

# Study on the Game of Payment Distribution Mode about AVEs

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**Abstract:** In this paper, the fixed-payment distribution mode of Agile Virtual Enterprises (AVEs) is studied based on game theory, and the result is compared with that of the share income mode. Finally, the paper proposes that the goal function of the game is changed thus promotes the Nash equilibrium effort level approaches or equals to the Pareto equilibrium effort level.

## 1 Problem Description

Fixed-payment distribution mode refers to the fixed payment by a member (usually the alliance leader) to other members based on the tasks shouldered by other members and according to the distribution amount previously agreed on (the payment can be done lump-sum or stage-by-stage) whereas the alliance leader obtain the total surplus and shoulder the total risk.

Let's assume that the agile virtual enterprise consists of two members, A and B, who are in line with the rational hypothesis and that the two members have different attitudes towards risk. Member A serves as the alliance leader and favors risk, who is willing to shoulder all the risk of the agile virtual enterprise and who thus obtain the total surplus. Member B, on the other hand, loathes risk and tries to avoid risk, who therefore is willing to obtain the fixed payment. Member A pays member B the fixed payment  $T$  (This can be the lump-sum payment or stage-by-stage payment). Further more, let's assume that  $a$  and  $b$  stand for the level of devotedness of A and B respectively, and that  $\alpha$  and  $\beta$  stand for the contribution co-efficiency of A and B whose quantity depends on the speciality and relative importance of the core ability of this enterprise.  $R(\alpha a, \beta b)$  stand for the total income of the agile virtual enterprise,  $A(\gamma a)$  and  $B(\delta b)$  stands for the cost of A and that of B respectively (here  $\gamma$  and  $\delta$  refer to the cost co-efficiency of A and that of B).  $P$  and  $Q$  stand for the net income of A and B respectively. Here,  $a$ ,  $b$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  are all bigger than 0.

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A further hypothesis includes  $R(\alpha a, \beta b) = 0.5(\alpha a + \beta b)^2 + (\alpha a + \beta b)$ ,  $A(\gamma a) = 0.5(\gamma a)^2$ ,  $B(\delta b) = 0.5(\delta b)^2$ , here  $\alpha < \gamma$ ,  $\beta < \delta$  so as to guarantee the convergence of the met return  $G$ , that is  $\frac{\partial^2 G}{\partial \alpha} < 0$ ,  $\frac{\partial^2 G}{\partial b} < 0$ .

Therefore, under the fixed payment distribution mode, the met return of the agile virtual enterprise and that of its members are respectively:

$$G = R(\alpha a, \beta b) - A(\gamma a) - B(\delta b), \quad P = R(\alpha a, \beta b) - T - A(\gamma a),$$

$$Q = T - B(\delta b)$$

Substitute the return function and the cost function with the above values, we have:

$$G = 0.5(\alpha a + \beta b)^2 + (\alpha a + \beta b) - 0.5(\gamma a)^2 - 0.5(\delta b)^2 \quad (1)$$

$$P = 0.5(\alpha a + \beta b)^2 + (\alpha a + \beta b) - T - 0.5(\gamma a)^2 \quad (2)$$

$$Q = T - 0.5(\delta b)^2 \quad (3)$$

Then, the questions are: what is the value domain of the fixed payment  $T$  and what is the devotedness of each member?

## 2 Determination of the Fixed Payment Value Domain

To guarantee the win-win of the members, the following conditions must be met:  $P > 0$ ,  $Q > 0$

Place the above two conditions in equations (2) and (3), we can obtain a value domain of the fixed payment  $T$ :

$$B(\delta b) < T < R(\alpha a, \beta b) - A(\gamma a)$$

Based on the above equation, we can draw the following conclusion:

**Conclusion 1:** Under the fixed payment mode, to guarantee the "win-win", the fixed payment to member B must be bigger than his cost while smaller than the difference between the total return of the agile virtual enterprise and the cost of member A (the alliance leader).

