Multi-attribute decision-making based on the SPIFGIA operators

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Abstract The work considers the prioritized relationships between different criteria with priority labels, and develops the scaled prioritized intuitionistic fuzzy geometric interaction averaging (SPIFGIA) operator, the scaled prioritized intuitionistic fuzzy weighted geometric interaction averaging (SPIFWGIA) operator, the scaled prioritized intuitionistic fuzzy hybrid geometric interaction averaging (SPIFHGIA) operator, and the scaled prioritized intuitionistic fuzzy hybrid weighted geometric interaction averaging (SPIFHWGIA) operator. We investigate the properties of the new SPIFGIA and SPIFWGIA operators, and the prioritized relationships between criteria in different situations for decision-making problems, and we also discuss the constructions of intuitionistic fuzzy sets with granularity. Then, we develop new approaches to intuitionistic fuzzy decision-making problems based on the presented SPIFGIA, SPIFWGIA, SPIFHGIA, and SPIFHWGIA operators. Finally, the paper shows the effectiveness and feasibility of the new methods by an example, and gives some comparisons.

Keywords Multi-attribute decision-making problems · Granularity · Scaled prioritized intuitionistic fuzzy geometric interaction operators

1 Introduction

Multiple attribute decision-making (MADM) problems are pervasive activities in many fields, such as management, database retrieval, and engineering (Saaty 1980; Yager and Filev 1999). Information aggregation plays an important part in MADM problems, to make decisions effectively, and correctly, the decision makers need to rank the schemes based on the existing information under different decision environments through different certain ways.

Fuzzy sets (FSs) are popular information forms for MADM problems which were first proposed by Zadeh (1965). Atanassov (1986) constructed the non-membership and hesitant functions and developed the intuitionistic fuzzy set (IFS), which is very useful in managing vagueness information and get a lot of attentions from literatures, such as the fundamental operational laws for IFSs defined in Atanassov (1994) and De et al. (2000), and the basic aggregation operators for IFSs proposed in Xu and Yager (2006).

MADM problems and granular computing have got more attentions from the literatures (Beliakov et al. 2011; Beliakov et al. 2010; Wei and Zhao 2012; Chen 2014; Bedregal et al. 2014; Livi and Sadeghian 2015; Pedrycz and Chen 2015; Chen and Chang 2015; Rodríguez et al. 2012, 2013, 2014; He et al. 2015; Chen et al. 2016; Apolloni et al. 2016; Antonelli et al. 2016; Ciucci 2016; Lingras et al. 2016; Loia et al. 2016; Maciel et al. 2016; Min and Xu 2016; Peters and Weber 2016; Skowron et al. 2016; Wilke and Portmann 2016; Xu and Wang 2016; Yao 2016). Considering the different backgrounds of experts, Xu and Wang (2016) gave an overview on managing multi-granularity linguistic term sets for MADM problems. Dubois and Prade (2016) highlighted the characteristics between the notion of formal concept and extensional

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fuzzy set regarding a similarity relation. Zhou and Chen (2011) developed the continuous generalized ordered weighted aggregation (CGOWA) operator. Kreinovich (2016) got different solutions to the same system of equations with the same granules on the example of interval uncertainty. To evaluating a word by an interval type-2 fuzzy set, Mendel (2016) gave a comparison of Hao–Mendel Approach, Enhanced Interval Approach, and Interval Approach. Wei (2012) studied the induced correlated geometric operator under intuitionistic fuzzy environment. Xu and Xia (2011) developed some new types of intuitionistic fuzzy aggregation operators on a basis of Dempster–Shafer theory of evidence. Zhang (2013) developed some approaches for *MADM* problems with the generalized power geometric operators under intuitionistic fuzzy environment. Zhao et al. (2010) studied the *MADM* problems based on the generalized aggregation operators for intuitionistic fuzzy information.

Considering various relationships among the attributes in *MADM* problems, to get reasonable conclusions, Yager (2008) developed the prioritized averaging (PA) operator. Yu et al. (2012) took account of the attributes in different priority levels and applied the prioritized operators to decision-making problems under interval-valued intuitionistic fuzzy environment. Wei (2012) investigated the *MADM* problems based on the prioritized hesitant fuzzy aggregation operators. Taking into account of the consistency of specific priority degrees of different attributes, He et al. (2016a, b) developed the scaled prioritized aggregation operators.

Considering that the aggregation operators (IFGA) developed in Xu and Yager (2006) under intuitionistic fuzzy environments are based on the traditional operations which do not consider the interactions of different IFNs, which is unsuitable to be used in the decision cases that the interactions of different IFNs exist, the intuitionistic fuzzy geometric interaction aggregation operator in He et al. (2014) fails to take into account of the priority levels of different attributes, and also motivated by the analyses of prioritized relationships between criteria in Yager (2008), Yu and Xu (2013), He et al. (2016a, b) and the interactions of different IFNs in He et al. (2014, 2016c), this paper discusses the constructions of intuitionistic fuzzy sets with granularity, which develops the scaled prioritized intuitionistic fuzzy geometric interaction averaging (SPIFGIA) operator and its weighted form, the SPIFWGIA operator, the SPIFHGIA, and SPIFHWGIA operators. We investigate the properties of the new SPIFGIA and SPIFWGIA operators, and the prioritized relationships between criteria in different situations for decision-making problems. Then, we develop new approaches to intuitionistic fuzzy

decision-making problems based on the presented SPIFGIA and SPIFWGIA operators. The effective and feasibility of the new methods is showed by an example and some comparisons are given.

We organize the rest of the paper in the following: In Sect. 2, some needed concepts are reviewed. Section 3 presents the SPIFGIA, SPIFWGIA, SPIFHGIA, and SPIFHWGIA operators. In Sect. 4, based on the presented SPIFGIA, SPIFWGIA, SPIFHGIA, and SPIFHWGIA operators, new approaches to intuitionistic fuzzy decision-making problems are investigated, and a numerical example shows the effective and feasibility of the developed methods with some comparisons. Finally, Sect. 5 concludes the paper.

2 Preliminaries

Zadeh (1965) developed the concept of *FS*. Atanassov (1986) extended it and proposed the *IFS*, which is more suitable to express and deal with vagueness.

Definition 1 (Atanassov 1986) $A = \{\langle x, u_A(x), v_A(x), \pi_A(x) \rangle | x \in X\}$ is an intuitionistic fuzzy set in an ordinary finite non-empty set X , where $u_A(x)$, $v_A(x)$, and $\pi_A(x)$ are the membership, non-membership, and hesitant functions, respectively. $u_A, v_A, \pi_A : X \rightarrow [0, 1]$ and $0 \leq u_A(x) + v_A(x) \leq 1$.

