Vectorial approximations of infinite-dimensional covariance descriptors for image classification

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The class of symmetric positive definite (SPD) matrices, especially in the form of covariance descriptors (CovDs), have been receiving increased interest for many computer vision tasks. Covariance descriptors offer a compact way of robustly fusing different types of features with measurement variations. Successful examples of applying CovDs addressing various classification problems include object recognition, face recognition, human tracking, texture categorization, visual surveillance, etc.

As a novel data descriptor, CovDs encode the second-order statistics of features extracted from a finite number of observation points (e.g., the pixels of an image) and capture the relative correlation of these features along their powers as a means of representation. In general, CovDs are SPD matrices and it is well known that the space of SPD matrices (denoted by Sym^+) is not a subspace in Euclidean space but a Riemannian manifold with nonpositive curvature. As a consequence, conventional learning methods based on Euclidean geometry are not the optimal choice for CovDs, as proven in several prior studies.

In order to better cope with the Riemannian structure of CovDs, many methods based on non-Euclidean metrics (e.g., affine-invariant metrics, log-Euclidean metrics, Bregman divergence, and Stein metrics) have been proposed over the last few years. In particular, the log-Euclidean metric possesses several desirable properties which are beneficial for classification: (i) it is fast to compute; (ii) it defines a true geodesic on Sym^+ ; and (iii) it comes up with

a broad variety of positive definite kernels.

More recently, infinite-dimensional RCovDs-based methods have shown encouraging results in image classification, as opposed to low-dimensional RCovDs [1]. It has been demonstrated that several types of Riemannian metrics for infinite-dimensional RCovDs can be computed with a mapping from the low-dimensional Euclidean space to a reproducing kernel Hilbert space (RKHS) and the corresponding Gram matrices. Normally, infinite-dimensional CovDs offer better discriminatory power than their low-dimensional counterparts. Nevertheless, all infinite-dimensional CovDs are rank-deficient because it is impossible to have an infinite number of observations in practice. Although this problem may, to some degree, be relieved by matrix-regularization (adding a small perturbation to avoid matrix singularity), it prevents the utilization of Riemannian geometry directly. Moreover, the computational complexity of analyzing infinite-dimensional CovDs is non-negligible.

Faraki et al. [2] used two specific feature mappings (random Fourier features and the Nyström method) for estimating infinite-dimensional CovDs, and derived resulting approximate infinite-dimensional CovDs as an alternative, to overcome the aforementioned issues. Motivated by this novel idea of estimating infinite-dimensional CovDs, we propose a method to obtain low-dimensional vectorial approximations of infinite-dimensional CovDs instead. We have two main contributions. Firstly, in contrast to the approximate infinite-dimensional CovDs generated from estimated data of certain kernels based on Euclidean distance, our vectorial approximations are directly estimated from valid kernels based on Riemannian metrics.

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More specifically, the proposed approach estimates data on an RKHS of the Riemannian manifold rather than an RKHS of the original vectorial feature space and thus preserves the Riemannian geometry information of infinite-dimensional CovDs better. Secondly, the subsequent vectorial approximations are in a more compact form than the approximate infinite-dimensional CovDs. The proposed approach can be considered as a dimensionality reduction from infinite-dimensional CovDs to low-dimensional vectors. In addition, classical learning methods based on Euclidean geometry can be directly implemented without difficulties. We use the Nyström method to learn the feature mapping from the RKHS of infinite-dimensional CovDs to a low-dimensional Euclidean space. With this feature mapping and valid kernels for infinite-dimensional CovDs, our approach not only preserves the intrinsic Riemannian geometry but also reduces the dimensionality, and is therefore better suited for classification. We have evaluated our approach on several standard datasets for image classification. Experimental results show the accuracy improvements of vectorial approximations over covariance approximations. Figure 1 shows the

entire procedure of our approach and Faraki et al.'s approach [2].

