A Novel Incremental Technique for Ultrasound to CT Bone Surface Registration Using Unscented Kalman Filtering

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Abstract. We propose a novel incremental surface-based registration technique that employs the Unscented Kalman Filter (UKF) to register two different data sets. The method not only reports the variance of the registration parameters but also has significantly more accurate results in comparison to the Iterative Closest Points (ICP) algorithm. Furthermore, it is shown that the proposed incremental registration algorithm is less sensitive to the initial alignment of the data sets than the ICP algorithm. We have validated the method by registering bone surfaces extracted from a set of 3D ultrasound images to the corresponding surface points gathered from the Computed Tomography (CT) data.

1 Introduction

Registration is a crucial step in applications of medical imaging in computer-assisted surgery. Two general methods for registration are intensity-based and feature-based approaches [1]. The former uses the intensity of two images or volumes of the targeted anatomy to calculate the registration parameters by using a variety of similarity measures, such as the mutual-information or normalized correlation. The latter, extracts corresponding geometric features in order to perform the registration. In the feature-based registration technique, if corresponding points between two data sets are available, then one could easily find the registration parameters by employing the closed-form solution provided by Horn [2]. However, the problem becomes more challenging when the corresponding points between the two cloud of points are unknown. In this case the most widely used registration method is the Iterative Closest Points (ICP) algorithm [3]. But the ICP algorithm is very sensitive to the initial alignment of data sets and easily gets trapped in a local minimum. Recently, Ma [4] has proposed a novel approach for estimating the registration parameters and visualizing the error distribution by using the Unscented Particle Filter (UPF). While excellent registration results are reported for a small size of data set (24 points), the algorithm converges very slowly due to the employment of 5000 particles. The need to employ a large number of particles makes this algorithm practically impossible to run for large data sets. The Particle Filtering is a powerful method when

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one is dealing with a nonlinear process or model corrupted by a non-Gaussian random noise. In the case of the Gaussian distribution assumption for the noise distribution, one could significantly reduce the computation time by using the Unscented Kalman Filtering (UKF), while achieving similar registration performance. Due to the significantly small computational requirements of the UKF algorithm in comparison to the UPF algorithm, it is possible to apply the UKF technique to large data sets.

In this paper, we propose a novel incremental registration algorithm based on the Unscented Kalman Filtering. It is shown that the proposed registration algorithm is less sensitive to the initial alignment and is more accurate than the ICP method. Finally, the robustness and accuracy of the proposed technique is examined by registering a 3D ultrasound data set of a Scaphoid bone surface to the corresponding 3D Computed Tomography (CT) data.

2 Method

2.1 Unscented Kalman Filter Registration

In 1960, R. E. Kalman published his famous paper describing a recursive and incremental solution to the discrete data linear filtering problem [5]. Since that time the Kalman Filtering (KF) has been the subject of research with several applications, specifically in the navigation and target tracking area [6]. The KF addresses the state vector estimation, $\mathbf{x} \in \Re^n$, of a discrete time control process model governed by a linear equation:

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{w}_{k-1}, \tag{1}$$

from the observation model which is a linear function as well:

$$\mathbf{z}_k = \mathbf{C}\mathbf{x}_k + \mathbf{v}_k, \tag{2}$$

where $\mathbf{A}$ and $\mathbf{C}$ are defined by the system dynamics, $\mathbf{z}_k \in \Re^m$ is the observation vector at time k, $\mathbf{w}_k$ and $\mathbf{v}_k$ represent the process and the observation noise at time k, respectively, and are independent Gaussian random vectors with distributions $\mathcal{N}(0, \mathcal{W})$ and $\mathcal{N}(0, \mathcal{V})$ respectively. The KF algorithm estimates the state vector by minimizing the mean square error. This estimation is optimum if the process and the observation models are defined by linear equations, and the process and the observation noises are independent Gaussian random variables. However, in a general case, the process and the observation models can be governed by non-linear equations:

