



Utilize freezing water to generate energy

Mohamad Kharseh¹ 

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Abstract

In a cold climate, the outdoor temperature is below zero most of the year. The low temperature can be used as a free energy resource by freezing water in a confined volume. It is well known that freezing water in a confined volume can create high pressure up to 220 MPa. This force might be utilized and used to generate mechanical energy using a hydraulic motor. The primary objective of the current study is to introduce readers to the concept of power generation using the freezing of water. This study also presents and discusses the impact of different factors on the amount of energy that can be generated by the freezing of water. For this aim, a computer model, which describes the principle, was developed and verified by comparison with the experimental data collected from a small go-kart built previously. Simulations and experimental results showed that there is a possibility to generate a considerable amount of energy by freezing water. The performed simulations showed that the amount of energy depends very much on the design and initial conditions. In other words, the better designed system can increase the generated power.

Keywords Water freezing · Thermodynamics · Low temperature · Energy generation

List of symbols

C_v	Specific heat capacity of the working gas at constant volume (J/kg K)	T_1	Freezer temperature or the temperature of the gas in the GOC at the beginning of the isotherm compression process (°K)
E	Extractable energy by freezing a confined water volume (Wh)	T_3	Ambient temperature or the temperature of the gas in GOC at the beginning of adiabatic expansion process (°K)
GOC	Gas oil container	T_4	Temperature of the gas in GOC at the end of the adiabatic expansion process (°K)
HM	Hydraulic motor	V_1	Volume of the gas oil container (m ³)
m	Mass of the working gas in GOC (kg)	V_2	Final volume of gas in the GOC when all water turned to ice (m ³)
N	Power generated by freezing a confined water volume (W)	V_w	Total volume of water in the WOC (m ³)
OC	Oil container	WOC	Water oil container
P_1	Initial gas pressure in the GOC (Pa)	γ	Adiabatic index of the gas, for Nertogine, $\gamma = 1.4$
P_2	Pressure in the GOC when all water turned to ice (Pa)	δ	Compression coefficient, $\delta = P_2/P_1$
R	Specific gas constant, for nitrogen $R = 296.8$ J/kg K	ε	Volumetric expansion coefficient of the water due to freezing (%)
t	Time of extraction work or time of the adiabatic expansion process(s)	θ	Heating coefficient, $\theta = T_3/T_1$

✉ Mohamad Kharseh, m.kharseh@gmail.com | ¹Department of Civil and Environmental Engineering, Chalmers University of Technology, Chalmersplatsen 4, 412 96 Gothenburg, Sweden.

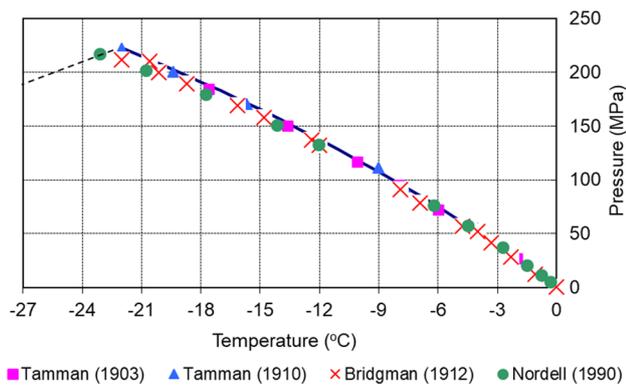


Fig. 1 Pressure–melting temperature of ice [4]. The line is a model estimate [5]

1 Introduction

Because of the hydrogen bonds, it is a well-known fact that water expands on freezing instead of contracting, which is insanely unusual [1, 2]. The combination of the low compressibility of the water (at 0 °C it is 5.1×10^{-4} 1/MPa) with its expansion upon freezing can be utilized to create a very high pressure by freezing water in a sturdy confined container [3]. Theoretically, the maximum pressure of 220 MPa can be reached at temperature – 22 °C (Fig. 1).