1 Review of covariance descriptor

Region covariance matrices were introduced as a novel region descriptor for human face detection and classification. The process of generating CovDs from images is illustrated in Fig. 2. The data matrix $X = [x_1, \dots, x_n]$ consists of n observations extracted from an image sample, $x_i \in \mathbb{R}^d$ denoting the i th observation. The CovDs is defined as

$$C = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})(x_i - \bar{x})^T = \frac{1}{n} X J_n X^T \quad (1)$$

where $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ is the mean of all n observations, and J_n is the centering matrix, defined by $J_n = I_n - \frac{1}{n} \mathbf{1}_n \mathbf{1}_n^T$ in which $\mathbf{1}_n$ as a column vector of n ones. J_n is a symmetric matrix, with $\text{rank}(J_n) = n - 1$, satisfying $J_n^2 = J_n$.

Assume we have an implicit mapping $\phi: \mathbb{R}^d \rightarrow \mathcal{H}$ from d -dimensional Euclidean space to the RKHS $\mathcal{H}$, whose dimensionality $|\mathcal{H}|$ could be considered as potentially infinite. With the mapped data matrix $X_\phi = [\phi(x_1), \dots, \phi(x_n)]$ and Eq. (1), a CovDs C_ϕ

Fig. 1 Conceptual comparison of our approach and Faraki et al.'s approach for image classification. Faraki et al.'s approach follows (a)–(d), (a) first mapping image features in low-dimensional Euclidean space to an infinite-dimensional Hilbert space (b) and then applying the Nyström method to learn a feature mapping from Hilbert space to another low-dimensional Euclidean space (c), which can be used to obtain the finite-dimensional approximations of infinite-dimensional CovDs (d). In contrast, the proposed approach follows the path (a)–(b)–(e)–(f) to learn a feature mapping from the Riemannian manifold of infinite-dimensional CovDs (e) to a new low-dimensional Euclidean space (f) to obtain the vectorial approximations.

Fig. 2 Conceptual illustration of generating CovDs from images. $F(x, y)$ is the feature vector at position (x, y) in the image sample, and is composed of information from different features such as intensity, gradient, Gabor wavelets, etc.

in the RKHS $\mathcal{H}$ is defined as

$$C_\phi = \frac{1}{n} X_\phi J_n X_\phi^T \quad (2)$$

This type of data embedding into an RKHS is common in many applications and seems to lead to impressive results. However, the use of infinite-dimensional CovDs is restricted in practice. Since $|\mathcal{H}| \gg n$, the CovDs C_ϕ is always rank-deficient and thus may only be positive semi-definite, with rank at most $n - 1$, which means it is on the boundary of the positive cone and at an infinite distance from any SPD matrices. In order to apply the theory of Sym^+ , regularization is applied as $C'_\phi = C_\phi + \lambda I$, with $\lambda > 0$. Now, C'_ϕ is strictly positive and invertible, which are necessary for Riemannian metrics.

2 Vectorial approximations of CovDs

In this section, we present the proposed approach for obtaining vectorial approximations of infinite-dimensional CovDs. The key idea is to estimate infinite-dimensional CovDs via valid kernels based on Riemannian metrics, resulting in low-dimensional vectors, to obtain better classification accuracy. Before delving into the detail, we start with a brief description of the metrics and corresponding kernels for CovDs.

2.1 Riemannian metrics

An SPD matrix is a symmetric matrix with the property that all its eigenvalues are positive. As mentioned above, the space of SPD matrices is not a Euclidean space but a Riemannian manifold. Each point on the Riemannian manifold has a well-defined continuous collection of scalar products defined on its tangent space and is endowed with an associated Riemannian metric. A Riemannian metric makes it possible to define various geometric notions on the Riemannian manifold, such as angles, distances, etc. Since the manifold is curved, the distances specify

the length of the shortest curve that connects the points, also known as geodesics.