$$\mathbf{x}_k = \mathbf{F}(\mathbf{x}_{k-1}, \mathbf{w}_{k-1}), \tag{3}$$

$$\mathbf{z}_k = \mathbf{H}(\mathbf{x}_k, \mathbf{v}_k). \tag{4}$$

In this case the KF estimation of the state vector is not optimum anymore. There are two well known solutions for dealing with nonlinearities in the process or the observation model. As a first solution, the non-linear function in the observation or process model can be linearized around a good initial guess, using the first order Taylor approximation, and then the KF algorithm is used to estimate the state vector. This method is called Extended Kalman Filtering (EKF) [6]. However, to use the EKF, one needs to have the first derivative of the nonlinear functions (Jacobian Matrix) for linearization. Finding the Jacobian matrix is usually cumbersome and makes the algorithm complicated.

As a second approach, one could use the true non-linear models and approximate the distribution of the state random variable rather than approximating the non-linear process or observation model. This method, called the Unscented Kalman Filtering (UKF) [7], uses the true non-linear models to approximate the distribution of the state or observation vector with the Gaussian distribution by using a set of deterministically chosen sample points. These sample points completely capture the true mean and covariance of the Gaussian random variables, and when propagated through the true non-linear system, accurately capture the posterior mean and covariance to the third-order of Taylor series expansion for any nonlinearity. The EKF algorithm, in contrast, only achieves first-order of Taylor series expansion accuracy, with the same order of complexity as that of the UKF algorithm [8]. Once distributions of the state and the observation vectors are estimated, one could easily estimate the state vector by using the KF technique.

The problem of estimating the state vector becomes more challenging by assuming that the observation and the process noise have non-Gaussian distributions. In this situation, where one is dealing with the system governed by nonlinearity functions and distorted by a non-Gaussian noise, the Particle Filter algorithm is used to estimate the state vector [9].

2.2 Incremental Registration Algorithm Based on UKF

Since we would like to estimate rigid transformation parameters which register a cloud of data set (register data set) to the desired data set (model data set), the state vector contains three rotational (θ_x, θ_y, θ_z) and three translational (t_x, t_y, t_z) parameters. We assume that the scale factors between two data sets are known. Therefore, the state space model can be defined as follows:

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \mathcal{N}(0, \mathcal{W}_k), \tag{5}$$

where, $\mathbf{x}_k = [t_x, t_y, t_z, \theta_x, \theta_y, \theta_z]^T \in \Re^6$ and $\mathcal{W}_k$ is the covariance matrix of the zero-mean Gaussian random vector. This covariance matrix allows the algorithm to move from a poor initial estimate to the successively better ones. However, the process noise variances should be small values, since the state vector (transformation parameters) is time invariant.

The observation model is defined as follows:

$$\mathbf{y}_{1:k} = \mathbf{R}_{[\theta_x,\theta_y,\theta_z]}(\mathbf{t}_{[t_x,t_y,t_z]} + \mathbf{u}_{1:k}) + \mathcal{N}(0, \mathcal{V}_k), \tag{6}$$

where $\mathbf{R}$ is an Euler rotation matrix about $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ axes, respectively, and $\mathbf{t}$ is a translation matrix along $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ coordinates. Furthermore, $\mathbf{u}_k$ is the

k_{th} registering point in the register data set and $\mathbf{y}_k$ is its corresponding point in the model data set; However, these correspondence points are unknown and we have used the well-known nearest-neighbor approximation method proposed by Besl and Mckay [3] for finding the correspondences. Finally, it is assumed that the observations are stimulated by a zero-mean Gaussian random vector with the covariance matrix of $\mathcal{V}_k$. This assumption can be verified by the following argument: Since the data sets' points are not accurately extracted because of the calibration and segmentation errors, it is logical to assume that they are degraded by a zero-mean uniformly distributed and independent random vector noise with a specific variance in each dimension. In the sequel, for simplicity, it is assumed that the variance of the noise is the same in each dimension; however in the reality they might be different. Also, different modality data sets have different uniformly distributed random noise vector characteristics. Hence, by assuming that the registration parameters (rotation matrix and translation vector) between two data sets are known, one can write:

$$\mathbf{y} + \mathbf{n}_y = \mathbf{R}(\mathbf{t} + \mathbf{u} + \mathbf{n}_u), \tag{7}$$

where $\mathbf{n}_y = [n_{1y}, n_{2y}, n_{3y}]^T$ and $\mathbf{n}_u = [n_{1u}, n_{2u}, n_{3u}]^T$ are zero-mean independent uniformly distributed random vectors in the two data sets with covariance matrices of $\sigma_y^2\mathbf{I}$ and $\sigma_u^2\mathbf{I}$ ($\mathbf{I}_{3\times3}$ is the identity matrix), respectively. $\mathbf{y} + \mathbf{n}_y$ is the selected point in the model data set and $\mathbf{u} + \mathbf{n}_u$ is its correspondence in the register data set. Equation (7) can be simplified as:

$$\mathbf{y} = \mathbf{R}(\mathbf{t} + \mathbf{u}) + \mathbf{R}\mathbf{n}_u - \mathbf{n}_y = \mathbf{R}(\mathbf{t} + \mathbf{u}) + \mathbf{n}, \tag{8}$$

where $\mathbf{n} = \mathbf{R}\mathbf{n}_u - \mathbf{n}_y$, is a random vector with three components. The first element of $\mathbf{n}$ can be written as:

$$n_1 = r_{11}n_{1u} + r_{12}n_{2u} + r_{13}n_{3u} + n_{1y}, \tag{9}$$

where r_{11}, r_{12} and r_{13} are the first row components of the rotation matrix $\mathbf{R}$. Using central limit theorem, it is easy to show that, with a good approximation, n_1 has a Gaussian distribution with the mean of zero and the variance of:

$$\sigma_{n_1}^2 = r_{11}^2\sigma_u^2 + r_{12}^2\sigma_u^2 + r_{13}^3\sigma_u^2 + \sigma_y^2 = \sigma_u^2 + \sigma_y^2. \tag{10}$$

In the same way, it can be verified that the two other components of $\mathbf{n}$ have the variance of $\sigma_u^2 + \sigma_y^2$ as well. Considering that the components of $\mathbf{n}$ are statistically independent, by a good approximation, $\mathbf{n}$ can be considered as a zero-mean independent Gaussian random vector with the covariance matrix of $(\sigma_u^2 + \sigma_y^2)\mathbf{I}$. Since the defined observation model is governed by a non-linear function distorted by a Gaussian observation noise, we have employed the UKF algorithm for estimating the registration parameters as follows:

In the first iteration, the state vector is initialized to zero, and only one random point is selected from the register data set. Next, the state vector is used to transfer the selected point to the model data set. Then the closest point in the model data set to the transferred selected point is transferred back to the register data set using the inverse of the transformation matrix represented by the state vector. The distance between this point and the original selected point is used to update the state vector and its covariance by using the UKF algorithm. The procedure is iteratively repeated by incrementally adding more points from the register data set to the algorithm in the next iterations.

3 Results

Two sets of experiments are performed to validate the accuracy of the proposed registration method. In addition, the registration results of the proposed method are compared to the well-known ICP algorithm. In the first experiment, a set of known random transformations are applied to a cloud of points and the proposed algorithm is employed to register the original point cloud to the transformed one. This experiment shows the performance of the proposed algorithm where there is no ultrasound calibration or segmentation error involved. In the second experiment, the point cloud representing the bone surface extracted from 3D ultrasound data is registered to its corresponding surface points extracted from the mesh data generated by segmenting CT images. This experiment shows the degradation of the registration parameters caused by segmentation and calibration errors.