## 3 The Devotedness Level under the Fixed Payment Mode

First of all, let's analyse the Nash equilibrium of the agile virtual enterprise. At present, each member maximizes his return, thus:

$$\frac{\partial P}{\partial \alpha} = 0, \quad \frac{\partial Q}{\partial b} = 0$$

Place the above conditions in equation (2) and (3) and the Nash equilibrium devotedness level under the fixed payment mode  $a_g^0$  and  $b_g^0$  are

$$a_g^0 = \frac{\alpha}{\gamma^2 - \alpha^2} \tag{4}$$

$$b_g^0 = 0 \tag{5}$$

Therefore, we can draw the following conclusion:

**Conclusion 2:** Under the fixed payment mode, the Nash equilibrium devotedness level of the member B (who obtains the fixed payment) is zero.

This conclusion indicates that under the condition where the fixed payment can be guaranteed, the member who obtains the fixed payment does not have the incentive to devotion during his working process, which can definitely influence his completion of the sub-task shouldered. Furthermore, the devotedness level is difficult to observe and measure. In order to solve this problem, the alliance leader of the agile virtual enterprise can set a precise standard concerning the work amount, quality and completion time for this member and can use this as the condition for fixed payment. That is to say, control and stimulus should be based upon the completion of work rather than the completion process. However, with regard to knowledge alliance, due to the facts that the resources input (experimental instrument, device and so on) and the imitativeness of knowledge-initiative work are not clear and that the responsibilities are not clear and that the results of each member are difficult to measure, the fixed payment distribution mode is suitable not for the knowledge-alliance agile virtual enterprise but for the product-alliance agile virtual enterprise.

If we compare the Nash equilibrium level under the fixed payment mode (see equation 4 and 5) with the Nash equilibrium level under the output-share mode(see equation 6 and 7 of literature [1]), we can draw the following conclusion:

**Conclusion 3:** without the restriction of the outside factors, the fixed payment mode stimulates the alliance leader more than the members whereas the output-share mode stimulates the members more than the alliance leader.

If the  $a_g^0$  and  $b_g^0$  of equation (4) and (5) are placed in equation (1), the Nash equilibrium total return of the agile virtual enterprise under the fixed payment mode is:

$$G_g^0 = \frac{\alpha^2}{2(\gamma^2 - \alpha^2)} \tag{6}$$

In the following part, we shall set the maximization of the total benefit of the agile virtual enterprise as the target and analyses the best devotedness level and the total return of the members of the agile virtual enterprise under the Pareto equilibrium. (Here, we hypothesize that every member meets the condition of collective rationality).

For this, we need use equation (1) to obtain the partial derivatives for a and b set them zero, thus:

$$\frac{\partial G}{\partial a} = \alpha(\alpha a + \beta b) + \alpha - \gamma^2 a = 0$$

$$\frac{\partial G}{\partial b} = \beta(\alpha a + \beta b) + \beta - \delta^2 b = 0$$

When we solve the above two equations, we obtain the best devotedness level of member A and B under the fixed payment mode:

$$a_{gp}^0 = \frac{\alpha \delta^2}{\delta^2 \gamma^2 - \alpha^2 \delta^2 - \beta^2 \gamma^2} \quad (7)$$

$$b_{gp}^0 = \frac{\beta \gamma^2}{\delta^2 \gamma^2 - \alpha^2 \delta^2 - \beta^2 \gamma^2} \quad (8)$$

$$G_{gp}^0 = \frac{\alpha^2 \delta^2 + \beta^2 \gamma^2}{2(\delta^2 \gamma^2 - \alpha^2 \delta^2 - \beta^2 \gamma^2)} \quad (9)$$

If we compare (7), (8), and (9) with (4), (5), and (6), we can see that under the fixed payment mode, the Pareto equilibrium devotedness level of each member is bigger than the Nash equilibrium devotedness level (the same as the output-share payment mode). Therefore, under the fixed payment distribution mode, the mechanism design that drives the Nash equilibrium devotedness level to the Pareto equilibrium devotedness level still exists.

If we compare the equations (7), (8), and (9) with the equations (2), (3), and (4), we can draw the conclusion:

**Conclusion 4:** Under the condition of Pareto maximization, the total net return of the agile virtual enterprise and the devotedness level of every member are not related to the distribution mode.

This conclusion shows that the total target of the agile virtual enterprise can always be met despite the interest distribution mode if the thought and action of each member is activated to transform from the individual rationality to collective rationality and if each member regards the total target and benefit as the guidance. This further shows the importance of the cooperation mechanism design concerning the members of the agile virtual enterprise.

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