, the human classification is not perfectly mimicked by the algorithm. The total amount of errors compared to the human observer classification is given by (0, 3, 6, 6, 12, 17, 24, 28, 42, 43, 58). If the three most clear occlusion-caused outliers are left out, this is (0, 0, 3, 5, 9, 15, 22, 25, 39, 41, 56).

As a way to avoid the removal of points in one of the sign of inner product diagram, one can obtain a parameterisation of exactly n points. This results in more or less the same outcome, since it still does not take into account the effects of occlusion. Secondly, forcing a standard number of points along the

0	598	662	713	721	724	726	734	734	752	761
۶	¥	¥	×	×	۳	*	キ	*	Ŧ	7
0	566	675	680	772	805	830	849	856 	874	877
~	F		790	7 04	•22	v7	842	V	250	054
*	X	755 X	×	1 94	835 W	855 X	042	842 1	₀52 ₩	834 *
0	360	618	655	680	691	707	730	747	780	833
×	×	×	¥	*	×	*	¥	*	笨	X
0	360	566	617	639	673	694	722	783	794	812
×	*	*	*	¥	*	*	¥	*	*	x
0	308	673	675	691	721	728	734	735	755	778
**	308	617	618	6 18	6 44	669	683	6 98	772	へ 826
*	×	×	×	¥	¥	*	*	*	*	۶
0	394	508	618	655	662	682	722	735	760	768
¥	¥	*	×	×	۲	*	×	×	X	X
0	394	533	598	632	639	644	721	730	761	761
0	508	632	683	680	700	704	707	709	713	722
*	*	*	*	*	*	*	*	*	*	X
0	533	682	689	694	698	728	747	778	780	791
×	¥	¥	*	×	×	×	×	×	×	-
0	3	253	309	488	560	780	705	848	848	855
aŭ	-		. W.	.de	>	5	185	فيه	-	-
*	۴	₩	♥	۲	>	1	785	Ħ	5	
0 0	3 04	309	358	443	646	763	795	798	818	825
° ∘ ♥	304 •	309	358	443	646	763	785 795	798	818 →	825
° ° ♥ °	304 ••• 842	255 ♥ 309 ♥ 900 ♥	358 358 3902	443 443 916	500 → 646 ↓ 917 ↓	763 918	783 795 795 954	798 798 795	818 ≫ 969 ₩	€25 € 969
	304 * 842 * 630	255 ♥ 309 ♥ 900 ♥ 715	358 358 358 3724	443 ♥ 916 ₩ 756	500 646 17 17 17 17 17 17 17 17 17	763 918	795 795 954 772	798 798 798 965 780	969 ₩ 787	 ★ ★ 969 ★ 805
	304 304 842 300 300	235 ³ 09 ³ 09 ⁹ 00 ⁷ 15 ⁷¹⁵	358 358 358 302 ₩ 724	443 ♥ 916 ₩ 756 ★	500 → 646 → 917 → 758	763 763 918 918 761	795 795 954 772	518 798 798 ₩ 965 780 780	818 ≫ 969 ≫ 787 ★	825 ♦ 969 ♥ 805
	304 42 42 530 530 544	255 ¥ 309 ¥ 900 ↑ 715 ≥ 644	358 → 902 → 724 → 681	443 ♥ 916 ₩ 756 ₹ 698	500 646 917 917 917 758 710	763 763 918 2761 721	795 795 795 954 772 743	798 798 798 798 798 798 7965 780 780 780	818 ≫ 969 ≫ 787 ★ 772	825 ♥ 969 ♥ 805 ₩ 786
	304 ♥ 842 1 630 ♥ 614 ♦	255 ¥ 309 ¥ 900 ↑ 15 ₹ 644 ★	358 358 358 358 902 ₩ 724 €81 ₩	443 ¥ 916 ₩ 756 ★ 698 ★	500 → 646 → 917 → 758 → 710 ★	1.00 ↑ 763 763 918 ♥ 761 ↑ 721 ★	795 795 795 7954 772 772 743	798 798 798 798 798 769 ★ 769 ★	818 ≫ 969 ≫ 787 ₹ 772 ≫	825 № 969 ♥ 805 № 786 ★
	304 ♥ 842 ♥ 630 ♥ 614 ♦ 358	235 309 900 115 ≥44 ★ 560	358 358 358 358 902 ♥ 724 ♥ 681 ♥ 562	443 443 916 第 756 大 698 大 591	546 参 646 参 917 学 758 710 大 694	763 763 918 918 761 721 721 737	763 795 795 954 772 743 743 781	798 798 965 780 769 769 782	818 818 → 969 → 787 ★ 772 → 840	825 ♥ 969 ♥ 805 ₹ 786 ₹ 849
	304 ♥ 842 ₩ 630 ♥ 614 ₹ 358 ♥	235 ¥ 309 ¥ 900 ↑ 15 644 ★ 560 ¥	358 358 358 902 ♥ 724 ♥ 681 ♥ 562 ♥	443 単 443 単 916 第 756 大 698 大 591 単	5.65 646 学 917 学 758 710 大 694 学	763 763 918 918 761 761 721 721 737	763 795 795 795 772 ₩ 743 781 781	798 798 798 798 965 780 大 769 大 782 デ	818 818 → 969 → 787 772 → 840 →	825 ♥969 ♥♥ 805 ₩ 786 ★ 849 ♥