This generated high pressure was used to split large stones during winter by drill a hole and fill it with water. As water freezes inside the hole, it expands and causes the rock to crash. Alternatively, the high pressure can be utilized somehow directly to drive a hydraulic motor and generate mechanical power. However, the review of the literature confirms that this phenomenon is a vastly untapped occurrence yet for energy generation. To our knowledge, the utilization of high pressure causing by freezing water to produce mechanical energy has received very little attention in the scientific literature. Kharseh and Al-Khawaja (2013) have investigated the possibility of utilizing water freezing to generate power [5]. The authors present a model simulating the icy rider (Fig. 2), which is a small go-kart built in the 1980s [6–8]. The vehicle is driven by a hydraulic motor which is powered by the pressure. The very high pressure is created by freezing 0.027 m³ of water in a confined volume and is used to power a hydraulic motor. The icy rider weighs ~ 200 kg including the driver. The hydraulic system shown in Fig. 5 transfers the created energy due to the freezing process. In this way, the icy rider reaches a maximum speed of ~ 70 km/h over a maximum driving distance of ~ 400 m.

However, the used approach may not be very accurate, and consequently, the obtained model Kharseh



Fig. 2 A small go-kart vehicle (icy rider) which was built and used to test the theoretical model

et al. may not mimic the icy rider. This can be proved by the significant deviance of the theoretical result, based on their proposed model, from experimental data.

In the current work, a theoretical model was developed to simulate the thermodynamic cycle of utilizing water freezing in a confined volume to generate mechanical power. The conceptual model was verified by comparison with the experimental data collected from running the icy rider. Then, the model was used to investigate the impact of different factors on the amount of energy that can be generated by the freezing of water.

2 Theoretical overview

2.1 Ice physics

Typically, increasing pressure leads to shifting the melting point to higher temperatures, but in the water, this is reversed. Namely, increasing the pressure results in reduced the melting temperature [9]. Figure 1 shows the experimental data of the relation between the pressure and melting temperature of ice [10–12]. In these measurements, the temperature of the ice was fixed at a constant value, while an increasing pressure was applied. The melting pressure was defined as the pressure when a sudden melting occurred. However, in a recent study conducted by Nordell, a different method was used to describe the relationship between pressure and melting temperature [4] as follows. A container filled with water was placed in a freezer of a constant temperature. Due to the expansion, freezing the water increases the pressure in the container. The pressure increases as long as the freezing of the water is occurring. The melting pressure point was defined as the point when increasing the pressure is stopped. These measurements were repeated for the different temperature of the freezer, and the results

are illustrated in Fig. 1. As shown, the maximum pressure of 220 MPa can be reached at the temperature $-22\text{ }^{\circ}\text{C}$. However, a new type of ice is formed at a lower temperature than $-22\text{ }^{\circ}\text{C}$, which is out the scope of the current work. A good approximation of the relationship between pressure and melting temperature is given as:

$$P = -1.3599 \cdot T^2 - 130.3 \cdot T + 9.3826 \quad (1)$$

Here, P stands for pressure (MPa), while T is melting temperature (in $^{\circ}\text{C}$).

2.2 Compression process

Unlike water, it is usual for liquids to contract on freezing and expand on melting. This contraction is because the molecules are in fixed positions within the solid but require more space to move around within the liquid. As shown in Fig. 3, water freezing at $0\text{ }^{\circ}\text{C}$ (at 1 atm. Pressure) results in a volume increase of about 9% [2, 13]. The expansion upon freezing comes from the fact that water crystallizes into an open hexagonal form. This hexagonal lattice contains more space than the liquid state. The ice then shrinks as the temperature decreases. However, the shrinkage is less than 0.4% going from -1 to $-22\text{ }^{\circ}\text{C}$ (Fig. 3). In other words, the contraction of the ice is insignificant and will be ignored in the current study.

