Median Associative Memories: New Results

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Abstract. Median associative memories (MEDMEMs) first described in [1] have proven to be efficient tools for the reconstruction of patterns corrupted with mixed noise. First formal conditions under which these tools are able to reconstruct patterns either from the fundamental set of patterns and from distorted versions of them were given in [1]. In this paper, new more accurate conditions are provided that assure perfect reconstruction. Numerical and real examples are also given.

1 Introduction

An associative memory (**M**) as described in [1] can be viewed as a device that relates input patterns and output patterns: $\mathbf{x} \rightarrow \mathbf{M} \rightarrow \mathbf{y}$, with **x** and **y**, respectively the input and output patterns vectors. Each input vector forms an association with a corresponding output vector. The associative memory **M** is represented by a matrix whose *ij*-th component is m_{ij}. **M** is generated from a finite a priori set of known associations, known as the *fundamental set of associations*, or simply the *fundamental set* (FS). If ξ is an index, the fundamental set is represented as: $\{(\mathbf{x}^{\xi}, \mathbf{y}^{\xi}) \mid \xi = 1,2,\ldots,p\}$ with p the cardinality of the set. Patterns that form the fundamental set are called *fundamental patterns*. If it holds that $\mathbf{x}^{\xi} = \mathbf{y}^{\xi}\ \forall\, \xi \in \{1,2,\ldots p\}$, then **M** is auto-associative, otherwise it is hetero-associative. A distorted version of a pattern $\mathbf{x}$ to be recalled will be denoted as $\tilde{\mathbf{x}}$. If when feeding a distorted version of $\mathbf{x}^{w}$ with $w \in \{1,2,\ldots,p\}$ to an associative memory **M**, then it happens that the output corresponds exactly to the associated pattern $\mathbf{y}^{w}$, we say that recalling is robust, if $\mathbf{x}^{w}$ is not distorted recalling is perfect. Several models for associative memories have emerged in the last 40 years. Refer for example to [3-6].

In [1] we first described an associative model based on the functioning of well-known median operator. Also in this paper was given a first set of formal conditions under which the proposed set of memories operate. In this paper, we provide more accurate conditions for the functioning of these memories. Examples with synthetic and real data are also given.

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2 Basics of Median Associative Memories

Two associative memories are fully described in [1]. Due to space limitations, only hetero-associative memories are described. Auto-associative memories can be obtained simple by doing $\mathbf{x}^{\xi} = \mathbf{y}^{\xi}\ \forall\, \xi \in \{1,2,\dots p\}$. Let us designate hetero-associative median memories as HAM-memories. Let $\mathbf{x} \in \mathbf{Z}^{n}$ and $\mathbf{y} \in \mathbf{Z}^{m}$ two vectors. To operate HAM memories two operations are required, one for memory training: $\Diamond_{\mathrm{A}}$ and one for pattern recall: $\Diamond_{\mathrm{B}}$.

2.1 Memory Construction

Two steps are required to build the HAM-memory:

Step 1: For each $\xi = 1,2,\cdots,p$, from each couple $\left(\mathbf{x}^{\xi},\mathbf{y}^{\xi}\right)$ build matrix: $\left[\mathbf{y}^{\xi} \Diamond_{\mathrm{A}} \left(\mathbf{x}^{\xi}\right)^{t}\right]_{m\times n}$ as:

$$\mathbf{y}\,\Diamond_{\mathrm{A}}\mathbf{x}^{t} = \begin{pmatrix} \mathrm{A}(y_1,x_1) & \mathrm{A}(y_1,x_2) & \cdots & \mathrm{A}(y_1,x_n) \\ \mathrm{A}(y_2,x_1) & \mathrm{A}(y_2,x_2) & \cdots & \mathrm{A}(y_2,x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{A}(y_m,x_1) & \mathrm{A}(y_m,x_2) & \cdots & \mathrm{A}(y_m,x_n) \end{pmatrix}_{m\times n} \tag{1}$$

Step 2: Apply the median operator to the matrices obtained in Step 1 to get matrix $\mathbf{M}$ as follows:

$$\mathbf{M} = \underset{\xi=1}{\overset{p}{\mathbf{med}}}\left[\mathbf{y}^{\xi} \Diamond_{\mathrm{A}} \left(\mathbf{x}^{\xi}\right)^{t}\right]. \tag{2}$$