	304 ₩ 842 ₩ 630 ₩ 614 € 358 358 3	309 309 900 715 644 560 3560 254	358 358 358 902 ♥ 724 € 81 ♥ 562 ♥ 304	443 単 443 単 443 単 443 単 916 デ 598 大 591 単 488	5.67 646 学 917 学 758 710 大 694 学 562	763 763 918 918 761 721 721 737 737 737 779	763 795 795 795 7954 7954 7954 772 ♥ 743 743 781 ₹781 ₹786	798 798 798 798 798 765 780 大 769 大 782 782 782 782 847	818 818 → 969 → 787 ★ 772 → 840 ★ 848	825 ♥ 969 ♥ ♥ 805 ₹ 849 ♥ 849 ♥ 855
	304 ₩ 842 ₩ 630 ₩ 614 ₹ 358 ₩ 3 ₩ 3 ₩	$\begin{array}{c} 2.55 \\ {}^{2} \\ 309 \\ {}^{3} \\ 900 \\ ^{715} \\ \hline \\ 644 \\ \bigstar \\ 560 \\ ^{6} \\ \swarrow \\ 254 \\ ^{2} \\ \swarrow \\ \end{array}$	358 358 358 902 ♥ 724 ♥ 681 ♥ 562 ♥ 304 ♥	443 単 443 単 443 単 916 756 大 698 大 591 単 488 単	5.65 646 学 917 学 758 デ 10 大 694 学 562 豪	763 763 918 ♥ 761 ♥ 721 ★ 737 ౫ 779 ♥	763 795 954 772 954 772 743 781 786	798 798 798 798 798 798 798 798	818 818 → 969 → 787 ★ 772 → 840 ★ 848 ★ ₩	825 ♥ 969 ♥ 805 786 ★ 849 ♥ 855 ▼
	304 304 842 14 630 144 358 358 3 3 2533	2.55 309 900 715 644 560 3254 254 254	358 358 358 902 ♥ 724 ★ 681 ♥ 562 ♥ 304 ♥ 304 ♥ 443	130 単 443 単 443 単 443 単 443 単 16 万 56 大 698 大 591 単 488 単 496	5.65 646 917 917 710 710 710 694 9262 591	763 763 918 ♥ 761 ♥ 761 ♥ 761 ♥ 761 ♥ 763 771 ♥ 779 ♥ 809	783 795 954 772 954 772 743 781 786 786 842	798 798 798 798 965 780 大 769 大 782 782 782 847 862	818 818 969 969 969 969 969 787 ₹772 840 848 848 883 883	825 ♥ 969 ♥ 805 786 ★ 849 ♥ 855 ₹ 897
• • ● • ● • ● • ● • ● • ● • ● • ● • ● •	304 304 842 630 614 358 358 3 253	$\begin{array}{c} 2.55 \\ {}^{3} \\ & 309 \\ \hline \\ & 309 \\ \hline \\ & 900 \\ \hline \\ & 715 \\ \hline \\ & 644 \\ \hline \\ & 560 \\ \end{matrix}$ $\begin{array}{c} 644 \\ \hline \\ & 560 \\ \end{matrix}$ $\begin{array}{c} 644 \\ \hline \\ & 560 \\ \end{matrix}$ $\begin{array}{c} 254 \\ \end{matrix}$ $\begin{array}{c} 254 \\ \end{matrix}$	358 358 902 902 902 724 681 481 562 304 443 443 443	1000 443 916 756 大 698 大 591 単 488 496 496	5.65 646 917 917 710 710 710 710 710 710 710 710 710 758 710 710 758 710 758 710 758 710 758 710 758 710 758 710 758 710 758 710 758 710 758 710 758 710 758 710 758 710 758 710 758 759	763 763 918 918 918 761 721 ★ 737 737 779 ★ 779 ★ 709 ★ 709 ★ 709 ★ 709 ★ 709 709 709 709 709 709 709 709	763 795 772 954 772 743 781 786 786 842 786	798 798 798 798 965 780 大769 大769 大782 782 782 782 782 782 782 782	818 818 969 969 969 969 787 ₹772 840 548 848 883 ₩ 883 ₩	825 969 ♥♥ 805 786 ★49 ¥55 \$897 \$97 ₩
* • * • * • * • * • * • * • * • * • * •	304 304 842 630 630 614 € 558 3 253 253 488 488	2.55 309 900 715 644 x 560 254 254 254 254 488 888	358 358 902 24 724 681 24 681 2562 304 304 443 443 496 696			763 763 918 918 918 918 761 721 721 721 727 737 779 779 709 795	783 795 954 772 172	デ 78 78 78 78 769 大 769 大 767 大 782 ポ 787 787 787 787 787 787 787	818 818 969 969 969 9787 ₹772 840 848 848 848 848 848 848 848	825 969 ♥♥ 805 786 ★ 849 ₩ 855 ₩ 897 ₩ 897 ₩ 848
