Seismic rotational displacement of retaining walls: a pseudo-dynamic approach

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Received: 12 May 2016 / Accepted: 11 July 2016 / Published online: 20 July 2016
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Abstract Design of earth retaining wall is an important problem in geotechnical engineering. A retaining wall may fail in sliding or rotation. In the present study, rotational mode of failure is considered. A new approach to compute the rotational displacement of gravity retaining wall under seismic condition is proposed. Seismic forces are computed using pseudo-dynamic method. Dry backfill soil is considered. The present solution is for horizontal ground surface. In the computation of rotational displacement, the location of rotating wall and shift in the point of application of all the associated forces after each time step is included which was ignored in earlier studies. From the present analysis, it is found that the rotational displacement depends on the characteristics of the input motion. A non-dimensional term is used to quantify the effect of input motion on the rotational displacement. Shear strength properties of the backfill soil and geometry of the wall plays very crucial role in computation of rotational displacement.

Keywords Retaining wall · Rotational displacement · Earthquake · Limit equilibrium method · Pseudo-dynamic approach

List of symbols

$a_{hs}(z, t), a_{vs}(z, t)$	Horizontal and vertical acceleration in backfill soil at depth z and time t
$a_{hw}(z, t), a_{vw}(z, t)$	Horizontal and vertical acceleration in wall at depth z and time t
b	b_w/H
b_w	Top width of the retaining wall
FS_{RD}	Dynamic factor of safety against rotation failure
f_a	Amplification factor for backfill soil
f_{aw}	Amplification factor for wall material
g	Acceleration due to gravity
G	Shear modulus
H	Height of retaining wall
I_c	Polar moment of inertia of the wall about centroid
k_h, k_v	Horizontal and vertical seismic acceleration coefficient at the base
$P_{ae}(t)$	Seismic active thrust
$Q_{hs}(t), Q_{hw}(t)$	Horizontal inertia forces in the active wedge and wall due to seismic acceleration
r_c	Radial distance between centroid and rotation center E
t	Time
t_s	Duration of input motion
T	Period of lateral shaking
V_s, V_{sw}	Shear wave velocity in the backfill and wall
V_p, V_{pw}	Primary wave velocity in the backfill and wall

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W_w	Weight of the wall
W_s	Weight of active soil wedge
z	Depth from the top of the backfill or wall
α	Rotational acceleration
α_f	Angle of inclination of failure plane with horizontal
β	Rotational velocity
$\gamma_{\text{soil}}, \gamma_{\text{wall}}$	Unit weight of the backfill and wall material
δ	Wall friction angle
ϕ	Soil friction angle
θ	Wall inclination angle
ω	Angular frequency of motion = $2\pi/T$
η	Angle of inclination of radial distance between centroid and rotation center E with horizontal
ψ	Rotational displacement

Introduction

Sliding and rotational failure are one of the two major failure modes of rigid gravity retaining wall. Safe weight of a retaining wall is obtained separately for sliding at the base and rotation about its toe. The design weight is the highest among these two. Majority of the practicing engineers use simple limit equilibrium method for analysis and design. If the driving force/moment is greater than the resisting force/moment, the section of the retaining wall becomes unsafe. Gravity retaining structures are heavy in section because it resists the thrust from the retained fill using its self-weight. And if we consider the earthquake forces, the design section of these walls become huge. So, if we can reduce the design section of this gravity retaining wall, it will be a way forward towards sustainability. A more realistic approach is to allow the wall to slide or rotate satisfying its serviceability criteria under seismic condition. As we all are aware that a very small quantity of displacement is sufficient to trigger active failure, a very strict serviceability criteria must be adopted in the design guideline.

Basha and Babu [2] computed the sliding displacement of bridge abutment under passive condition using pseudo-dynamic method. Relative acceleration of the failure wedge was integrated to compute the sliding displacement. Huang et al. [7] conducted shake table experiments on rigid retaining walls. Experimental observations were compared with the theoretical computation using Newmark's method. Conti et al. [6] proposed a theory for sliding displacement. A corrective factor was suggested for the horizontal relative acceleration of the wall and that factor is related to the properties of backfill soil and wall material.