It is worth mentioning that if increased pressure lowers the melting point, the growth in volume on freezing is even more significant. Namely, the expansion volume of the water because of the freezing is 16.8% at $-20\text{ }^{\circ}\text{C}$ and pressure 200 MPa compared to 9% at $0\text{ }^{\circ}\text{C}$ and pressure 1 atm. [3]. Figure 4 shows the molar volumes of ice and water along the melting point curve.

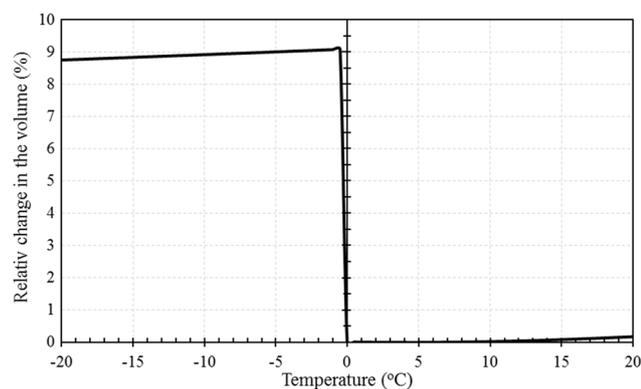


Fig. 3 Relative change in the liquid water and ice versus the temperature at normal pressure

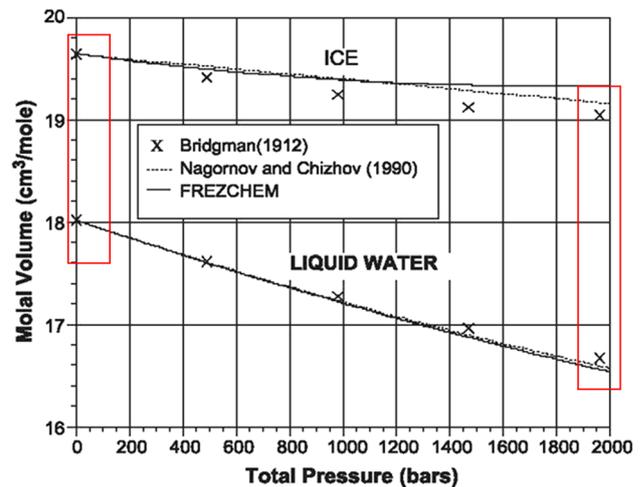


Fig. 4 Molar volumes of ice and liquid water along the gage pressure ice/water melting curve from [3]

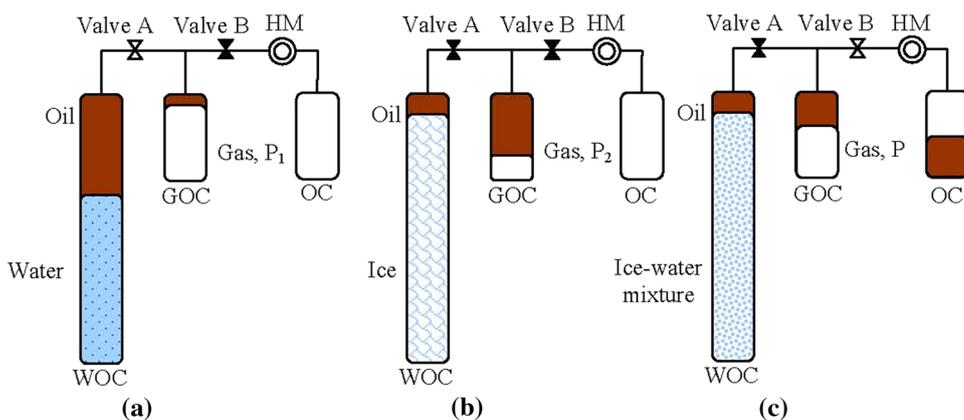
3 Energy Regimen

Figure 5 illustrates the hydraulic system used to extract the mechanical energy of freezing water. The system consists of a water–oil container (WOC), a gas oil container (GOC) which acts as a pressure accumulator, an oil collector (OC), a hydraulic motor and two valves those are used to control the flow of oil between the components. The operation cycle is composed of four primary processes:

1. Charging process (Fig. 5a): valve A is open, and valve B is closed. Due to the freezing of water in WOC, oil will be pushed into GOC, and consequently, the pressure in the GOC increases.
2. Storing process (Fig. 5b): valve A is closed, i.e., all valves are closed at this moment, and pressurized gas may be retained until it is needed. During this time, the gas might absorb heat from its surrounding of higher temperature, and hence, the gas pressure increases.
3. Extracting process (Fig. 5c): valve B is opened (valve A is still closed), and the oil flows through a hydraulic motor generating energy, E. The used oil is collected in the OC.
4. Sucking process: by opening valve A (valve B is still open) and melting the ice in WOC, oil returns to WOC.

The volume expansion of the freezing water forces the oil out of the WOC into the GOC. Consequently, the gas pressure increases from the initial pressure P_1 to the final pressure P_2 . It is worth mentioning that pressure P_2 can be controlled by the volume of WOC, GOC and amount of the water in the WOC. When the GOC is fully charged, and all water has turned to ice, valve A is closed, and the pressurized oil is stored until it is released by opening valve

Fig. 5 Hydraulic system of the proposed system divided into three parts: **a** charging, **b** storing and **c** extraction modes. WOC water–oil container, GOC gas oil container (pressure accumulator), OC oil collector, HM hydraulic motor



B and driving the hydraulic motor. Definitely, during the storing period, the gas vessel will exchange heat with the surrounding, which might have a higher temperature. Consequently, the pressure inside the gas vessel increases until thermal equilibrium between the GOC and its surrounding is reached. Since the freezing process is slow, the compression process is considered to be isothermal, i.e., $T_1 = T_2$, while the fast expansion process is seen as adiabatic, i.e., $s_3 = s_4$, as shown in the thermodynamic cycle in Fig. 6. As mentioned above, during the storing period the compressed gas receives heat from its surroundings. Since the gas vessel is closed, the exchanging heat with the surrounding occurs at constant volume. During the work generation process, the compressed gas expands until the gas in the GOC returns to its initial volume. Recall that the compression process is isotherm and expansion process is adiabatic. Furthermore, under the operation conditions of the icy rider in the current study the pressure in GOC at the end of expansion process is lower than the initial pressure, i.e., $P_1 > P_4$, as will be shown in the following analyses. Thus, due to the temperature difference between the gas and its surrounding, at the end of the expansion process, the gas absorbs heat from the surrounding at constant volume to return to the initial pressure. The performed derivation of thermodynamic processes follows basic rules found in any textbook on thermodynamics [14] as follows. Based

on the first law of thermodynamics, the work done by the adiabatic expansion equals the change in internal energy ΔU of the gas:

$$E = -\Delta U = c_v \cdot m \cdot (T_3 - T_4) \tag{2}$$

where T_3 and T_4 are the temperatures of the gas at the beginning and the end of the expansion process, respectively; m is the mass of gas (kg); and c_v is the specific heat capacity of the gas at constant volume (J/kg K), which is defined as:

$$c_v = \frac{R}{\gamma - 1} \tag{3}$$

where γ is the adiabatic index of the gas and R the specific gas constant (J/kg K).

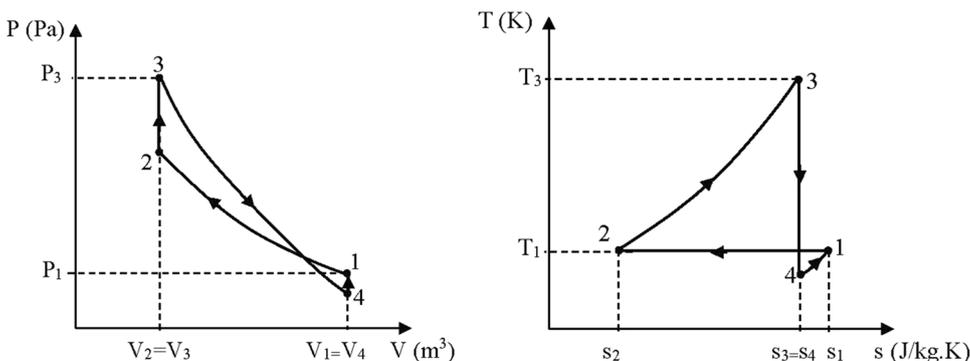
The next analysis aims to eliminate the unknown terms (i.e., T_4 and m) from Eq. 2.

