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ORIGINAL ARTICLE

The Imperialist Competitive Algorithm for Optimal Multi-Objective Location and Sizing of DSTATCOM in Distribution Systems Considering Loads Uncertainty

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Abstract The optimal location and sizing of distribution static compensator (DSTATCOM) in distribution systems is a complex nonlinear problem. This problem is constrained by various technical limits and can offer different objectives that would provide many benefits to the network. These include minimization of power losses, index of voltage profile, load balancing index, and annual cost saving index which have been considered in this paper. In the present work, the Imperialist Competitive Algorithm (ICA) is employed for optimizing the distribution systems where an optimal location and sizing of DSTATCOM is investigated. In this study, an aggregating operator named Max-geometric mean is used for combination of objectives and providing overall objective function. The scaling of objectives is performed in the fuzzy framework. The proposed algorithm is implemented in 33 and 69 buses IEEE test systems. Furthermore, the uncertainty of the loads of the balanced system is modeled by using a fuzzy technique. Based on the numerical results of this work, one can extract that the performance of the ICA is slightly higher than other meta-heuristic algorithms; hence the introduced approach can be used by utility services for optimal DSTATCOM allocation and sizing in the distribution systems.

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Introduction

Recently, using modern techniques and power electronic devices such as flexible AC transmission systems (FACTS) in distribution systems have increased obviously. This increase can be justified by factors such as environmental concerns, the restructuring of the electricity market, complete utilization of lines capacity, improvement power quality and enhancement of voltage stability (Kumar and Mishra 2014). Researches have shown that installation of FACTS based equipment in the power distribution system could lead to achieve many benefits such as voltage profile improvement, reduction in lines losses, security enhancement for critical loads, reduction in the on-peak operation cost and improvement in the power quality and reliability of supply (Patel et al. 2016). This equipment includes solid state transfer switch (SSTS), dynamic voltage restorer (DVR), distribution static compensator (DSTATCOM) and unified power quality conditioner (UPQC) (Chandrasekaran and Ramachandaramurthy 2016; Vinkovic and Mihalic 2008). In this paper, DSTATCOM as a shunt connected voltage sourced converter (VSC) is employed to enhance of objectives. In order to maximize the benefits of installation of DSTATCOM, it is essential to determine the optimal size of units and their best location in distribution systems; otherwise, it could lead to adverse effects such as increase in power losses and network costs (Yuvaraj et al. 2015a). In recent years, many various methods have been reported to solve this problem. Some of the different methodologies used in this concern have been published in Duan et al. (2016) and Kavousi-Fard and Akbari-Zadeh (2014).

Different methods have been used to solve the optimal location and sizing of DSTATCOM problem. Authors in Devi and Geethanjali (2014) have used particle swarm optimization (PSO) algorithm for obtaining the optimal location and sizing of distributed generation (DG) and DSTATCOM. The objectives include reducing the total power losses along with voltage profile improvement of the radial distribution system. In Taher and Afsari (2014), an immune Algorithm (IA) is used to search the optimal location and sizing of DSTATCOM for obtaining power loss reduction and improvement of current and voltage profile in distribution systems. Authors in Jazebi et al. (2011) have used differential evolution (DE) algorithm for DSTATCOM allocation in distribution networks considering reconfiguration. The main goals include mitigating losses and improving voltage profile. A stochastic framework based on the point estimate method (PEM) load flow to consider the uncertainty effects of loads in DSTATCOM allocation and sizing problem is investigated in Khorram-Nia et al. (2013). Authors in Akbari-Zadeh et al. (2014) have studied a new stochastic structure to simulate the uncertainty of loads for optimal allocation and sizing of DSTATCOM. The objectives are minimizing the total active power losses and diminishing the voltage deviation of the buses. The suggested structure in Mahendra Repalle et al. (2014) includes a fuzzy logic based optimal location and sizing of DSTATCOM in radial distribution systems for voltage profile improvement. Authors in Li and Li (2014) have determined the optimal location of DSTATCOM based on the dynamic analytic method and the trajectory sensitivity index for improvement of nodes voltage stability. In Bagheri Tolabi et al. (2015), a combination of a fuzzy multi-objective approach and ant colony optimization (ACO) algorithm to solve the simultaneous reconfiguration and optimal allocation (size and location) of photovoltaic (PV) arrays and DSTATCOM in a distribution system is proposed. The objectives of the mentioned research include loss reduction, voltage profile improvement, and increase in the feeder load balancing.

Considering the above-discussed literature, the contribution of this paper is to introduce imperialist competitive algorithm (ICA) as an evolutionary optimization tools to solve proposed multi-objective nonlinear problem. The proposed algorithm is implemented for the optimal allocation of DSTATCOM in distribution systems using a fuzzy-based multi-objective programming method. The considered objective functions include the minimization of total real power losses, index of voltage profile (IVP), load balancing in the feeders, and annual cost. A fuzzy-based framework is used to transform objective functions into fuzzy memberships and then finally to combine them into a single-objective function, which is optimized subject to a variety of power system operational constraints. The uncertainty of the loads is modeled by using a fuzzy approach. The proposed method is tested on balanced 33-bus and 69-bus distribution systems. Numerical results show the efficiency of the ICA algorithm compared to the other algorithms.

The rest of this paper is organized as follows. In Sect. 2, the proposed formulation for optimal location and sizing of DSTATCOM in distribution systems is presented. In Sect. 3, the basic principle of the proposed algorithm is described. Application of the proposed method to the problem is elaborated in Sect. 4. Section 5 models the uncertainty of the loads. Section 6 details test results. Finally, Sect. 7 concludes the paper.

Proposed Formulation for Optimal Location and Sizing of DSTATCOM

In this section, the proposed formulation for optimal location and sizing of DSTATCOM in distribution systems is elaborated with its objective functions and constraints.

Objective Functions

Minimization of Power Losses

Minimizing active power losses has been one of the decisive issues in distribution systems. It is calculated as sum of power losses of branches as:

$$P_{loss} = \sum_{k=1}^{N_{br}} R_k |I_k|^2 \tag{1}$$

$$\text{Min } f_1 = IPL = \frac{P_{loss_{after}}}{P_{loss_{before}}}, \tag{2}$$

where R_k and I_k represent the resistance and current of branch k, respectively; N_{br} is the total number of branches in the system; IPL is Index of Power Loss; $P_{loss_{before}}$ is the real power loss before allocation of DSTATCOM; $P_{loss_{after}}$ is the real power loss of the radial system after allocation.