Zeng and Steedman [10] proposed rotating block method for computation of rotational displacement of gravity retaining wall under seismic condition. Horizontal seismic inertia force was assumed to be pseudo-static in nature. Effect of vertical seismic inertia force was ignored. It is well known, that a pseudo-static analysis does not consider the effects of time, frequency and dynamic properties of the backfill material. Choudhury and Nimbalkar [4, 5] computed the rotational displacement of gravity retaining wall using rotating block method and pseudo-dynamic seismic inertia forces for both passive and active earth pressure conditions. Basha and Babu [3] used a non-linear failure surface in the computation of rotational displacement for passive condition. Pain et al. [9] proposed to update the location of the rotating wall and its influence on the directions of external forces during each time increment of the calculation of rotational displacement.

First the expression of rotational acceleration proposed by Choudhury and Nimbalkar [5] are reviewed and rectified, and then the rotational displacement of gravity retaining wall is computed for the active case. An improvement over the rotating block method is proposed.

Methodology

Seismic inertia forces

Pseudo-dynamic method is extensively used in the solution of earth pressure problem under seismic condition. Description of pseudo-dynamic method is not given in this paper to avoid similarity. The expression of acceleration at any instance was proposed by Choudhury and Nimbalkar [5]. The expression of acceleration distribution for the backfill soil and the wall are as follows:

$$a_{hs}(z, t) = \left\{ 1 + \frac{H-z}{H} (f_a - 1) \right\} k_h g \cdot \sin \omega \left(t - \frac{(H-z)}{V_s} \right) \quad (1)$$

$$a_{vs}(z, t) = \left\{ 1 + \frac{H-z}{H} (f_a - 1) \right\} k_v g \cdot \sin \omega \left(t - \frac{(H-z)}{V_p} \right) \quad (2)$$

$$a_{hw}(z, t) = \left\{ 1 + \frac{H-z}{H} (f_{aw} - 1) \right\} k_h g \cdot \sin \omega \left(t - \frac{(H-z)}{V_{sw}} \right) \quad (3)$$

$$a_{vw}(z, t) = \left\{ 1 + \frac{H-z}{H} (f_{aw} - 1) \right\} k_v g \cdot \sin \omega \left(t - \frac{(H-z)}{V_{pw}} \right) \quad (4)$$

Fig. 1 Details of forces acting on the soil wedge and the wall

Backfill soil is assumed to be isotropic, homogeneous and dry. Present solution is for rigid retaining structure. The basic soil properties: friction angle and unit weight are assumed to be constant during shaking. Shear modulus (G) is constant within the backfill. Details of the failure surface are shown in Fig. 1. ABEF is a gravity retaining wall. Angle of inclination is θ . H is the vertical height of the wall.

α_f is the inclination angle of the assumed failure surface with the horizontal. Mass of a thin element dz of the active soil wedge ABD at depth z is

$$m_s(z) = \frac{\gamma_{\text{soil}}(H-z)[\tan \theta + \cot \alpha_f]}{g} dz \quad (5)$$

$Q_{hs}(t)$ and $Q_{vs}(t)$ are the seismic inertia forces in the active soil wedge ABD

$$Q_{hs}(t) = \int_0^H m_s(z) a_{hs}(z, t) dz \quad (6)$$

$$Q_{vs}(t) = \int_0^H m_s(z) a_{vs}(z, t) dz \quad (7)$$

Total seismic active earth pressure, $P_{ac}(t)$ may be computed by solving the equilibrium equation of the failure wedge ABD,

$$P_{ac}(t) = \frac{W_s \sin(\alpha_f - \phi) + Q_{hs}(t) \cos(\alpha_f - \phi) - Q_{vs} \sin(\alpha_f - \phi)}{\cos(\phi + \delta + \theta - \alpha_f)}, \quad (8)$$

where $W_s = 0.5\gamma H^2(\tan \theta + \cot \alpha_f)$.

However, the point of application of $P_{ac}(t)$ has to be determined. The expression of $P_{ac}(t)$ is in terms of the height of the wall, H . First the generic expressions of these integrals are obtained in terms of variable z instead of H . And then the expression of $P_{ac}(t)$ is differentiated with respect to z . This will give us the distribution of $P_{ac}(t)$ with respect to depth. The total moment of the seismic active

earth pressure with reference to the point E may be calculated as:

$$h_d(t) = \frac{\int_0^H P_{ac}(t) \cos \theta (H-z) dz}{P_{ac}(t) \cos \delta}; \quad (9)$$

$$\text{where } p_{ac}(t) = \frac{\partial P_{ac}(t)}{\partial z}$$

Wall is also subjected to harmonic base shaking. The mass of a thin element dz of the wall AFEB at depth z is

