Intell Ind Syst (2016) 2:107–117
DOI 10.1007/s40903-016-0044-y

ORIGINAL PAPER

A PEM Fuel Cells Control Approach Based on Differential Flatness Theory

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Received: 14 February 2016 / Revised: 25 April 2016 / Accepted: 4 May 2016 / Published online: 20 May 2016

Abstract The article presents an approach to nonlinear control of fuel cells using differential flatness theory and Kalman filtering. First, it is proven that the dynamic model of fuel cells is a differentially flat one which means that all its state variables and control inputs can be expressed as differential functions of specific stare variables which are the so-called flat outputs of the system. By exploiting the differential flatness properties of the model its transformation to an equivalent linear form (canonical Brunovsky form) becomes possible. For the latter description of the system's dynamics the design of a state-feedback controller is achieved. This control scheme should be also robust to model uncertainties and external perturbations. To cope with this problem the state-space description of the PEM fuel cells is extended by considering as additional state variables the derivatives of the aggregate disturbance input. Next, a Kalman filter-based disturbance observer is applied to the linearized extended model of the fuel cells. This estimation method enables to identify the disturbance and model uncertainty terms that affect the system and to introduce a complementary control element that compensates for the perturbations' effects. The efficiency of the proposed control scheme is evaluated through simulation experiments.

Keywords PEM fuel cells · Nonlinear control · Differential flatness theory · Global linearization · Kalman filtering · Disturbances estimator

1 Introduction

A fuel cell is an electrochemical energy device that converts the chemical energy of the reaction between hydrogen and oxygen into electricity and heat giving also water as by product of the reaction [1–5]. Fuel cells are a renewable power generation source and their use gets widely deployed in the smart grid [6–10]. In this article a nonlinear feedback control method that is based on differential flatness theory is developed for proton exchange membrane (PEM) Fuel Cells [11–13]. First it is proven that the dynamic model of the fuel cells is a differentially flat one. This means that all its state variables and its control inputs can be expressed as differential functions of a primary variable which is the so-called flat output [14–18]. Differential (linear) independence is another property that holds between the flat output and its derivatives. By exploiting differential flatness properties the fuel cells' model can be transformed into an equivalent linearized description which is the canonical Brunovsky form [19–23]. In the latter representation of the system the design of a stabilizing feedback controller becomes possible.

Another problem that has to be dealt with in the deign of the fuel cells' nonlinear controller is that the system is subjected to model uncertainties and external perturbations. To compensate for these disturbances it is proposed to use a Kalman Filter-based disturbance observer in the control loop [24,25]. The state-space description of the PEM fuel cells is extended by considering as additional state variables the derivatives of the aggregate disturbance input. Through the Kalman Filter recursion and by processing exclusively mea-

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surements of the system's output it becomes also possible to identify the perturbation input. The applied Kalman filter method, also known as derivative-free nonlinear Kalman filter consists of the Kalman Filter algorithm applied on the input–output linearized model of the PEM fuel cells. Next, the feedback control law for the system is modified with the inclusion of an additional element which annihilates the aggregate disturbances effects. The performance of this differential flatness theory-based control and estimation scheme is confirmed through simulation experiments.

The structure of the paper is as follows: in "Nonlinear Dynamics of the Fuel Cells" section the nonlinear dynamics of the PEM fuel-cells model is analyzed and the associated states-space description is obtained. In "Linearization of the Fuel Cells Dynamics Using Differential Flatness Theory" section it is proven that the dynamic model of the PEM fuel cells is a differentially flat one. Moreover, by exploting differential flatness properties an input–output linearized description of the system is obtained. In "Linearization of the Fuel Cells Dynamics Using Lie Algebra" section linearization of the PEM fuel cells model is performed using a Lie algebra-based approach. In "Flatness-Based Control of the Nonlinear Fuel Cells Dynamics" section flatness-based control is developed for the PEM fuel cells dynamics while the Kalman Filter is used in the control loop as a disturbance observer. In "Simulation Tests" section the stability properties and good transient performance of the proposed control scheme are confirmed through simulation experiments. Finally, in "Conclusions" section concluding remarks are provided.

