Reconstruction of Orthogonal Polygonal Lines

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Abstract. An orthogonal polygonal line is a line consisting of adjacent straight segments having only two directions orthogonal to each other. Because of noise and vectorization errors, the result of vectorization of such a line may differ from an orthogonal polygonal line.

This paper contains the description of an optimal method for the restoration of orthogonal polygonal lines. It is based on the method of restoration of arbitrary ground truth lines from the paper [1]. Specificity of the algorithm suggested in the paper consists of filtering vectorization errors using a priori information about orthogonality of the ground truth contour.

The suggested algorithm guarantees that obtained polygonal lines will be orthogonal and have minimal deviations from the ground truth line. The algorithm has a low computational complexity and can be used for restoration of orthogonal polygonal lines with many vertices. It was developed for a raster-to-vector conversion system ArcScan for ArcGIS and can be used for interactive vectorization of orthogonal polygonal lines.

Keywords: Polygonal line, orthogonality, line drawings, maps, vectorization, error filtering.

1 Introduction

The term *orthogonal polygonal lines* will be used to refer to polygonal lines consisting of orthogonal straight segments. There are only two permissible directions for these segments. These are called cardinal directions. Any two segments of an orthogonal polygonal line are either parallel or perpendicular to each other. Any two successive segments are perpendicular to each other. A rectangle is an example of an orthogonal polygonal line.

Orthogonal polygonal lines can be seen at different line drawings, for example, in architectural plans, engineering drawings, and electrical schematics. Fig. 1 shows a fragment of a city map. Most of the building outlines are orthogonal lines.

The results of vectorization of lines from monochrome images usually are corrupted. Because of scanning noise, discretization, binarization, and vectorization errors, even straight lines are converted to polygonal lines after raw vectorization. Fig. 2 shows a monochrome image obtained by scanning a straight line and the result of raw vectorization. The number of segments in a resulting polygonal line and the deviations of these segments from the ground truth straight lines are sometimes used to evaluate vectorization error [2].

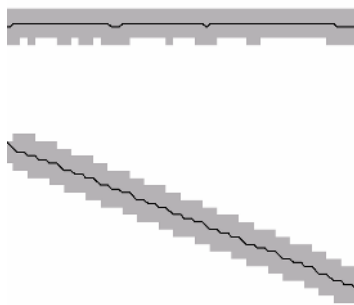**Fig. 1.** A fragment of a city map with buildings. Many of the building borders are orthogonal lines.

Fig. 2. The monochrome image of straight lines and polygonal lines as a result of raw vectorization

Post-processing usually follows raw vectorization. One of the tasks of post-processing is defragmentation. The goals of defragmentation are data compression and increasing precision. In the past, data compression was more important. The most widely used compression methods solve the problem of data compression by removing some source polygonal line vertices (see, for example, the Douglas-Peucker method in [3]).

The main criterion for removing a vertex is the distance from the vertex of the source polygonal lines to the polygonal line that is a result of compression. Because the location errors of the remaining vertices are not corrected, the precision of vectorization may not be enhanced. In spite of this the Douglas-Peucker compression method is used till now for defragmentation and simplification of polygonal lines.

Recently, because of considerable reduction in price and increasing capacity of computer memory, the problem of data compression has become less important, while the problem of enhancing the precision of vectorization has become more critical.

One approach to the problem of increasing vectorization precision of polylines consisting of geometric primitives follows. A source polygonal line obtained by raw vectorization is divided into nonoverlapping fragments such that each could be approximated with good precision using some geometric primitive (for example, a straight segment, or a circle arc). By finding the optimal approximations of the primitives and the intersections of adjacent primitives, it is possible to build a sequence of primitives that is the restoration of ground truth line. The source polygonal line must be divided in such a way that some functional that is a measure of approximation error will be minimized. The vital importance of such an approach has a definition of the functional.

In [1], such an approach is used with one restriction (after dividing the source line into fragments, they are approximated only with straight segments). The functional value depends not only on the precision of the approximation of fragments of source lines but also on the number of fragments or the number of straight segments of the resulting polygonal line. This method uses only one parameter – the penalty for each segment of the resulting polygonal line.

If the ground truth line is an orthogonal line, the method suggested in [1] does not guarantee that the resulting polygonal line will be an orthogonal line. It is possible to resolve the problem by taking into account geometrical constraints (in this case it is an orthogonality) after a polygonization of the result of raw vectorization (for example, with a beautification method from [4]). But the suggested method resolves the problem with simultaneous polygonization and taking into account geometric constraints. It provides the capability to dramatically increase the accuracy of resulting polylines.