Log-Euclidean metric. One of the most commonly encountered Riemannian metrics on Sym^+ is the log-Euclidean metric, which results in classical Euclidean computations as

$$d_{LEM}(C_1, C_2) = \|\log(C_1) - \log(C_2)\|_F \quad (3)$$

where $C_1, C_2 \in Sym^+$, and $\|\cdot\|_F$ denotes the matrix Frobenius form. $\log(\cdot)$ is the common matrix logarithm operator. The eigen-decomposition of a CovDs C is given by $C = U \Sigma U^T$, and the logarithm is calculated using $\log(C) = U \log(\Sigma) U^T$. As shown in Ref. [3], Sym^+ endowed with a log-Euclidean metric is also an inner product space, with inner product $\langle C_1, C_2 \rangle_{\log} = \text{tr}(\log(C_1) \log(C_2))$. A broad variety of kernels can be developed.

Log-Hilbert-Schmidt metric. A generalization of the log-Euclidean metric on the Riemannian manifold of positive definite matrices to the infinite-dimensional setting is the log-Hilbert-Schmidt metric. It is applied in particular to compute distances between CovDs in an RKHS, for which explicit formulae and the inner product are obtained via the Gram matrices induced by an implicit mapping ϕ and the corresponding positive definite kernel K . Let $X = [x_1, \dots, x_n]$, $Y = [y_1, \dots, y_n]$ be two data matrices, and $C'_{\phi(x)} = C_{\phi(x)} + \alpha I$, $C'_{\phi(y)} = C_{\phi(y)} + \beta I$ are CovDs in the RKHS $\mathcal{H}$, as defined in Section 2. Let K_x , K_y , and $K_{x,y}$ be the $n \times n$ Gram matrices of the positive definite kernel K . Let $A = \frac{1}{\sqrt{\lambda n}} X_\phi J_n$ and $B = \frac{1}{\sqrt{\lambda n}} Y_\phi J_n$, so that $A^T A = \frac{1}{\lambda n} J_n K_x J_n$, $B^T B = \frac{1}{\lambda n} J_n K_y J_n$, $A^T B = \frac{1}{\lambda n} J_n K_{x,y} J_n$. Let N_A and N_B be the numbers of nonzero eigenvalues of spectral decompositions $A^T A = U_A \Sigma_A U_A^T$ and $B^T B = U_B \Sigma_B U_B^T$. The log-Hilbert-Schmidt metric between $C'_{\phi(x)}$ and $C'_{\phi(y)}$ is defined as

$$\begin{cases} d_{LHSM}(C'_{\phi(x)}, C'_{\phi(y)}) = \text{tr}[\log(I_{N_A} + \Sigma_A)]^2 \\ \quad + \text{tr}[\log(I_{N_B} + \Sigma_B)]^2 - 2C_{AB} + (\log \alpha - \log \beta)^2 \\ C_{AB} = 1_{N_A}^T \log(I_{N_A} + \Sigma_A) \Sigma_A^{-1} \\ \quad \cdot (U_A^T A^T B U_B \circ U_A^T A^T B U_B) \Sigma_B^{-1} \log(I_{N_B} + \Sigma_B) 1_{N_B} \end{cases} \quad (4)$$

where $\circ$ denotes the Hadamard (element-wise) product and the log-Hilbert-Schmidt inner product is defined as $\langle C'_{\phi(x)}, C'_{\phi(y)} \rangle_{\log HS} = C_{AB} + (\log \alpha)(\log \beta)$.

2.2 Nyström method

The Nyström method was used in Ref. [2] for learning a feature mapping $\zeta: \mathbb{R}^d \rightarrow \mathbb{R}^D$ to obtain $D \times D$ infinite-dimensional CovDs estimates. Let $X_\phi = [\phi(x_1), \dots, \phi(x_n)]$ be the mapped data matrix defined in Section 2, so that $\langle \phi(x_i), \phi(x_j) \rangle_{\mathcal{H}} \simeq \zeta(x_i)^T \zeta(x_j)$. In other words, a rank D approximation of kernel $k(x, y) = \langle \phi(x), \phi(y) \rangle_{\mathcal{H}}$ can be computed as $Z^T Z$, $Z = [\Sigma^{1/2} V]$. Σ and V are the top D eigenvalues and corresponding eigenvectors of $K = [k(x_i, x_j)]$, $d \leq D \leq n$. Estimates of $\phi(x_i)$ can be obtained with the mapping $\zeta(\cdot)$ by

$$\zeta(x_i) = \Sigma^{1/2} V (k(x_i, x_1), \dots, k(x_i, x_n))^T \quad (5)$$

The corresponding estimates of CovDs in $\mathcal{H}$ are obtained as

$$C_\zeta = \frac{1}{n} X_\zeta J_n X_\zeta^T \quad (6)$$

where $X_\zeta = [\zeta(x_1), \dots, \zeta(x_n)]$.

