A New Performance Parameter for IEEE 802.11 DCF*

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Abstract. In this paper, we define a new performance parameter, named *PPT*, for 802.11 DCF, which binds successful transmission probability and saturation throughput together. An expression of optimal minimum contention windows (CW_{min}) is obtained analytically for maximizing PPT. For simplicity, we give a name DCF-PPT to the 802.11 DCF that sets its CW_{min} according this expression. The simulation results indicate that, compared to 802.11 DCF, DCF-PPT can significantly increase the PPT and successful transmission probability (about 0.95) in condition that the saturation throughput is not decreased.

1 Introduction

Much research has been conducted on the performance of IEEE802.11 DCF[1]. In [2] and [3], the author gave a Markov chain model for the backoff procedure of 802.11 DCF and studied its saturation throughout. Haitao Wu *et al.* [4] considered the maximum retransmit count and improved the model given in [3]. In [5], the authors evaluated the performance of 802.11 DCF in terms of the spatial reuse. Wang C. et al. [6] proposed a new efficient collision resolution mechanism to reduce the collision probability. In [7], an enhancement for DCF is proposed to augment the saturation throughput by adaptively adjusting the contention window.

Although saturation throughput is an important performance parameter for 802.11 DCF because enhancing saturation throughput can utilizes the channel more efficiently, increasing the successful transmission probability is also important for 802.11 DCF. In this paper, we define a novel performance parameter, named Product of successful transmission Probability and saturation Throughput (PPT), for 802.11 DCF. The analysis is given to maximize PPT.

The rest of this paper is organized as follows: In section 2, we define PPT, and analyze how to maximize PPT. In section 3, the performance of DCF-PPT is simulated with different stations on terms of saturation throughput, successful transmission probability and PPT. We conclude this paper in section 4.

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2 PPT: Defining and Maximizing

Before defining PPT, we give the same definition of saturation throughput as in [3] as follows:

Definition 1: The saturation throughput of 802.11 DCF, *S,* is the limit throughput reached by the system as the offered load increase, which represent the maximum throughput in system's stable condition.

Definition 2: The system's stable condition is the condition on which the transmission queue of each station is nonempty.

We define the successful transmission probability as follows:

Definition 3: The successful transmission probability *P* is the probability that a given transmission occurring on a slot is successful.

Based on Definition 1 and Definition 3, we define PPT as follows:

Definition 4: The PPT is the product of successful transmission probability and saturation throughput, that is

$$PPT = S \times P \tag{1}$$

The definition of PPT binds saturation throughput and successful transmission probability together. Maximizing PPT can increases the saturation throughput while keeping high successful transmission probability, which is illustrated in the following.

In [3], the author gave a two-dimensional Markov chain $\{b(t),s(t)\}$ to analyze the performance of 802.11 DCF, and obtained the saturation throughput *S* as follows:

$$S = \frac{P_s \cdot P_{tr} \cdot E[P]}{(1-P_{tr})\cdot \sigma + P_{tr} \cdot P_s \cdot T_s + P_{tr}(1-P_s)\cdot T_c} \tag{2}$$

where, *E[P]* is the average packet payload size, T_s is the average time the channel is sensed busy because of a successful transmission, T_c is the average time the channel is sensed busy during a collision, σ is the duration of an empty slot time, P_{tr} is the probability that there is at least one transmission in the considered slot time, P_s is the probability that a transmission occurring on the channel is successful, and

$$P_{tr} = 1-(1-\tau)^n \tag{3}$$

$$P_s = \frac{n\tau \cdot (1-\tau)^{n-1}}{1-(1-\tau)^n} \tag{4}$$

where, τ is the probability that a station transmits in a randomly chosen slot, which can be expressed as follows[3]:

$$\tau = \frac{2\cdot(1-2p)}{(1-2p)\cdot(w+1)+p\cdot w\cdot(1-(2p)^m)} \tag{5}$$

where, w is the contention windows, m is the maximum backoff stage, p is the probability that a transmitted packet encounters a collision, which is expressed as

$$p = 1 - (1 - \tau)^{n-1} \quad (6)$$

Note that in definition 3, P is the probability that a given transmission occurring on a slot is successful, and a given transmission occurring on a slot is successful if and only if the *n-1* remaining stations don't transmit in the same slot, so it is easy to obtain that

$$P = (1 - \tau)^{n-1} \quad (7)$$

Plugging expression (2) and (7) into (1), we obtain

$$PPT = \frac{P_s \cdot P_{tr} \cdot E[P]}{(1 - P_{tr}) \cdot \sigma + P_{tr} \cdot P_s \cdot T_s + P_{tr}(1 - P_s) \cdot T_c} \cdot (1 - \tau)^{n-1} \quad (8)$$

