On Shape Orientation When the Standard Method Does Not Work

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Abstract. In this paper we consider some questions related to the orientation of shapes when the standard method does not work. A typical situation is when a shapes under consideration has more than two axes of symmetry or if the shape is n-fold rotationally symmetric, when $n > 2$. Those situations are well studied in literature. Here, we give a very simple proof of the main result from [11] and slightly adapt their definition of principal axes for rotationally symmetric shapes. We show some desirable properties that hold if the orientation of such shapes is computed in such a modified way.

Keywords: Shape, orientation, image processing, early vision.

1 Introduction

The computation of a shape's orientation is a common task in the area of computer vision and image processing, being used for example to define a local frame of reference, and helpful for recognition and registration, robot manipulation, etc. It is also important in human visual perception; for instance, orientable shapes can be matched more quickly than shapes with no distinct axis [8]. Another example is the perceptual difference between a square and a diamond (rotated square) noted by Mach in 1886 [6], which can be explained by their multiple reference frames, i.e. ambiguous orientations [8]. There are situations (see Fig. 1 (a), (b), (c)) when the orientation of the shapes seems to be easily and naturally determined. On the other hand, a planar disc could be understood as a shape without orientation.

Most situations are somewhere in between. For very non-regular shapes it could be difficult to say what the orientation should be – see Fig. 6**(a),(b)**. Rotationally symmetric shapes can also have poorly defined orientation – see Fig 2 (d). Moreover, even for regular polygons (see Fig. 2 (a) and (b)) is debatable whether they are orientable or not. For instance, is a square an orientable shape?

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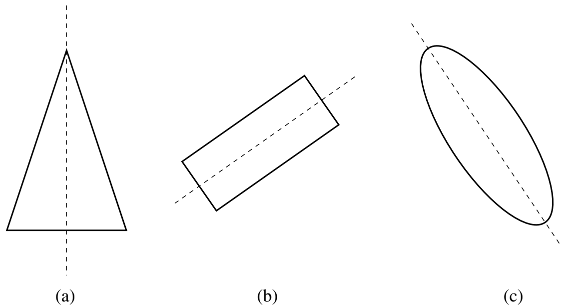

Fig. 1. It is reasonable to say that the orientation of the presented shapes coincide with the dashed lines

The same question arises for any regular n-gon, but also for shapes having several axes of symmetry, and n-fold ($n > 2$) rotational symmetric shapes – see shapes from Fig. 2. It is known ([11]) that the standard method, based on computing the axis of the last second moment, does not suggest any answer what the shape orientation should be if applied to n-fold ($n > 2$) rotationally symmetric shapes.

An compromised answer could be that such shapes are orientable but they do not have the unique orientation. Naturally, if a n-fold rotationally symmetric shape is considered as an orientable shape, than it should be n lines (making mutual angles that are multiplication of $\frac{2\pi}{n}$) that define its orientation. If a shape has n axes of symmetry than it is reasonable to use such axes to represent the shape orientation. Some solutions are proposed in [5,9,11], for example.

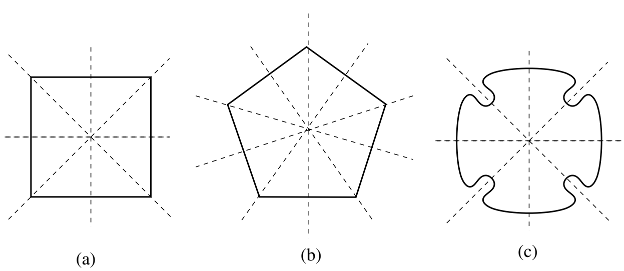

Fig. 2. The dashed lines seem to be reasonable candidates to represent the orientation of the shapes (a), (b), and (c). It is not quite clear what the orientation of 4-fold rotationally symmetric shape (d) should be.

2 Standard Method for Computing Orientation

In this section we give a short overview of the method which is mostly used in practice for computing orientation.