A new method of line fragmentation suggested in this paper differs from the method described in [1] by using *a priori* information that the ground truth line is an orthogonal line.

2 Statement of the Problem

Let p_i, where $i = 0,...,n$, be vertices of polygonal line P_n. Let q_j-th vertices p_{q_j} divide P_n into a set of nonoverlapping polygonal fragments, and $Q_n = \{q_0 = 0, q_1,..., q_m = n\}$ be a set of indexes of the decomposition points of a source polygonal line, where m is a number of segments.

Suppose that cardinal directions of the sought orthogonal polygonal line are horizontal (H) and vertical (V) directions. Let X be one of the cardinal directions and $\perp X$ be a direction perpendicular to X.

Let L_j^X and $L_j^{\perp X}$ be lines having cardinal directions X and $\perp X$ and minimal integral standard deviations $\varepsilon_{q_{j-1},q_j}^X$ and $\varepsilon_{q_{j-1},q_j}^{\perp X}$ from the corresponding

fragment $(p_{q_{j-1}}, p_{q_j})$ limited with q_{j-1}-th and q_j-th vertices, where $j = 1,...,m$ of the source polygonal line (see Fig. 3). In Appendix 1, there is an algorithm for building such lines.
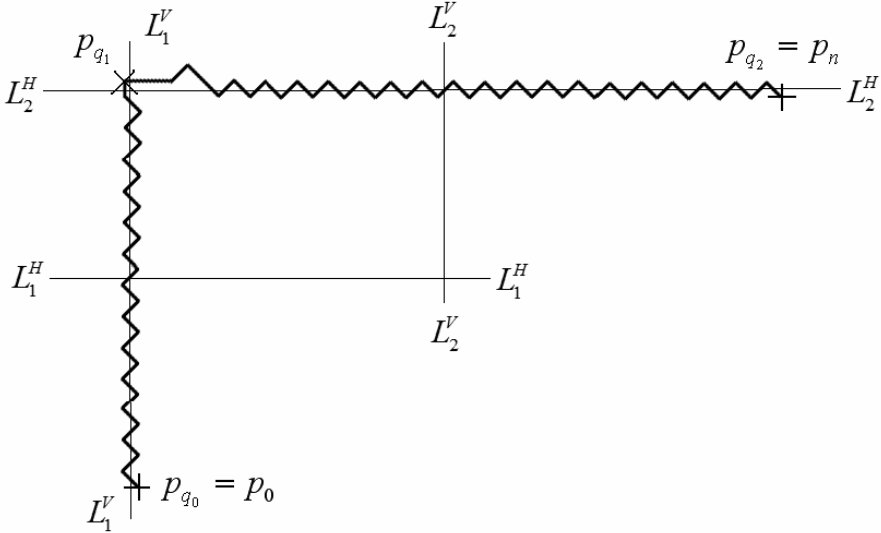

Fig. 3. Polygonal line P_n and horizontal and vertical lines (L_i^H and L_i^V, $i = 1,2$) are approximations of fragments (p_{q_0}, p_{q_1}) and (p_{q_1}, p_{q_2}), where $q_0 = 0$, $q_2 = n$

The measure of the error of the orthogonal polygonal line approximation is

$$F(Q_n, X, Y, \Delta, m) = m \cdot \Delta + (\varepsilon_{q_0, q_1}^X + \varepsilon_{q_1, q_2}^{\perp X} + ... + \varepsilon_{q_{m-1}, q_m}^Y), \qquad (1)$$

where Δ is a penalty for each straight segment of the resulting orthogonal polygonal line, the second item is the sum of the integral standard deviations, X is the direction of the first segment, and Y is the direction of the last segment of the orthogonal polygonal line.

Lines L_j^X and $L_j^{\perp X}$ are used for building orthogonal polygonal lines. The vertices of the orthogonal polygonal lines are the intersections of adjacent perpendicular lines L_j^X and $L_{j+1}^{\perp X}$. The beginning and the end of the orthogonal polygonal lines are projections of points p_0 and p_n on lines L_1^X and L_m^Y.