The *ij*-th component **M** is given as follows:

$$m_{ij} = \underset{\xi=1}{\overset{p}{\mathbf{med}}}\,\mathrm{A}\left(y_i^{\xi}, x_j^{\xi}\right). \tag{3}$$

2.2 Pattern Recall

We have two cases:

Case 1: Recall of a fundamental pattern. A pattern $\mathbf{x}^{w}$, with $w \in \{1,2,\cdots,p\}$ is presented to the memory **M** and the following operation is done:

$$\mathbf{M} \Diamond_{\mathrm{B}} \mathbf{x}^{w}. \tag{4}$$

The result is a column vector of dimension n, with i-th component given as:

$$\left(\mathbf{M}\Diamond_{\mathrm{B}} x^{w}\right)_i = \underset{j=1}{\overset{n}{\mathbf{med}}}\,\mathrm{B}\left(m_{ij}, x_j^{w}\right). \tag{5}$$

Case 2: Recall of a pattern from an altered version of it. A pattern $\tilde{\mathbf{x}}$ (altered version of a pattern $\mathbf{x}^{w}$ is presented to the HAM memory $\mathbf{M}$ and the following operation is done:

$$\mathbf{M}\Diamond_{\mathrm{B}}\tilde{\mathbf{x}}. \tag{6}$$

Again, the result is a column vector of dimension n, with i-th component given as:

$$\left(\mathbf{M}\Diamond_{\mathrm{B}}\tilde{\mathbf{x}}\right)_i = \underset{j=1}{\overset{n}{\mathbf{med}}}\,\mathrm{B}\left(m_{ij}, \tilde{x}_j\right). \tag{7}$$

Operators A and B might be chosen among those already proposed in the literature. In this paper we adopt operators A and B used in [6]. Operators A and B are defined as follows:

$$\mathrm{A}(x, y) = x - y \tag{8.a}$$

$$\mathrm{B}(x, y) = x + y \tag{8.b}$$

Conditions, for perfect recall of a pattern of the FS or from an altered version of them, according to [1] follow:

Theorem 1 [1]. Let $\left\{\left(\mathbf{x}^{\alpha}, \mathbf{y}^{\alpha}\right) \mid \alpha = 1,2,\ldots,p\right\}$ with $\mathbf{x}^{\alpha} \in \mathbf{R}^{n}$, $\mathbf{y}^{\alpha} \in \mathbf{R}^{m}$ the fundamental set of an HAM-memory $\mathbf{M}$ and let $\left(\mathbf{x}^{\gamma}, \mathbf{y}^{\gamma}\right)$ an arbitrary fundamental couple with $\gamma \in \{1,\cdots,p\}$. If $\underset{j=1}{\overset{n}{\mathbf{med}}}\,\varepsilon_{ij} = 0$, $i = 1,\cdots,m$, $\varepsilon_{ij} = m_{ij} - \mathrm{A}\left(y_i^{\gamma}, x_j^{\gamma}\right)$ then $\left(\mathbf{M}\Diamond_{\mathrm{B}}\mathbf{x}^{\gamma}\right)_i = \mathbf{y}_i^{\gamma}, i = 1\ldots m$.

Corollary 1 [1]. Let $\left\{\left(\mathbf{x}^{\alpha}, \mathbf{y}^{\alpha}\right) \mid \alpha = 1,2,\ldots,p\right\}$, $\mathbf{x}^{\alpha} \in \mathbf{R}^{n}$, $\mathbf{y}^{\alpha} \in \mathbf{R}^{m}$. A HAM-median memory $\mathbf{M}$ has perfect recall if for all $\alpha = 1,\cdots,p$, $\mathbf{M}^{\alpha} = \mathbf{M}$ where $\mathbf{M} = \mathbf{y}^{\xi}\Diamond_{\mathrm{A}}\left(\mathbf{x}^{\xi}\right)^{t}$ is the associated partial matrix to the fundamental couple $\left(\mathbf{x}^{\alpha}, \mathbf{y}^{\alpha}\right)$ and p is the number of couples.