The standard approach defines the orientation by the so called axis of the least second moment ([1,2]). That is the line which minimizes the integral of the squares of distances of the points (belonging to the shape) to the line. The integral is

$$I(\delta, \rho, S) = \iint\limits_S r^2(x, y, \delta, \rho) dxdy \tag{1}$$

where $r(x, y, \delta, \rho)$ is the perpendicular distance from the point (x, y) to the line given in the form

$$x \cdot \cos\delta - y \cdot \sin\delta = \rho.$$

It can be shown that the line that minimizes $I(S, \delta, \rho)$ passes through the centroid $(x_c(S), y_c(S))$ of the shape S where $(x_c(S), y_c(S)) = \left(\frac{\iint_S x dxdy}{\iint_S dxdy}, \frac{\iint_S y dxdy}{\iint_S dxdy} \right)$. In other words, without loss of generality, we can assume that the origin is placed at the centroid. Since required line minimizing $I(S, \delta, \rho)$, passes through the origin we can set $\rho = 0$. In this way, the shape orientation problem can be reformulated to the problem of determining δ for which the function $I(S, \delta)$ defined as

$$I(\delta, S) = I(\delta, \rho = 0, S) = \iint\limits_S (-x \cdot \sin\delta + y \cdot \cos\delta)^2 dxdy^1$$

reaches the minimum.

Further, if the central geometric moments $\overline{m}_{p,q}(S)$ are defined as usually

$$\overline{m}_{p,q}(S) = \iint\limits_S (x - x_c(S))^p \cdot (y - y_c(S))^q \; dx \; dy,$$

and by the assumed $(x_c(S), y_c(S)) = (0, 0)$, we obtain

$$I(\delta, S) \;=\; (\sin\delta)^2 \cdot \overline{m}_{2,0}(S) - \sin(2\cdot\delta) \cdot \overline{m}_{1,1,}(S) + (\cos\delta)^2 \cdot \overline{m}_{0,2}(S). \tag{2}$$

The minimum of the function $I(\delta, S)$ can be computed easily. Setting the first derivative $I'(x, S)$ to zero, we have

$$I'(\delta, S) = \sin(2\delta) \cdot (\overline{m}_{2,0}(S) - \overline{m}_{0,2}(S)) - 2 \cdot \cos(2\delta) \cdot \overline{m}_{1,1}(S) = 0.$$

That easily gives that the required angle δ, but also the angle $\delta + \pi/2$, satisfies the equation

$$\frac{\sin(2\delta)}{\cos(2\delta)} = \frac{2 \cdot \overline{m}_{1,1}(S)}{\overline{m}_{2,0}(S) - \overline{m}_{0,2}(S)}. \tag{3}$$

Thus, the maximum and minimum of $I(\delta, S)$ are easy to compute.

Let us mention that, when working with digital objects which are actually digitizations of real shapes, then central geometric moments $\overline{m}_{p,q}(S)$ are replaced with their discrete analogue, i.e., with the so called *central discrete moments.*

[1] The squared distance of a point (x, y) to the line $X \cdot \cos\delta - Y \cdot \sin\delta = 0$ is $(-x\sin\delta + y\cos\delta)^2$.

Since the digitization on the integer grid $\mathbf{Z}^2$ of a real shape S consists of all pixels whose centers are inside S it is natural to approximate $\overline{m}_{p,q}(S)$ by the central discrete moment $M_{p,q}(S)$ defined as

$$M_{p,q}(S) = \sum_{(i,j)\in S\cap\mathbf{Z}^2} (i - x_{cd}(S))^p \cdot (j - y_{cd}(S))^q$$

where $(x_{cd}(S), y_{cd}(S))$ is the centroid of the discrete shape $S \cap \mathbf{Z}^2$. For some details about the efficiency of the approximation $\overline{m}_{p,q}(S) \approx M_{p,q}(S)$ see [3].

If the all geometric moments in (3) are replaced with the corresponding discrete moments we have the equation

$$\frac{\sin(2\delta)}{\cos(2\delta)} = \frac{2 \cdot M_{1,1}(S)}{M_{2,0}(S) - M_{0,2}(S)} \tag{4}$$

which describes the angle δ which is used to describe the orientation of discrete shape $S \cap \mathbf{Z}^2$.

So, the standard method is very simple (in both “real” and “discrete” versions) and it comes from a natural definition of the shape orientation. However, it is not always effective. Indeed, if $I(\delta, S)$ is a constant function then the method does not work – i.e., it does not tell us what the angle should be used to define the orientation of S. $I(\delta, S) = contsnt$ can happen for very non regular shapes but perhaps the most typical situation is when the considered shape S has more than two axes of symmetry, or more generally, if S is an n-fold rotationally symmetric shape (with $n > 2$).

