Research Article

$N = Z$ even–even proton-rich nuclei and nuclear structure

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Abstract
Nuclear systems near nucleon drip lines have great importance in studying nucleon–nucleon force properties. However, such unstable systems have few experimental data. Due to technical development in recent years, new experimental data assembly provides huge information to analyze possible observation of new phenomena. In the aim of exploring proton-rich nuclei near closed shells, we have studied the even–even $Z = N$ nuclei in $28 < Z < 50$, $50 < N < 82$ mass region. This study is based on the investigation of the core polarization and the monopole effect originated from the interactions between the magic core and the valence nucleons. In this context, this work includes the calculation of the energy spectra of Zr, Mo, Ru, Pd and Cd nuclei. These calculations were performed in the framework of the nuclear shell model using *NuShellX@MSU* code. Using recent single-particle energies, the three-body effect consideration introduces some modifications on the effective interaction. By means of the modified interaction, some nuclear properties of the studied nuclei have been evaluated. The results show a reasonable agreement with the available experimental data. The monopole effect is an important component of nucleon–nucleon interaction, in which its consideration gives a reasonably agreement with available data.

Keywords Nuclear structure · NuShellX@MSU code · Proton-rich isotopes · $28 < Z < 50, 50 < N < 82$ mass region · $N = Z$ nuclei · Monopole interaction

1 Introduction

Similarly to the atomic shell model, nuclei are assumed to consist of orbits filled by nucleons obeying quantum mechanical restrictions. Large gaps can be observed between some of these orbits, which indicates the existing of shells with high binding energy. In such shell, the maximum number of nucleons filling orbits from the first subshell to the corresponding one is the so-called magic number.

This model, the nuclear shell model, is one of the most successful theoretical model that gives the opportunity to understand and explain nuclear structure properties [1–7]. In scope of this model, an appropriate nucleus is considered as inert core which has magic numbers for proton and/or neutron parts. Only valence nucleons outside the core are considered to be active and taken into account in the used model space. In a very successful approximation in nuclear shell model theory, interactions between the core, which has been excited in this approximation, and the valence nucleons can be taken into account. This is known as *monopole effect* [8–10]. Identification of the spectroscopic properties of nuclei is an important way to investigate the nuclear properties. The $28 < Z < 50$, $50 < N < 82$ mass region contains exotic nuclei that lie near astrophysical processes paths (r and rp processes) ($Z = 28 - 48$) [11]. Noted that the region contains rp-process waiting points for which the process has to wait for several successive beta decays in order to achieve an isotopic chain with low proton separation energy S_p [12]. Neergard has studied ^{88}Ru and ^{92}Pd ground states in $g_{9/2}$ shell model [13]. The obtained results in his work were in

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agreement with the literature. In 2016, Qi and Wyss [14] studied $N = Z$ nuclei, in which the evidence of np pairing has been studied. Their results show good agreement with the experimental data for the first and the second excited states in ^{92}Pd. However, their calculations overestimate the other excited states. In this paper, shell model calculations have been performed for $N = Z$ nuclei: ^{80}Zr, ^{84}Mo, ^{88}Ru, ^{92}Pd and ^{96}Cd, near ^{100}Sn doubly magic core. They are close to the proton drip line and the rp-astrophysical process path, and more precisely, they present the *process* waiting points in the studied region [11] (Fig. 1).

For these nuclei lying in this path, few experimental data exist in the literature. In order to investigate shell evolution in this region, monopole effect has been considered. The calculations are realized in the framework of the nuclear shell model, by means of *NuShellX@MSU* nuclear structure code [15]. We have performed modifications on *jj44bpn* original interaction [16, 17] based on the monopole effect. The jbpnme-modified interaction has been used in order to reproduce energetic spectra of the studied nuclei.

In the first section, some theoretical features of monopole interaction will be presented following with the used calculation method. Then, the calculation results will be exposed and discussed. Some concluded remarks will be given at the end of this paper.

2 Materials and methods

The monopole effect is one of the most important phenomena in studying nuclear structure. After new nuclei discovering especially in the exotic regions and the appearance of new magic numbers as a result of shell evolutions, this phenomenon—which results from the interactions between valence nucleons and the nucleons in the core [9, 18]—becomes an interesting topic [8, 19]. Therefore, the nuclear system Hamiltonian is composed of two terms as given below in Eq. 1.

$$H = H_{\text{monopole}} + H_{\text{multipole}} \tag{1}$$

where the monopole part is expressed in its explicit form as in Eq 2.

$$H_{\text{monopole}} = \sum_{s} n_s \varepsilon_s + \sum_{s \le t} \left(a_{st} n_{st} + b_{st} T_{st} \right) \tag{2}$$