Theorem 2 [1]. Let $\left\{\left(\mathbf{x}^{\alpha}, \mathbf{y}^{\alpha}\right) \mid \alpha = 1,2,\ldots,p\right\}$, $\mathbf{x}^{\alpha} \in \mathbf{R}^{n}$, $\mathbf{y}^{\alpha} \in \mathbf{R}^{m}$ a FS with perfect recall. Let $\eta^{\alpha} \in \mathbf{R}^{n}$ a pattern of mixed noise. A HAM-median memory $\mathbf{M}$ has perfect recall in the presence of mixed noise if this noise is of median zero, this is if $\underset{j=1}{\overset{n}{\mathbf{med}}}\,\eta_j^{\alpha} = 0, \forall\alpha$.

2.3 Case of a General Fundamental Set

In [2], it was shown that due to, in general, a fundamental set (FS) does not satisfy the restricted conditions imposed by Theorems 1 and its Corollary. In [2] it is proposed the following procedure to transform a general FS into an auxiliary FS' satisfying the desired conditions:

TRAINING PHASE:

Step 1. Transform the FS into an auxiliary fundamental set (FS') satisfying Theorem 1:

1) Make $D = cont$, a vector.
2) Make $(\bar{\mathbf{x}}^1, \bar{\mathbf{y}}^1) = (\mathbf{x}^1, \mathbf{y}^1)$.
3) For the remaining couples do {
   For $\xi = 2$ to p {
   $\bar{\mathrm{x}}^{\xi} = \bar{\mathrm{x}}^{\xi-1} + D$; $\hat{\mathbf{x}}^{\xi} = \bar{\mathbf{x}}^{\xi} - \mathbf{x}^{\xi}$; $\mathrm{y}^{\xi} = \bar{\mathrm{y}}^{\xi-1} + D$; $\hat{\mathbf{y}}^{\xi} = \bar{\mathbf{y}}^{\xi} - \mathbf{y}^{\xi}$.
   }

Step 2. Build matrix **M** in terms of set FS': Apply to FS' steps 1 and 2 of the training procedure described at the beginning of this section.

RECALLING PHASE:

We have also two cases, i.e.:

Case 1: Recalling of a fundamental pattern of FS:

1) Transform $\mathbf{x}^{\xi}$ to $\bar{\mathbf{x}}^{\xi}$ by applying the following transformation: $\bar{\mathbf{x}}^{\xi} = \mathbf{x}^{\xi} + \hat{\mathbf{x}}^{\xi}$.
2) Apply equations (4) and (5) to each $\bar{\mathbf{x}}^{\xi}$ of FS' to recall $\bar{\mathbf{y}}^{\xi}$.
3) Recall each $\mathbf{y}^{\xi}$ by applying the following inverse transformation: $\mathbf{y}^{\xi} = \bar{\mathbf{y}}^{\xi} - \hat{\mathbf{y}}^{\xi}$.

Case 2: Recalling of a pattern $\mathbf{y}^{\xi}$ from an altered version of its key: $\tilde{\mathbf{x}}^{\xi}$:

1) Transform $\tilde{\mathbf{x}}^{\xi}$ to $\bar{\mathbf{x}}^{\xi}$ by applying the following transformation: $\bar{\mathbf{x}}^{\xi} = \tilde{\mathbf{x}}^{\xi} + \hat{\mathbf{x}}^{\xi}$.
2) Apply equations (6) and (7) to $\bar{\mathbf{x}}^{\xi}$ to get $\bar{\mathbf{y}}^{\xi}$, and
3) Anti-transform $\bar{\mathbf{y}}^{\xi}$ as $\mathbf{y}^{\xi} = \bar{\mathbf{y}}^{\xi} - \hat{\mathbf{y}}^{\xi}$ to get $\mathbf{y}^{\xi}$.