The next lemma (it is a particular case of Theorem 1 from [11]) proves easily that the standard method cannot be used if the measured shape has more than two symmetry axes.

Lemma 1. *If a given shape has more than two axes of symmetry then $I(\delta, S)$ is a constant function.*

Proof. From (2) it is obvious that $I(\delta, S)$ can have no more than one maximum and one minimum on the interval $[0, \pi)$ or it must be a constant function. Trivially $I(0, S) = I(\pi, S)$. So, if S has more than two axes of symmetry then $I(\delta, S)$ must be constant since the first derivative $I'(\delta, S)$ does not have more than two zeros on the interval $[0, \pi)$. ∎

Remark 1. Lemma 1 implies $I(S, \delta) = \frac{1}{2} \cdot (\overline{m}_{2,0}(S) + \overline{m}_{0,2}(S))$ (for all $\delta \in [0, \pi)$) if S has more than two symmetry axes. The standard method does not tell us what the orientation should be in such a situation. Obviously, the standard method is limited by the simplicity of the function $I(\delta, S)$.

3 High-Order Principal Axes

In [11] it has been noted that the standard method does not work if applied to n-fold ($n > 2$) rotationally symmetric shapes. As usual, rotationally symmetric shapes are such shapes which are identical to itself after being rotated through any multiple of $\frac{2\pi}{n}$ (the problem of detecting number of folds but also the problem of detecting symmetry axes are well studied – see [4,7,10], for example). So, if a discrete point set S is n-fold rotationally symmetric then it is of the form

$$S = \left\{ (r_i, \theta_{i,j}) \mid i = 1, \ldots, m, \;\; j = 1, \ldots, n, \text{ and } \theta_{i,j} = \theta_{i,1} + (j-1)\frac{2\pi}{n} \right\} \quad (5)$$

where points $(r_i, \theta_{i,j})$ from S are given in polar coordinates.

As mentioned, the function $I(\delta, S)$ is not a strong enough mathematical tool to be used for the defining the orientation of n-fold ($n > 2$) rotationally symmetric shapes. In order to overcome such a problem, the authors of [11] proposed the use of the N^{th}*-order central moments of inertia.* A precise definition follows.

Definition 1. *Let a shape S whose centroid coincide with the origin. Then, the N-order central moment of inertia, denoted as $I_N(\delta, S)$ about a line going through the shape centroid with slope* $\tan\delta$ *is defined as*

$$I_N(\delta, S) = \sum_{(x,y)\in S} (-x\sin\delta + y\cos\delta)^N. \quad (6)$$

In other words, the authors suggest that a more complex function than (2) should be used. Obviously, if $N = 2$ we have the standard method.

A nice result, related to n-fold rotationally symmetric shapes and their corresponded N^{th}-order central moments has been proven in [11]. The proof presented in [11] is pretty long. Here, we give a very elemental proof.

Theorem 1. *(*[**11**]*) For an n-fold rotationally symmetric shape S, having the centroid coincident with the origin, its N^{th}-order central moment of inertia $I_N(\delta, S)$ is constant about any line going through its centroid for all N less than n.*

Proof. Let an n-fold rotationally symmetric shape S, with the centroid placed at the origin. Setting the first derivative of $I_N(\delta, S)$ to be equal to zero, we can derive that there are not more than $2N$ values of δ for which $dI_N(\delta, S)/d\delta$ vanishes, if $I_N(\delta, S)$ is not a constant function. Indeed, starting form

$$\frac{dI_N(\delta, S)}{d\delta} = \sum_{(x,y)\in S} N \cdot (-x\sin\delta + y\cos\delta)^{N-1} \cdot (-x\cos\delta - y\sin\delta) \quad (7)$$

we will distinguish two situations – denoted below by (**i**) and (**ii**).

(**i**) – If $\delta = 0$ and $\delta = \pi$ (i.e. $\sin\delta = 0$) are not solution of $dI_N(\delta, S)/d\delta = 0$, then (from (8))

$$\frac{dI_N(\delta, S)}{d\delta} = 0 \quad \Leftrightarrow \quad (\sin\delta)^N \cdot \sum_{(x,y)\in S} (-x \; + y\cot\delta)^{N-1} \cdot (x\cot\delta + y) = 0.$$

Since the quantity

$$\sum_{(x,y)\in S} (-x \; + y\cot\delta)^{N-1} \cdot (x\cot\delta + y)$$

is an N-degree polynomial on $\cot\delta$ it cannot have more than N real zeros

$$\cot\delta = z_1, \;\; \cot\delta = z_2, \;\; \ldots, \;\; \cot\delta = z_k, \qquad (k \leq N).$$

In other words, because of $\cot\delta = \cot(\delta+\pi)$ the equation

$$\frac{dI_N(\delta,S)}{d\delta} = 0$$

has no more than $2N$ solutions.