The task is to find such set $\hat{Q}_n = \{\hat{q}_0, \hat{q}_1,...,\hat{q}_{\hat{m}}\}$ and values $\hat{X}$ and $\hat{Y}$ that do the value of the functional (1) minimal. This set $\hat{Q}_n$, directions $\hat{X}$ and $\hat{Y}$, and corresponding orthogonal polygonal line $\hat{R}_n$ are optimal ones.

3   Iterative Algorithm of Decomposition of a Polygonal Line into Fragments

Let M_k^Y be a minimal error of approximation of polygonal line P_k with orthogonal polygonal line R_k^Y when Δ and Y are fixed. The corresponding set of indices of decomposition points of polygonal line P_k is Q_k^Y. The upper index Y is an orientation of the last straight segment of the orthogonal polygonal line. The minimum value of approximation error of the polygonal line P_k if the index of the next to last element i, is $M_{i,k}^Y$. The corresponding set and orthogonal polygonal line are denoted $Q_{i,k}^Y$ and $R_{i,k}^Y$.

Obviously

$$M_{i,k}^Y = M_i^{\perp Y} + \varepsilon_{i,k}^Y + \Delta, \tag{2}$$

where $i = 0,...,k-1$.

Therefore M_k^Y is the minimal value of $M_{i,k}^Y$ for $0 \le i < k$.

If the orientation of the first segment of the sought orthogonal polygonal line $\hat{R}_n$ is unknown and so can be horizontal or vertical, then

$$M_0^H = 0, \ M_0^V = 0. \tag{3}$$

Suppose that all minimal errors of approximation M_i^H and M_i^V of polygonal line P_{k+1} and corresponding sets Q_i^H and Q_i^V, where $i = 1,...,k$ are known. If minimum errors M_{k+1}^H and M_{k+1}^V of P_{k+1} and corresponding set Q_{k+1}^H and Q_{k+1}^V could be found, an iterative algorithm can be build to evaluate an optimal set $\hat{Q}_n$ and optimal orthogonal polygonal line $\hat{R}_n$.

Using the method of least squares, a horizontal or vertical line can be built through the fragment of the source polygonal line between vertices p_i and p_{k+1} and the standard deviation $\varepsilon_{i,k+1}^Y$ and the minimal value of approximation error $M_{i,k+1}^Y$ can be calculated.

By analyzing $M_{i,k+1}^H$, where $i = 0,...,k$, the minimum value of $M_{i,k+1}^H$ and corresponding value $i = i^H$ can be found. Similarly it can be found $i = i^V$.

Sets Q_{k+1}^H and Q_{k+1}^V can be found by adding $k+1$ to sets $Q_{i^H}^V$ and $Q_{i^V}^H$ respectively (attention must be paid to the sequence of superscripts). Thus the problem is resolved.

Repeating this procedure n times sets Q_n^H and Q_n^V will be obtained that provide the minimum values of functional (1) for given values of Δ and Y.

If the last segment of the resulting orthogonal line must be horizontal or vertical then the optimal sets are Q_n^H or Q_n^V respectively. If there is no such requirement for the last segment of the resulting orthogonal line, then the optimal set is

$$\hat{Q}_n = \begin{cases} Q_n^H, M_n^H < M_n^V; \\ Q_n^V, \text{otherwise.} \end{cases} \tag{4}$$

Fig. 4 shows the source polygonal line and orthogonal line obtained using this method.

a)   b)   c)
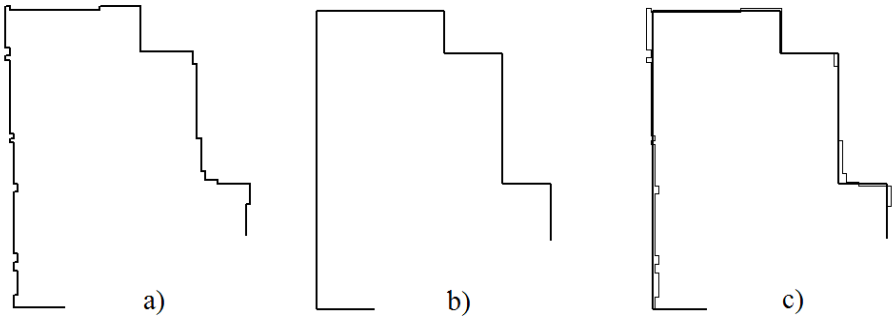
Fig. 4. Orthogonal polygonal line obtained using the described method for $\Delta = 30$ (*a - the source polygonal line, b - orthogonal polygonal line, c - the source and the result line together*)

4   Optimization of the Iterative Algorithm

The algorithm described above has the square calculating complexity. It can be used when the source polygonal line does not have many vertices, for example, an outline of buildings in maps of middle scale. In the case in which the source line has many vertices, the suggested algorithm can cause an essential delay. It is especially inadmissible for interactive modes.