The score function $S(A) = u_A - v_A$ is proposed in Chen and Tan (1994) to express the suitable degree of an alternative satisfies a decision maker's demand. And the accuracy function $H(A) = u_A + v_A$ is defined in Hong and Choi (2000) to express the accurate degree of an intuitionistic fuzzy set.

Intuitionistic fuzzy numbers (IFNs) is denoted as $A = \langle u_A, v_A \rangle$ for convenience. Xu and Yager (2006) proposed the following method to compare different IFNs.

Definition 2 (Xu and Yager 2006) Let $A = \langle u_A, v_A \rangle$ and $B = \langle u_B, v_B \rangle$ be two IFNs. Then

1. If $S(A) < S(B)$, then $A < B$.
2. If $S(A) > S(B)$, then $A > B$.
3. If $S(A) = S(B)$ and $H(A) < H(B)$, then $A < B$.
4. If $S(A) = S(B)$ and $H(A) > H(B)$, then $A > B$.

Some interactive operational laws on IFNs based on the probability theory are proposed in He et al. (2014, 2015) as follows.

Definition 3 Let $A = \langle u_A, v_A \rangle$ and $B = \langle u_B, v_B \rangle$ be two IFNs, then

$$A \oplus B = \langle 1 - (1 - u_A) \cdot (1 - u_B), (1 - u_A) \cdot (1 - u_B) - (1 - (u_A + v_A)) \cdot (1 - (u_B + v_B)) \rangle \quad (1)$$

$$\lambda A = \langle 1 - (1 - u_A)^\lambda, (1 - u_A)^\lambda - (1 - (u_A + v_A))^\lambda \rangle, \lambda > 0 \quad (2)$$

$$A \otimes B = \langle (1 - v_A)(1 - v_B) - (1 - (u_A + v_A))(1 - (u_B + v_B)), 1 - (1 - v_A)(1 - v_B) \rangle \quad (3)$$

$$A^\lambda = \langle (1 - v_A)^\lambda - (1 - (u_A + v_A))^\lambda, 1 - (1 - v_A)^\lambda \rangle, \lambda > 0. \quad (4)$$

The scaled prioritized relationships between different attributes in He et al. (2016a, b) are defined by priority labels in the following.

Let $H_1, H_2, \dots, H_q$ be a set of attributes categories and $H_i = \{a_{i1}, a_{i2}, \dots, a_{in_i}\}$. Suppose the priority level of attributes in H_i is higher than those in H_k for $i \leq k$. The scaled prioritized relationships between different attributes in He et al. (2016a, b) are defined by priority labels and priority coefficient $d_{i,i+1}$ in the following.

If $d_{i,i+1} = 0.1$, then the label of priority degree of H_i and H_{i+1} is extreme priority.

If $d_{i,i+1} = 0.4$, then the label of priority degree of H_i and H_{i+1} is strong priority.

If $d_{i,i+1} = 0.6$, then the label of priority degree of H_i and H_{i+1} is slight priority.

If $d_{i,i+1} = 0.8$, then the label of priority degree of H_i and H_{i+1} is weak priority.

If $d_{i,i+1} = 1$, then the label of priority degree of H_i and H_{i+1} is equal priority.

The weight based on priority coefficient obtained in He et al. (2016a, b) is as follows.

$$w_i = \begin{cases} d_{0,1} = 1, & i = 1 \\ w_{i-1} \cdot d_{i-1,i} = \prod_{k=1}^i d_{k-1,k}, & i = 2, 3, \dots, q \end{cases} \quad (5)$$

And the normalized weight is

$$\tilde{w}_i = \frac{w_i}{\sum_{i=1}^q (n_i \cdot w_i)}. \quad (6)$$

He et al. (2016a, b) pointed it out that the weights can be got by the formula (7) in the case that the priority relationship of H_i and H_{i+1} is unknown:

$$h_i = \begin{cases} 1, & i = 0 \\ \theta(\phi(a_{i1}(x_1), \dots, a_{in_i}(x_1)), \dots, \phi(a_{i1}(x_m), \dots, a_{in_i}(x_m))), & i = 1, 2, \dots, q \end{cases} \quad (7)$$

where $x_i (i = 1, 2, \dots, m)$ is the possible alternatives, ϕ or θ is the aggregation function such as the OWA, minimum, and maximum functions.

The weights based on Eq. (7) are obtained in He et al. (2016a, b) as follows.

$$w_i = \prod_{k=1}^i h_{k-1}, \quad i = 1, 2, \dots, q. \quad (8)$$

The normalized weight is defined by the following equation (He et al. 2016a, b):

$$\tilde{w}_i = \frac{\prod_{k=1}^i h_{k-1}}{\sum_{i=1}^q \left(n_i \cdot \prod_{k=1}^i h_{k-1} \right)}, \quad i = 1, 2, \dots, q. \quad (9)$$

3 The SPIFGIA and SPIFHGIA Operators

This section introduces the SPIFGIA and SPIFHGIA operators and investigates their properties.

3.1 The SPIFGIA operator

Definition 4 Suppose the priority relationship of g_i and g_{i+1} in the given set of attributes $G = \{g_1, g_2, \dots, g_n\}$ is reflected by the priority coefficient $d_{i,i+1} (i = 1, 2, \dots, q-1)$, with $d_{0,1} = 1$. The involved decision information is evaluated by a set of IFNs $A_i (i = 1, \dots, n)$, and $A_i = \langle u_{A_i}, v_{A_i} \rangle$. The SPIFGIA operator is defined by the following equation:

$$SPIFGIA(A_1, \dots, A_n) = \bigotimes_{i=1}^n A_i^{\prod_{k=1}^i d_{k-1,k}} / \sum_{j=1}^n \prod_{k=1}^j d_{k-1,k}. \quad (10)$$

Theorem 1 Let $A_i = \langle u_{A_i}, v_{A_i} \rangle (i = 1, \dots, n)$ be a set of IFNs. Then, $SPIFGIA(A_1, \dots, A_n) \in IFNs(X)$ and

$$SPIFGIA(A_1, \dots, A_n) = \left\langle \prod_{i=1}^n (1 - v_{A_i})^{\prod_{k=1}^i d_{k-1,k}} / \sum_{j=1}^n \prod_{k=1}^j d_{k-1,k} \right. \\ \left. - \prod_{i=1}^n (1 - (u_{A_i} + v_{A_i}))^{\prod_{k=1}^i d_{k-1,k}} / \sum_{j=1}^n \prod_{k=1}^j d_{k-1,k} \right. \\ \left. \times 1 - \prod_{i=1}^n (1 - v_{A_i})^{\prod_{k=1}^i d_{k-1,k}} / \sum_{j=1}^n \prod_{k=1}^j d_{k-1,k} \right\rangle. \quad (11)$$

Proof Equation (11) can be proved with using the mathematical induction as follows.