(ii) – If $\delta = 0$ and $\delta = \pi$ (i.e. $\sin\delta = 0$) are solution of $dI_N(\delta,S)/d\delta = 0$, then easily (see (8))

$$\sum_{(x,y)\in S} x \cdot y^{N-1} = 0. \tag{8}$$

But, in such a situation

$$P(\cot\delta) = \sum_{(x,y)\in S} (-x \; + y\cot\delta)^{N-1} \cdot (x\cot\delta + y)$$

is an $(N-1)$-degree polynomial on $\cot\delta$ (see (10), the coefficient of $(\cot\delta)^N$ vanishes). Consequently, $P(\cot\delta)$ cannot have more than $N-1$ real zeros:

$$\cot\delta = z_1, \;\; \cot\delta = z_2, \;\; \ldots, \;\; \cot\delta = z_k, \qquad (k \leq N-1),$$

i.e. there are no more than $2(N-1)$ values of δ for which $P(\cot\delta)$ vanishes. So, again, $dI_N(\delta,S)d/\delta = 0$ has at most $2N$ solutions, including $\delta = 0$ and $\delta = \pi$.

Thus, the number of zeros that could have $dI_N(\delta,S)/d\delta$ is not bigger than $2N$.

On the other side, if S is a fixed n-fold rotationally symmetric shape, then $I_N(\delta,S)$ must have (because of the symmetry) at least n local minima and n local maxima (one minimum and one maximum on each interval of the form $[\beta, \beta+2\pi/n)$, or it must be a constant function. That means, $dI_N(\delta,S)/d\delta$ must have (at least) $2n$ zeros $\delta_1, \delta_2, \ldots, \delta_{2n}$.

Since the presumption $N < n$ does not allow $2n$ zeros of $dI_N(\delta,S)/d\delta$ if $I_N(\delta,S)$ is not a constant functions, we just derived a contradiction. Thus $I_N(\delta,S)$ must be a a constant function for all N less than n. ∎

4 Comments on High-Order Principal Axes

The computing orientation is not always easy and straightforward. As shown by Lemma 1, even the orientation of a square cannot be computed if the standard method is applied. Once again, the standard method, if works, gives only one line which should represent the shape orientation. Lemma 1 is related to the shapes having more than two axes of symmetry but there are also irregular shapes whose orientation is not computable by the standard method. Since it is clear that the function (2) (that uses the second degree moments only) is not powerful enough to define the orientation of any shape, [11] involves more complex functions $I_N(\delta, S)$ that should be used to define the orientation of n-fold rotationally symmetric shapes.

Precisely, [11] defines an N-th order principal axis of a degenerate shape S (a shape for which the standard method does not work) as a line going through the centroid of S about which the $I_N(\delta, S)$ is minimized. Then, the orientation of S is defined by one of N-th order principal axes. Of course, for any fixed N there are still shapes whose orientation cannot be computed in this generalized manner – it is enough to consider an n-fold rotationally symmetric shape with $n > N$ (see Theorem 1).

Theorem 1 gives a clear answer that for an n-fold rotationally symmetric shape, the N-th order principal axes cannot be determined for all $N < n$. On the other side, even Theorem 1 says nothing about the existence of minima (maxima) of $I_{N=n}(\delta, S)$ it seems that $N = n$ could be an appropriate choice of the order to define the high order principal axes for an n-fold rotationally symmetric shape. If n-th order principal axes of an n-fold rotationally symmetric shape S exist, then they can be computed easily, as given by the next lemma.

Lemma 2. *(**[11]**) The directions, δ, of the N^{th}-order principal axes of an n-fold rotationally symmetric S satisfy following equations:*

$$\tan(n\delta) = \begin{cases} \dfrac{n \cdot M_{n-1,1}(S)}{M_{n,0}(S) - (n-1) \cdot M_{n-2,2}(S)} & \text{if } n \text{ is even} \\[2ex] \dfrac{-M_{n,0}(S)}{M_{n-1,1}(S)} & \text{if } n \text{ is odd.} \end{cases}$$

Remark 2. It is important to notice that Lemma 2 does say nothing if S is not n-fold rotationally symmetric.