$$m_w(z) = \frac{\gamma_{\text{wall}}[b_w + 2z \tan \theta]}{g} dz. \quad (10)$$

$Q_{hw}(t)$ and $Q_{vw}(t)$ are the seismic inertia forces in the wall AFEB

$$Q_{hw}(t) = \int_0^H m_w(z) a_{hw}(z, t) dz \quad (11)$$

$$Q_{vw}(t) = \int_0^H m_w(z) a_{vw}(z, t) dz \quad (12)$$

Critical acceleration coefficient for rotation

FS_{RD} is a dynamic factor of safety against rotational mode of failure.

$$FS_{RD} = \frac{\text{Resisting moment}}{\text{Driving moment}} = \frac{P_{ac} \sin(\delta + \theta) \times (B_w - h_d \tan \theta) + W_w r_c \cos \eta_0}{P_{ac} \cos(\delta + \theta) h_d + Q_{hw} r_c \sin \eta_0 + Q_{vw} r_c \cos \eta_0} \quad (13)$$

The critical seismic acceleration coefficients at the base are evaluated using Eq. (13). A trial and error procedure is adopted. And in that process a seismic acceleration coefficient value is assumed to compute the value of FS_{RD} . The minimum value of seismic acceleration coefficient for which FS_{RD} becomes 1.0 is categorized as critical seismic acceleration coefficient.

The rotation displacement (ψ) may be obtained as:

$$\psi = \int_0^t \beta \, dt \quad (18)$$

A procedure is developed to update the shift in the point of application of all the associated forces after each time step. Total input motion is divided into n number of equal segments each of Δt duration. When the ground acceleration just exceed the critical base acceleration, the wall will start rotating, and at that time step seismic active thrust is $P_{ae}(t)$ for the wall inclination θ , the point of application of the seismic active earth pressure is $h_d(t)$, $\eta = \eta_0$ and $\psi = 0$. Angular acceleration α and angular velocity β may be determined from the Eqs. (16) and (17a, 17b). Calculate the rotational displacement ψ from the Eq. (18). For the next time step, i.e., $t + \Delta t$, update the wall inclination to $\theta + \psi$, solve for the seismic active earth thrust $P_{ae}(t + \Delta t)$, $h_d(t + \Delta t)$ may be determined using Eq. (9), update η to $\eta_0 + \psi$. Obtain α , β and ψ . This procedure is repeated until the expression in Eq. (17a) is satisfied.

Results and discussion

A computer program is written with the assumption of Zeng and Steedman [10] and seismic forces are considered to be pseudo-static in nature. Figure 3 shows the validation of the MATLAB program with the results reported by Zeng and Steedman [10]. All the input data are kept exactly same as given in Example 1 of Zeng and Steedman [10]. The purpose of Fig. 3 is to demonstrate the accuracy of the developed program. And it is worth mentioning that the developed program matches well with the results of Zeng

Fig. 3 Validation of the developed MATLAB program

Fig. 4 Influence of soil friction angle on rotational displacement

and Steedman [10]. After this validation, pseudo-static seismic forces are replaced by pseudo-dynamic seismic forces mentioned in the Eqs. (6), (7), (11) and (12) and the proposed procedure to update the shift in the point of application of all the associated forces after each time step is implemented in the MATLAB program.

Following input parameters are considered $b_w/H = 0.2$ and 0.3 ; $\phi = 20^\circ, 30^\circ$ and 40° ; $\delta/\phi = 0.0, 0.5$ and 1.0 ; $\theta = 0, 3, 5$ and 7° ; duration of shaking (t_s) = 1–5 s; $f_{aw} = 1.0$; $T = 0.33$ s; $\gamma_{soil} = 17$ kN/m³; $\gamma_{wall} = 24$ kN/m³; $V_p/V_s = 1.87$ and $V_{pw}/V_{sw} = 1.56$.

Rotational displacement (ψ) decrease with the increase in the soil friction angle value (ϕ) (refer Fig. 4). ψ decreases by about 83 and 81 % when ϕ increases from 20° to 30° and 30° to 40° , respectively, for $k_h = 0.20$. Rotational displacement is decreasing due to the reduction in the seismic active earth pressure. Rotational displacement (ψ) decrease with the increase in wall friction angle value (δ) (refer Fig. 5). ψ decreases by about 54 and 73 % when δ increases from 0 to 0.5ϕ and 0.5ϕ to ϕ for $k_h = 0.30$.