1. Suppose $n = 1$, $d_{0,1} = 1$, we have

$$\begin{aligned}
SPIFGIA(A_1) &= \langle u_{A_1}, v_{A_1} \rangle = \\
&\left\langle 1 - v_{A_1}^1 - (1 - (u_{A_1} + v_{A_1}))^1, 1 - (1 - v_{A_1})^1 \right\rangle \\
&= \left\langle (1 - v_{A_1})^{d_{0,1}/d_{0,1}} - (1 - (u_{A_1} + v_{A_1}))^{d_{0,1}/d_{0,1}}, 1 \right. \\
&\quad \left. - (1 - v_{A_1})^{d_{0,1}/d_{0,1}} \right\rangle.
\end{aligned}$$

Namely, Eq. (11) holds.

2. Suppose Eq. (11) holds for $n = l$, if $n = l + 1$, with Eq. (1), we have

$$\begin{aligned}
&SPIFGIA(A_1, \dots, A_{l+1}) \\
&= SPIFGIA(A_1, \dots, A_l) \otimes A_{l+1} \frac{\prod_{k=1}^{l+1} d_{k-1,k}}{\sum_{j=1}^{l+1} \prod_{k=1}^j d_{k-1,k}} \\
&= \left\langle \prod_{i=1}^l (1 - v_{A_i}) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^{l+1} \prod_{k=1}^j d_{k-1,k}} \right. \\
&\quad \left. - \prod_{i=1}^l (1 - (u_{A_i} + v_{A_i})) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^{l+1} \prod_{k=1}^j d_{k-1,k}}, 1 \right. \\
&\quad \left. - \prod_{i=1}^l (1 - v_{A_i}) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^{l+1} \prod_{k=1}^j d_{k-1,k}} \right\rangle \\
&\oplus \left\langle (1 - v_{A_i}) \frac{\prod_{k=1}^{l+1} d_{k-1,k}}{\sum_{j=1}^{l+1} \prod_{k=1}^j d_{k-1,k}} \right. \\
&\quad \left. - (1 - (u_{A_i} + v_{A_i})) \frac{\prod_{k=1}^{l+1} d_{k-1,k}}{\sum_{j=1}^{l+1} \prod_{k=1}^j d_{k-1,k}}, 1 \right. \\
&\quad \left. - (1 - v_{A_i}) \frac{\prod_{k=1}^{l+1} d_{k-1,k}}{\sum_{j=1}^{l+1} \prod_{k=1}^j d_{k-1,k}} \right\rangle \\
&= \left\langle \prod_{i=1}^{l+1} (1 - v_{A_i}) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^{l+1} \prod_{k=1}^j d_{k-1,k}} \right. \\
&\quad \left. - \prod_{i=1}^{l+1} (1 - (u_{A_i} + v_{A_i})) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^{l+1} \prod_{k=1}^j d_{k-1,k}}, 1 \right. \\
&\quad \left. - \prod_{i=1}^{l+1} (1 - v_{A_i}) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^{l+1} \prod_{k=1}^j d_{k-1,k}} \right\rangle.
\end{aligned}$$

Thus, Eq. (11) holds for $n = l + 1$.

Therefore, using mathematic induction on n , Eq. (11) holds for n .

Next, we prove that $PIFGIA(A_1, \dots, A_n) \in IFNs(X)$.

By Definition 1, noting that $A_i = \langle u_{A_i}, v_{A_i} \rangle \in IFNs(X)$, $i = 1, 2, \dots, n$, we have $0 \leq u_{A_i}, v_{A_i} \leq 1$ and $0 \leq u_{A_i} + v_{A_i} \leq 1$.

Then,

$$0 \leq 1 - \prod_{i=1}^n (1 - v_{A_i}) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^n \prod_{k=1}^j d_{k-1,k}} \leq 1,$$

$$\begin{aligned}
0 &\leq \prod_{i=1}^n (1 - v_{A_i}) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^n \prod_{k=1}^j d_{k-1,k}} \\
&\quad - \prod_{i=1}^n (1 - (u_{A_i} + v_{A_i})) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^n \prod_{k=1}^j d_{k-1,k}} \leq 1
\end{aligned}$$

and

$$\begin{aligned}
&\left(1 - \prod_{i=1}^n (1 - v_{A_i}) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^n \prod_{k=1}^j d_{k-1,k}} \right) \\
&\quad + \left(\prod_{i=1}^n (1 - v_{A_i}) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^n \prod_{k=1}^j d_{k-1,k}} \right. \\
&\quad \left. - \prod_{i=1}^n (1 - (u_{A_i} + v_{A_i})) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^n \prod_{k=1}^j d_{k-1,k}} \right) \\
&= 1 - \prod_{i=1}^n (1 - (u_{A_i} + v_{A_i})) \frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^n \prod_{k=1}^j d_{k-1,k}} \in [0, 1].
\end{aligned}$$

Thus, according to Definition 1, we have $SPIFGIA(A_1, \dots, A_n) \in IFNs(X)$.

Theorem 2 Let $A_i = \langle u_{A_i}, v_{A_i} \rangle, B_i = \langle u_{B_i}, v_{B_i} \rangle \in IFNs(X)$, $i = 1, 2, \dots, n$. Then

1. **Idempotency:** If $A_i = A_0 = \langle u_{A_0}, v_{A_0} \rangle$ for all i , then $SPIFGIA(A_1, \dots, A_n) = A_0$.
2. **Boundedness:** Let $A^- = \langle \max\{0, (\min(u_{A_i} + v_{A_i}) - \max(v_{A_i}))\}, \max(v_{A_i}) \rangle$, $A^+ = \langle \max(u_{A_i} + v_{A_i}) - \min(v_{A_i}), \min(v_{A_i}) \rangle$. Then, $A^- \leq SPIFGIA(A_1, \dots, A_n) \leq A^+$.
3. **Monotonicity:** If $v_{A_i} \geq v_{B_i}$, $u_{A_i} + v_{A_i} \leq u_{B_i} + v_{B_i}$ for all i , we have $SPIFGIA(A_1, \dots, A_n) \leq SPIFGIA(B_1, B_2, \dots, B_n)$.