Some examples of shape orientations obtained by a use of higher order principal axes are given Fig. 3. In the presented cases, the method satisfies the basic request for which it was involved - i.e. it suggests a precise answer what the orientation of n-fold symmetric shapes should be. That could be enough for, let say, an image normalization task. Also, a very nice property is given by Lemma 2 – i.e. in the case when S is an n-fold rotationally symmetric shape (with a known n) then the computation of principal axes is very simple.

On the other side, just looking at the presented example, we can see that sometimes (even case) the orientation coincide with one of symmetry axes, but sometimes (odd case) does not. That could be a strong objection. This disadvantage is caused by the fact that there is no a good enough “geometric” motivation for a use of centralized geometric moments having an odd order. The preference that the shape orientation coincides with one of its symmetry axes (if any) seems to be very reasonable.

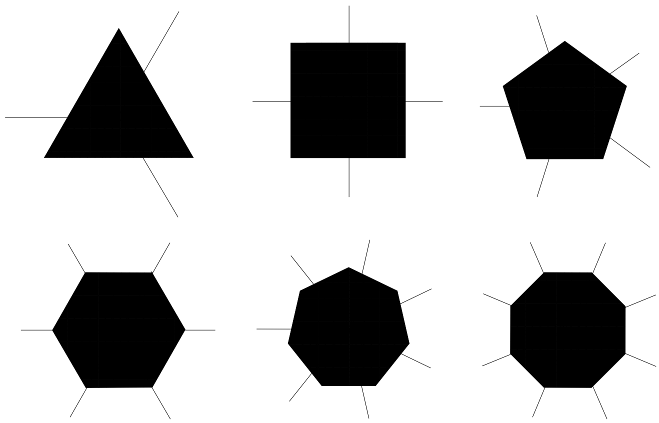

Fig. 3. The principal axes (obtained as suggested in [11]) for regular $3, 4, 5, 6, 7$, and 8-gons are presented

The situation is even worse. If a shape S has at least one symmetry axis and if the orientation is computed as the line that minimizes $I_{2k+1}(\beta, S)$ then very likely such a line will not coincide with any axis of symmetry of S. Indeed, let an axis-symetric set S. Without loss of generality we can assume that this axis coincides with the x-axis. So, S is the union of the sets:

- Set A which consists of all points from S that have a positive y coordinate;
- set B which consists of all points from S that have a negative y coordinate;
- set C which consists of all points from S that have y coordinate equal to zero.

Since x-axis is a symmetry axis of S, we have $(x, y) \in A \Leftrightarrow (x, -y) \in B$. Thus, we can write:

$$I_{2k+1}(\delta, S) = \sum_{(x,y)\in A} (-x\sin\delta + y\cos\delta)^{2k+1} + \sum_{(x,y)\in B} (-x\sin\delta + y\cos\delta)^{2k+1}$$
$$+ \sum_{(x,y)\in C} (-x\sin\delta + y\cos\delta)^{2k+1} =$$
$$= \sum_{(x,y)\in A} \left((-x\sin\delta + y\cos\delta)^{2k+1} + (-x\sin\delta - y\cos\delta)^{2k}\right)$$
$$+ \sum_{(x,0)\in C} 2k(-x\sin\delta)^{2k+1}.$$

The first derivative is

$$\frac{dI_{2k+1}(\delta, S)}{d\delta} = \sum_{(x,y)\in A} (2k+1)\cdot(-x\sin\delta + y\cos\delta)^{2k} \cdot (-x\cos\delta - y\sin\delta)$$

$$+ \sum_{(x,y)\in A} (2k+1)\cdot(-x\sin\delta - y\cos\delta)^{2k}\cdot(-x\cos\delta + y\sin\delta)$$

$$+ \sum_{(x,0)\in C} (2k+1)\cdot(-x\sin\delta)^{2k}(-x\cos\delta).$$

From the last equality we obtain

$$\frac{d^2 I_{2k+1}(0,S)}{d\delta^2} = -(4k+2)\sum_{(x,y)\in A} xy^{2k} = -(4k+2)M_{1,2k}(S).$$

Thus, $d^2 I_{2k+1}(0,S)/d\delta^2$ is not necessarily equal to zero and, consequently, a maximum is not guaranteed.