To achieve the aim of effective and compact infinite-dimensional CovDs estimation, we propose a revised scheme. Rather than kernels based on Euclidean distance, we use kernels based on Riemannian metrics to learn a mapping $\zeta': \text{Sym}^+ \rightarrow \mathbb{R}^{D'}$ with the Nyström method. Given a set $\mathcal{C} = \{C_1, \dots, C_m\}$ of m CovDs, we obtain a D' -dimensional vector representation by modifying Eq. (5) to

$$\zeta'(C_i) = \Sigma_{\text{LEM}}^{1/2} V_{\text{LEM}} (k_{\text{LEM}}(C_i, C_1), \dots, k_{\text{LEM}}(C_i, C_m))^T \quad (7)$$

where k_{LEM} is a valid kernel based on the log-Euclidean metric, Σ_{LEM} and V_{LEM} are the top D' eigenvalues and corresponding eigenvectors of the $m \times m$ kernel matrix $K_{\text{LEM}} = [k_{\text{LEM}}(C_i, C_j)]$, $D' \leq m$. This gives vectorial approximations of CovDs in set $\mathcal{C}$ as $\mathcal{V} = \{\zeta'(C_1), \dots, \zeta'(C_n)\}$, which can be conveniently used for classification in Euclidean space.

Obviously, we can extend this approach to estimate infinite-dimensional CovDs. Given a set of m infinite-dimensional CovDs, $\mathcal{C}' = \{C'_{\phi,1}, \dots, C'_{\phi,m}\}$, its D' -dimensional vectorial approximation $\mathcal{V}' = \{\zeta'(C'_{\phi,1}), \dots, \zeta'(C'_{\phi,m})\}$ can be obtained as the same in Eq. (7) by

$$\zeta'(C'_{\phi,i}) = \Sigma_{\text{LHSM}}^{1/2} V_{\text{LHSM}} (k_{\text{LHSM}}(C'_{\phi,i}, C'_{\phi,1}), \dots, k_{\text{LHSM}}(C'_{\phi,i}, C'_{\phi,m}))^T \quad (8)$$

and k_{LHSM} is a valid kernel based on the log-Hilbert-Schmidt metric, Σ_{LHSM} and V_{LHSM} are the top D' eigenvalues and corresponding eigenvectors of the $m \times m$ kernel matrix $K_{\text{LHSM}} = [k_{\text{LHSM}}(C'_{\phi,i}, C'_{\phi,j})]$, $D' \leq m$. The process of estimating infinite-dimensional CovDs using the Nyström method and kernels based on log-Hilbert-Schmidt metric is schematically represented in Algorithm 1.

3 Evaluation

The proposed approach has been evaluated on standard datasets for three image classification tasks: material texture categorization, virus cell identification, and human face recognition. Figure 3 shows image samples from different datasets.

3.1 Implementation details

In order to demonstrate the empirical performance of our approach, we compare the accuracy of low-dimensional CovDs, approximate infinite-dimensional CovDs and vectorial approximations of infinite-dimensional CovDs with a simple nearest neighbor classifier, and we also provide classification results of partial least squares (PLS) based