The accumulation of oil in the GOC (as a result of freezing the water in the WOC) leads to a gradual decrease in the gas volume. When all water has turned into ice, the final gas volume in the GOC, say V_2 , becomes:

$$V_2 = V_1 - \varepsilon \cdot V_w \tag{4}$$

Here, V_1 is the initial gas volume in the GOC (the size of the gas oil container), ε is the volumetric expansion coefficient of the water due to freezing, and V_w is the total volume of

Fig. 6 Thermodynamic cycle of the proposed system on P - V and T - S diagram



water in the WOC. Recall that the freezing process is prolonged, and the whole system during the freezing process is located inside the freezer. Thus, compression gas process during water freezing is an isothermal process, and therefore, one can write:

$$P_1 \cdot V_1 = P_2 \cdot V_2 \quad (5)$$

After the whole water turned into ice, the icy rider is taken out the freezer and placed in warmer space. The comprised gas in GOC will absorb heat from its surrounding, and consequently, the pressure of the gas will increase at constant volume (confined contained). From the definition of the isochoric process between states 2–4, one can write:

$$\frac{P_2}{P_3} = \frac{T_2}{T_3} \quad (6)$$

Energy extraction occurs when valve B is opened (see Fig. 5), and the oil is pushed through the hydraulic motor due to the expansion of the pressurized gas in the GOC. Since the gas expansion (between stages 3 and 4 in Fig. 6) is fast, the expansion process can be treated as an adiabatic process. Hence, one can write:

$$P_3 \cdot V_3^\gamma = P_4 \cdot V_4^\gamma \quad (7)$$

Because the initial gas pressure is higher than the atmosphere pressure (in the current case study, $P_1 = 100$ MPa), the volume of the gas at the end of adiabatic expansion equals the original volume. Finally, the thermodynamic cycle will be closed, and the gas returns to the initial conditions at the isochoric process (process 4–1 in Fig. 6) due to exchanging heat with the surrounding. The closing process can be expressed by:

$$\frac{P_1}{P_4} = \frac{T_1}{T_4} \quad (8)$$

Indeed, the ideal gas law or what is known as the Clapeyron relation is applicable at each stage of the cycle; see Fig. 6:

$$P \cdot V = m \cdot R \cdot T \quad (9)$$

Solving the above equations yields:

$$T_4 = T_3 \cdot \delta^{1-\gamma} \quad (10)$$

$$m = \frac{\varepsilon \cdot \delta \cdot P_1 \cdot V_w}{T_1 \cdot R \cdot (\delta - 1)} \quad (11)$$

Substituting Eqs. 3, 10 and 11 in Eq. 2 yields

$$E = \frac{\varepsilon \cdot \delta \cdot P_1 \cdot V_w \cdot (1 - \delta^{1-\gamma})}{(\gamma - 1) \cdot (\delta - 1)} \cdot \theta \quad (12)$$

The maximum pressure that can occur during the storing period is given by:

$$\frac{P_3}{P_1} = \delta \cdot \theta \quad (13)$$

where γ is the adiabatic index of the gas (in the current case, where the nitrogen is the working gas in the GOC, $\gamma = 1.4$). $\delta = P_2/P_1$ is compression coefficient, which is defined as the ratio of the pressure in the gas container at the end of the freezing process to the initial gas pressure; see Fig. 6. V_w is the total volume of the liquid water in the WOC. ε is the volumetric expansion coefficient of the water due to freezing. $\theta = T_3/T_1$ is heating coefficient, which is defined as the ratio of the high reservoir temperature (in the current case T_3 is the ambient temperature ≈ 21 °C) to the low reservoir temperature (in the present situations T_1 is the freezer temperature $= -11$ °C).