Given the expression of (3) and (4), (8) can be rewritten as:

$$PPT = \frac{n\tau \cdot (1-\tau)^{n-1} \cdot E[P]}{(1-\tau)^n \cdot \sigma + n\tau \cdot (1-\tau)^{n-1} \cdot T_s + \left[1 - (1 - \tau + n\tau) \cdot (1-\tau)^{n-1}\right] \cdot T_c} \cdot (1-\tau)^{n-1} \quad (9)$$

Expressions (2) and (7) denote that S and P are the function of τ, but the curves of S vs. τ and P vs. τ, which are shown in Fig. 1, are very different. Maximizing S does not means maximizing P simultaneously. However, maximizing *PPT* can obtain high S and P simultaneously because PPT is their product.

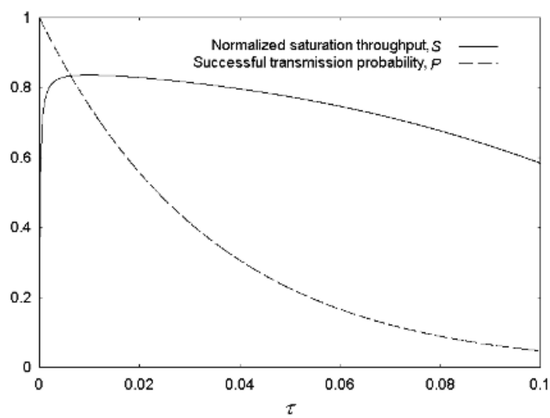

Fig. 1. S vs. τ, P vs. τ, $0 \le \tau \le 0.1$, n=30

Fig.2 indicates that *PPT* has a maximum value. We will deduce the optimal τ in the following.

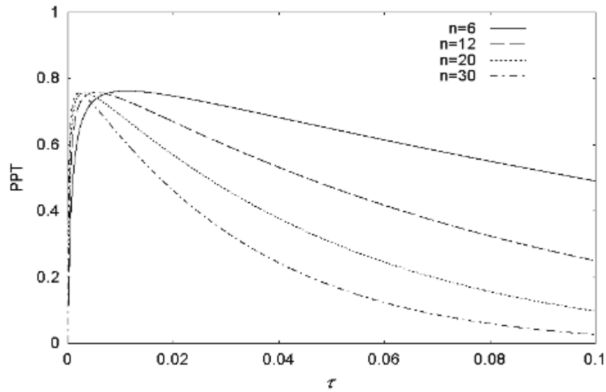

Fig. 2. *PPT* vs. τ, $0 \le \tau \le 0.1$

Taking the derivative of (1) with respect to τ, and imposing it equal to 0, we obtain the following equation:

$$\frac{d(PPT)}{d\tau} = \frac{d(S \cdot P)}{d\tau} = \frac{dS}{d\tau} \cdot P + \frac{dP}{d\tau} \cdot S = 0 \tag{10}$$

Note that

$$S = \frac{n\tau \cdot (1-\tau)^{n-1} \cdot E[P]}{(1-\tau)^n \cdot \sigma + n\tau \cdot (1-\tau)^{n-1} \cdot T_s + \left[1 - (1-\tau+n\tau) \cdot (1-\tau)^{n-1}\right] \cdot T_c} \tag{11}$$