Minimization of Index of Voltage Profile (IVP)

For the purpose of minimizing the buses voltage deviation, the IVP is defined as follows:

$$\text{Min } f_2 = \text{IVP} = \frac{\sum_{i=1}^{N_{bus}} \left|1 - V_{i_{after}}\right|}{\sum_{i=1}^{N_{bus}} \left|1 - V_{i_{before}}\right|}, \quad (3)$$

where $V_{i_{after}}$ and $V_{i_{before}}$ are the values of bus voltage for each configuration after and before allocation, respectively; N_{bus} is the number of buses.

Minimization of Load Balancing Index (LBI)

For the purpose of load balancing, first, an appropriate parameter is defined, indicating the loaded portion of the branches. This portion is defined as the line usage index for the *i*th branch, calculated as follows (Saffar et al. 2011):

$$Line\ Usage\ Index = \frac{I_k}{I_k^{max}}, \quad (4)$$

where I_k represents the current of branch k; I_k^{max} is the permitted rating of branch k.

LBI is calculated and parameter of *Y* can be expressed as follows:

$$Y = \left[\frac{I_1}{I_1^{max}} \frac{I_2}{I_2^{max}} \frac{I_3}{I_3^{max}} \cdots\cdots \frac{I_{N_{br}}}{I_{N_{br}}^{max}}\right]. \quad (5)$$

So the *LBI* is expressed as follows:

$$LBI = Var(Y) \quad (6)$$

$$\text{Min } f_3 = \frac{LBI_{after}}{LBI_{before}}, \quad (7)$$

where *Var* represents the variance operation; LBI_{after} and LBI_{before} are the values of the *LBI* for each configuration after and before allocation, respectively. The smaller value of the *LBI* index indicates that the load balancing has been conducted more efficiently.

Minimization of Annual Cost Saving Index (ACSI)

The investment cost for a DSTATCOM per year is calculated according to the following equation:

$$COST_{DSTATCOM,year} = COST_{DSTATCOM} \times Q_{inst} \times \frac{(1+B)^n \times B}{(1+B)^n - 1}, \quad (8)$$

where $COST_{DSTATCOM,year}$ is annual cost of DSTATCOM; $COST_{DSTATCOM}$ is cost of investment in the year of allocation; B is asset rate of return; Q_{inst} is capacity of installed DSTATCOMs; n represents the lifelong of DSTATCOM.

The *ACSI* for a DSTATCOM is defined as:

$$\text{Min } f_4 = ACSI = \frac{K_L\left(T \times P_{TLOSS}^{WITH}\right) + \left(K_{TDP} \times COST_{DSTATCOM,year}\right)}{K_L\left(T \times P_{TLOSS}^{WITHOUT}\right)}, \quad (9)$$

where K_L is energy cost of losses; T is hours per year; K_{TDP} is time duration proportion; $P_{TLOSS}^{WITHOUT}$ and P_{TLOSS}^{WITH} represent total power loss before and after installation of DSTATCOM, respectively.

Fuzzy-Based Combination of Objective Functions

In order to find a solution in which all objective functions are optimized, a multi-objective programming method should be used. Due to the fact that the four considered objective functions have different scales, using the simple method of combining them into one objective function results in scaling problems. In order to transform objective functions into the same range, the fuzzification method is used (Akorede et al. 2011). Applying this method, all objective functions are fuzzified and transformed into the same range of [0, 1]. The trapezoidal fuzzy membership function for objective function i is defined as:

$$\rho_i = \begin{cases} 1 & f_i \le f_i^{min} \\ \dfrac{f_i^{max} - f_i}{f_i^{max} - f_i^{min}} & f_i^{min} \le f_i \le f_i^{max}, \\ 0 & f_i \ge f_i^{max} \end{cases} \quad (10)$$

where f_i^{min} and f_i^{max} represent the ideal and nadir values for objective function i, respectively; f_i is objective function value; ρ_i is its fuzzy membership value.

Ideal and nadir values represent the best and worst accessible values of each objective function in the solution space of the problem, respectively. The ideal value for each objective function is obtained by individually optimizing that objective function regardless of other objective functions. Then, we should carry out four individual single-objective optimization tasks to get the ideal value of four objective functions described in the previous subsection. By individually optimizing each objective function, the values of other objective functions are also obtained and they may not be optimal if objective functions are competing; i.e. optimizing one objective function causes others to be deteriorated. Among the obtained values from individual optimizations, the worst value of each objective

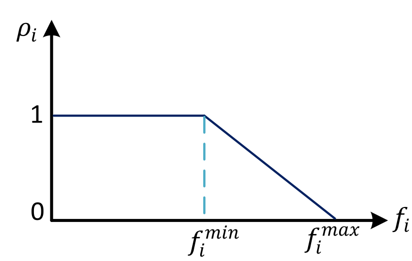

Fig. 1 Trapezoidal fuzzy membership function for objective functions

function gives its nadir value. More details can be found in Akorede et al. (2011).

Fuzzy membership as a function of objective function is depicted in Fig. 1. In this figure, a smaller value of the objective function leads to a larger membership function, which is more preferred when the objective function is for minimization. In the proposed method, four memberships of ρ_{Loss}, ρ_{IVP}, ρ_{LBI}, and ρ_{ACSI} are calculated for the objective functions of loss, IVP, LBI, and $ACSI$, respectively. There are several methods to combine these memberships and constitute an overall fuzzy satisfaction function representing the fitness of the solution of the multi-objective problem. If the combination of objective functions is done carefully without scaling problems, Pareto optimality of the solution can be guaranteed (Esmaili 2013) and, at the same time, it has less computation burden than Pareto-based methods (Esmaili 2013). This type of combining objective functions has been already used in some papers such as Akorede et al. (2011) using some operators. In Gupta et al. (2010) introduced a newer operator named "max geometric mean" that provided better performance than other techniques of combining objective functions. Using this technique, the degree of overall fuzzy satisfaction is computed as follows:

$$\mu_f = (\rho_{Loss} \cdot \rho_{IVP} \cdot \rho_{LBI} \cdot \rho_{ACSI})^{1/4}, \tag{11}$$

where μ_f represents the overall fitness function of the solution. This overall fitness function is the objective function that is maximized in the DSTATCOM optimal allocation and sizing multi-objective problem.

Constraints

The proposed multi-objective problem for DSTATCOM allocation is optimized subject to following constraints.