3 New Results About MEDMEMs

In general, noise added to a pattern does not satisfy the conditions imposed by Theorem 2. The following new results (in the transformed domain) state the conditions under which MEDMEMS present perfect recall under general mixed noise:

Theorem 3. Let $\{(\mathbf{x}^{\alpha},\mathbf{y}^{\alpha}) \mid \alpha = 1,2,\ldots,p\}$, $\mathbf{x}^{\alpha} \in \mathbf{R}^{n}$, $\mathbf{y}^{\alpha} \in \mathbf{R}^{m}$ a fundamental set $\mathbf{x}^{\xi+1} = \mathbf{x}^{\xi} + D$, $\mathbf{y}^{\xi+1} = \mathbf{y}^{\xi} + D$, $\xi = 1,2,\ldots,p$, $D = (d,\cdots,d)^{T}$, $d = Const$. Without lost of generality suppose that is p odd. Thus the associative memory $\mathbf{M} = \mathbf{y}^{\xi} \Diamond_{\mathrm{A}} (\mathbf{x}^{\xi})^{T}$ has perfect recall in the presence of noise if less than $\frac{n+1}{2} - 1$ of the elements of any of the input patterns are distorted by mixed noise.

In other words, it is enough that less than 50% of the elements of a pattern of the FS be distorted by mixed noise of any level so that the pattern is perfectly recalled. Let us verify this with an example:

Example 1. Let us suppose the following fundamental set of patterns in the transformed domain, obtained from a general FS as explained in section 2.3:

$$\mathbf{x}^{1} = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 3 \\ 1 \end{pmatrix}, \ \mathbf{y}^{1} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}; \ \mathbf{x}^{2} = \begin{pmatrix} 7 \\ 6 \\ 5 \\ 8 \\ 6 \end{pmatrix}, \ \mathbf{y}^{2} = \begin{pmatrix} 7 \\ 6 \\ 8 \end{pmatrix} \text{ and } \mathbf{x}^{3} = \begin{pmatrix} 12 \\ 11 \\ 10 \\ 13 \\ 11 \end{pmatrix}, \ \mathbf{y}^{3} = \begin{pmatrix} 12 \\ 11 \\ 13 \end{pmatrix}.$$

According to Corollary 1, one can easily verify that:

$$\mathbf{M} = \mathbf{y}^{1} \Diamond_{\mathrm{A}} (\mathbf{x}^{1})^{T} = \begin{pmatrix} 0 & 1 & 2 & -1 & 1 \\ -1 & 0 & 1 & -2 & 0 \\ 1 & 2 & 3 & 0 & 2 \end{pmatrix}.$$

In this case, as $n = 5$, then according to Theorem 3 it is enough that no more than $\frac{5+1}{2} - 1 = 2$ elements of any of the patterns keys is contaminated with mixed noise for perfect recall of its corresponding pattern. Let us verify this with an example. Let us suppose the following distorted version of key $\mathbf{x}^{2}$, where second and fifth components (underlined) have been highly modified:

$$\tilde{\mathbf{x}} = (7 \quad \underline{33} \quad 5 \quad 8 \quad \underline{12})^{T}.$$

When applying equations (6) and (7), we have that:

$$\mathbf{M} \Diamond_{B} \tilde{\mathbf{x}} = \begin{pmatrix} \mathbf{med}(7,34,7,7,13) \\ \mathbf{med}(6,33,6,6,12) \\ \mathbf{med}(8,35,8,8,14) \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ 8 \end{pmatrix}$$

As we can appreciate the recalled pattern exactly corresponds to the pattern: $\mathbf{y}^2$.

In case more than 50% of the elements of a key pattern are distorted by mixed noise, the recalled pattern in the transformed domain is an additive multiple of the corresponding pattern $\mathbf{y}^\xi$. Let us verify this with the following example:

Example 2. Let us suppose the following distorted version of key pattern $\mathbf{x}^2$ of example 1, where as reader can appreciate four components (underlined) appear modified:

$$\tilde{\mathbf{x}} = \begin{pmatrix} \underline{9} & 6 & \underline{7} & \underline{10} & \underline{8} \end{pmatrix}^T.$$

When applying equations (6) and (7), we have:

$$\mathbf{M}\Diamond_B \tilde{\mathbf{x}} = \begin{pmatrix} \mathbf{med}(9,7,9,9,9) \\ \mathbf{med}(8,6,8,8,8) \\ \mathbf{med}(10,8,10,10,10) \end{pmatrix} = \begin{pmatrix} 9 \\ 8 \\ 10 \end{pmatrix}.$$

Note how in this case the recalled version of $\mathbf{y}^2$, $\mathbf{y}^\xi_{recalled}$, in the transformed domain differs in 2 with respect two the original one.