The monopole part contains single-particle energies ε_s, occupation number n_{st} and isospin operators T_{st}. a_{st} and b_{st} operators can be expressed in terms of $V_{j_s j_t}^{\tau\tau'}$ which present an energy average over the relative orientations of the orbits is given in Eq. 3 [9, 20] (see Ref. [21] for more details). In this expression V_J is weighted by the degeneracy of $(2J+1)$. The mean energy corresponds to a certain proton–neutron interaction which is considered as monopole term. It does not depend on the J values. j_τ and $j_{\tau'}$ are the spins of interacting nucleons in the considered orbits, which interact with the inert core.

$$V_{j_s j_t}^{\tau\tau'} = \frac{\sum_J (2J+1) V_J(j_s j_t)}{\sum_J (2J+1)} \tag{3}$$

In the shell model calculations, two-body matrix elements (TBMEs) are modified by using the monopole effect (Fig. 2). For even–even $N = Z$ proton-rich nuclei laying in *rp-process* path, the proton–proton, neutron–neutron and proton–neutron monopole effects have been considered.

Using Eq. 3, we have calculated the monopole terms for $\pi 1g_{9/2}$ and $\nu 1g_{9/2}$.

$$V_{g_{9/2} g_{9/2}}^{\tau\tau'} = \frac{\sum_J (2J+1) V_J\left(g_{9/2}, g_{9/2}\right)}{\sum_J (2J+1)} \tag{4}$$

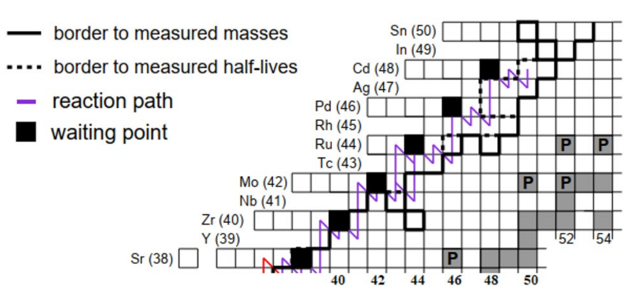

Fig. 1 $N = Z$ rp-process waiting points near proton drip line in the studied mass region [11]

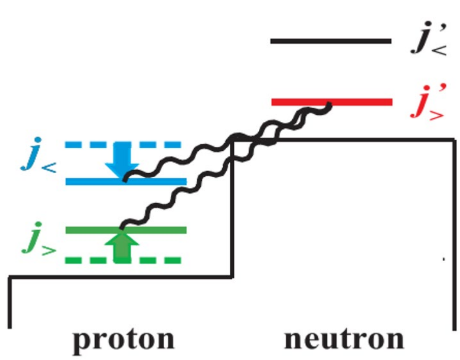

Fig. 2 Monopole effect produced by interaction between a proton in $j_{>,<} = l \mp \frac{1}{2}$ and a neutron in $j'_{>,<} = l' \mp \frac{1}{2}$ [22]