Taking the derivative of S with respect to τ, and making some simplification, we obtain

$$\frac{dS}{d\tau} = \frac{\left[n \cdot (1-\tau)^{n-1} - n\tau(n-1) \cdot (1-\tau)^{n-2}\right] \cdot f(\tau) - n\tau \cdot n \cdot (1-\tau)^{n-1} \cdot f'(\tau)}{f^2(\tau)} \cdot E[P] \tag{12}$$

where,

$$f(\tau) = (1-\tau)^n \cdot \sigma + n\tau \cdot (1-\tau)^{n-1} \cdot T_s + \left[1 - (1-\tau+n\tau) \cdot (1-\tau)^{n-1}\right] \cdot T_c \tag{13}$$

$$f'(\tau) = -n(1-\tau)^{n-1}\sigma + (n - n^2\tau) \cdot (1-\tau)^{n-2} T_s + n\tau \cdot (n-1) \cdot (1-\tau)^{n-2} T_c \tag{14}$$

Taking the derivative of P with respect to τ, we obtain

$$\frac{dP}{d\tau} = -(n-1) \cdot (1-\tau)^{n-2} \tag{15}$$

Plugging expression (12) and (15) into (10), and making some simplification, we obtain

$$(1+\tau-2n\tau) \cdot f(\tau) - \tau \cdot (1-\tau) \cdot f'(\tau) = 0 \tag{16}$$

Moreover, plugging expression (13) and (14) into (16), and making some simplification, we obtain

$$\left(1+\tau-n\tau-n\tau^2\right)\sigma+\left(n\tau^2-n^2\cdot\tau^2\right)T_s + \left[\frac{1+\tau-2n\tau}{(1-\tau)^{n-1}}-\left(1-n\tau-\tau^2-n^2\tau^2+2n\tau^2\right)\right]T_c=0 \tag{17}$$

(17) is an equation in one variable of degree *n*. Noting that $\tau<<1$, we can obtain $(1-\tau)^{n-1}\approx 1$. Moreover, ignoring the τ^2 items, (17) was simplified to a linear equation as follows:

$$(n-1)\cdot T_c\cdot\tau+(n-1)\cdot\sigma\cdot\tau-\sigma=0 \tag{18}$$

The approximate optimal τ, denoted as τ_{opt}, can be obtain from (18) as follows:

$$\tau_{opt}=\frac{1}{(n-1)\cdot T_c^*+n-1} \tag{19}$$

where, $T_c^*=\frac{T_c}{\sigma}$, which is the duration of a collision measured in slot time unit σ.

Expression (5) and (6) show that for given *n*, τ depends on the system parameters *m* and *w*. In [1], the default value of *m* is 5. In [3], the author have point out that the saturation throughput don't change obviously after the value of *m* is beyond 5. So, we let *m* keep on its default value 5, and only consider how to adjust *w* to maximize *PPT*.

Plugging (19) into (6), we can obtain

$$p=1-\left(1-\frac{1}{(n-1)\cdot T_c^*+n-1}\right)^{n-1} \tag{20}$$

From (5), we can obtain

$$w=\frac{(1-2p)\cdot(2-\tau)}{\tau-p\cdot\tau-p\cdot\tau\cdot(2p)^m} \tag{21}$$

Plugging (19) and (20) into (21), the expression of optimal *w*, denoted as w_{opt}, can be written as

$$w_{opt}=\frac{\left(-1+2\cdot\left(1-\frac{1}{(n-1)\cdot T_c^*+n-1}\right)^{n-1}\right)\cdot\left(2-\frac{1}{(n-1)\cdot T_c^*+n-1}\right)}{\frac{1}{(n-1)(T_c^*+1)}\left(\left(1-\frac{1}{(n-1)(T_c^*+1)}\right)^{n-1}-2^m\left(1-\left(1-\frac{1}{(n-1)(T_c^*+1)}\right)^{n-1}\right)^{m+1}\right)} \tag{22}$$

Expression (22) shows that we can adjust the values *w* (and consequently τ) to maximize the *PPT*.

In order to simplify the computation of w_{opt}, we approximate the expression (22) in the following.

Let

$$x = \left(1 - \frac{1}{(n-1)\cdot T_c^* + n - 1}\right)^n = \left(1 - \frac{1}{n\cdot\left(\frac{n-1}{n}T_c^* + \frac{n-1}{n}\right)}\right)^n = \left(1 - \frac{1}{nk}\right)^n$$

where, $k = \frac{n-1}{n}T_c^* + \frac{n-1}{n}$.