Power Flow Equations

Active and reactive power balance at each node of the network should be observed using following constraints:

$$PG_i - PD_i = \sum_{j=1}^{N_{bus}} |V_i||V_j||Y_{ij}| \cos(\delta_i - \delta_j - \varphi_{ij}) i = 1, \ldots, N_{bus} \tag{12}$$

$$QG_i - QD_i = \sum_{j=1}^{N_{bus}} |V_i||V_j||Y_{ij}| \sin(\delta_i - \delta_j - \varphi_{ij}) i = 1, \ldots, N_{bus}, \tag{13}$$

where PG_i and QG_i are active and reactive generations at bus i; PD_i and QD_i are active and reactive demands at bus i; V_i and δ_i represent the magnitude and angle of voltage phasor at bus i; $|Y_{ij}|$ and φ_{ij} are the magnitude and angle of ij entry from the bus admittance matrix.

Branch Current Limits

In order to protect cables and feeders against excessive currents, their rating should be taken into account:

$$|I_k| \leq I_k^{max} \quad k = 1, \ldots, N_{br} \tag{14}$$

Bus Voltage Permissible Range

Bus voltages, after the DSTATCOM allocation and sizing problem should remain in their permissible range specified by the system operator:

$$V_{min} \leq V_j \leq V_{max} \quad j = 1, \ldots, N_{bus}, \tag{15}$$

where V_{min} and V_{max} are minimum and maximum allowable voltages, respectively, which are considered $V_{min} = 0.95pu$ and $V_{max} = 1.05pu$.

DSTATCOM Reactive Generation Limits

DSTATCOM output reactive power should be kept within their operational limits given by its manufacturer:

$$Q_{DSTATCOM}^{min} \leq Q_{DSTATCOM} \leq Q_{DSTATCOM}^{max}, \tag{16}$$

where $Q_{DSTATCOM}^{min}$ and $Q_{DSTATCOM}^{max}$ are minimum and maximum generated reactive power by DSTATCOM, respectively, which are considered $Q_{DSTATCOM}^{min} = 0kVAr$ and $Q_{DSTATCOM}^{max} = 10000kVAr$.

DSTATCOM Modeling

The static model of DSTATCOM should consider losses such as transformer and inverter losses. The basic principles and mathematical model of STATCOM and DSTATCOM are similar, therefore the power flow model of STATCOM seems to be suitable for power flow studies of DSTATCOM. In steady state conditions, DSTATCOM behaves similarly to a shunt reactive power source that adjusts the voltage magnitude of the bus where it is to be installed. If the bus i is a load bus of the system with a consumption equal to $P_{Li} + jQ_{Li}$, the model of DSTATCOM on bus i can be considered as a new PV bus j that is added to bus i with its active power set to zero (Kazemtabrizi and Acha 2014). The transformer is modeled by its leakage resistance and reactance; $R_T + jX_T$. This model is illustrated in Fig. 2.

Imperialist Competitive Algorithm (ICA)

The policy of extending the power of an imperial beyond its own boundaries is named imperialism. An imperialist uses different policy for dominating other countries include direct rule or by less clear tools such as control of the

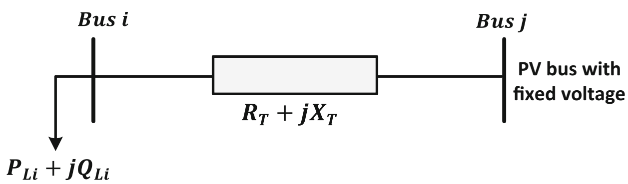

Fig. 2 Static model for DSTATCOM installed in bus *i*

market for goods or raw materials. Atashpaz-Gargari and Lucas (2007) proposed the ICA in 2007. This algorithm as a socio-politically motivated global search strategy has recently been used for optimizing different optimization tasks (Ali 2015; Khabbazi et al. 2009).

Initialization Phase

This algorithm similar to other evolutionary algorithms is initialized by the population of P countries which are generated randomly within the search space. Each country is defined by $contry_i = [p_1, p_2, \ldots, p_{nvar}]$ which $nvar$ is number of decision variables. The best countries with the best fitness function in the initial population are chosen as the imperialists and other countries are as the colonies of these imperialists. The initial empires are built by dividing colonies among imperialists based on imperialist's power. To divide the colonies among imperialists proportionally, the following normalized cost of an imperialist is defined (Khabbazi et al. 2009):

$$C_n = c_n - max\{c_i\}, \tag{17}$$

where c_n and C_n are the cost of nth imperialist and its normalized cost, respectively. The normalized power of each imperialist can be calculated by the normalized cost of all imperialists according to the following equation (Khabbazi et al. 2009):

$$P_n = \left| \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \right|. \tag{18}$$

The empires should be divided into the initial colonies based on their powers; consequently, the initial number of colonies belonging to the nth empires is as follows (Khabbazi et al. 2009):

$$N.C._n = round\{P_n \times N_{col}\}, \tag{19}$$

where $N.C._n$ is the initial number of colonies belonging to the nth empires; N_{col} is the total number of initial colonies. The initial number of colonies are randomly allocated to the nth imperialist. In this algorithm, the bigger empires have more number of colonies, while the weaker ones have less (Khabbazi et al. 2009).

Assimilation Phase

After clustering colonies among the imperialists, the assimilation phase is started. In this phase, the colonies start moving toward their empire. A vector x from the colony to the imperialist is determined for direction of the movement. The vector x is a random variable with having uniform distribution.

$$x \sim U(0, \beta \times d), \tag{20}$$

where d is the distance between the colony and the imperialist state; β is a number greater than one. The reason behind using $\beta > 1$ is that the colonies to get closer to the imperialist state from both sides. For realization this phase, it is not necessary that the colonies movement toward imperialists be done in straight line due to limitations of the searching capability (Khabbazi et al. 2009). For movement toward imperialists, the colonies can be diverted from straight line equal to θ degree. This fact increases the ability of searching more area around the imperialist. θ is a random angle with uniform distribution:

$$\theta \sim U(-\gamma, \gamma), \tag{21}$$

where γ is parameter for regulating the deviation from the original direction. The amounts of β and γ are arbitrary. However, for good convergence of countries to the global minimum, a value of about two for β and about $\pi/4(rad)$ for γ are used in most of implementations (Khabbazi et al. 2009).