A technique to reduce the calculating complexity of the described algorithm is suggested. There are inequalities that can be used for defining if a given part of the polygonal line P_k can contain the next to last point of decomposition that minimizes $M_{i,k}^Y$. This makes it unnecessary to analyze every vertex p_i, where $i = 0,...,k-1$ of the polygonal line, while finding a new optimal point i^Y.

This technique is based on two obvious inequalities.

The first arises from the fact that an error of the optimal approximation of a polygonal line with two straight segments is not greater than the error of the optimal approximation of the same polygonal line with one straight segment.

$$\varepsilon^X_{q_1,q_2} + \varepsilon^X_{q_2,q_3} \le \varepsilon^X_{q_1,q_3}, \text{ where } 0 \le q_1 \le q_2 \le q_3 \le n. \tag{5}$$

The second inequality arises from the fact that the minimum error of the approximation of some part of the polygonal line is not greater than the minimum error of the approximation of the whole polygonal line.

$$\min\{M^Y_{q_1}, M^{\perp Y}_{q_1} + \Delta\} \le M^Y_{q_2}, \tag{6}$$

where $0 \le q_1 \le q_2 \le n$, and $\perp Y$ is a direction orthogonal to the direction Y.

From inequalities (5) and (6) it follows (see an Appendix 2) that for $0 \le q_1 \le q < q_2 \le k+1$:

$$M^Y_{q,k+1} \ge \tilde{M}^Y_{q_1,q_2,k+1}, \tag{7}$$

$$M^Y_{q,k+1} \ge \tilde{\tilde{M}}^Y_{q_2,k+1}, \tag{8}$$

where

$$\tilde{M}^Y_{q_1,q_2,k+1} = \min\{M^{\perp Y}_{q_1}, M^Y_{q_1} + \Delta\} + \varepsilon^Y_{q_2-1,k+1} + \Delta, \tag{9}$$

$$\tilde{\tilde{M}}^Y_{q_2,k+1} = \min\{M^Y_{q_2-1}, M^{\perp Y}_{q_2-1} + \Delta\} + \varepsilon^Y_{q_2-1,k+1}. \tag{10}$$

Denote

$$\hat{M}^Y_{q_1,q_2,k+1} = \max\{\tilde{M}^Y_{q_1,q_2,k+1}, \tilde{\tilde{M}}^Y_{q_2,k+1}\}. \tag{11}$$

Suppose the value of the functional of some decomposition of the polygonal line P_{k+1}, where the next to last point of the decomposition does not belong to the half-interval $[q_1, q_2)$ was calculated. Denote this functional F^Y. The next to last point q of decomposition for which $M^Y_{q,k+1}$ becomes minimal can be located inside half-interval $[q_1, q_2)$ only if

$$\hat{M}^Y_{q_1,q_2,k+1} \le F^Y. \tag{12}$$

If this condition is not met, the next to last point of decomposition does not belong to $[q_1, q_2)$ and this half-interval can be skipped. Using this, it is possible to accelerate a search at each step of the described iterative algorithm.

5 Building a Close Orthogonal Polygonal Line

The task of analyzing a case when the source polygonal line is closed as in, for example, the borders of buildings or other area objects can be resolved by reducing it to the previous one.

First, it is necessary to open a source polygonal line, in other words, to find the beginning. The first point and the end point of the source line coincide $p_0 = p_n$.

Let the first point be the upper-left vertex of the source line. Because of such choice of the beginning of the polygonal line the error of approximation is not minimal but the additional error is small. Reorder the vertices so that a new source polygonal line passes around the area object in a clockwise direction.

In this case the first segment of the orthogonal polygonal line is horizontal. Therefore the last segment must be vertical because the line is closed.