Bearing in mind the assumption made to build the simulation model, the pressure of the gas in the GOC at the end of the expansion process is lower than the initial pressure. Following analyses is to verify this assumption made. From the above equations, it is possible to derive the relationship between the initial pressure and the final pressure as follows

$$\frac{P_4}{P_1} = \frac{\theta}{\delta^{1-\gamma}} \quad (14)$$

Finally, it might be of great interest to calculate the pure power, N , generated by freezing a bound water volume. To achieve this goal, the energy generation must be divided by the extraction time, namely:

$$N = \frac{E}{t} = \frac{\varepsilon \cdot \delta \cdot P_1 \cdot V_w \cdot (1 - \delta^{1-\gamma})}{(\gamma - 1) \cdot (\delta - 1) \cdot t} \cdot \theta \quad (15)$$

As shown by Eq. 15, the produced power is inversely proportional to the extraction time. The latter strongly depends on the design of the hydraulic system including the hydraulic motor and the transforming pipes.

4 Results and discussion

Equation 12 is used to calculate the extractable energy due to freezing water in a confined volume. The theoretical model was verified by comparison with the experimental data collected from running the icy rider. The specifications of the icy rider are given in Table 1. In icy rider, for the selected containers, the amount of the water was chosen based on the strength of the vessels so that the maximum pressure in the GOC does not exceed the maximum allowed pressure of the components with enough safety margin.

Table 1 Specification of the icy rider

Weight	200 kg	Initial pressure	10 MPa	Speed	70 km/h
Water volume	27 l	Pressure at the end of freezing	25 MPa	Freezer temperature	-10
Volumetric expansion coefficient	9%	Adiabatic index	1.4	Ambient temperature	21 °C

Based on Eq. 12, the extractable mechanical energy, E , is 34.9 kJ. If the approximated weight and maximum speed (200 kg and 70 km/h) are trustable, then the kinetic energy of the vehicle would be 37.8 kJ. So, the deviance from experimental kinetic energy is 8%. The difference between the extractable energy obtained from Eq. 12 and kinetic energy derived from the preliminary results can be explained as follows:

- The weight of the icy rider with a driver was overestimated in the empirical measurement, and the speed of the vehicle was overestimated in the experimental analysis. In other words, to achieve a realistic match between the simulation and empirical results, the weight of the system (icy rider with a drive) or the speed of the vehicle should be modified to 185 kg and 69 km/h, respectively. It is worth mentioning that the icy rider and the experimental measurements were taken as inexpensively as possible. This means that

there is a significant potential for uncertainty in the experimental results.

- Alternatively, the used volumetric expansion coefficient of the water, $\epsilon = 9\%$, is a lower estimation of the real expansion coefficient. It is a well-known fact that the increase in volume of freezing is higher if the melting point is lowered due to the increased pressure (for example, at -20 °C $\epsilon = 16.8\%$) [3]. In other words, for the given weight and speed of the go-kart equal 200 kg and 70 km/h, respectively, the expansion coefficient must be modified to $\epsilon = 9.8\%$ to achieve a realistic match between the simulation and experimental results.

Hence, the developed conceptual model is an acceptable model to simulate the utilization of water freezing for creating mechanical energy. Another important result can be obtained from the developed model here is that the maximum amount of energy that can be generated by the freezing of 1 l of water can be up to 22.1 kJ. This result