Exchanging Phase

When the colonies move toward imperialist, it is possible that a colony get a position with lower cost than the imperialist. In this situation, the colony and the imperialist exchange their positions. After that, the algorithm will progress by the imperialist in the new position and the imperialist will assimilate the colonies in its new position.

Calculation of Total Power of an Empire

Total power of an empire is summation of the power of imperialist country and a percentage of the power of the colonies of an empire as follows:

$$\begin{aligned} T.C. = {} & Cost(imprialist_n) \\ & + \delta\ mean\{(colonies\ of\ empire_n)\}, \end{aligned} \tag{22}$$

where $T.C.$ is the total cost of the nth empire; δ is participation factor of colonies in the total power of empire and it is a positive small number; δ represents good results with a value 0.1 in most of the implementations (Khabbazi et al. 2009).

Imperialistic Competition

This phase is started by selecting a colony of the weakest empire and then finding the possession probability of each empire. The possession probability P_P and the total power of the empire have a proportional relation. The normalized total cost of an empire is obtained as follows:

$$N.T.C._n = T.C._n - max\{T.C._i\}, \tag{23}$$

where $N.T.C._n$ and $T.C._n$ are the normalized total cost and the total cost of nth empire, respectively. The possession probability of each empire is determined by having the normalized total cost as follows:

$$P_{P_n} = \left| \frac{N.T.C._n}{\sum_{i=1}^{N_{imp}} N.T.C._i} \right|. \tag{24}$$

The vector P is generated to cluster the mentioned colonies among empires, this vector is as follows:

$$P = \left[P_{p1}, P_{p2}, P_{p3}, \ldots, P_{pN_{imp}}\right]. \tag{25}$$

After that, the vector R is generated in the same size with P. The elements of this vector are uniformly distributed random numbers.

$$R = \left[r_1, r_2, r_3, \ldots, r_{N_{imp}}\right] r \in U(O, 1). \tag{26}$$

Finally, the vector D is created by subtracting R from P. The mentioned colony (colonies) belonging to an empire which relevant index in D has maximum value. The process of selecting an empire is similar to the roulette wheel process in Genetic Algorithm (GA) (Khabbazi et al. 2009).

Application of the Proposed Algorithm to DSTATCOM Allocation

In this problem, the size and the placement of DSTATCOMs are selected as decision variables. Figure 3 shows the structure of countries in the employed ICA.

As shown in Fig. 3, the countries are in the form of strings the number of cells of which corresponds to the number of buses, N_{bus}. A random number multiple of 5 between 0 and 10,000 kVAr is attributed to each cell with the value of $Dsize_i$. The algorithm then updates the values at each iteration. Since the updated values are most likely not to be multiples of 5, the values are rounded to the closest number which is a multiple of 5. After the convergence, those buses for which a zero value is obtained are not considered for equipment installation.

$Dsize_1$	$Dsize_2$	$Dsize_3$	………	$Dsize_{N_{bus}}$

Fig. 3 Structure of a country

Figure 4 shows the flowchart of the proposed method. To applying the proposed ICA to this problem, the following steps have been considered:

1. Define the input data. The input data which should be provided include system base configuration, line impedances, characteristics of DSTATCOM, the number of countries, the number of empires, number of decision variables in the country, the maximum number of iterations, and the internal parameters of the algorithm (assimilation coefficient and assimilation angle).
2. Generate the initial population (countries). A random number multiple of 5 between 0 and 10,000 kVAr is attributed to each cell.
3. Perform the load flow and calculate the objective functions. A so-called direct approach which is proposed in Jen-Hao (2003) is used to provide load flow solutions. For each individual in the population, based on power flow results, the overall fitness function is calculated by using (11). The violation of the constraints is also verified in this step. The fitness function value associated to a solution is considered to be zero if the constraints are violated for that solution.
4. Selecting the colonies and imperialists and forming empires.
5. Moving the colonies toward their relevant imperialist (assimilation).
6. Exchanging the position of the colony with its imperialist if the cost of the mentioned colony is lower than that in the imperialist.
7. Computing the total cost of all empires, regarding the power of the imperialist and its colonies.
8. Selecting the weakest colony (colonies) from the weakest empires and give it (them) to the empire that has the most likelihood to possess it (imperialistic competition).
9. Removing the powerless empires. The updated values are rounded to the closest number which is a multiple of 5.
10. Check the termination criterion. The termination criterion is the maximum number of iterations. If the number of iterations is equal to the predefined number, go to the step 11; otherwise, go to the step 5.
11. Determine the best solution. After meeting the termination criterion, the process is finished.

Those buses for which a zero value is obtained are not considered for equipment installation.

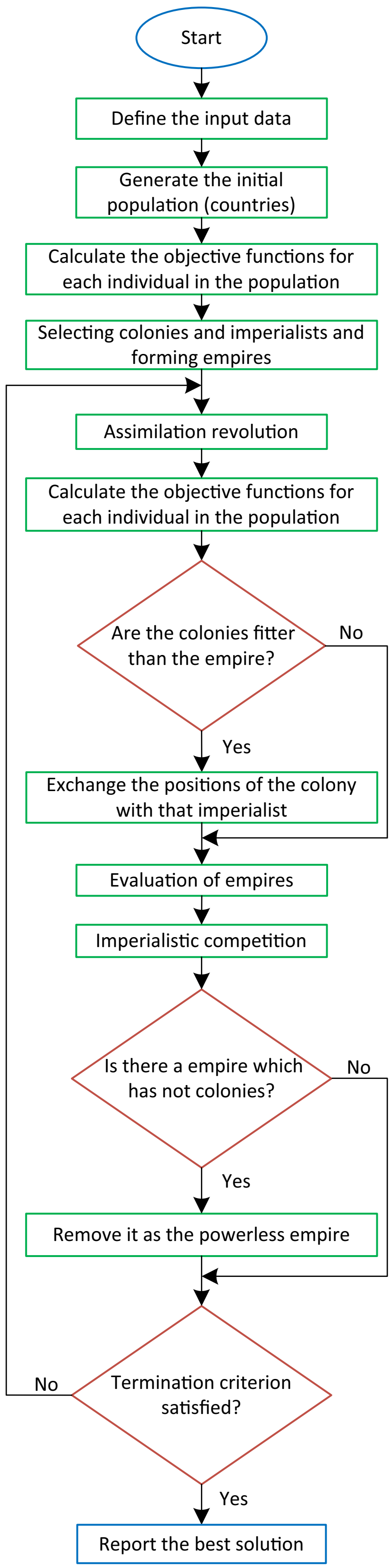

Fig. 4 Flowchart of the proposed ICA for allocation of DSTATCOM

Fuzzy Modeling of Uncertainties

In the third case of simulation, loads of networks are considered uncertain instead of the constant power. Therefore, the principle of uncertain modeling of the networks loads is clarified in this section.