Algorithm 1 Estimating infinite-dimensional CovDs

Input: A set of infinite-dimensional CovDs $\mathcal{C} = \{C'_{\phi,1}, \dots, C'_{\phi,m}\}$, a kernel k_{LHSM} based on the log-Hilbert-Schmidt metric and the target dimensionality D' .
Output: Vectorial approximations $\mathcal{V}' = \{\zeta'(C'_{\phi,1}), \dots, \zeta'(C'_{\phi,m})\}$.
 Compute the kernel matrix $K_{\text{LHSM}} = [k_{\text{LHSM}}(C'_{\phi,i}, C'_{\phi,j})]$;
 Compute the top D' eigenvalues Σ_{LHSM} and corresponding eigenvectors V_{LHSM} of K_{LHSM} ;
for $i = 1 \rightarrow m$ **do**
 Compute $\zeta'(C'_{\phi,i})$ using Eq. (8);
end for

Fig. 3 Several image samples from 3 different datasets: UIUC Material, Virus, and FERET.

techniques. In particular, low-dimensional CovDs combined with kernelized PLS is similar to the method of covariance discriminant learning (CDL) [3], which can be considered as the state-of-the-art for CovDs-based classification. Different algorithms compared in our experiments are as follows:

- $\text{Cov} \cdot \text{NN}_{\text{LEM}}$: Low-dimensional CovDs with log-Euclidean metric based NN classifier.
- $\text{Cov}'_{\text{Inf}} \cdot \text{NN}_{\text{LEM}}$: Approximate infinite-dimensional CovDs obtained by the Nyström method with log-Euclidean metric based NN classifier, which is the same as the method in Ref. [2] but replacing the affine-invariant metric with the log-Euclidean metric.
- $\text{Vec}'_{\text{Inf}} \cdot \text{NN}_{\text{EM}}$: Vectorial approximations of infinite-dimensional CovDs with Euclidean metric based NN classifier.
- $\text{Cov} \cdot \text{KPLS}$: Low-dimensional CovDs with kernelized PLS.
- $\text{Cov}'_{\text{Inf}} \cdot \text{KPLS}$: Approximate infinite-dimensional CovDs with kernelized PLS.
- $\text{Vec}'_{\text{Inf}} \cdot \text{PLS}$: Vectorial approximations of infinite-dimensional CovDs with PLS.

Following standard practice [2], we extracted original features from each image sample and utilized a Gaussian kernel to generate infinite-dimensional CovDs. Gaussian kernels based on the log-Euclidean metric and log-Hilbert–Schmidt metric were adopted as valid Riemannian kernels for low-dimensional CovDs and infinite-dimensional CovDs respectively. All parameters and the target dimensionality were determined by cross-validation. All algorithms were performed with MATLAB software and a quad-core 3.0 GHz CPU.

3.2 Material texture categorization

For the first task, we used the UIUC Material dataset [4], which contains 18 categories of materials taken in the wild. Each category has 12 images of various sizes with various geometric details included. For each pixel at coordinate (x, y) in an image sample I , we extract a feature vector:

$$F_{x,y} = \left[I_{x,y}^R, I_{x,y}^G, I_{x,y}^B, \left| \frac{\partial I}{\partial x} \right|, \left| \frac{\partial I}{\partial y} \right|, \left| \frac{\partial^2 I}{\partial x^2} \right|, \left| \frac{\partial^2 I}{\partial y^2} \right|, |G_{x,y}^{0,0}|, \dots, |G_{x,y}^{u,v}| \right] \quad (9)$$

where $I_{x,y}^R, I_{x,y}^G, I_{x,y}^B$ are the color intensities, followed by the magnitude of intensity gradients and the

magnitude of Laplacians, and is the response of a 2D Gabor wavelet with orientation u and scale v . We set $u = 4, v = 3$ here, and therefore the dimensionality of $F_{x,y}$ is 19. Following standard practice, we randomly chose half of the images from each category for training, and the rest were used for testing. This random selection procedure was repeated 10 times and the average classification accuracy with standard deviation is reported.