Proof Theorem 2 can be proved by Theorem 1 and Definition 3 and omitted here.

3.2 The SPIFWGIA operator

Definition 5 Assume the detailed priority relationship of H_i and H_k in the given set of attributes $\{H_1, H_2, \dots, H_q\}$ is unknown except that $H_1 \succ H_2 \succ \dots \succ H_q$, and $H_i = \{H_{i1}, H_{i2}, \dots, H_{in_i}\}$. The involved decision information is evaluated by a set of IFNs $A_{ijt} = \langle u_{A_{ijt}}, v_{A_{ijt}} \rangle (i =$

$1, 2, \dots, q; j = 1, 2, \dots, q_i; t = 1, 2, \dots, m$). The *SPIFGIA* operator is then defined by the following equation:

$$\begin{aligned} & SPIFGIA(A_{11t}, \dots, A_{1q_1t}, \dots, A_{q1t}, \dots, A_{qqqt}) \\ &= \bigotimes_{i=1}^q \bigotimes_{j=1}^{q_i} A_{ijt}^{\frac{1}{n_k} \sum_{k=1}^{n_k} \sum_{m=1}^m \frac{u_{A_{kjt}} - v_{A_{kjt}} + 1}{2}} \bigg/ \sum_{i=1}^q n_i \prod_{k=1}^{n_k} \frac{1}{n_k} \sum_{m=1}^m \frac{u_{A_{kjt}} - v_{A_{kjt}} + 1}{2}, \end{aligned} \quad (12)$$

where $d_{i-1,i}$ is evaluated as

$$d_{i-1,i} = \frac{1}{n_i \cdot m} \sum_{j=1}^{n_i} \sum_{t=1}^m \frac{u_{A_{ijt}} - v_{A_{ijt}} + 1}{2}. \quad (13)$$

Similar to Theorem 1, we have

$$SPIFGIA(A_{11t}, \dots, A_{1n_1t}, \dots, A_{q1t}, \dots, A_{qnqt}) \in IFNs(X)$$

and

$$\begin{aligned} & SPIFGIA(A_{11t}, \dots, A_{1q_1t}, \dots, A_{q1t}, \dots, A_{qqqt}) \\ &= \left\langle \prod_{i=1}^q \prod_{j=1}^{q_i} (1 - v_{A_{ijt}})^{\frac{1}{n_k} \sum_{k=1}^{n_k} \sum_{m=1}^m \frac{u_{A_{kjt}} - v_{A_{kjt}} + 1}{2}} \bigg/ \sum_{i=1}^q n_i \prod_{k=1}^{n_k} \frac{1}{n_k} \sum_{m=1}^m \frac{u_{A_{kjt}} - v_{A_{kjt}} + 1}{2} \right. \\ &\quad - \prod_{i=1}^q \prod_{j=1}^{q_i} (1 - (u_{A_i} + v_{A_i}))^{\frac{1}{n_k} \sum_{k=1}^{n_k} \sum_{m=1}^m \frac{u_{A_{kjt}} - v_{A_{kjt}} + 1}{2}} \bigg/ \sum_{i=1}^q n_i \prod_{k=1}^{n_k} \frac{1}{n_k} \sum_{m=1}^m \frac{u_{A_{kjt}} - v_{A_{kjt}} + 1}{2}, \\ &\quad \left. 1 - \prod_{i=1}^q \prod_{j=1}^{q_i} (1 - v_{A_{ijt}})^{\frac{1}{n_k} \sum_{k=1}^{n_k} \sum_{m=1}^m \frac{u_{A_{kjt}} - v_{A_{kjt}} + 1}{2}} \bigg/ \sum_{i=1}^q n_i \prod_{k=1}^{n_k} \frac{1}{n_k} \sum_{m=1}^m \frac{u_{A_{kjt}} - v_{A_{kjt}} + 1}{2} \right\rangle. \end{aligned} \quad (14)$$

In addition, we have the properties of *SPIFGIA* operator as follows:

- Idempotency:** If $A_{ijt} = A_0 = \langle u_{A_0}, v_{A_0} \rangle$ for all i, j , and t , then $SPIFGIA(A_{11t}, \dots, A_{1n_1t}, \dots, A_{q1t}, \dots, A_{qnqt}) = A_0$.
- Boundedness:** Let $A_t^- = \left\langle \max_{ij} \left\{ 0, (\min_{ij} (u_{A_{ijt}} + v_{A_{ijt}}) - \max_{ij} (v_{A_{ijt}})) \right\}, \max_{ij} (v_{A_{ijt}}) \right\rangle$, $A_t^+ = \left\langle \max_{ij} (u_{A_{ijt}} + v_{A_{ijt}}) - \min_{ij} (v_{A_{ijt}}), \min_{ij} (v_{A_{ijt}}) \right\rangle$. Then, $A_t^- \leq SPIFGIA(A_{11t}, \dots, A_{1n_1t}, \dots, A_{q1t}, \dots, A_{qnqt}) \leq A_t^+$.
- Monotonicity:** If $v_{A_{ijt_1}} \geq v_{A_{ijt_2}}$, $u_{A_{ijt_1}} + v_{A_{ijt_1}} \leq u_{A_{ijt_2}} + v_{A_{ijt_2}}$ for all i and j , then $SPIFGIA(A_{11t_1}, \dots, A_{1n_1t_1}, \dots, A_{q1t_1}, \dots, A_{qnqt_1}) \leq SPIFGIA(A_{11t_2}, \dots, A_{1n_1t_2}, \dots, A_{q1t_2}, \dots, A_{qnqt_2})$.

3.3 The SPIFHGIA operators

Taking into account that the *SPIFGIA* and *SPIFGIA* operators weight only the intuitionistic fuzzy numbers by the priority levels of the attributes, and in many practical situations, the intuitionistic fuzzy numbers themselves are

also needed to be weighted, for example, the intuitionistic fuzzy number A is much accurate than intuitionistic fuzzy number B , and then, the intuitionistic fuzzy number A should be given larger weight than B before using the *SPIFGIA* and *SPIFGIA* operators. Also motivated by the intuitionistic fuzzy hybrid aggregation operators (Xu 2007), we propose the scaled prioritized intuitionistic fuzzy hybrid geometric interaction averaging (*SPIFHGIA*) operator and the scaled prioritized intuitionistic fuzzy hybrid weighted geometric interaction averaging (*SPIFHWGIA*) operator in this subsection.