Fuzzy Load Modeling

Using fuzzy numbers is one of the effective methods to model load uncertainty in a distribution system. For representation of uncertainty in load, Triangular Fuzzy Number (TFN) is often used, because it is the most suitable to handle while providing a high-quality explanation of the load uncertainties in networks (Haghifam et al. 2008). In this study, the uncertainty of the load is presented as TFN. $\tilde{P}$ is shown in Fig. 5, which can be written in the form of $\tilde{P} = (P_L, P_M, P_R)$. The likelihoods of the load is described through TFN as a triangular possibility distribution where the load is estimated to be in the vicinity of mean value P_M, no more than P_R and no less than P_L.

Voltage and Current Constraints Modeling

Since the loads are modeled using fuzzy numbers, system variables are considered as TFNs which may have real or imaginary parts; therefore fuzzy domain is utilized to employ mathematical operators. For this reason, result of the system load flow is calculated in the fuzzy domain. Voltage at node k is described as TFN while in this node, deterministic values are used for the upper and lower voltage limits (V_{max} and V_{min}, respectively). Voltage constraint in the fuzzy domain is defined as (Haghifam et al. 2008):

$$\mathrm{V}_{min} \tilde{\leq} \tilde{V}_k \tilde{\leq} \mathrm{V}_{max} \quad k = 1, \ldots, N_{bus}. \tag{27}$$

In this fuzzy equation, simple 'true' or 'false' values cannot be related to voltage. However voltage constraint is violated only if definite degree of possibility expressed as follows is obtained:

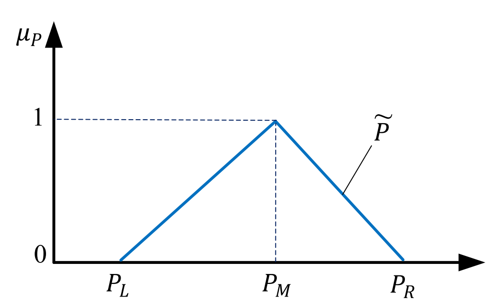

Fig. 5 Triangular membership function for power load

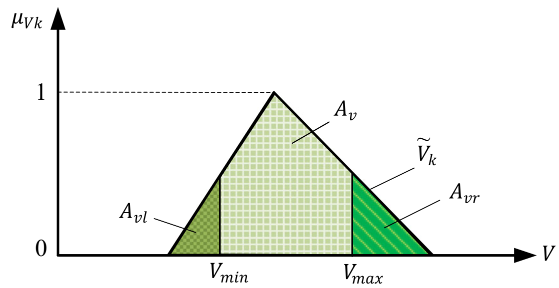

Fig. 6 Voltage constraint in fuzzy domain

$$S_{V_k} = \frac{A_{vl} + A_{vr}}{A_v + A_{vr} + A_{vl}}, \tag{28}$$

where A_v is the area under the membership function between the V_{min} and V_{max}. Moreover, regarding Fig. 6, the A_{vl} and A_{vr} are the areas under the membership function, at the left side of V_{min} and the right side of V_{max}, respectively. As it is mentioned earlier, the voltage constraint is violated only if definite degree of possibility S_{V_k} is obtained. In order to change the possibility S_{V_k}, the parameters of the $\tilde{V}_k$ must be changed. This can be achieved by changing the ratio between areas (A_v, A_{vl}, and A_{vr}).

For a specified condition in a distribution network, by running the fuzzy load flow, the currents of the network branches are also happen to be fuzzy numbers. The TFN ($\tilde{I}_k$) is used in Fig. 7 to represent the current of branch k, where permissible rate of currents in this branch is specified by I_k^{max}. Alike to the fuzzy description of the voltage constraint, same concepts are utilized for the definition of current constraint in the fuzzy domain as follows (Haghifam et al. 2008):

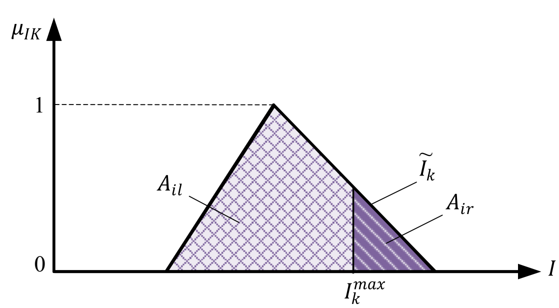

Fig. 7 Current constraint in fuzzy domain

$$\tilde{I}_k \tilde{\leq} I_k^{max} \quad k = 1, \ldots, N_{br}. \tag{29}$$

In addition to the fuzzy description of current constraint, this current constraint is violated only if definite degree of possibility is obtained. Degree of possibility for violation of the current constraint can be defined as:

$$S_{I_k} = \frac{A_{ir}}{A_{il} + A_{ir}}, \tag{30}$$

where A_{il} and A_{ir} are the areas under the membership function at the left side and right side of the permissible rating of currents in branch k, i.e. I_k^{max}, respectively. Normally, greater possibility in violation of voltage constraint at node k and current of branch k is presumable by larger values of S_{V_k} and S_{I_k}.

With fuzzy modeling of the loads in which variables expressed as TFNs having real or imaginary parameters, the objective functions are obtained as fuzzy numbers. The objective functions relating triangular fuzzy values are to be compared and classified to investigate various planning solutions. In this paper, the defuzzification of the objective function value is implemented using removal function $R(\tilde{b})$, which for a TFN (b_L, b_M, b_R) is defined as:

$$R(\tilde{b}) = (b_L + 2b_M + b_R)/4. \tag{31}$$

Simulation Results

The proposed method has been tested on two different systems: a 33-bus and a 69-bus balanced distribution system. The simulations have been implemented using MATLAB 7.10.0 (R2010a; The MathWorks, Natick, Massachusetts, USA) on an Intel(R) Core(TM) i5, 2.40-GHz, PC with 4-GB RAM. The constant power load model has been considered for simulations of both distribution test systems and three load levels are defined as follows:

$$P_{L,new} + Q_{L,new} = LF \times (P_L + Q_L), \tag{32}$$

where LF is Load Factor and equal to 0.5, 1.0 and 1.6 for light, medium and peak load, respectively. The parameters of optimization algorithm and objective function for the examined test systems are shown in Tables 1 and 2. The parameters in Table 1 are chosen from literature (Ali 2015; Khabbazi et al. 2009) and are further improved by a trial and error process.