The preceding fact can be formally expressed by the following:

Proposition 1. If a distorted key pattern $\tilde{\mathbf{x}}^\xi$ does satisfies the conditions imposed by Theorem 3, the recalled version, $\mathbf{y}^\xi_{recalled}$, is additive multiple of the corresponding pattern $\mathbf{y}^\xi$.

The following result provides sufficient conditions under which a fundamental pattern has perfect recall if more than 50% of its elements are distorted by mixed noise:

Theorem 4. Let $\{(\mathbf{x}^\xi, \mathbf{y}^\xi) \mid \xi = 1,2,\ldots,p\}$, $\mathbf{x}^\alpha \in \mathbf{R}^n$, $\mathbf{y}^\alpha \in \mathbf{R}^m$ a fundamental set $\mathbf{x}^{\xi+1} = \mathbf{x}^\xi + D$, $\mathbf{y}^{\xi+1} = \mathbf{y}^\xi + D$, $\xi = 1,2,\ldots,p$, $D = (d,\cdots,d)^T$, $d = Const.$ Without lost of generality suppose that is p odd. Thus the associative memory $\mathbf{M} = \mathbf{y}^\xi \Diamond_{\mathrm{A}} (\mathbf{x}^\xi)^T$ has perfect recall in the presence of noise if more than $\frac{n+1}{2} - 1$ of its components are distorted by noise with absolute magnitude less than $d/2$. The index of its corresponding pattern is given by: $i = \arg\min_l d(m_{1j} \Diamond_B x_j^\xi, y_1^l), \xi = 1,\cdots,p$.

Example 3. The absolute magnitude of the noise added to key pattern $\mathbf{x}^2$ in example 2 is less than $d/2=5/2=2.5$ unities, thus according to Theorem 4:

$$i=\arg\min_l d\left(m_{1j}\Diamond_B x_j^k, y_1^l\right)=\arg\min_l\left(|9-2|,|9-7|,|9-12|\right)=2\,.$$

Thus, the pattern associated to the distorted key is $\mathbf{y}^2$ as predicted by Theorem 4. A special case of Theorem 4 is given by the following:

Corollary 2. Let $\left\{\left(\mathbf{x}^\xi,\mathbf{y}^\xi\right)\mid \xi=1,2,\ldots,p\right\}$, $\mathbf{x}^\alpha\in\mathbf{R}^n$, $\mathbf{y}^\alpha\in\mathbf{R}^m$ a fundamental set $\mathbf{x}^{\xi+1}=\mathbf{x}^\xi+D$, $\mathbf{y}^{\xi+1}=\mathbf{y}^\xi+D$, $\xi=1,2,\ldots,p$, $D=\left(d,\cdots,d\right)^T$, $d=Const.$ Without lost of generality let us suppose that p is odd. Thus the associative memory $\mathbf{M}=\mathbf{y}^\xi\Diamond_{\mathrm{A}}\left(\mathbf{x}^\xi\right)^T$ has perfect recall in the presence of noise if all the components of any pattern are distorted but the absolute magnitude of the noise added to them is less than $d/2$. The index of its corresponding pattern is given by: $i=\arg\min_l d\left(m_{1j}\Diamond_B x_j^\xi, y_1^l\right), \xi=1,\cdots,p$.

Example 4. Let us suppose the following distorted version of key $\mathbf{x}^2$ of example 1 where now all elements have been modified:

$$\tilde{\mathbf{x}}=\begin{pmatrix}9 & 5 & 3 & 10 & 7\end{pmatrix}^T.$$

By applying equations (6) and (7), we have:

$$\mathbf{M}\Diamond_B\tilde{\mathbf{x}}=\begin{pmatrix}\mathbf{med}(9,6,5,9,8)\\ \mathbf{med}(8,5,4,8,7)\\ \mathbf{med}(10,7,6,10,9)\end{pmatrix}=\begin{pmatrix}8\\7\\9\end{pmatrix}.$$

The index of the corresponding pattern is obtained as:

$$i=\arg\min_l d\left(m_{1j}\Diamond_B x_j^k, y_1^l\right)=\arg\min_l\left(|8-2|,|8-7|,|8-12|\right)=2\,.$$

Thus, the pattern associated to the distorted key is $\mathbf{y}^2$ as expected.