Definition 6 Suppose the priority relationship of g_i and g_{i+1} in the given set of attributes $G = \{g_1, g_2, \dots, g_n\}$ is reflected by the priority coefficient $d_{i,i+1}$ ($i = 1, 2, \dots, q - 1$) with $d_{0,1} = 1$. The involved decision information is evaluated by a set of IFNs A_i ($i = 1, \dots, n$), and $A_i = \langle u_{A_i}, v_{A_i} \rangle$, where $\omega = (\omega_1, \dots, \omega_n)^T$ is the weight vector of A_i ($i = 1, 2, \dots, n$), satisfying that $\omega_i \in [0, 1]$ and $\sum_{i=1}^n \omega_i = 1$, denoting the different accuracy degree of A_i ($i = 1, 2, \dots, n$), and $\tilde{A}_i = n\omega_i A_i$ ($i = 1, \dots, n$), and n is the balancing coefficient that plays a role of balance. Then, the *SPIFHGIA* operator is defined by the following equation:

$$SPIFHGIA(A_1, \dots, A_n) = \bigotimes_{i=1}^n (n\omega_i A_i)^{\frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^n \prod_{k=1}^j d_{k-1,k}}}. \quad (15)$$

Theorem 3 Let $A_i = \langle u_{A_i}, v_{A_i} \rangle$ ($i = 1, \dots, n$) be a set of IFNs. Then, $SPIFHGIA(A_1, \dots, A_n) \in IFNs(X)$ and

$$\begin{aligned} & SPIFHGIA(A_1, \dots, A_n) = \\ & \left\langle \prod_{i=1}^n (1 - (1 - u_{A_i})^{n\omega_i} + (1 - (u_{A_i} + v_{A_i}))^{n\omega_i})^{\frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^n \prod_{k=1}^j d_{k-1,k}}} \right. \\ & \quad - \prod_{i=1}^n ((1 - (u_{A_i} + v_{A_i}))^{n\omega_i})^{\frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^n \prod_{k=1}^j d_{k-1,k}}}, \\ & \quad \left. \times 1 - \prod_{i=1}^n (1 - (1 - u_{A_i})^{n\omega_i} + (1 - (u_{A_i} + v_{A_i}))^{n\omega_i})^{\frac{\prod_{k=1}^i d_{k-1,k}}{\sum_{j=1}^n \prod_{k=1}^j d_{k-1,k}}} \right\rangle. \end{aligned} \quad (16)$$

Theorem 4 (Idempotency) Let $A_i = \langle u_{A_i}, v_{A_i} \rangle$, $B_i = \langle u_{B_i}, v_{B_i} \rangle \in IFNs(x)$, $i = 1, 2, \dots, n$. If $A_i = A_0 = \langle u_{A_0}, v_{A_0} \rangle$ for all i , then $SPIFHGIA(A_1, \dots, A_n) = A_0$.

Definition 5 Assume that the detailed priority relationship of H_i and H_k in the given set of attributes $\{H_1, H_2, \dots, H_q\}$ is unknown except that $H_1 \succ H_2 \succ \dots \succ H_q$, and $H_i = \{H_{i1}, H_{i2}, \dots, H_{in_i}\}$. The involved decision information is evaluated by a set of IFNs $A_{ijt} = \langle u_{A_{ijt}}, v_{A_{ijt}} \rangle$ ($i = 1, 2, \dots, q$; $j = 1, 2, \dots, q_i$; $t = 1, 2, \dots, m$). Where $\omega = (\omega_1, \dots, \omega_n)^T$ is the weight vector

of $A_{ijt} (i = 1, 2, \dots, q)$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^q \omega_i = 1$, denoting the different accuracy degree of $A_{ijt} (i = 1, 2, \dots, q)$, and $\hat{A}_{ijt} = n\omega_i A_{ijt} (i = 1, \dots, n)$, where n is the balancing coefficient that plays a role of balance. The *SPIFHWGIA* operator is then defined by the following equation:

$$\begin{aligned} & \text{SPIFHWGIA}(A_{11t}, \dots, A_{1q1t}, \dots, A_{q1t}, \dots, A_{qqqt}) \\ &= \bigotimes_{i=1}^q \bigotimes_{j=1}^{q_i} n\omega_i A_{ijt} \left/ \prod_{k=1}^i \frac{1}{n_k^m} \sum_{j=1}^{n_k} \sum_{t=1}^m \frac{u_{A_{kjt}}^{-v_{A_{kjt}}+1}}{2} \right/ \left/ \sum_{i=1}^q n_i \prod_{k=1}^i \frac{1}{n_k^m} \sum_{j=1}^{n_k} \sum_{t=1}^m \frac{u_{A_{kjt}}^{-v_{A_{kjt}}+1}}{2} \right/ \end{aligned} \quad (17)$$

Similar to Theorem 1, we have

$$\text{SPIFHWGIA}(A_{11t}, \dots, A_{1n1t}, \dots, A_{q1t}, \dots, A_{qnqt}) \in \text{IFNs}(X)$$