Table 1 Parameters of the proposed algorithm for the examined test systems

Test system	Number of countries	Number of empires	β	γ	Max. iterations	$Trial^{Max}$
IEEE 33-bus test system	50	6	2	0.5	60	30
IEEE 69-bus test system	50	6	2	0.5	100	30

Table 2 Parameters of the objective function for the examined test systems

$COST_{DSTATCOM}\left(\frac{\$}{kVAR}\right)$	N (year)	$K_L\left(\frac{\$}{kWh}\right)$	B	K_{TDP}	T
50	30	0.06	0.1	1	8760

Case study 1: IEEE 33-bus balanced test system

This is a 12.66 kV radial distribution system that has 33 buses and 32 branches and system data is derived from Taher and Afsari (2014). The total real and reactive power loads of this radial system that is shown in Fig. 8 are 3715 kW and 2300 kVAr, respectively.

This system has the initial power loss equal to 202.677 kW and minimum bus voltage equal to 0.91 *pu*. The multi-objective fitness function is optimized by ICA and the parameters of the algorithm were selected in accordance with Table 1. The Results derived by optimizing the multi-objective fitness function for this test system are shown in Table 3. Since the proposed algorithm has a random number generation basis, several runs should be done to obtain a group of solutions and finally the best solution is selected. In this case study, the best solution is chosen after 30 runs.

According to Table 3, application of the DSTATCOM for all the load levels has led to improvement of the

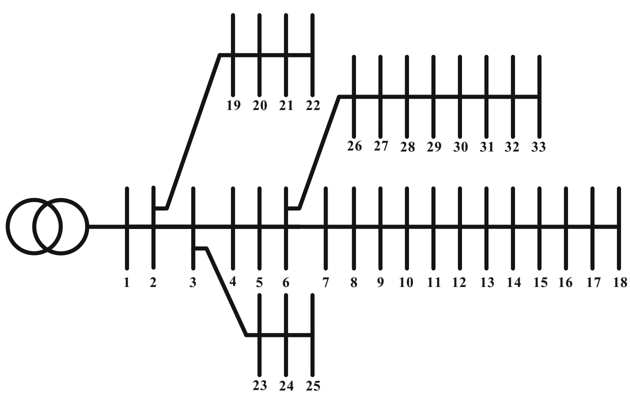

Fig. 8 Single line diagram of IEEE 33-bus system

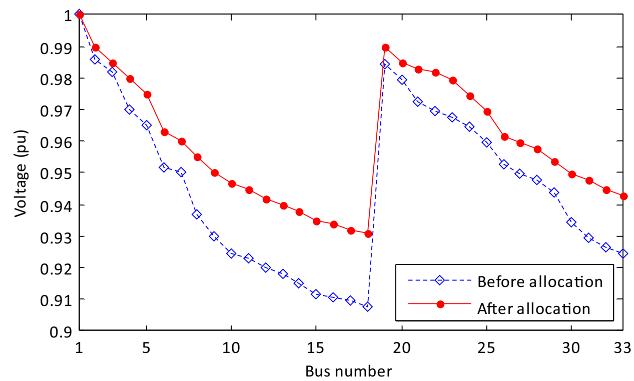

Fig. 9 Bus voltage profile of the 33-bus test system with multiple DSTATCOMs

objectives of the optimization including loss reduction, voltage profile improvement and *LBI* enhancement. In this table, the base case results which are the results of the system without installation of the DSTATCOM are extracted using the medium load level. By comparison of the medium load and the base case results, it is seen that loss is reduced up to 31 percent, the minimum voltage has 1.8 percent increase and *LBI* has experienced 73 percent reduction.

Figure 9 shows the bus voltage profile before and after optimal DSTATCOM placement for a medium load. As shown in this figure, the bus voltage profile is obviously improved with optimal allocation of DSTATCOM using the ICA.

In order to evaluate the performance of the ICA with other algorithms, results of the proposed algorithm are compared with Bacterial Foraging Optimization Algorithm (BFOA), Bat Algorithm (BA), Harmony Search Algorithm (HSA) and IA. The results of the comparison are given in Table 4. According to the results, all three algorithms have proposed the same location for installation of the DSTATCOM, i.e. bus 10 and bus 30, however sizes of the DSTATCOMs are different. The largest size of the DSTATCOM is proposed by BFOA which is 1800 kVAr and the smallest size is proposed by IA. Moreover, the proposed ICA has offered less kVAr in comparison with

Table 3 Results of multi-objective optimization by the proposed algorithm for 33-bus test system under different types of load factor

	Load factor			
	Base case	Light load	Medium load	Peak load
Optimal size (kVAr) and location	–	195 (10), 475 (30)	455 (10), 1005 (30)	790 (10), 1670 (30)
Total kVAr	–	670	1460	2460
P_{loss}(kW)	202.67	30.766	140.24	366.324
Q_{loss}(kVAr)	135.24	19.786	93.67	236.897
$V_{\min}(pu)$	0.9131	0.9789	0.9301	0.9001
LBI (*pu*)	0.1575	0.0427	0.0435	0.0476

Table 4 Comparison of the results of ICA with other algorithms for the 33-bus test system

	Proposed algorithm	BFOA (Yuvaraj et al. 2015b)	BA (Yuvaraj et al. 2015b)	HSA (Yuvaraj et al. 2015a)	IA (Taher and Afsari 2014)
Optimal size (kVAr) and location	455 (10), 1005 (30)	600 (10), 1200 (30)	450 (10), 995 (30)	1150 (30)	962.49 (12)
Total kVAr	1460	1800	1445	1150	962.49
P_{loss} (kW)	140.24	137.50	146.73	143.97	171.79
Reduction in P_{loss} (%)	30.31	32.15	27.60	28.97	15.23
Q_{loss}(kVAr)	93.67	92.01	95.63	96.47	115.26
Reduction in Q_{loss} (%)	30.73	31.96	29.28	28.67	14.77
$V_{\min}$ (pu)	0.9301	0.9789	0.9299	0.9236	0.9258
$LBI(pu)$	0.0435	–	–	–	–
$ACSI$	0.7684	0.7718	0.7998	–	–
Computational time (s)	8.86	11.06	9.85	–	–

BFOA. Therefore, the percentage of active and reactive power loss reduction and also the minimum voltage are slightly smaller than the BFOA. However, regarding the economical index (ACSI), the proposed ICA method provides improvements over the BFOA and BA. In addition, in terms of the calculation time, the proposed algorithm is remarkably faster than the two other algorithms.