Note that $nk >> 1$, x can be approximate as $x = e^{-\frac{1}{x}}$. As $p = x / \left(1 - \left(\frac{1}{nk}\right)\right)$, p can be approximated as

$$p = 1 - \frac{1}{e^{1/k}\cdot\left(1 - \frac{1}{nk}\right)} \tag{23}$$

Plugging (19) and (23) into (21), we can obtain the approximated expression of w_{opt} as follows:

$$w_{opt} = \frac{\left(-1 + \frac{2}{e^{1/k}\cdot\left(1-\frac{1}{nk}\right)}\right)\cdot\left(2 - \frac{1}{(n-1)\cdot T_c^* + n - 1}\right)}{\frac{1}{(n-1)(T_c^*+1)}\left(\frac{1}{e^{1/k}\cdot\left(1-\frac{1}{nk}\right)} - 2^m\left(1 - \frac{1}{e^{1/k}\cdot\left(1-\frac{1}{nk}\right)}\right)^{m+1}\right)} \tag{24}$$

where,

$$k = \frac{n-1}{n}T_c^* + \frac{n-1}{n} \tag{25}$$

Moreover, if $n >> 1$, then $\frac{n-1}{n} \approx 1$, $k = T_c^* + 1$, w_{opt} can be further approximated as

$$w_{opt} = \frac{\left(-1 + \frac{2}{e^{1/k}\cdot\left(1-\frac{1}{nk}\right)}\right)\cdot\left(2 - \frac{1}{(n-1)\cdot T_c^* + n - 1}\right)}{\frac{1}{(n-1)(T_c^*+1)}\cdot\left(\frac{1}{e^{1/k}\cdot\left(1-\frac{1}{nk}\right)} - 2^m.\left(1 - \frac{1}{e^{1/k}\cdot\left(1-\frac{1}{nk}\right)}\right)^{m+1}\right)} \tag{26}$$

where, $k = T_c^* + 1$.

The computation of expression (24) and (26) is less complex than expression (22) after approximating the expression *p*. If the network size is small $(n \le 10)$, we make use of (24) to calculate w_{opt}, and if the network size is huge $(n > 10)$, we make use of (26) to calculate w_{opt}. For simplicity, in section 3, we only make use of expression (24) to calculate w_{opt}.

3 Simulation

In this section, we firstly simulate the PPT, which is maximized by adjusting w_{opt} according to expression (24), and compare the simulated result to the numerically calculated maximum PTT. Then, we compare DCF-PPT to 802.11 DCF in terms of PPT, successful transmission probability and saturation throughput. The simulation platform is NS-2 [8]. The physical layer is DSSS. The stations transmit packets by means of RTS/CTS mechanism, and the simulation parameters are shown in table 1.

Table 1. Simulation Parameters

Channel Bit Rate	2Mbit/s
Slot Time	20μs
SIFS	10μs
DIFS	50μs
PHYHeader	192bits
MACHeader	144bits
RTS Length	160bits
CTS Length	112bits
CW_{min}	32
CW_{max}	1024
CBR Packet Size	1024Bytes

To calculate w_{opt} from expression (24), we must obtain the T_c. T_c is the average time the channel is sensed busy during a collision. In [3], when 802.11 DCF transmit packet by means of RTS/CTS mechanism, the author gave the expression of T_c as $T_c = RTS + DIFS + \sigma$ .In this paper, we revised expression of T_c as

$$T_c = RTS + EIFS + \sigma \qquad (27)$$

where, $EIFS = SIFS + ACK + DIFS$.

Making use of the expression (24), (25) and (27), we calculated the w_{opt} as in Table 2, according to different number of stations.

Table 2. Calculated w_{opt} for different number of stations

Number of stations	w_{opt}
6	147
12	386
20	696
30	1083

3.1 Comparing Simulated PPT to Numerical PPT

The maximum numerical PPT curve and the simulated PPT curve are drawn in Fig. 3, in different number of stations. In the simulation, we select the minimum contention windows (CW_{min}) according to table 2.