	304 304 842 300 414 € 358 358 3 3 3 3 3 3 3 3 488 488 488	255 309 309 900 715 644 560 254 254 488 488 488	358 358 902 24 681 304 562 304 443 496 496 496	443 443 916 第756 大 698 大 591 単 488 単 496 単 496 単 466 単		763 763 918 918 918 761 721 721 727 737 779 779 779 795 795 75	783 795 954 772 954 772 954 772 954 772 954 772 954 772 954 772 743 781 786 842 807 \$\$07	2日本 198 198 198 198 198 198 198 198 198 198	818 818 969 969 969 9787 次772 ● 840 第787 840 第883 第883 第839 常7 第7 第7 第7 第7 第7 第7 第7 第7 第7 第	825 ♥ 969 ♥ 805 786 ★ 849 ♥ 849 ♥ 855 \$ 897 ♥ 848 ♥
· • • • • • • • • • • • • • • • • • • •	304 304 842 630 614 358 358 3 253 488 324 488 334	2.55 309 900 715 644 560 254 254 488 488 488 630	358 358 902 902 724 681 304 443 496 443 496 443 496	443 単443 単443 916 756 大 698 大 591 単88 単96 単96 単96 単96 単 488 単 496 単 496 単 496 単 497 757 591 単 488 単 495 757 591 単 488 単 595 591 191 191 191 191 191 191	5.67 646 917 710 710 710 710 710 710 710 710 710 710 710 710 710 710 7000 7000 7000 7000 7000 7000 7000 7000	763 763 918 918 918 761 721 721 727 737 779 709 709 709 709 709 709 70	763 795 954 772 954 772 743 743 781 786 842 807 807 789	· · · · · · · · · · · · · · · · · · ·	818 818 969 969 969 9787 大772 840 条83 米83 米83 米83 米83 米83 米83 米83 米	825 ♥969 ♥♥ 805 786 ★49 ₩ 848 ₩ 897 ♥ 848 ♥ 848 ♥ 809
★ • ★ • ★ • ★ • ★ • ★ • ★ • ★ • ★ • ★ •	$\begin{array}{c} & & \\$	$\begin{array}{c} 2.55 \\ {}^{3} \\ \swarrow \\ 309 \\ 900 \\ \fbox \\ 715 \\ \hline \\ 644 \\ \bigstar \\ 560 \\ \end{matrix}$ $\begin{array}{c} 254 \\ ^{5} \\ \checkmark \\ 254 \\ \swarrow \\ 488 \\ \end{matrix}$ $\begin{array}{c} 630 \\ \swarrow \\ 300 \\ \end{matrix}$	358 358 902 902 724 681 304 443 496 443 496 496 496 481 496	443 443 916 756 大 698 大 591 488 496 496 496 496 496 496 496 496	5.67 646 917 7100 7100 7100 7100 7100 7100 7100 7100	763 918 918 918 761 721 737 737 737 737 709 789 789 789 789	783 795 772 743 781 786 789 789 789 789	いお 798 780 大 787 大 782 782 1 802 1	818 818 969 ₩ 787 772 ₩ 840 548 883 883 883 839 ₹ 803 €	825 ♥969 ♥♥ 805 ₩ 786 ★ 849 ♥ 855 ₩ 897 ♥ 848 ♥ 809 ★ 809 ★
× ∘ ¥ ∘ ¥ ∘ ¥ ∘ * ∘ * ∘ * ∘ * ∘ * ∘ * ∘ *	$\begin{array}{c} & & \\$	$\begin{array}{c} 2.55 \\ {}^{2.55} \\ \hline \\ 309 \\ \hline \\ 900 \\ \hline \\ 715 \\ \hline \\ 644 \\ \hline \\ 560 \\ \end{matrix}$ $\begin{array}{c} 644 \\ \hline \\ 560 \\ \end{matrix}$ $\begin{array}{c} 644 \\ \hline \\ 560 \\ \end{matrix}$ $\begin{array}{c} 254 \\ \end{matrix}$ $\begin{array}{c} 254 \\ \end{matrix}$ $\begin{array}{c} 488 \\ \end{matrix}$ $\begin{array}{c} 630 \\ \end{matrix}$ $\begin{array}{c} 614 \\ \hline \\ 630 \\ \end{matrix}$	358 358 902 902 902 724 681 304 443 496 443 496 496 496 496 811 496 812	$\begin{array}{c} & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ &$		763 763 918 918 918 918 918 918 761 721 大737 大737 779 789 795 789 787 789 787 789 787 789	763 795 954 772 954 772 743 781 786 782 781 786 782 787 787 789 772 772 743 781 786 785 789 772 772 789 772	いが 798 780 780 780 780 782 782 782 782 782 782 782 782 782 782	818 ■ 18 ● 969 ● 969 ● 787 〒 772 ● 840 ● 848 第 883 〒 839 ● 839 ● 803 ● 91 ● 191 ●	$ \begin{array}{c} 825 \\ 969 \\ 969 \\ 969 \\ 805 \\ 786 \\ 786 \\ 786 \\ 849 \\ 9 \\ 855 \\ 897 \\ 855 \\ 897 \\ 848 \\ 809 \\ 848 \\ 809 \\ 818 \\ $