Table 2 Pressure ratio between the initial and final pressure

	δ					
	1.5	2	2.5	3	3.5	4
T_3						
-11	0.85	0.76	0.69	0.64	0.61	0.57
-5	0.87	0.78	0.71	0.66	0.62	0.59
0	0.89	0.79	0.72	0.67	0.63	0.60
5	0.90	0.80	0.74	0.68	0.64	0.61
10	0.92	0.82	0.75	0.70	0.65	0.62
20	0.95	0.85	0.78	0.72	0.68	0.64
30	0.98	0.88	0.80	0.75	0.70	0.66
40	1.02	0.91	0.83	0.77	0.72	0.69
50	1.05	0.93	0.85	0.79	0.75	0.71
60	1.08	0.96	0.88	0.82	0.77	0.73
70	1.11	0.99	0.91	0.84	0.79	0.75
80	1.15	1.02	0.93	0.87	0.82	0.77
90	1.18	1.05	0.96	0.89	0.84	0.80
100	1.21	1.08	0.99	0.92	0.86	0.82
110	1.24	1.11	1.01	0.94	0.89	0.84
120	1.28	1.14	1.04	0.97	0.91	0.86
130	1.31	1.17	1.07	0.99	0.93	0.88
140	1.34	1.19	1.09	1.02	0.96	0.91
150	1.37	1.22	1.12	1.04	0.98	0.93

is obtained from Eq. 12 by substituting the volume and P_1 and P_2 by 1 l, 219 MPa and 220 MPa, respectively. It is worth mentioning that the latter represents the maximum pressure of 220 MPa can be reached; see Fig. 1.

Recall that to drive the theoretical model of the icy rider it was assumed that the pressure at the end of the expansion pressure was considered to be lower than the initial pressor (i.e., $P_4 < P_1$). For a verification purpose, Eq. 14 was used to calculate the ratio between the initial and final pressure for different working parameters, and the results are given in Table 2. As shown, for the given operating conditions of the icy rider (i.e., $T_3 = 20$ and $\delta = 2.5$), the pressure at the end of the expansion process is lower than the initial pressure ($P_4 = 0.78 P_1$). Thus, the assumption is legal, and consequently, the developed model is valid.

Toward investigating the effect of different working parameters on the extractable energy, the calculation was carried out for different values of specific parameters including freezing and ambient temperature, the adiabatic index of the gas, initial pressure and volumetric expansion

coefficient of the water. For this objective, the considered factors in this study were assumed to vary between 25% higher and lower than the nominal values given in Table 1. Table 3 shows the recognized elements and their ranges (i.e., 25% higher and smaller than the theoretical values listed in Table 1). The impact of the parameters was calculated as the changes in the extractable energy due to variations in the considered factors, and the results are illustrated in Fig. 7. As shown, the selected elements have a different impact on the extractable energy. However, the volumetric expansion coefficient of the water (or it could be any other liquid) and the initial pressure seem to be the most critical factors. The significance of these two factors is because the gas pressure at the end of the freezing process depends on them. This means replacing the water with another substance of higher volumetric expansion coefficient results in an increase in the generated energy. In hot climates, melting the paraffin wax, for instance, which has a higher volumetric expansion coefficient as compared to water, can be used to generate energy. Building a stronger system allows us to increase the initial pressure that leads to increasing the amount of generated energy. In other words, there might be different ways to increase the generated energy due to phase change.

Finally, Eq. 15 was used to calculate the extractable mechanical power under the current specifications of the icy rider. Figure 8 illustrates power generation for different extraction time. Since the extracted energy was used for ~ 25 s, i.e., the duration of the ride, the average power was about 1.4 kW. However, shortening the extraction time by improving the hydraulic system (e.g., wider pipe and a better motor with higher flow capacity) increases the generation power significantly. Figure 8 shows the mechanical

Table 3 Parameter considered in the sensitivity analysis

Factor	Range	Factor	Range
Freezing temperature	-13.8–8.3	The adiabatic index of the gas	1.1–1.8
Ambient temperature	15.8–26.3	Initial pressure	7.5–12.5
Volumetric expansion coefficient			8.6–11.3%