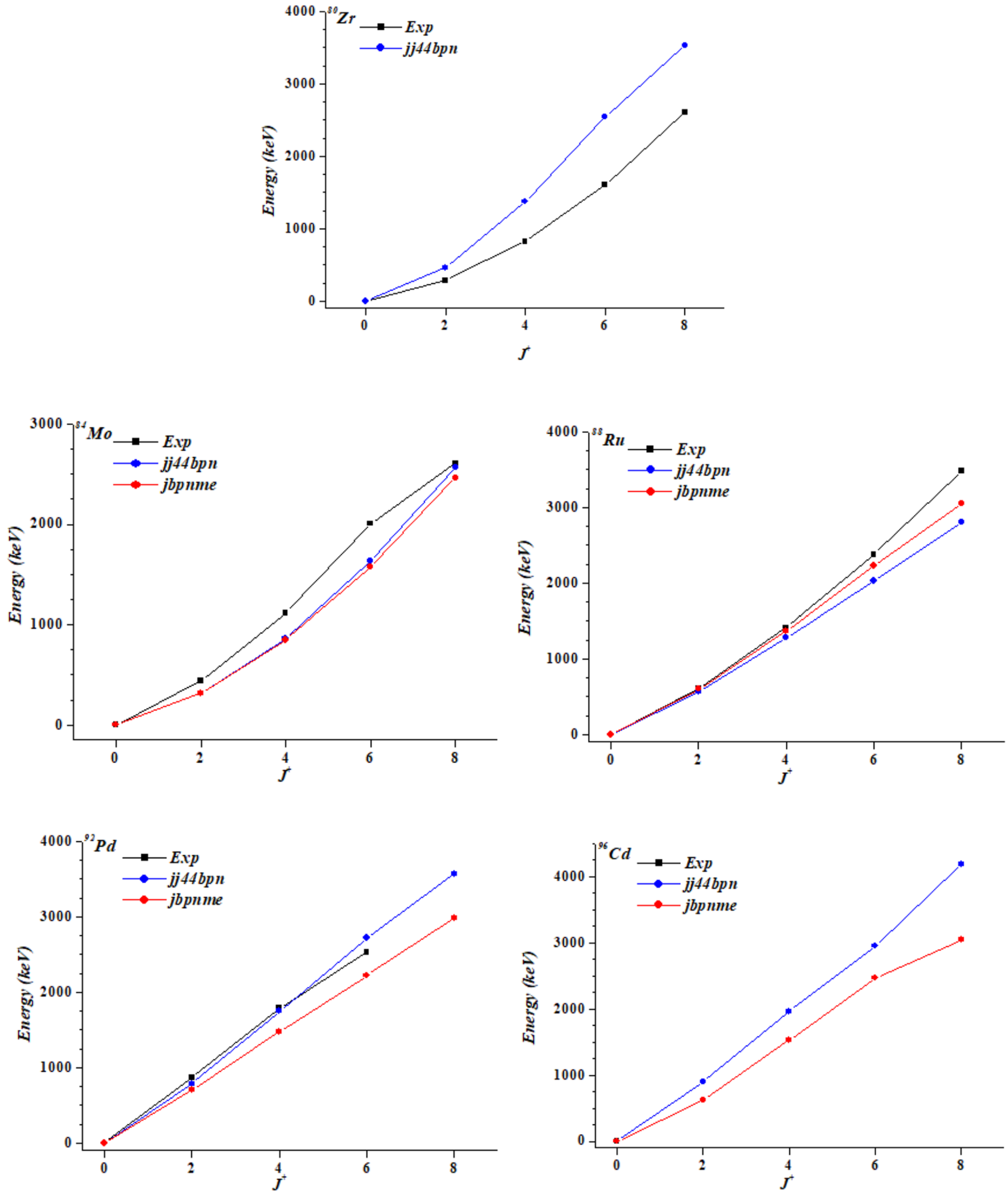

Fig. 3 Calculated energetic spectra of $N = Z$ nuclei near *rp-process* path in $28 < Z < 50$, $50 < N < 82$ mass region in comparison with the experimental data

This centroid can be calculated for $\pi\pi$, $\nu\nu$ and $\pi\nu$ parts of the effective interaction. For V_J terms, two-body matrix elements (TBMEs) of *jj44bpn* have been used. The monopole terms were estimated to be $V_{\pi\pi} \sim 300$ keV, $V_{\pi\nu} \sim 600$ keV and $V_{\nu\nu} \sim 600$ keV. These values are used to generate the new TBMEs using Eq. 5. TBMEs and a new interaction named *jbpnme* are introduced. Some calculations are released by means of this new interaction.

$$\langle j_\tau j_{\tau'} | V_{jbpnme} | j_\tau j_{\tau'} \rangle_J = \langle j_\tau j_{\tau'} | V_{jj44bpn} | j_\tau j_{\tau'} \rangle_J + \text{monopole term} \quad (5)$$

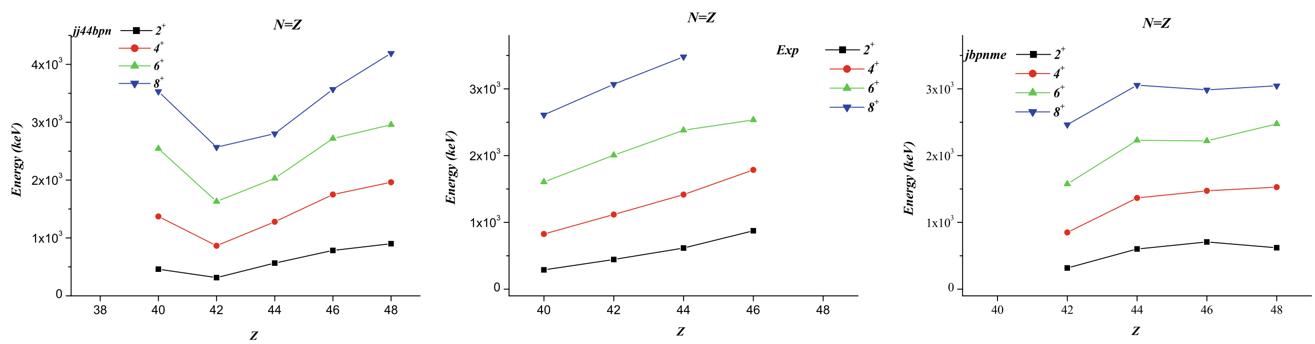

Fig. 4 Calculated systematics of first excited states by means of jj44bpn (left) and jbpnme (right) in comparison with the experimental data (center) for proton-rich $N = Z$ nuclei