Fig. 6. Classes 4 and 5

0	196	215	215	219	234	241	262	327	361	527
0	227	283	301	327	372	376	381	397	471	583
0	•	260	•	•- 274	• 342	347	• 348	•- 361	◆ 411	↓ 471
•	♦-	◆ 214	◆- 221	◆	← 260	◆ 262	◆- 201	◆- 200	◆- 376	↓18
•	190 ♦	±14 ◆	•	€	±200	.202	€		•-	+ 10 ♦
0 •	221	234	262	289	291	301 •-	301 •-	307	342	473
0	180	194	214	215	236	266	274	307	372	452
0	219	221	236	236	262	274	283	296	4 8	502
•	◆ 215	◆ 221	◆ 227	◆- 236	♦-	ا ♦	◆- 283	◆- 200	◆- 411	♦ 518
•	215 ◆	•	•	€	€	£85 €	±285	<u>∠</u> 99	+11 ◆	÷
0 ◆	347 •	373	405 •	418	452	473	502	518	527 •	583
0	130	180	196	198	247	274	283	289	381	405
0	130	194	236	241	267	283	296	301	373	397
•	+	+	•	+	•	•	+	•	•	•
0	317	357	410	501	533	573	623	688	708	732
1	Ť	1	Ŧ		*	T		*		×,
0	317	481	543	610	630	663	712	719	724	727
Ť	1	T	1	T	*		X	*		X
0	357	428	467	483	484	541	543	619	655	656
1	1		T	T	*		Ť	**	N,	-
0	344	428	489	501	547	560	594	602	609	614
1		1	*	1	T	-	-	-	T	-
0	375	425	483	486	573	583	592	600	609	610
T	T	*	1	1	1	●-	ŧ	-	1	Ŧ
0	344	408	477	486	490	503	530	541	608	615
1		1	*	T	- 1	T	ŧ	1	-	-
0	562	636	641	670	681	687	689	696	697	697
₹	ŧ	1	1	T	-	*	1	X	×.	*
0	346	375	410	467	481	503	547	635	636	665
Ŧ	*	T	1	T	T			-	ŧ	•
0	346	425	477	484	489	505	533	580	620	630
*	T	T	1	1	1	-	1	ŧ	-	Ŧ
0	356	530	562	611	612	615	628	648	659	659
ŧ	ŧ	1	Ŧ	N 44	×.	۰.	-	-	▶.	-
0	356	408	577	580	592	628	636	638	641	641
ŧ	ŧ	1	-	*	T	1	Ŧ	-	×.	Ŧ