and

$$\begin{aligned} & \text{SPIFHWGIA}(A_{11t}, \dots, A_{1q1t}, \dots, A_{q1t}, \dots, A_{qqqt}) \\ &= \left\langle \prod_{i=1}^q \prod_{j=1}^{q_i} \left((1 - (1 - u_{A_{ijt}})^{n\omega_i} + (1 - (u_{A_{ijt}} + v_{A_{ijt}}))^{n\omega_i}) \prod_{k=1}^i \frac{1}{n_k^m} \sum_{j=1}^{n_k} \sum_{t=1}^m \frac{u_{A_{kjt}}^{-v_{A_{kjt}}+1}}{2} \right) \right/ \sum_{i=1}^q n_i \prod_{k=1}^i \frac{1}{n_k^m} \sum_{j=1}^{n_k} \sum_{t=1}^m \frac{u_{A_{kjt}}^{-v_{A_{kjt}}+1}}{2} \\ & \quad - \prod_{i=1}^q \prod_{j=1}^{q_i} \left((1 - (u_{A_{ijt}} + v_{A_{ijt}}))^{n\omega_i} \right) \prod_{k=1}^i \frac{1}{n_k^m} \sum_{j=1}^{n_k} \sum_{t=1}^m \frac{u_{A_{kjt}}^{-v_{A_{kjt}}+1}}{2} \right/ \sum_{i=1}^q n_i \prod_{k=1}^i \frac{1}{n_k^m} \sum_{j=1}^{n_k} \sum_{t=1}^m \frac{u_{A_{kjt}}^{-v_{A_{kjt}}+1}}{2}, \\ & \quad \times 1 - \prod_{i=1}^q \prod_{j=1}^{q_i} \left((1 - (1 - u_{A_{ijt}})^{n\omega_i} + (1 - (u_{A_{ijt}} + v_{A_{ijt}}))^{n\omega_i}) \prod_{k=1}^i \frac{1}{n_k^m} \sum_{j=1}^{n_k} \sum_{t=1}^m \frac{u_{A_{kjt}}^{-v_{A_{kjt}}+1}}{2} \right) \right/ \sum_{i=1}^q n_i \prod_{k=1}^i \frac{1}{n_k^m} \sum_{j=1}^{n_k} \sum_{t=1}^m \frac{u_{A_{kjt}}^{-v_{A_{kjt}}+1}}{2} \right\rangle. \end{aligned} \quad (18)$$

Theorem 5 (Idempotency) If $A_{ijt} = A_0 = \langle u_{A_0}, v_{A_0} \rangle$ for all i, j , and t , then $\text{SPIFHWGIA}(A_{11t}, \dots, A_{1n1t}, \dots, A_{q1t}, \dots, A_{qnqt}) = A_0$.

4 New MADM methods and numerical example

Many applications in real word, such as information retrieval, multi-attribute decision-making, and database retrieval, are the process of finding the best alternatives from a set of candidate alternatives based on their satisfaction to a collection of criteria. The central task of these problems is the information aggregation. The developed SPIFGIA, SPIFWGIA, and SPIFHGIA operators in this paper will provide effective information aggregation techniques under intuitionistic fuzzy environments and thus can be used to solve MADM problems in real word.

The validity of the new methods is showed by an example with some comparisons.

4.1 New approaches based on the SPIFGIA SPIFWGIA and SPIFHGIA operators

Let $x_t (t = 1, 2, \dots, m)$ be m possible alternatives in MADM problems. Assuming that the considered attributes $G = \{g_1, g_2, \dots, g_n\}$ for m possible alternatives are classed into q sorts $\{H_1, H_2, \dots, H_q\}$ and $H_i = \{H_{i1}, H_{i2}, \dots, H_{in_i}\}$. The priority level of attributes in the sort H_i is higher than those in H_k if $i \leq k$. Based on which, the involved decision information is evaluated by a set of IFNs $A_{ijt} = \langle u_{A_{ijt}}, v_{A_{ijt}} \rangle (t = 1, 2, \dots, m; j = 1, 2, \dots, q_i; i = 1, 2, \dots, q)$, where $u_{A_{ijt}}$ is constructed by decision makers based on the degree that x_t satisfies H_{ij} . $v_{A_{ijt}}$ is constructed based on the degree that x_t do not satisfy H_{ij} . We then have the new algorithms for MADM problems in the following.

4.1.1 Algorithm 1

Step 1 Construct intuitionistic fuzzy numbers $A_{ijt} = \langle u_{A_{ijt}}, v_{A_{ijt}} \rangle (t = 1, 2, \dots, m; j = 1, 2, \dots, q_i; i = 1, 2, \dots, q)$ to evaluate the degrees of considered m alternatives $x_t (t = 1, 2, \dots, m)$ satisfying attributes $H_{ij} (j = 1, 2, \dots, q_i; i = 1, 2, \dots, q)$. The decision makers give their evaluations that x_t meets the criterion H_{ij} as $\hat{u}_{A_{ijt}}$, the x_t do not satisfy the criterion H_{ij} as $\hat{v}_{A_{ijt}}$, and hesitant degree as $\hat{\pi}_{A_{ijt}}$. Then, $u_{A_{ijt}} = \hat{u}_{A_{ijt}} / (\hat{u}_{A_{ijt}} + \hat{v}_{A_{ijt}} + \hat{\pi}_{A_{ijt}})$ and $v_{A_{ijt}} = \hat{v}_{A_{ijt}} / (\hat{u}_{A_{ijt}} + \hat{v}_{A_{ijt}} + \hat{\pi}_{A_{ijt}})$. And the constructions of intuitionistic fuzzy sets with granularity please refer to Sect. 4.3.

Step 2 Suppose that the priority levels of different attributes $H_{ij} (i = 1, 2, \dots, q; j = 1, 2, \dots, q_i)$ are known,

Table 1 Priority relationship matrix

	$LP(H_1, H_2)$	$LP(H_2, H_3)$	$LP(H_3, H_4)$	$LP(H_4, H_5)$
Priority relationship	Strong priority	Slight priority	Weak priority	Extreme priority
Priority coefficient	0.4	0.6	0.8	0.1

construction project is selected as the most important part which will be evaluated under five attributes, aiming to obtain the best bidding scheme. H_i ($i = 1, 2, 3, 4, 5$) are the involved five attributes listed in the following.

H_1 : The quotation of the project.

H_2 : The period of the construction.

H_3 : The construction project's quality.

H_4 : The technology of the construction.

H_5 : The reputation of the company's business.

Assume that x_1, x_2, x_3, x_4 are chosen as the four possible construction organizations. The priority levels of five attributes are $H_1 \succ H_2 \succ H_3 \succ H_4 \succ H_5$, and the detailed relationships are reflected by $d_{i,i+1}$ ($i = 1, 2, 3, 4$) in Table 1, which is the same as in He et al. (2016c).

4.2.1 Method 1

Step 1 According to the construction method of IFNs in Method 1, the decision makers construct intuitionistic fuzzy numbers to evaluate the degrees of involved four alternatives satisfying the considered five attributes as Table 2. It should be noted that the evaluated IFNs in Table 2 are the same as those in He et al. (2016c) for a fair comparison.

It should be noted that the invited experts are familiar with the iron and steel research work, the development of the industry and the environmental equality requirements of the city, after investigating the importance of the involved five attributes and the characteristics of the four possible construction organizations, they give the priority relationship matrix of the five attributes as Table 1, and the decision information matrix of involved four alternatives satisfying the considered five attributes as Table 2. For the constructions of intuitionistic fuzzy sets in Table 2 with granularity, please refer to Sect. 4.3.