It is interesting to note $d^2 I_{2k+1}(\pi/2,S)/d\delta^2 = 0$. So, if $I_{2k+1}(\delta,S)$ reaches the maximum for an angle δ_0, then it seems to be more reasonable to define the orientation of S by the angle $\pi/2+\delta_0$, rather than by the angle δ_0, as suggested by [11].

5 Modified Use of High Order Principal Axes

Here, we use a modified approach to the problem. We accept that we have to use a more complex method than the standard one. So, we are going to use N^{th}-order central moments with $N > 2$ and will try to make a compromise between the following requests:

- **(c1)** The method should have a reasonable geometric motivation;
- **(c2)** The method should give some answer what orientation should be even for rotationally symmetric shapes;
- **(c3)** The method should give reasonably good results if applied to non regular shapes;
- **(c4)** The orientation should be relatively easy to compute.

If we go back to the standard definition of shape orientation, we can see that it is defined by the line that minimizes the sum of squares of distances of the points to this line. The squared distance (rather than the pure Euclidean distance) has been taken in order to enable an easy mathematical calculation. Following this initial idea and taking into account the problems explained by Theorem 1, we suggest that the orientation should be defined as a line which minimizes the total sum of a (suitably chosen) even-power of distances of the points to the line. We give a formal definition.

Definition 2. *Let a given integer k and let a given shape S whose centroid coincide with the origin. Then, the orientation of S is defined by an angle δ that minimizes*

$$I_{2k}(\delta,S) = \sum_{(x,y)\in S} (-x\sin\delta + y\cos\delta)^{2k}. \tag{9}$$

Now, we show a desirable property of the orientation computed in accordance with Definition 2. Let an axis-symetric set S. Because of the definition of $I_{2k}(\delta, S)$, without loss of generality we can assume that this axis coincides with the x-axis. Again, if S is represented as the union of:

- set A consisting of all points from S that have a positive y coordinate,
- set B consisting of all points from S that have a negative y coordinate,
- set C consisting of all points from S that have y coordinate equal to zero,

and if the x-axis is a symmetry axis of S, we have $(x, y) \in A \Leftrightarrow (x, -y) \in B$. Thus, we can write:

$$\begin{aligned} I_{2k}(\delta, S) &= \sum_{(x,y)\in A} (-x\sin\delta + y\cos\delta)^{2k} + \sum_{(x,y)\in B} (-x\sin\delta + y\cos\delta)^{2k} \\ &\quad + \sum_{(x,y)\in C} (-x\sin\delta + y\cos\delta)^{2k} \\ &= \sum_{(x,y)\in A} \left((-x\sin\delta + y\cos\delta)^{2k} + (-x\sin\delta - y\cos\delta)^{2k}\right) \\ &\quad + \sum_{(x,0)\in C} 2k(-x\sin\delta)^{2k} \end{aligned}$$

$$\begin{aligned} \frac{dI_{2k}(\delta, S)}{d\delta} &= \sum_{(x,y)\in A} 2k(-x\sin\delta + y\cos\delta)^{2k-1}(-x\cos\delta - y\sin\delta) \\ &+ \sum_{(x,y)\in A} 2k(-x\sin\delta - y\cos\delta)^{2k-1}(-x\cos\delta + y\sin\delta) \\ &+ \sum_{(x,0)\in C} 2k(-x\sin\delta)^{2k-1}(-x\cos\delta). \end{aligned}$$

From the last equality we have that the first derivative of I_{2k} vanishes if $\delta = 0$, but also if $\delta = \pi/2$, i.e.,

$$\frac{dI_{2k}(0, S)}{d\delta} = \frac{dI_{2k}(\pi/2, S)}{d\delta} = 0.$$

The above equality shows that a symmetry axis (if any) has a "good chance" to be coincident with the computed orientation if Definition 2 is applied.

Since naturally defined, the orientation computed in proposed manner should performs well if applied to non regular shapes – that is illustrated by a few examples on Fig. 6.