It is to be noted that in the considered objective function, in comparison to Yuvaraj et al. (2015b), as well as the cost associated with loss and installation of DSTATCOM, the LBI and IVP indices are also involved. It is clear that inclusion of additional terms in a goal function deviates the solution from that which is provided when these terms had not been considered.

Figure 10 indicates the convergence characteristic of the ICA for the multi-objective function in case study 1. It is shown that after 28 iterations, the ICA reaches to full convergence and fitness function value remains constant at approximately 0.852.

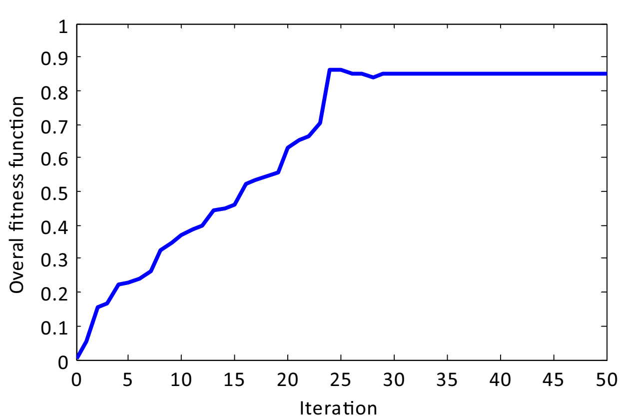

Fig. 10 Convergence rate of the multi-objective function in case study 1 using ICA

Case Study 2: IEEE 69-Bus Balanced Test System

The second case study is a large-scale test system with 69 buses and 68 branches. Details for the line and load data of the system can be found in Taher and Afsari (2014). This system is shown in Fig. 11 with total real and reactive power loads of 3.80 MW and 2.69 MVAr, respectively. This system has the initial power loss equal to 225 kW and minimum bus voltage equal to 0.9090 *pu*.

The optimal solutions for optimizing the multi-objective function are presented in Table 5. The results indicated for the multi-objective function were the best results obtained after 30 instances of running the proposed method. Results presented in Table 5 are the results of the optimization for three levels of the loading of the system. As it is seen in this table, in all the load levels, the same location is offered for DSTATCOM installation. However, the size of the DSTATCOM is increased by the growth of the loading of the system. On the other hand, the active and reactive power losses during the medium loading of the system, which is dominant loading for the distribution system, have experienced 34.6 percent reduction compared to the base case. Installation of the DSTATCOM has led to 2.5 percent enhancement of the minimum voltage and improvement of the load balancing in feeders up to 57.1 percent.

For assessing the performance of the proposed algorithm in comparison with other algorithms, results of the proposed algorithm are compared with the BFOA and BA. The results of the comparison are given in Table 6. Based on the results, all three algorithms have proposed the same location for installation of the DSTATCOM, i.e. bus 15 and bus 61, but sizes of the DSTATCOMs are different. The BFOA has proposed the greatest size of the DSTATCOM, which is 1910 kVAr and the least size is proposed by BA.

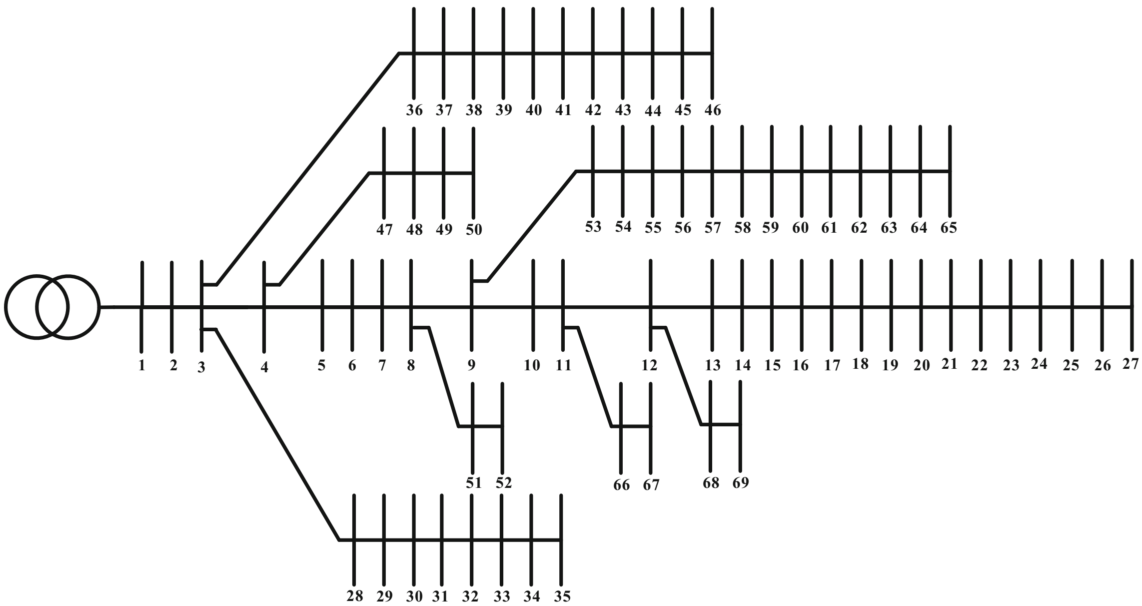

Fig. 11 Single line diagram of IEEE 69-bus system

Table 5 Results of multi-objective optimization by the proposed algorithm for 69-bus test system under different types of load factor

	Load factor			
	Base case	Light load	Medium load	Peak load
Optimal size (kVAr) and location	–	165 (15), 885 (61)	375 (15), 1280 (61)	685 (15), 2350 (61)
Total kVAr	–	1050	1655	3035
P_{loss}(kW)	225	35.974	147.35	412.657
Q_{loss}(kVAr)	102.2	18.603	72.382	195.235
$V_{\min}$ (pu)	0.9090	0.9634	0.9324	0.9001
LBI (pu)	0.1345	0.0567	0.0577	0.0589

Table 6 Comparison of the results of ICA with other algorithms for the 69-bus test system