Step 2 Because the priority levels of different attributes H_i ($i = 1, 2, 3, 4, 5$) are known, we use the *SPIFGIA*

Table 2 Decision information matrix

	H_1	H_2	H_3	H_4	H_5
x_1	<0.6,0.2>	<0.4,0.2>	<0.6,0.3>	<0.7,0.1>	<0.6,0.2>
x_2	<0.5,0.3>	<0.5,0.3>	<0.5,0.2>	<0.7,0.2>	<0.5,0.2>
x_3	<0.5,0.3>	<0.4,0.2>	<0.5,0.1>	<0.6,0.2>	<0.5,0.1>
x_4	<0.5,0.4>	<0.4,0.3>	<0.5,0.4>	<0.6,0.3>	<0.6,0.3>

operator to aggregate the decision information and have the collective ones as follows.

$$A_1 = SPIFGIA(A_{11}, \dots, A_{15}) = \langle 0.5836, 0.2041 \rangle,$$

$$A_2 = \langle 0.5262, 0.2768 \rangle$$

$$A_3 = \langle 0.5007, 0.2433 \rangle, \quad A_4 = \langle 0.5045, 0.3687 \rangle.$$

Step 3 Based on the score function developed in Chen and Tan (1994), we have $S(A_1) = 0.3795$, $S(A_2) = 0.2493$, $S(A_3) = 0.2574$, and $S(A_4) = 0.1359$.

Step 4 Based on the scores of A_t ($t = 1, 2, 3, 4$) in Step 3, we obtain that $x_1 \succ x_3 \succ x_2 \succ x_4$, x_1 has the maximum score and thus is the best one.

If the priority levels of different attributes H_i ($i = 1, 2, 3, 4, 5$) are unknown, we can use the *SPIFWGIA* operator to aggregate the decision information and have the decision procedure as follows.

4.2.2 Method 2

Step 1 is the same as that in Method 1.

Step 2 Use the *SPIFWGIA* operator to aggregate the decision information in Table 2 and get the collective ones as follows.

$$A_1 = SPIFWGIA(A_{11}, \dots, A_{15}) = \langle 0.5792, 0.2058 \rangle,$$

$$A_2 = \langle 0.5286, 0.2684 \rangle,$$

$$A_3 = \langle 0.4987, 0.2231 \rangle, \quad A_4 = \langle 0.5095, 0.3584 \rangle.$$

Step 3 Based on the score function defined in Chen and Tan (1994), we have $S(A_1) = 0.3735$, $S(A_2) = 0.2602$, $S(A_3) = 0.2756$, and $S(A_4) = 0.1511$.

Step 4 Based on the scores of A_t ($t = 1, 2, 3, 4$) in Step 3, we obtain that $x_1 \succ x_3 \succ x_2 \succ x_4$, x_1 has the maximum score and thus is the best one.

If the different accuracy degrees of evaluated IFNs are measured by the weight vector $\omega = (0.2, 0.18, 0.22, 0.19, 0.21)^T$, we can use the *SPIFHGIA* and *SPIFHWGIA* operators to aggregate the decision information and have the decision procedure as follows.

4.2.3 Methods 3

Step 1 is the same as that in Method 1.

Step 2 Because the priority levels of different attributes and the different accuracy degrees of evaluated IFNs are known, we use the *SPIFHGIA* operator to aggregate the

decision information and have the collective ones as follows:

$$\begin{aligned} A_1 &= SPIFHGIA(A_{11}, \dots, A_{15}) = \langle 0.5876, 0.2006 \rangle, \\ A_2 &= \langle 0.5192, 0.2777 \rangle, \\ A_3 &= \langle 0.4977, 0.2423 \rangle, \quad A_4 = \langle 0.5062, 0.3661 \rangle. \end{aligned}$$

Step 3 Based on the score function developed in Chen and Tan (1994), we have $S(A_1) = 0.3870$, $S(A_2) = 0.2414$, $S(A_3) = 0.2554$, and $S(A_4) = 0.1401$.

Step 4 Based on the scores of A_t ($t = 1, 2, 3, 4$) in Step 3, we obtain that $x_1 \succ x_3 \succ x_2 \succ x_4$, x_1 has the maximum score and thus is the best one.

4.2.4 Methods 4

Step 1 is the same as that in Method 2.

Step 2 Use the *SPIFWGIA* operator to aggregate the decision information in Table 2 and get the collective ones as follows.

$$\begin{aligned} A_1 &= SPIFWGIA(A_{11}, \dots, A_{15}) = \langle 0.5852, 0.2016 \rangle, \\ A_2 &= \langle 0.5212, 0.2694 \rangle, \\ A_3 &= \langle 0.4957, 0.2218 \rangle, \quad A_4 = \langle 0.5129, 0.3549 \rangle. \end{aligned}$$

Step 3 Based on the score function defined in Chen and Tan (1994), we have $S(A_1) = 0.3836$, $S(A_2) = 0.2518$, $S(A_3) = 0.2739$, and $S(A_4) = 0.1580$.

Step 4 Based on the scores of A_t ($t = 1, 2, 3, 4$) in Step 3, we obtain that $x_1 \succ x_3 \succ x_2 \succ x_4$, x_1 has the maximum score and thus is the best one.

4.3 Construct intuitionistic fuzzy sets with granularity

Pedrycz and Chen (2015) pointed out that granularity can help decision makers come to an agreement for imprecise information based on fuzzy techniques and fuzzy logic. Compared with the “crisp” properties that are either true or false, to describe the properties H that decision makers are not completely sure whether a fixed value is big or small, $u_H(x)$ is assigned to each alternative to indicate the degree which it satisfies this property (Lorkowski and Kreinovich 2015): (1) the degree $u_H(x) = 1$ indicates that the decision makers are completely sure that the value x satisfies the property H ; (2) $u_H(x) = 0$ indicates that the decision makers are completely sure that the value x does not meet H ; (3) if $0 < u_H(x) < 1$, then the decision makers are not completely sure that the value x satisfies the property H or not.

By requesting decision makers to mark their preference degrees of x meets the property H based on a Likert scale,

Lorkowski and Kreinovich (2015) used granules to draw forth $u_H(x)$ from experts. For example, given a scale from 0 to n , the decision makers can choose their preferences from the labels of 0, 1, ..., n , if the decision maker choose m , then $u_H(x)$ will be m/n . Lorkowski and Kreinovich (2015) also stated that the decision makers can choose their description of uncertainty based on a larger value n if a more detailed scale is needed and given.