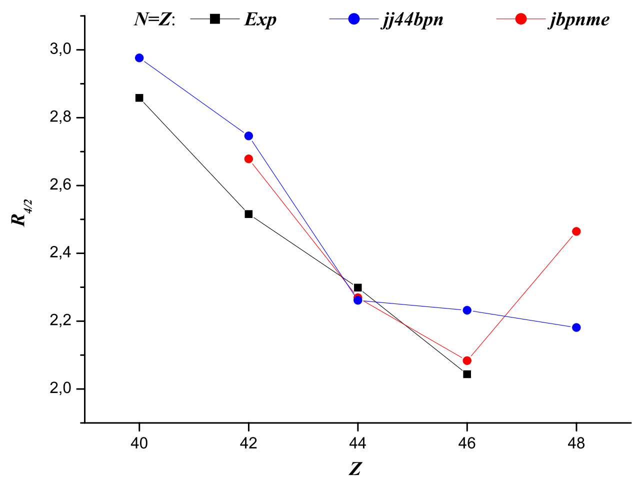

Fig. 5 Calculated and experimental $R_{4/2}$ ratios for proton-rich $N = Z$ nuclei

3 Results and discussion

In the calculations, we have used *jj44pn* single-particle space model (*SPS*) in ^{78}Ni mass region, which contains $\pi(1f_{5/2}, 2p_{3/2}, 2p_{1/2}$ and $1g_{9/2}$ orbits for protons and $\nu(1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$ and $1g_{9/2})$ for neutrons, as a single-particle space *SPS*. The single-particle energies (*SPE*s) were taken from the experimental data for protons using ^{79}Cu spectrum [23, 24] , from Grawe et al. [25], for neutron part.

$$\varepsilon_s = B(CS+1) - B(CS) + E_j^*(CS+1) \quad (6)$$

Here, $B(CS/CS+1)$ denotes experimental binding energy of closed core/closed core with an additional particle $(CS/CS+1)$. $E_j^*(CS+1)$ are excited energies of nucleus with a particle outside the core, with $J = j_s$ the spin of the subshell s [26].

The used interaction is modified by adding monopole term to original *jj44bpn* interaction [16, 17]. One of the well-known nuclear shell model codes (*NuShellX@MSU* [15]) has been used to perform spectroscopic calculations in this work [15].

In Fig. 3, we have shown the energy spectra of ^{80}Zr, ^{84}Mo, ^{88}Ru, ^{92}Pd and ^{96}Cd, respectively, for jbpnme and jj44bpn interactions in comparison with the available experimental data [24]. The low laying states in the studied nuclei are dominated by the configuration $\pi\nu\left(g_{9/2}g_{9/2}\right)^{mn}$, where m and n are, respectively, valence proton and neutron numbers outside the used magic core in the considered nuclei.

For ^{84}Mo isotope, both interactions give the energy values close to each other and experimental data. But for ^{88}Ru isotope, *jbpnme* elaborated interaction gives closer results to the experimental data. For ^{96}Cd isotope, although no experimental data exist in the literature, theoretical estimations on energy values have been performed by both interactions.

As it is seen in Fig. 4, the first 2^+, 4^+, 6^+ and 8^+ levels show slight increasing trends as mass number increases for experimental values (center), whereas these increasing trends are lost after mass number 88 and stay almost constant in the calculated spectra from *jbpnme* interaction (right). For the original interaction (left), the energies have decreasing trends for the two first nuclei (^{80}Zr and ^{84}Mo), which is not the case for experimental data.

In Fig. 5, we have given the experimental and the calculated $R_{4/2}$ ratios. Due to the absence of experimental data for ^{96}Cd, only calculated ratios have been given. As can be clearly seen in Fig. 5, all factors are in the range 2–3. The calculated ratios using our elaborated interaction (*jjbpnme*) and the experimental ones show the same behavior. According to the ratios, the most spherical nucleus among them is ^{92}Pd, indicating a pure harmonic vibrator and all the others are deformed showing transitional behavior. This is not well reproduced by the original interaction (*jj44bpn*). Furthermore, ^{80}Zr is candidate for X(5) with about 2.9 $R_{4/2}$ value.

4 Conclusion

This work is based on the energetic spectra calculations for even–even proton-rich $N = Z$ nuclei. By this purpose, the calculations are realized in the framework of the nuclear shell model by means of *NuShellX@MSU* nuclear structure code. Therefore, we have considered *jj44bpn* original interaction and we carried out some modifications based on the monopole interaction to get *jbpnme* one. According to the results, all the calculated spins and parities of the studied nuclei are in good agreement with the experimental ones. In addition, our new interaction reproduces well the $R_{4/2}$ ratio behavior in the studied region. As it is shown, the monopole interaction consideration has an important effect on the nuclear spectroscopic properties calculation and leads to reproduce them for exotic nuclei.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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