	Proposed algorithm	BFOA (Yuvaraj et al. 2015b)	BA (Yuvaraj et al. 2015b)
Optimal size (kVAr) and location	375 (15), 1280 (61)	480 (15), 1430 (61)	330 (15), 1220 (61)
Total kVAr	1655	1910	1550
P_{loss} (kW)	147.35	148.07	146.73
Reduction in P_{loss} (%)	34.6	34.19	34.78
Q_{loss} (kVAr)	72.382	68.76	68.43
Reduction in Q_{loss} (%)	29.17	32.72	33.04
$V_{\min}$ (pu)	0.9324	0.9332	0.9299
LBI (pu)	0.0577	–	–
$ACSI$	0.8125	0.8236	0.8312
Computational time (s)	9.37	11.06	9.85

However, the proposed ICA has offered less MVAr in comparison with BFOA. As a result, the percentage of active and reactive power loss reduction, as well as the minimum voltage, are a bit smaller than the BFOA. Moreover, regarding the ACSI which is an economical index, the proposed ICA has a better performance. Besides, in terms of the calculation time, the proposed algorithm is remarkably faster than two other algorithms.

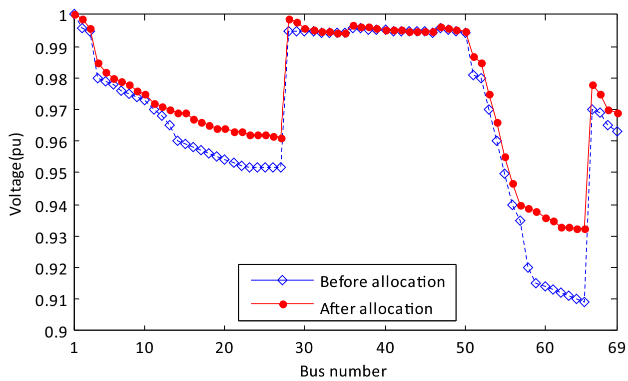

Fig. 12 Bus voltage profile of the 69-bus test system with multiple DSTATCOMs

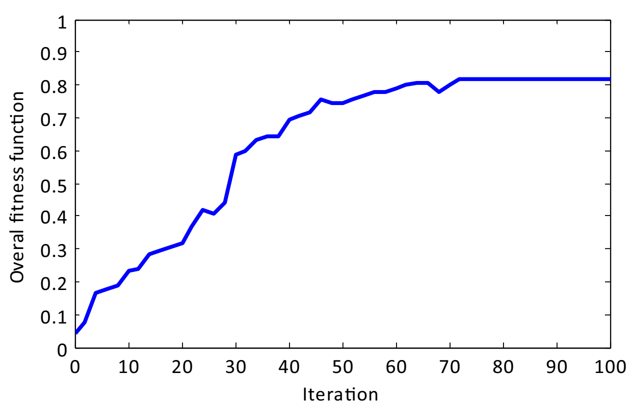

Fig. 13 Convergence rate of the multi-objective function in case study 2 using ICA

Figure 12 shows the bus voltage profile before and after optimal DSTATCOM placement. Regarding this figure, by installation of the DSTATCOM, the optimum voltage profile at all the busbars of the distribution system is improved. Furthermore, Fig. 13 shows the convergence characteristic of the ICA for case study 2. As shown in Fig. 13, fitness function after 73 iterations converged to 0.82.

Case Study 3: the 33-Bus Balanced Test System with Load Uncertainty

In the third case study of the paper, the earlier discussed 33-bus distribution system is utilized with taking into account the fuzzy modeling of network loads uncertainty which is explained in Sect. 5. The simulation results of these scenarios including the distribution system with light, medium, and peak load are presented in Table 7. As previously mentioned, since loads of the network are modeled using fuzzy numbers, the objective functions are obtained in the form of TFNs and defuzzification of the objective function values are implemented using removal function, hence they are transformed into integers. Regarding the Table 7, the objective functions of base case are 210.30 kW (P_{loss}), 141.67 kVAr (Q_{loss}), 0.9037 (V_{min}), and 0.1597 (*LBI*). By comparison of these values with the corresponding values in Table 3, it can be concluded that in the uncertainty situation, the objective functions are weaker. According to the Table 7, the location of DSTATCOM has not changed in comparison to the Table 3, which uncertainties are not considered, however the size of DSTATCOM is slightly reduced. Since the loads are modeled in fuzzy domain, the variables are expressed as TFNs having real or imaginary parameters instead of integers, the objective functions are obtained as fuzzy numbers and they get small distance from their suitable values.

Conclusions

This paper has presented a new long-term planning for optimal location and sizing of the DSTATCOMs in radial distribution networks using the imperialist competitive algorithm. The multi-objective optimization problem includes loss reduction, voltage profile improvement, feeder load balancing, and cost reduction. Considering the proposed long-term planning, the costs of distribution system are declined; therefore it can provide more interests

Table 7 Results of multi-objective optimization by the proposed algorithm for the 33-bus test system under different types of load factor considering load uncertainty

	Load factor			
	Base case	Light load	Medium load	Peak load
Optimal size (kVAr) and location	–	185 (10), 465 (30)	435 (10), 995 (30)	775 (10), 1660 (30)
Total kVAr	–	650	1430	2435
P_{loss} (kW)	210.30	33.876	142.77	375.127
Q_{loss} (kVAr)	141.67	22.648	95.87	241.925
$V_{\min}$ (pu)	0.9037	0.9425	0.9189	0.9000
LBI (pu)	0.1597	0.0465	0.0477	0.0498

for utility services. The proposed algorithm is implemented in a 33-bus and a 69-bus radial distribution system and the results are compared with other meta-heuristic optimization algorithms. Based on the numerical results, the optimal placing and sizing of the DSTATCOM in distribution systems improve all of the mentioned objectives; hence the proposed approach can be used by utility services for optimal DSTATCOM allocation and sizing in the distribution systems. According to obtained results from case study 1, the improvements in the active power loss, reactive power loss, V_{min} and *LBI* indices are 30.31, 30.37, 1.8, and 72.3%, respectively. Also in case study 2, the considered objectives such as active power loss, reactive power loss, V_{min} and *LBI* are improved by 34.6, 30.73, 0.085, and 53.13%, respectively. Finally, considering the results of case study 3, the improvements in the active power loss, reactive power loss, V_{min} and *LBI* indices are 32.11, 32.32, 1.6, and 70.13%, respectively.

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