Considering that the IFSs can be seen as special cases of type-2 fuzzy sets, Naim and Hagrass (2015) proposed interval type-2 fuzzy sets based on hesitation index from intuitionistic evaluation, where the interval type-2 membership function is illustrated by the left and right end-points, for example, $[u_H(x), \overline{u}_H(x)]$ in intuitionistic value is $\langle u_H(x), 1 - \overline{u}_H(x) \rangle$.

Therefore, with the methods in Pedrycz and Chen (2015), Lorkowski and Kreinovich (2015) and Naim and Hagrass (2015), i.e., Granular Computing, we can construct the IFNs in this paper. For example, experts marks her confidence by left membership value 6 and right membership value 8 on a scale from 1 to 10 to indicate that the degrees x_1 satisfy the property H_1 , and thus, A_{11} in the numerical example is $\langle \frac{6}{10}, 1 - \frac{8}{10} \rangle = \langle 0.6, 0.2 \rangle$. And other intuitionistic fuzzy sets in Table 2 can be constructed similarly.

4.4 Comparative analysis

From the final results in above four methods in Sect. 4.2, we have that the results got in the four developed methods are consistency.

From $H_1 \succ H_2 \succ H_3 \succ H_4 \succ H_5$, we have the attribute H_1 is most important. Since $A_{11} = \langle 0.6, 0.2 \rangle > A_{21} = A_{31} = \langle 0.5, 0.3 \rangle > A_{41} = \langle 0.5, 0.4 \rangle$, the most reasonable result is that x_1 is the best one, this is consistent with the result got by this paper, which also illustrates the effective of proposed methods in this paper.

Compared with the existing methods in Xu and Yager (2006), Zhou and Chen (2011), and He et al. (2014, 2015a, 2016a, c) for decision-making problems, we have the following conclusions.

Comparison 1 Since the *SPIFGIA* in this paper is based on the interactive addition, scalar multiplication, multiplication, and power operations in Definition 3, which consider the interactions of different IFNs, while the aggregation operators (IFGA) developed in Xu and Yager (2006) under intuitionistic fuzzy environments are based on the traditional operations which do not consider the interactions of different IFNs, and thus, the characteristic of the developed *SPIFGIA* operator in this work for MADM problems is that it is suitable to be used in the

decision cases that the interactions of different IFNs exist compared with the IFGA operators in Xu and Yager (2006).

Comparison 2 Compared with the CGOWA operator developed in Zhou and Chen (2011), the new methods consider the consistency of priority levels of different attributes for MADM problems and are suitable for intuitionistic fuzzy environment.

Comparison 3 If we apply the intuitionistic fuzzy geometric interaction aggregation operator in He et al. (2014) for MADM problems to solve the numerical example in Sect. 4.2, we will fail to take into account of the priority levels of different attributes, namely the important information in Table 1 will be disregarded.

Comparison 4 If we use the scaled prioritized intuitionistic fuzzy interaction averaging (*SPIFIA*) operator proposed in He et al. (2016c) to aggregate the information in the above example, namely, using the method in He et al. (2016c) to solve the numerical example in this paper, then we have

$$A_1 = \langle 0.5762, 0.2114 \rangle, \quad A_2 = \langle 0.5258, 0.2772 \rangle \\ A_3 = \langle 0.4918, 0.2522 \rangle, \quad A_4 = \langle 0.4930, 0.3802 \rangle.$$

Then

$$S(A_1) = 0.3648, \quad S(A_2) = 0.2486, \quad S(A_3) = 0.2396, \\ S(A_4) = 0.1128.$$

Thus, x_1 is the best one.

Thus, the final result got in this paper is the same to that obtained in He et al. (2016c); this illustrates the effective of proposed method in this paper.

However, the *SPIFIA* operator proposed in He et al. (2016c) weights only the intuitionistic fuzzy numbers by the priority levels of the attributes, and in many practical situations, the intuitionistic fuzzy numbers themselves are also needed to be weighted, for example, if the different accuracy degrees of evaluated IFNs measured by the weight vector in numerical example in Sect. 4.2 are $\omega = (0.01, 0.37, 0.22, 0.19, 0.21)^T$, with the *SPIFHGIA* operator proposed in this paper to aggregate the decision information, we have

$$A_1 = \langle 0.4821, 0.1170 \rangle, \quad A_2 = \langle 0.5406, 0.1260 \rangle \\ A_3 = \langle 0.4020, 0.1058 \rangle, \quad A_4 = \langle 0.4789, 0.1960 \rangle.$$

Then

$$S(A_1) = 0.3651, \quad S(A_2) = 0.4146, \quad S(A_3) = 0.2962, \\ S(A_4) = 0.2829.$$

Thus, x_2 is the best one.

Which shows that the low accuracy degrees of evaluated IFNs A_{i1} ($i = 1, 2, 3, 4$) will weak the effects of attributes H_1 correspondingly, and thus, the result obtained is more rational and conforms to reality.

Thus, the difference between the *SPIFIA* operator in He et al. (2016c) and the *SPIFHGIA* operator in this paper is that the *SPIFIA* operator in He et al. (2016c) is based on the interactive addition and scalar multiplication operations; the *SPIFHGIA* operator in this paper is based on the interactive multiplication and power operations. In addition, this paper provides new methods for MAGDM taking into account of the consistency of the priority levels of different attributes.

In addition, the advantage of the proposed *SPIFHGIA* operator in this paper over the *SPIFIA* operator in He et al. (2016c) is that the proposed *SPIFHGIA* operator in this paper takes into account the intuitionistic fuzzy numbers themselves by the accuracy degrees measured by the weight vector and get much reasonable result in practical situations that the accuracy degrees of the evaluated the intuitionistic fuzzy numbers themselves are also needed to be weighted.

5 Conclusions

The work develops the *SPIFGIA*, *SPIFWGIA*, *SPIFHGIA*, and *SPIFHWGIA* operators, considering the prioritized levels of different attributes with priority labels. We investigate the properties of the *SPIFGIA* and *SPIFWGIA* operators, and discuss the priority levels between criteria in different situations for decision-making problems. The construction of intuitionistic fuzzy sets with granularity is discussed. Then, we develop new approaches to intuitionistic fuzzy decision-making problems based on the presented *SPIFGIA*, *SPIFWGIA*, *SPIFHGIA*, and *SPIFHWGIA* operators. Finally, the effectiveness and feasibility of the new methods are showed by an example and some comparisons are given.

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