Of course, the main disadvantage of the modified method is a higher computation complexity caused by the size of coefficient $2k$ from (9). It is not expected that a closed formula (as it is the formula (3) in the case of $2k = 2$) could

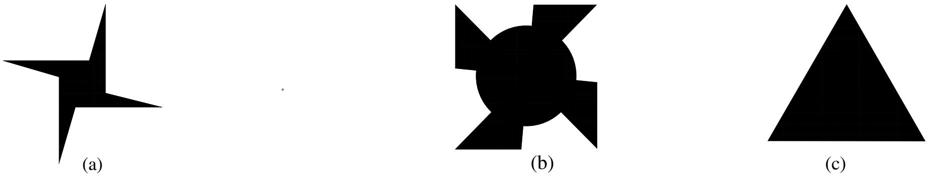

Fig. 4. (a) I_2 is nearly a constant value. The minimum of I_4 is reached for 44 degrees, while I_8 has the minimun for 42 degrees. (b) I_2 is nearly a constant value. The minimum of I_4 is reached for 11 degrees, while I_8 has the minimum for 8 degrees. (c) I_2 and I_4 are nearly constants. The minimum of I_6 is reached for 150 degrees – as preferred.

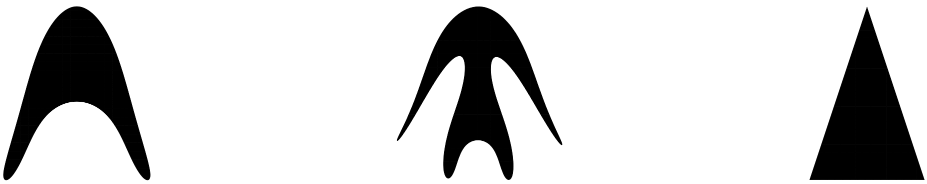

Fig. 5. The presented figures have exactly one axis of symmetry. In all presented cases the minimum of I_2, I_4, I_6, I_8, and I_{10} is obtained to be very close to 90 degrees.

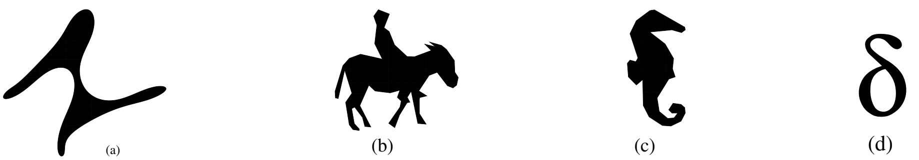

Fig. 6. Minimum values for I_2, I_4, I_6, I_8, and I_{10} are obtained for the following angle values: (a) 48, 56, 61, 63, and 64, respectively. (b) 114, 131, 32, 31, and 31, respectively. (c) 96, 95, 94, 92, and 92, respectively. (d) 87, 88, 88, 88, and 88, respectively.

be derived. But, the formula (9) enables an easy and straightforward numerical computation. Several examples are given on Fig. 4-6.

Rotationally symmetric shapes are presented on Fig. 4. The obtained results are in accordance with the previous theoretical observations. Particularly, the obtained minimum of I_6 says that the orientation of a regular triangle is coincident with one of its symmetry axes.

On Fig. 5 the orientation is measured for shapes having one symmetry axis. In all cases the computed minimal values for I_2, I_4, I_6, I_8, and I_{10} are obtained for an angle of 90 degrees – as preferred.

Fig. 6 displays non symmetric shapes. It may be assumed that the orientation is not well-defined for the shapes presented on Fig. 6 (a) and Fig. 6 (b). Indeed, when measure the orientation as the minimum of I_N we obtain different angle values for different values of N.

On the other side, since the shape on Fig. 6(c) and Fig. 6 (d) seems to be "well orientable" we obtain almost same angle values that should represent the orientation.

6 Concluding Remarks

In this paper we consider some problems related to the shape orientation. The most studied situation when such problems arise, is when working with shapes having many axes of symmetry and with n-fold rotationally symmetric shapes. The paper is mainly based on the results presented in [11]. A very short proof of the main result from [11] is presented. It is clarified that the most of of problems come from the fact that the function (2) is not complex enough to be used to compute orientation of an arbitrary shape. As an solution, a use of higher moments is suggested in [11]. Some disadvantages of such a proposal are discussed here as well. The main of them is that shapes having an odd number of axes of symmetry could have the computed orientation that does not coincide with any of symmetry axes. This paper suggest a modified use of the higher order moments that should avoid this disadvantage.

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