3.3 Virus cell identification

For this task, we used the Virus dataset [5], which contains images of 15 different virus classes. Each class has 100 segmented grayscale images, with a resolution of 41×41 pixels, all formed by transmission electron microscopy. We applied a similar setting as used for the previous dataset in Eq. (9), extracting 25-dimensional feature vectors at each pixel of an image to generate CovDs given by

$$F_{x,y} = \left[I_{x,y}, \left| \frac{\partial I}{\partial x} \right|, \left| \frac{\partial I}{\partial y} \right|, \left| \frac{\partial^2 I}{\partial x^2} \right|, \left| \frac{\partial^2 I}{\partial y^2} \right|, |G_{x,y}^{0,0}|, \dots, |G_{x,y}^{u,v}| \right] \quad (10)$$

where $I_{x,y}$ is the intensity and the 2D Gabor wavelet was used at 4 orientations and 5 scales here. We employed the original 10 splits provided by the Virus dataset and performed experiments with 9 splits for training and 1 split for testing. The average classification accuracy with standard deviation for all 10 gallery/probe combinations is reported.

3.4 Human face recognition

The last dataset we used is the FERET dataset [6]. We selected the b subset of FERET dataset for evaluation, which comprises 200 persons, each having 7 images with expression and illumination variations. We again applied a similar setting as used for the previous dataset in Eq. (9), extracting 43-dimensional feature vectors with 5 orientations and 8 scales for 2D Gabor wavelets to generate CovDs given by

$$F_{x,y} = \left[I_{x,y}, x, y, |G_{x,y}^{0,0}|, \dots, |G_{x,y}^{u,v}| \right] \quad (11)$$

We randomly selected 3 images of each person for training and used the remaining images for testing, repeating the entire procedure 10 times.

3.5 Discussion of results

Tables 1–3 give the classification accuracies of all algorithms on these three datasets. As we

Table 1 Classification accuracies for the UIUC Material dataset

Method	UIUC
Cov · NN _{LEM}	27.8% ± 3.3%
Cov' _{Inf} · NN _{LEM}	36.3% ± 3.6%
Vec' _{Inf} · NN _{EM}	37.1% ± 2.8%
Cov · KPLS	37.4% ± 2.4%
Cov' _{Inf} · KPLS	45.6% ± 2.7%
Vec' _{Inf} · PLS	47.7% ± 3.0%

Table 2 Classification accuracies for the Virus dataset

Method	Virus
Cov · NN _{LEM}	60.2% ± 4.0%
Cov' _{Inf} · NN _{LEM}	67.7% ± 4.3%
Vec' _{Inf} · NN _{EM}	68.9% ± 3.8%
Cov · KPLS	69.3% ± 2.9%
Cov' _{Inf} · KPLS	77.6% ± 3.1%
Vec' _{Inf} · PLS	79.4% ± 3.3%

Table 3 Classification accuracies for the FERET dataset

Method	FERET
Cov · NN _{LEM}	93.8% ± 3.2%
Cov' _{Inf} · NN _{LEM}	95.5% ± 3.7%
Vec' _{Inf} · NN _{EM}	96.1% ± 3.0%
Cov · KPLS	97.2% ± 3.4%
Cov' _{Inf} · KPLS	98.4% ± 2.9%
Vec' _{Inf} · PLS	98.2% ± 3.5%

can see, the vectorial approximations, estimated from infinite-dimensional CovDs, are superior to low-dimensional CovDs and approximate infinite-dimensional CovDs for all datasets, using the nearest neighbor classifier. The improvement of the vectorial approximations over the low-dimensional CovDs is up to 9.3% for these datasets, which is consistent with the claim that infinite-dimensional CovDs offer better discriminatory power over their low-dimensional counterparts. Our approach also outperforms the method of Ref. [2] based on the log-Euclidean metric by at least 0.6%. We believe that estimating the infinite-dimensional CovDs with kernels based on proper Riemannian metrics can better reveal the intrinsic geometry than kernels based on a Euclidean metric, as the empirical results demonstrate. Interestingly, our approach even obtained comparable performance to more complicated kernelized PLS learning. With PLS, the classification accuracies of our vectorial approximations were slightly boosted to 47.7% ±

3.0% and 79.4% ± 3.3% for UIUC and Virus datasets, respectively. This indicates that vectorial approximations are well-suited for CovDs-based classification.

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