3.1 Two Point Clouds Registration

By using the ultrasound calibration and segmentation methods proposed in [10], the 2D ultrasound images of a phantom Scaphoid bone surface are transferred to the 3D real-world coordinates. Then a set of random transformations are applied to the 3D ultrasound point cloud, and for each transformation, registration parameters are estimated between the original point cloud and the transferred one. For constructing the 3D ultrasound data set, 280 points of the bone surface

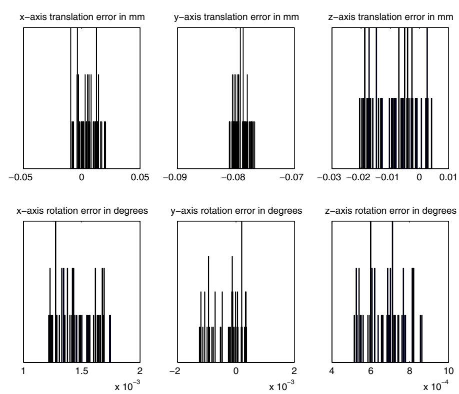


Fig. 1. Error distributions for 50 UKF registrations; standard deviations are $2.6° \times 10^{-8}$, $2.7° \times 10^{-7}$ and $9.5° \times 10^{-9}$ for **x**, **y** and **z** axis rotation errors, and 6.8×10^{-5}mm, 1.4×10^{-6}mm and 5.3×10^{-5}mm for the **x**, **y** and **z** axis translation errors, respectively.

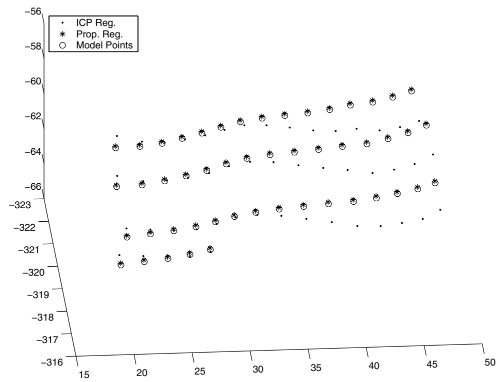


Fig. 2. Registration of model and registered data sets using ICP and our proposed method with the same initial conditions (units are in millimeters)

from 14 ultrasound images are selected. By drawing from the uniform distribution $\mathcal{U}(\pm 10mm, \pm 10mm, \pm 10mm, \pm 10°, \pm 10°, \pm 10°)$, 50 random transformations are generated and each transformation is applied to the 3D ultrasound point cloud to construct the model data set. Next, the proposed registration method is used to register the original data set (register data set) to the model data set. The distribution of the rotation and translation errors are shown in Figure 1. As expected, the variance and the mean of errors are very small (almost zero), since there is no calibration or segmentation errors in the data sets. For the same set of data and initial conditions, the registration results for the ICP algorithm and our proposed registration method are compared as well. The same uniform distribution mentioned above is used to generate the initial conditions. Figure 2 shows the registration result for the ICP algorithm and the proposed registration method for one of the simulations. It is seen that the proposed method's performance is significantly higher than that of the ICP algorithm. On average, the maximum distance error using the ICP registration algorithm is 1.5mm, whereas this error reduces to 0.8mm using the proposed registration method.

3.2 CT to Ultrasound Registration

In this experiment the constructed 3D ultrasound cloud is registered to the bone mesh surface, extracted from the CT data. Here, the two data sets have different number of points and are extracted from different imaging modalities, therefore containing different inherent calibration and segmentation errors. At first the 3D ultrasound data is manually aligned to the CT mesh using the fiducial points mounted on the Scaphoid bone. We were able to align two data sets manually within the range of 2mm fiducial registration error. Then a set of random transformations are applied to the 3D ultrasound point cloud, and for each transformation, registration parameters are estimated between the CT mesh data and the transferred 3D ultrasound points. The 3D ultrasound data set is constructed by selecting 450 points from the bone surface within 30 ultrasound images. Obviously, due to the uncertainty caused by the thickness of the bone response in each ultrasound image, these points are just approximations to the location of the bone within these images. As before, 45 random transformations are produced by drawing from the uniform distribution $\mathcal{U}(\pm 10mm, \pm 10mm, \pm 10mm, \pm 10°, \pm 10°, \pm 10°)$ and each transformation is applied to the randomly selected 3D ultrasound points. Then the proposed registration method is employed to register the transformed 3D ultrasound point cloud to the CT mesh data set. The distribution of the rotation and translation error of the estimated transformation parameters are shown in Figure 3. For the similar data sets and the same initial conditions, the registration result for the ICP algorithm and the proposed registration method is compared in Figure 4. As shown, the proposed method registers the two data sets much