Wall inclination angle (θ) also has significant influence on the rotational displacement (ψ) (refer Fig. 6). When θ increases from 0° to 3° and 3° to 5° , ψ decreases by about 81 and 79 %, respectively, for $k_h = 0.30$. But the wall stops rotation when θ increases from 5° to 7° . So unlike the previous cases (for soil friction angle and wall friction angle), where only the seismic active earth pressure plays the key role, here the geometry of the wall plays a pivotal role in the rotational displacement. For higher values of θ , weight of the wall is more so the extra restoring moment is reducing the rotational displacement. Rate of increase in the rotational displacement is observed to be more for lower values of wall inclination angle.

Vertical seismic acceleration reduces the total resisting moment which ultimately leads to a higher value of

Fig. 5 Influence of wall friction angle on rotational displacement

Fig. 7 Influence of vertical seismic acceleration on rotational displacement

Fig. 6 Influence of wall inclination angle on rotational displacement

rotational displacement for the same amount of horizontal seismic inertia force. Rotational displacement decreases by about 30 and 26 % when k_v increases from 0 to $0.5 k_h$ and $0.5 k_h$ to k_h , respectively, for $k_h = 0.30$ (refer Fig. 7).

In the present study, finite values of seismic wave velocities are considered for both soil and wall material. Materials used for the construction of rigid retaining walls generally have high values of seismic wave velocities because of which phase change and amplification of input acceleration are not generally encountered in the wall. Amplification of acceleration is commonly encountered in the soil deposit. Present method allows us to explore the influence of amplification of acceleration through a very simple model. In that model a linear variation of acceleration is considered and the acceleration is assumed to be amplified towards the ground surface. The amount of amplification needs to be assumed and it is done with the

Fig. 8 Influence of amplification of acceleration in the backfill soil on rotational displacement

help of an amplification factor value. Rotational displacement increases as the amplification factor increases (refer Fig. 8). Seismic active earth pressure is more for higher amplification factor value. ψ increases by approximately 42 % when amplification factor is increased from 1.0 to 1.6 for $k_h = 0.30$.

The amount of rotational displacement largely depends on the duration of input motion (t_s). For the input parameters mentioned in the Fig. 9, it is seen that for a 5-m-high retaining wall when t_s increases from 3 to 5 s, the rotational displacement increases by about 99 %.

Referring to Fig. 9, a typical comparison is made between the rotational displacement computed using the rotating block method and the proposed method for $\phi = 40^\circ$, $\delta = \phi/2$, $\theta = 0^\circ$, $b_w/H = 0.3$, $k_h = 0.3$, $k_v = 0$, $f_a = f_{aw} = 1.0$, $T = 0.33$ s, $V_s = 200$ m/s, $V_p = 374$ m/s,

Fig. 9 Comparison of rotational displacement for $\phi = 40^\circ$, $\delta = 0.5\phi$, $\theta = 0^\circ$, $b_w/H = 0.3$, $k_h = 0.3$, $k_v = 0$, $f_a = f_{aw} = 1.0$, $T = 0.33$ s, $V_s = 200$ m/s, $V_p = 374$ m/s, $V_{sw} = 2500$ m/s, $V_{pw} = 3900$ m/s $\gamma_{soil} = 17$ kN/m³, and $\gamma_{wall} = 24$ kN/m³

Fig. 10 Variation of seismic inertia forces in soil and wall for 1-m-high retaining wall having the same input parameters mentioned in Fig. 9

$V_{sw} = 2500$ m/s, $V_{pw} = 3900$ m/s $\gamma_{soil} = 17$ kN/m³, and $\gamma_{wall} = 24$ kN/m³. Three different retaining walls with height, $H = 1, 5$ and 10 m are considered keeping rest of the input parameters same. Intentionally the amplification factor values are kept equal to 1.0 to understand the effect of phase difference of acceleration and also the consequence of the assumption of Zeng and Steedman [10]. Figure 9 clearly shows the importance of the proposed method when the walls are subjected to longer duration of shaking. The values of rotational displacement computed using the proposed method and the rotating block method are comparable when $\psi < 1^\circ$.