While the deriving the above algorithm to build an orthogonal polygonal line, it is assumed that the first segment of the orthogonal line can be either horizontal or vertical. Substituting

$$M_0^H = \infty, \ M_0^V = 0 \tag{13}$$

instead of condition (3) the orthogonal polygonal line with horizontal first segment $X = H$ is obtained. Because the last segment must be vertical, the condition $\hat{Q} = Q_n^V$ is used instead of condition (4).
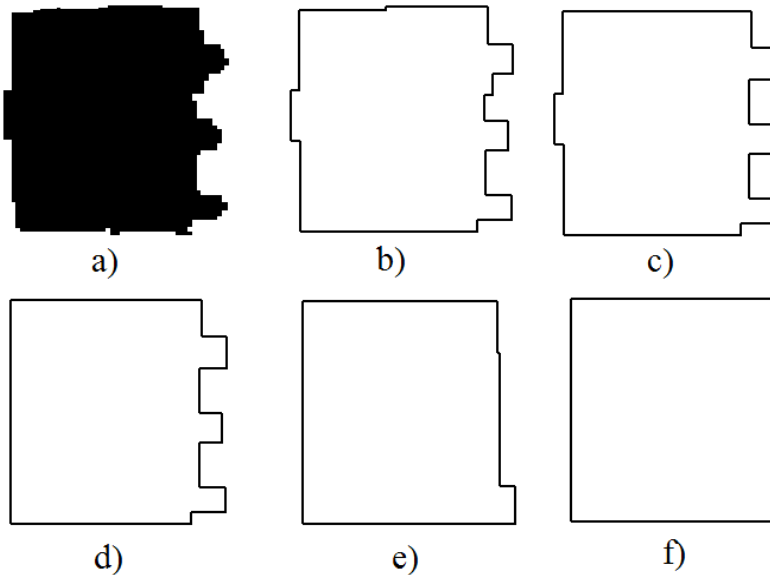

Fig. 5. A building and approximations of its border with orthogonal polygonal lines (*a - raster image of the building with noise and b-f - approximations of building borders obtained with* $\Delta = $ *30, 100, 300, 1000, 3000 accordingly*)
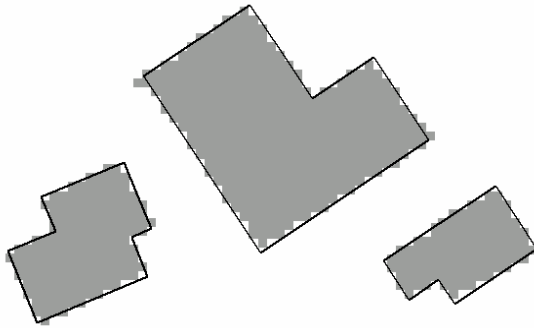
Fig. 6. Image of three buildings and corresponding orthogonal polygonal lines

Fig. 5 shows a monochrome image of a building (with noise) and orthogonal polygonal lines obtained with different values of Δ .

Fig. 6 shows a fragment of a scanned map with three buildings and orthogonal polygonal lines obtained with the suggested method.

6   How to Find Cardinal Directions

Usually cardinal directions are not known in advance. Sometimes different objects have different cardinal directions (see, for examples, the borders of buildings in Fig. 6). In these cases, the orthogonal polygonal lines are built N times with one of the cardinal directions

$$\alpha_i = h \cdot i \text{, where } h = 90^\circ / N \text{ ; } i = 0,...,N-1. \tag{14}$$

The coordinate system is rotated to the angle α_i and the task is resolved with horizontal and vertical cardinal directions. The orthogonal polygonal line with minimal error of approximation is the desired solution. Then it is necessary only to turn it back through angle $-\alpha_i$.

The value of N depends on the required precision. Using dichotomy it is possible to increase the precision with the same N .

7   Conclusion

In this paper, the optimal method is suggested to reconstruct orthogonal polygonal lines after vectorization. This method is based on the dynamic programming technique.

Because of errors caused by scanning, binarization, vectorization, and other processes, even straight lines become polygonal lines. One of the goals of post-processing is noise filtering.

In [1], a new method was suggested for filtering errors of vectorization.

A polygonal line obtained as a result of the raw vectorization is divided into no overlapping fragments.[1] The method guarantees the minimum value of the functional that depends on precision of approximation of resulting parts with straight segments and on the number of parts, or the number of segments of the result polygonal line. This method uses one parameter – a penalty for each straight segment of the resulting polygonal line. The error of approximation is calculated as integral standard error. It is possible to modify the method using another measure of the error.

By finding intersections of straight lines obtained as an optimal approximation of fragments a new polygonal line can be built.

Fig. 7. A fragment of the city map from Fig. 1 with borders of orthogonal buildings (the result of processing by ArcScan for ArcGIS)

The method described in this paper is a modification of the method from [1]. Modifying its method with *a priori* information that the sought polygonal line is an orthogonal line provides the method described in this paper. This method guarantees that the resulting polygonal line will be an orthogonal line with almost minimal error compared to the source orthogonal polygonal line.