4 Experiments with Real Patterns

In this section, it is shown the applicability of the results given in section 3. Experiments were performed on different sets of images. In this paper we show the results obtained with photos of five famous mathematicians. These are shown in

Figure 1. The images are 51×51 pixels and 256 gray-levels. To build the memory, each image $f_{51\times 51}(i,j)$ was first converted to a pattern vector $\mathbf{x}^{\xi}$ of dimension 2,601 (51×51) elements by means of the standard scan method, giving as a result the five patterns $\mathbf{x}^{\xi} = \begin{bmatrix} x_1^{\xi} & x_2^{\xi} & \cdots & x_{2601}^{\xi} \end{bmatrix}, \xi = 1,\ldots,5$. It is not difficult to see that this set of vectors does not satisfy the conditions established by Theorem 1 and its Corollary. It is thus transformed into an auxiliary FS by means of the transformation procedure described in section 2.3, giving as a result the transformed patterns: $\mathbf{z}^{\xi} = \begin{bmatrix} z_1^{\xi} & z_2^{\xi} & \cdots & z_{2601}^{\xi} \end{bmatrix}, \xi = 1,\ldots,5$. It is not difficult to see in the transformed domain, each transformed pattern vector is an additive translation of the preceding one.

(a) (b) (c) (d) (e)

Fig. 1. Images of the five famous people used in the experiments. (a) Descartes. (b) Einstein. (c) Euler. (d) Galileo, and (e) Newton. All Images are 51×51 pixels and 256 gray levels.

First pattern vector z^1 was used to build matrix M. Any other pattern could be used due to according to Corollary 1: $\mathbf{M}^1 = \mathbf{M}^2 = \ldots = \mathbf{M}^5 = \mathbf{M}$. To build matrix M, equations 1-3 were used.

4.1 Recalling of the Fundamental Set of Images

Patterns z^1 to z^5 were presented to matrix M for recall. Equations 6 and 7 were used for this purpose. In all cases, as expected, the whole FS of images was perfectly recalled.

4.2 Recalling of a Pattern from a Distorted Version of It

Three experiments were performed. In the first experiment the effectiveness of Theorem 3 was verified when less than 50% of the pixels of an image was distorted by mixed noise. In the second experiment the effectiveness of Theorem 4 was verified when all pixels of an image were distorted with noise but with absolute magnitude less than $d/2$. In the third experiment, the pixels of an image were distorted in such a way that they do not satisfy Theorems 3 to 4, no perfect recall should thus occur.

4.2.1 Effectiveness of Theorem 3

In this case the five images shown in Figure 1 were corrupted with mixed noise in such a way that less than half of its pixels changed in their values. For each photo several noisy versions with different levels of salt and pepper noisy were generated. Figure 2 shows 5 of these noisy images. Note the level of added distortion. When applying the recalling procedure described in Section 2.3, as specified by Theorem 3 in all cases as shown in Figure 2(b) the desired image was of course perfectly recalled.

(a)

(b)

Fig. 2. (a) Noisy images used to verify the effectiveness of Theorem 3 when less than 50% of the elements of patterns are distorted by noise. (b) Recalled images.

4.2.2 Effectiveness of Theorem 4

In this case all elements of the five images shown in Figure 1 were distorted with mixed noise but respecting the restriction that the absolute magnitude of the level of noise added to a pixel is inferior to $d/2$. For each image a noisy version was generated. The five noisy versions are shown in Figure 3(a). When applying the recalling procedure described in section 2.3, as expected in all cases the desired image was perfectly recalled. Figure 3(b) shows the recalled versions.

4.2.3 Results When Recalling Conditions Are Not Satisfied

One experiment was performed. In the case more than 50% of the elements of each pattern were distorted with saturating salt and pepper noise. In this case mainly salt noise was added to the images. One noisy version for each image was generated in each case. Figure 4(b) show the noisy versions for each image. Figure 4(c) show the corresponding recalled versions. As can be appreciated in some cases, the desired image is correctly recalled. In others it is associated to other image.