Fig. 7. Classes 6 and 7

shape wipes out the complexity of shape, so the matching actually yields worse results.

First attempts have been made in order to remove a set of $\frac{m}{n_2}$ locally neighbouring points. For the occluded human figure, this yielded a better matching to other human shapes. It is, however, computationally very expensive implemented. To compare two shapes takes approximately tens of minutes, compared

0	124	318	353	358	404	441	446	556	578	585
-		-	٠.	_	-	-	-		*	Ņ
0	537	556	575	580	583	584	592	637	642	649
-	- 1	-	-		٠.	-	-	х.		Ì
0	490	505	549	577	600	602	635	671	685	689
÷.	1		-	1	T	1	T	1	*	×,
0	124	338	362	379	430	448	520	537	547	554
\$	-	_	-	٠.	-	-	•	-	*	Y
0	505	561	587	591	615	632	642	686	689	700
-	-		-	-	ŧ	T	-	1	X	*
0	310	318	338	362	379	449	491	505	592	613
-	-	-	•	-	٠	-	-	-	-	×,
0	338	338	358	439	469	494	567	587	601	606
9	•	-	-	-	-	-	•		*	-
0	446	449	494	498	520	561	561	590	614	615
	-	-		-		•		-	Ţ	Į.
0	425	430	441	491	514	549	551	560	575	590
		•	-	-	-			Ţ	-	
0	353	366	379	379	425	469	561	583	589	629
				-		-			7	•
0	310	366	404	439	448 •	498	514	584	591	608
-	-		-	-	•			-	-	•
0	189	193	219	330	417	456	515	533	573	578
~	~	1	1	ľ	1		-	5	1	~
0	189	225	262	320	373	406	438	489	493	502
~	~	1	1	I	1		-	1	1	1
0	219	225	228	337	339	395	438	457	459	471
~	~	~	1	1	ĩ	-	~	5	1	~
0	255	393	406	413	427	438	453	455	456	484
	ĩ	1	~	1	5	/	1	~	~	-
0	193	228	262	278	393	445	519	549	577	592
1	~	1	~	I		1	-	1	~	1
0	160	230	243	266	455	471	502	505	577	578
~	1	1	-	1		/	~	I	1	~
0	160	230	259	293	453	459	493	497	573	592
1	~	1	1	-		1	2	I	~	1
0	216	228	259	266	337	373	413	417	445	478
1	-	1	1	1	1	2		~	1	I
0	255	278	320	330	339	478	497	505	520	535
ľ		1	~	\sim	1	1	1	1	-	-
0	216	243	287	293	395	438	484	515	519	520
-	1	~	1	1	~	~		\sim	1	ľ
0	228	230	230	287	427	457	489	533	535	549
1	1	1	1	-		/	2	~	ľ	1

Fig. 8. Classes 8 and 9

to several seconds in the equidistant case. However, as the optimal match is a summation of a set of multiplications, a fast dynamic program may be available. In this case the task would be to find a shortest manifold in 4D.

