PREFACE

Preface

Shigeaki Koike[1]

This is a special issue dedicated to Doctor Hitoshi Ishii (Research Fellow, Tsuda University / Professor Emiritus, Waseda University) in commemoration of Kodaira Kunihiko Prize from the Mathematical Society of Japan.

In 2019, "the Mathematical Society of Japan has set up Kodaira Kunihiko Prize with the aim of honoring the members of the Mathematical Society of Japan who made excellent lifetime achievements on mathematics. Received bequest money from the late Mr. Takao Okada, and respecting the will of the deceased, this prize is aimed at implementing projects that contribute to the promotion of mathematics. Awards are given every 4 years".[1]

Hitoshi Ishii received the first Kodaira Kunihiko Prize for the study of viscosity solution theory for fully nonlinear partial differential equations in 2019. At the same time, the same prize was also presented to Hiroshi Fujita (Professor Emeritus, the University of Tokyo) for the study of functional analytic methods in nonlinear partial differential equations, Toshikazu Sunada (Professor, Meiji University / Professor Emeritus, Tohoku University) for the study on geometric analysis and related fields, and Shigefumi Mori (Director-General, Distinguished Professor, Kyoto University Institute for Advanced Study) for birational classification of algebraic varieties.

Hitoshi Ishii is a leading researcher who completed the basic theory of viscosity solutions introduced by Pierre-Louis Lions and Michael G. Crandall [1] in 1981. We refer to [2] and references therein for the basic theory of viscosity solutions to fully nonlinear possibly degenerate elliptic/parabolic partial differential equations. For more detailed history concerning on Hitoshi Ishii, we refer to the preface in [6].

Since his research covers all aspects of viscosity solution theory and its various applications, we are not able to make a review of all of them. Here we only highlight a few of his significant works.

The first remarkable achievement was the existence result via the so-called Perron's method in [3]. It has turned out to show the existence of viscosity solutions, it is sufficient to construct only viscosity subsolutions and supersolutions, separately. Furthermore,

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its proof is extremely original and completely different from those of other existence results for solutions of partial differential equations.

The next outstanding achievement is the comparison principle to derive the uniqueness of viscosity solutions for degenerate second-order elliptic equations in [4]. This is nowadays called Ishii's lemma, where matrix inequalities also lead to the regularity of viscosity solutions from their algebraic structure in [5]. For these research, he received the Autumn Prize from Mathematical Society of Japan in 1994, which is the one for the members who have made exceptional contributions in the field of research.

In 2006, Hitoshi Ishii was selected as an invited speaker in the International Congress of Mathematicians at Madrid. In 2007, he was also selected as an invited speaker in the 6th International Congress on Industrial and Applied Mathematics at Zürich. In 2013, he was awarded as the first Fellow in American Mathematical Society.

Hitoshi Ishii has been an editor of several international journals:

Advances in Mathematical Sciences and Applications (1992–present)
Funkcialaj Ekvacioj (1996–2018)
Nonlinear Differential Equations and Applications (2000–present)
Zeitschrift für Analysis und ihere Anwendungen (2006–2010)
Advances in Calculus of Variations (2008–present)
Journal de Mathématiques Pures et Appliquées (2011–present)
Bulletin of Mathematical Sciences (2011–2018)
Palestine Journal of Mathematics (2012–present)
Minimax Theory and its Applications (2021–present)

At the end of this preface, we would like to thank the referees for their critical reviews and useful comments on the original papers. We would really like to thank Hitoshi Ishii for an everlasting friendship and important mathematics.

Guest editors:
@Kazuhiro Ishige
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