The method is a combinatorial one and has the quadratic computation complexity. There was a suggested optimization that reduces the number of analyzed solutions, which essentially increases the speed of resolving the task. After optimization, the algorithm can be used for reconstruction of the orthogonal polygonal lines from the source polygonal lines with a large number of vertices, which is common for polygonal lines obtained with vectorization.

The method has been generalized for closed orthogonal polygonal lines, for example, borders of buildings in maps.

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[1] The polygonal line must have segments roughly equal in size to a pixel; otherwise, it is necessary to perform densification.

The method can also be generalized for the following cases:

- Decomposition of the source polygonal lines into fragments some of which are singular (with zero length)
- Polygonal lines with a fixed angle between adjacent segments differing from 90°
- Polygonal lines with the arbitrary number of permissible directions
- M -dimensional polygonal lines, where $M > 0$

The method has been implemented in ArcScan for ArcGIS. Examples in Fig. 4-7 show the results obtained with the suggested method.

References

1. Gribov A., Bodansky E.: A New Method of Polyline Approximation. Structural, Syntactic, and Statistical Pattern Recognition, Portugal, LNCS 3138, Springer (August 2004) 504-511
2. Phillips I.T., Chhabra A.K.: Empirical Performance Evaluation of Graphics Recognition Systems. IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 21, No. 9 (September 1999) 849-870
3. David H. Douglas and Thomas K. Peucker: Algorithms for the Reduction of the Number of Points Required to Represent a Digitized Line or Its Caricature. Canadian Cartographer', Vol. 10, No. 2 (December 1973) 112-122
4. Pavlidis T., VanWyk C.J.: An Automatic Beautifier for Drawings and Illustrations. Computer Graphics, Vol. 19, No. 3, ACM Press (July 1985) 225-234

Appendix 1: Horizontal and Vertical Lines Approximating Some Part of the Polygonal Line

Let t be a parameter equal to the distance from the beginning of the polygonal line till the current point along this line. Let l_{j_1} and l_{j_2} be values of the parameter defining the beginning and the end of the analyzed part of the polygonal line. The horizontal line approximating a given part of the polygonal line can be defined as

$$y = \frac{V_y}{l_{j_2} - l_{j_1}}, \text{ where } V_y = \int_{l_{j_1}}^{l_{j_2}} y(t)\,dt \,.$$

The vertical line can be found similarly

$$x = \frac{V_x}{l_{j_2} - l_{j_1}}, \text{ where } V_x = \int_{l_{j_1}}^{l_{j_2}} x(t)\,dt \,.$$

Integral standard deviations of these straight lines are defined as

$$\varepsilon_{j_1,j_2}^H = \int_{l_{j_1}}^{l_{j_2}} y^2(t)dt - V_y^2 / \left(l_{j_2} - l_{j_1}\right),$$

$$\varepsilon_{j_1,j_2}^V = \int_{l_{j_1}}^{l_{j_2}} x^2(t)dt - V_x^2 / \left(l_{j_2} - l_{j_1}\right).$$

Appendix 2: Derivation of Inequalities (7) and (8)

Let $0 \le q_1 \le q < q_2 \le k+1$.

From inequalities (2) and (6), and obvious inequality $\varepsilon_{q,k+1}^Y \ge \varepsilon_{q_2-1,k+1}^Y$, it is possible to obtain $M_{q,k+1}^Y \ge \min\left\{M_{q_1}^{\perp Y}, M_{q_1}^Y + \Delta\right\} + \varepsilon_{q_2-1,k+1}^Y + \Delta$, in other words,

expression (9).

From inequalities (2) and (5) follows $M_{q,k+1}^Y \ge M_q^{\perp Y} + \varepsilon_{q,q_2-1}^Y + \varepsilon_{q_2-1,k+1}^Y + \Delta$. From an obvious inequality $M_q^{\perp Y} + \varepsilon_{q,q_2-1}^Y + \Delta \ge \min\left\{M_{q_2-1}^Y, M_{q_2-1}^{\perp Y} + \Delta\right\}$ it

follows $M_{q,k+1}^Y \ge \min\{M_{q_2-1}^Y, M_{q_2-1}^{\perp Y} + \Delta\} + \varepsilon_{q_2-1,k+1}^Y$, or expression (10).