6 Summary and Conclusions

A new shape descriptor is introduced. It is based on pairs of points on the shape that lie on a circle that is tangent to the shape at these points. It is therefore closely related to both Medial Axis and Symmetry Set methods. Each point on the shape is compared to all other points on the shape regarding a geometrical relation. Based on this, to each pair of points a value +1 or -1 is assigned. This yields an efficient data structure.

Secondly, shapes can be compared using this data structure. As test, a general data base [16] was used, containing shapes in different classes. Some of the shapes are severely occluded. To compare two data structures, the used approach removed a set of equidistant points along the shape, thus enforcing two shapes parameterised with the same number of points. This allows simple comparison of two data structures.

Although this matching assumption is very general and a priori not suited for occluded shapes, results were relatively good. The comparison of two shapes can be done in few seconds, using non-optimised Mathematica code. Some shape classes were completely correct classified, while other had a correct score for most of the shapes. The shapes that significantly scored bad were shapes with a large blocked occlusion, or with a locally removed part. These parts cannot be matched correctly by definition with the used method. We note that these deformed shapes give a relatively simple different Medial Axis. Secondly, we only matched one shape to another, allowing the changes to appear in only one shape. In general, the matching involves changes to both shapes, for example in matching the hands of class 5 (see Fig. 6, bottom) with different occluded fingers.

An obvious amendment of the matching algorithm is the possibility of removing a set of neighbouring points. This will solve the problem of occluded parts, both where a part of the shape is removed, and where a part (a block) is added. Second, the method is to be designed to find the optimal solution allowing both data structures to be changed. As the optimal match is a summation of a series of multiplications, a fast shortest-path based dynamic program may be available to incorporate these two amendments simultaneously.

References

- Blum, H.: Biological shape and visual science (part i). Journal of Theoretical Biology 38 (1973) 205–287
- 2. Kimia, B.: On the role of medial geometry in human vision. Journal of Physiology - Paris 97 (2003) 155–190
- Ogniewicz, R.L., Kübler, O.: Hierarchic voronoi skeletons. Pattern Recognition 28 (1995) 343–359

- Sebastian, T., Kimia, B.B.: Curves vs. skeletons in object recognition. Signal Processing 85 (2005) 247–263
- Siddiqi, K., Kimia, B.: A shock grammar for recognition. Proceedings CVPR '96 (1996) 507–513
- Sebastian, T., Klein, P., Kimia, B.B.: Recognition of shapes by editing shock graphs. IEEE Transactions on Pattern Analysis and Machine Intelligence 26 (2004) 550–571
- Pelillo, M., Siddiqi, K., Zucker, S.: Matching hierarchical structures using association graphs. IEEE Transactions on Pattern Analysis and Machine Intelligence 21 (1999) 1105–1120
- Bruce, J.W., Giblin, P.J., Gibson, C.: Symmetry sets. Proceedings of the Royal Society of Edinburgh 101 (1985) 163–186
- Giblin, P.J., Kimia, B.B.: On the local form and transitions of symmetry sets, medial axes, and shocks. International Journal of Computer Vision 54 (2003) 143–156
- Blake, A., Taylor, M., Cox, A.: Grasping visual symmetry. Proceedings Fourth International Conference on Computer Vision (1993) 724–733
- 11. Blake, A., Taylor, M.: Planning planar grasps of smooth contours. Proceedings IEEE International Conference on Robotics and Automation (1993) 834–839 vol.2
- Kuijper, A., Olsen, O.: On extending symmetry sets for 2D shapes. In: Proceedings of S+SSPR. (2004) 512–520 LNCS 3138.
- Kuijper, A., Olsen, O., Giblin, P., Bille, P., Nielsen, M.: From a 2D shape to a string structure using the symmetry set. In: Proceedings of the 8th European Conference on Computer Vision. Volume II. (2004) 313–326 LNCS 3022.
- Kuijper, A., Olsen, O.: Transitions of the pre-symmetry set. In: Proceedings of the 17th International Conference on on Pattern Recognition. Volume III. (2004) 190–193
- Sharvit, D., Chan, J., Tek, H., Kimia, B.: Symmetry-based indexing of image databases. Journal of Visual Communication and Image Representation 9 (1998) 366–380
- Sebastian, T., Klein, P., Kimia, B.B.: Recognition of shapes by editing shock graphs. In: Proceedings of the 8th ICCV. (2001) 755–762