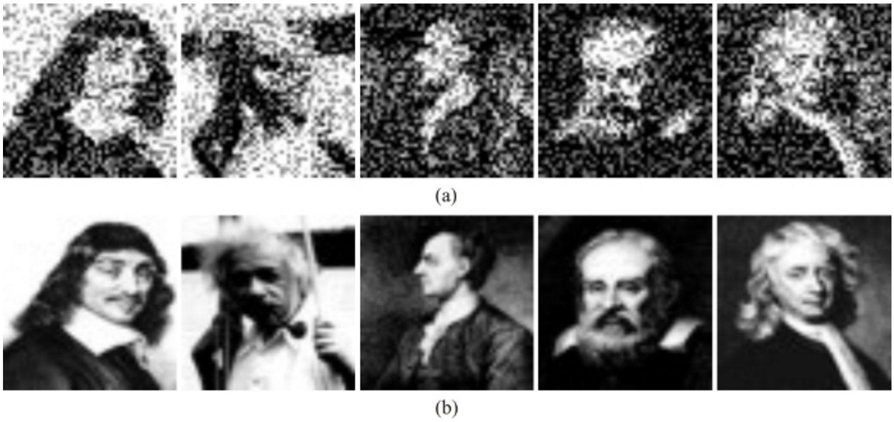

Fig. 3. (a) Noisy images used to verify the effectiveness of Corollary 2 when the absolute magnitude of the noise added to the pixels is less than $d/2$. (b) Recalled versions.

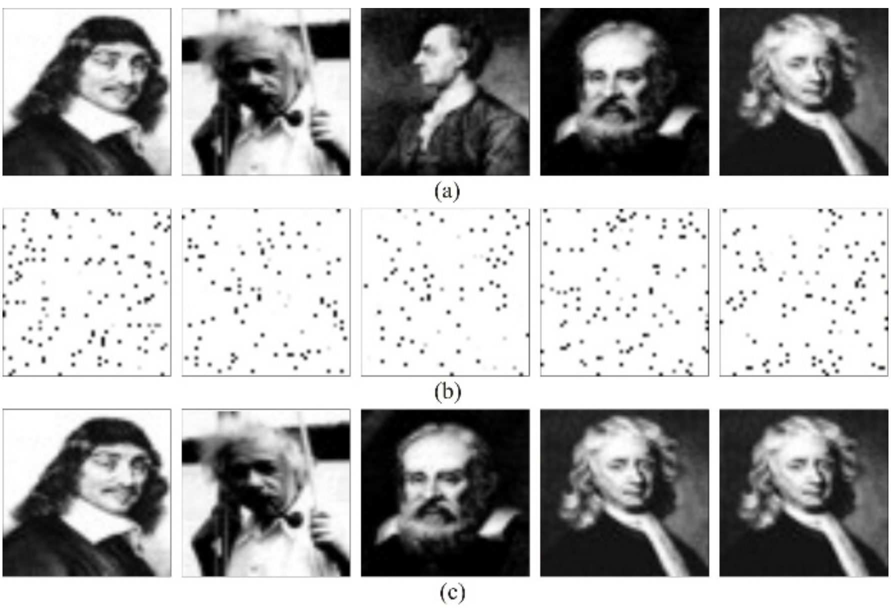

Fig. 4. (a) Original images. (b) Noisy images used to verify the effectiveness of the propositions when more than 50% of the elements of patterns are distorted by noise with absolute magnitude of noise added greater than $d/2$. In this case 99% of the elements of the images were altered. The ratio between salt noise and pepper noise added to the elements was 99 to 1. (c) Recalled images.

5 Conclusions and Present Research

In this paper we have presented some new results about median memories recently introduced in [1]. The new propositions provide new conditions under which the proposed memories can perfectly recalled a pattern of a given fundamental set in the presence of mixed noise.

Actually, we are looking for more general results for perfect recall. We are also investigating the performance of the proposed memories with other operators different form **min**, **max** and **median**. We are also searching for more efficient methods to speed up the learning and recalling procedures.

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