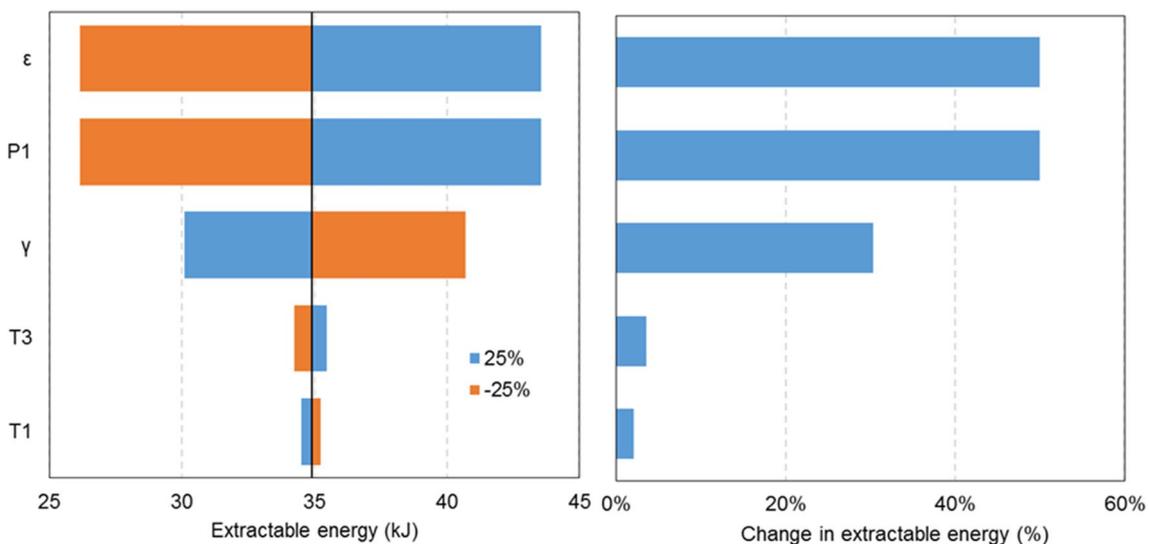


Fig. 7 Impact of different factors in the term of the extractable energy from the cycle in Fig. 5

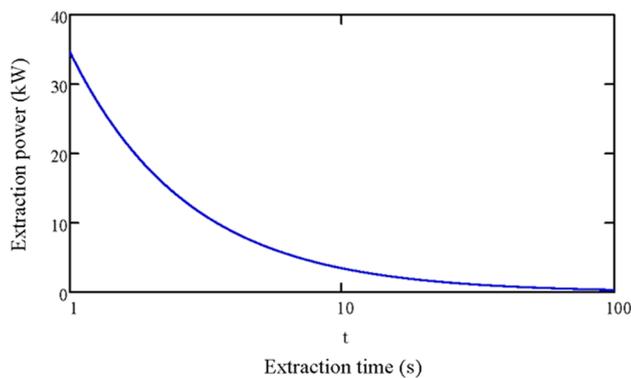


Fig. 8 Power of the proposed system as a function of the extraction time

power that can be generated from the specification of the studied icy rider as a function of the extraction time.

5 Conclusions

The current work presents a concept for power generation using the freezing of water. For this aim, theoretical model was developed. The conceptual model was verified by using the experimental data collected from a small go-kart built previously. The results prove that in cold regions, there is a potential to generate high-quality energy from low-quality thermal energy by freezing water in a confined volume.

The developed model showed that there is a possibility to generate about 22 kJ by freezing 1 l of water. Although this concept can produce a small amount of energy, one can imagine other applications that could make use of such a system. However, this amount of energy depends very much on the design and initial conditions. In other words, better designed system can increase the generated energy.

It is worth mentioning that in the current study, water was used to generate high pressure due to the frizzing. However, any other substance, which might have a higher volumetric expansion coefficient, can be used to generate high pressure. Also, in hot climates, the frizzing process can be replaced with the melting process to generate the high pressure. For instance, melting paraffin wax, which

has a higher volumetric expansion coefficient as compared to water, can be used in the same way shown above. In other words, there might be different ways to increase the generated energy due to phase change.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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