Rotational displacement computed using the proposed methodology is 34% higher than that obtained using rotating block method for 1 -m-high retaining wall subjected to 1 s of ground shaking. For 1 m height retaining wall, horizontal seismic inertia force in soil and the wall are in phase, i.e., they are acting simultaneously. Figure 10

Fig. 11 Variation of seismic inertia forces in soil and wall for 5-m-high retaining wall having the same input parameters mentioned in Fig. 9

shows the variation of these two seismic inertia forces with time and it is clear that there is no time lag or phase difference between these two seismic inertia forces. When the wall rotation reaches a critical value after which the point of application of gravitational acceleration crosses the toe of the wall, the resisting moment becomes a part of driving moment and the wall topples about the toe. But rotating block method does not consider this phenomenon, because the method does not consider the change in location of the rotating wall. For 1 -m-high retaining wall and the data mentioned in the Fig. 9, if the duration of input motion is greater than 1.966 s the wall will fail in rotation about its toe according to the proposed methodology.

Rotational displacement computed using the proposed methodology is only 3% higher than that obtained using rotating block method for the 5 -m-high retaining wall subjected to 1 s of ground shaking. But the difference in rotation displacement is as high as 43% for 5 s duration of shaking. Present results are higher than the results of rotating block method for the 5 -m-high retaining wall. Whereas rotational displacement computed using the proposed methodology is comparable with the results of rotating block method for 10 m height of the wall for even 5 s duration of shaking. Figures 11 and 12 show the variation of horizontal seismic inertia force in soil and the wall for wall height 5 and 10 m, respectively, and a clear phase difference may be observed. The phase difference is more for 10 m height of retaining wall. This is the difference between the pseudo-static and pseudo-dynamic approach. Pseudo-dynamic approach demonstrates that both these seismic inertia forces may or may not act simultaneously.

This time lag or phase difference may be understood with the help of a non-dimensional term H/TV_s . Phase difference is more for higher values of H/TV_s . In other words, for the same period of ground shaking (T) and same velocity of shear wave (V_s), phase difference will be marginal for shorter walls and for comparatively higher

Fig. 12 Variation of seismic inertia forces in soil and wall for 10-m-high retaining wall having the same input parameters mentioned in Fig. 9

walls the phase difference have significant influence on rotational displacement. If we look into the present set of data, the value of H/TV_s is 0.03, 0.15 and 0.30 for wall height of 1, 5 and 10 m, respectively. And as discussed the phase difference is significant for 10 m height of retaining wall (refer Fig. 12).

A pseudo-static-based approach does not account this phase difference phenomenon. Centrifuge experiments conducted by Nakamura [8] also highlighted that wall inertia and dynamic earth pressure do not occur simultaneously. Athanasopoulos-Zekkos et al. [1] numerically validated the experimental results reported by Nakamura [8]. Numerical simulations were carried out in commercially available Finite Element software. Reduction in the value of dynamic active thrust, acting at the instant peak wall inertia was also reported by Athanasopoulos-Zekkos et al. [1]. Finding of the present study is in line with the conclusion of Nakamura [8] and Athanasopoulos-Zekkos et al. [1].

The assumption of Zeng and Steedman [10] is found to be valid only for very small values of rotational displacement ($\psi < 1^\circ$). It is also observed that, the rotational displacement obtained using both the methodologies reduce with an increase in the wall height. These reductions in the value of rotational displacement may be attributed to the increase in the value of the polar moment of inertia.

Conclusions

A solution is proposed for the calculation of rotational displacements of gravity retaining walls. Importance of change in location of the rotating wall and the shift in the point of application of seismic active earth pressure after each time step is highlighted. From the analysis, it is clear that the rotational displacement not only depends on the

properties of the backfill, it depends on the characteristics of the input motion, wall geometry and the properties of the wall material. In the present analysis, a time lag or phase difference is observed between the seismic inertia force in the soil and seismic inertia force in the wall. And this observation is in line with the earlier experimental and numerical studies. Pseudo-static method considers that these two seismic forces act simultaneously which is certainly not the case. Design of retaining wall against rotational failure considering pseudo-static seismic forces may be over conservative for certain set of input parameters. Present method may be a good alternative to pseudo-static approach in terms of sustainability. Soil friction angle, wall friction angle and wall inclination angle have significant influence on the rotational displacement. Using the present methodology one can optimize the design section, i.e., width of the wall, inclination of the wall, backfill material, etc. The findings of the present study are helpful in designing retaining wall which is seismically stable.

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