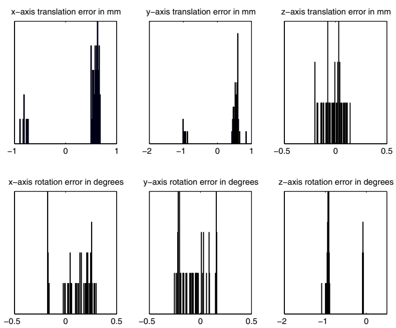
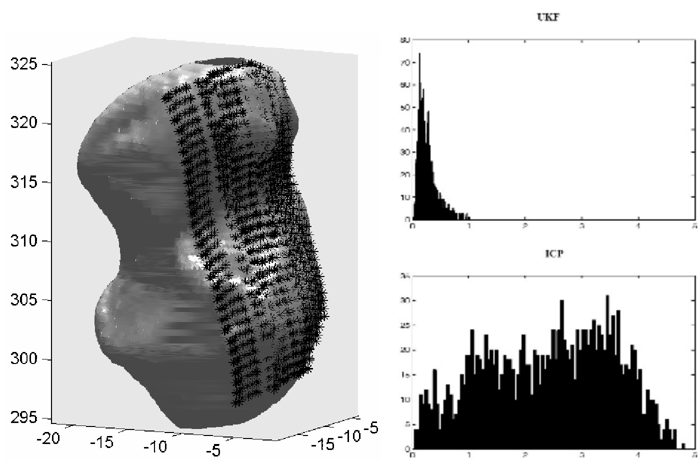


Fig. 3. Error distributions for 45 UKF registrations; standard deviations are 0.022°, 0.018° and 0.1° for **x**, **y** and **z** axes rotation errors, and 0.26mm, 0.32mm and 0.008mm for the **x**, **y** and **z** axes translation errors, respectively.



(a) (b)

Fig. 4. Comparison of the proposed method to ICP: a) The black star points represent the bone surface extracted from the ultrasound images, overlaid on the 3D surface mesh extracted from CT using UKF; b) Surface registration error (units are in mm).

more accurately than the ICP algorithm which is trapped in a local minimum. Figure 4(b) shows the distance error histogram between the 3D ultrasound points cloud and the corresponding CT data set after registration. The maximum and the root mean square distance errors using the ICP algorithm are 6mm and 3.7mm, respectively, whereas these errors reduce to 1.4mm and 0.31mm using the UKF method.

4 Discussion and Conclusions

A novel incremental surface-based registration technique based on the UKF is proposed. It is shown that the proposed method accurately registers the bone surface points extracted from the 3D ultrasound images to the ones from CT images. This method offers notable advantages to the other approaches such as the ICP and UPF algorithms. The proposed registration method not only is less sensitive to the initial guess or alignment, but also is significantly more accurate than the ICP algorithm as shown in Section 2. Moreover, against the UPF algorithm, the proposed registration method has less limitation on the size of the registration points due to the significantly less computational complexity. On the average, the complexity of the proposed method is $\mathcal{O}(N^2)$, N is the number of points in the register data set, while the complexity of the ICP algorithm and the UPF registration method with P particles, in the best case, are $\mathcal{O}(N \log N)$ and $\mathcal{O}(PN^2)$, respectively. In the future, further simulations and experiments will be performed to validate the UKF algorithm and its capture range under different measurement noise conditions.

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