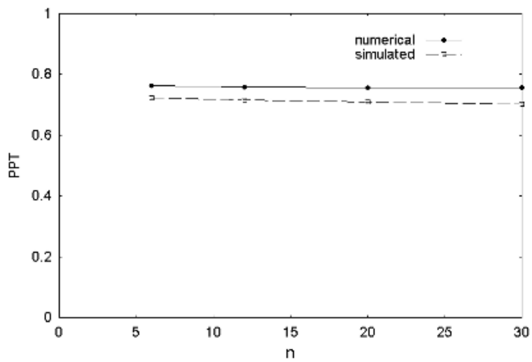

Fig. 3. Maximum PPT vs. n

Fig.3 shows that the simulated PPT is smaller than numerical PPT because we make some approximation to obtain the expression of w_{opt} in section 2, as make the calculated w_{opt} departure from the ideal value a bit . But the difference between the simulated PPT and the numerical PPT is less than 8%.

3.2 Comparing DCF-PPT to 802.11 DCF

According to different number of stations, the saturation throughput, successful transmission probability and PPT of DCF-PPT and 802.11 DCF are drawn in Fig. 4, Fig. 5 and Fig. 6, respectively.

Fig. 4 shows that the saturation throughput of DCF-PPT for all selected number of stations, except 12, is higher than 802.11 DCF. This is due to that we adaptively adjust CW_{min} according to the number of stations.

Fig. 5 shows that the successful transmission probability of DCF-PPT (about 0.95) is much higher than 802.11 DCF, and it does not decrease obviously with the number of stations increasing, as is also attributed to that we adjust CW_{min} adaptively according to the number of stations.

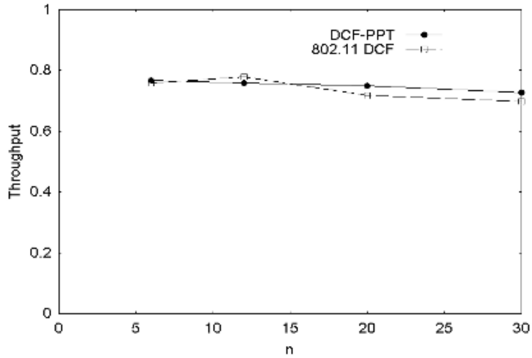

Fig. 4. The normalized throughput of DCF-PPT and 802.11 DCF

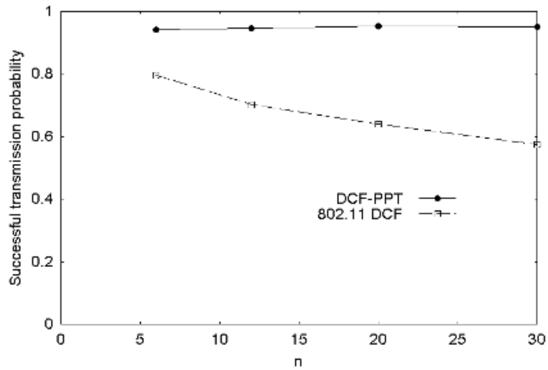

Fig. 5. The successful transmission probability of DCF-PPT and 802.11 DCF

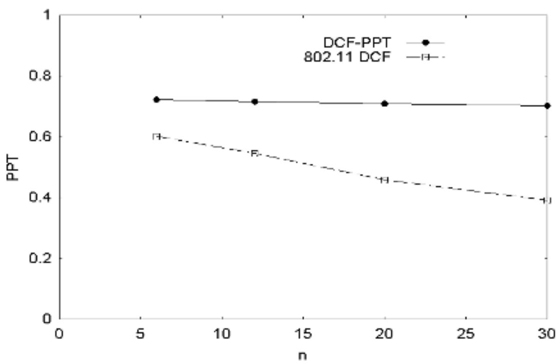

Fig. 6. The successful transmission probability of DCF-PPT and 802.11 DCF

Fig. 6 shows that the PPT of DCF-PPT is much higher than 802.11 DCF, and it does not decrease obviously when the number of stations increases.

4 Conclusion

In this paper, we define a novel performance parameter for 802.11 DCF, which binds successful transmission probability and saturation throughput together. The analysis is given to maximize PPT.

The performance of DCF-PPT is simulated with different stations on terms of saturation throughput, successful transmission probability and PPT. The simulation results indicate that DCF-PPT can largely increase the PPT and successful transmission probability in the condition that the saturation throughput is not decreased, comparing to 802.11 DCF.

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