2 Nonlinear Dynamics of the Fuel Cells

2.1 Nonlinear Dynamics of PEM Fuel Cells

The PEM fuel cell consists of a polymer electrolyte membrane which is placed between the electrodes (anode and cathode), as shown in Fig. 1. Ions can be diffused through the membrane. If an electrical circuit is established between the anode and the cathode, there will be also a flow of electrons and a potential will appear between the electrodes.

In the considered PEM fuel cell the anode is supplied with gas that contains hydrogen while the cathode is supplied with gas which contains oxygen. The overall electrochemical dynamics is

$$\begin{array}{ll} \text{Anode} & 2H_2 \leftrightarrow 4H^+ + 4e^- \\ \text{Cathode} & O_2 + 4H^+ + 4e^- \leftrightarrow 2H_2O \\ \text{Overall} & 2H_2 + O_2 \leftrightarrow 2H_2O + \text{electricity} + \text{heat} \end{array} \tag{1}$$

The anode can either be supplied with H_2 under pressure or can be supplied with hydrogen by the *reformer* which generates H_2 from methane or other natural gas. The cathode

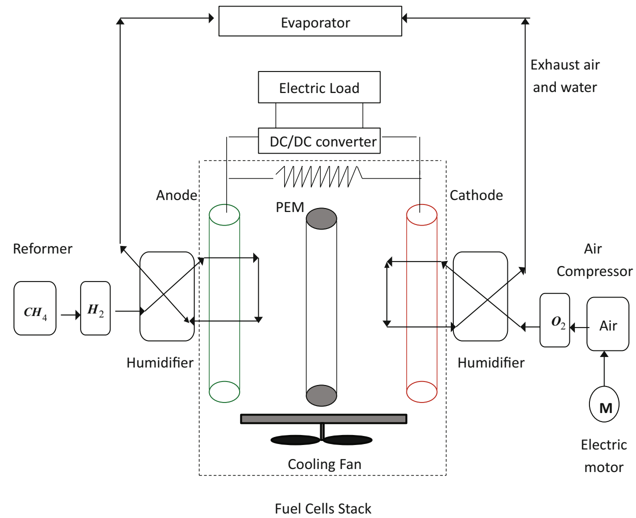


Fig. 1 A PEM fuel cells model

is supplied with oxygen through an air compressor connected to an air filter and finally connected to an air flow controller (valve). On both sides a humidifier is used to prevent drying of the PEM. To produce a higher voltage, multiple cells are connected in series and this forms a stack of fuel cells. A singe cell provides voltage between 0 and 1 V.

Conditions about the PEM fuel cells functioning are outlined as follows: (1) The temperature of the fuel cells both at the anode's and at the cathode's side is assumed to remain constant, (2) The anode and the cathode are sufficiently humidified, (3) it is assumed that the produced water is evaporated (4) the inlet reactants are assumed to be supplied in constant mole fractions. This means that pure hydrogen 100 % is fed to the anode. The air supply to the cathode consists of nitrogen and oxygen at ratios 79 and 21 %, respectively, (5) the gases are assumed to follow the ideal gases low.

The dynamics of the fuel cells system is given through the following two sets of differential equations [2]:

Anode mole conservation:

$$\frac{dP_{H_2}}{dt} = \frac{RT}{V_a}[H_{2in} - H_{2used} - H_{2out}] \tag{2}$$

$$\frac{dP_{H_2O_A}}{dt} = \frac{RT}{V_a}[H_2O_{in} - H_2O_{used} - H_2O_{out}] \tag{3}$$

Cathode mole conservation:

$$\frac{dP_{O_2}}{dt} = \frac{RT}{V_c}[O_{2in} - O_{2used} - O_{2out}] \tag{4}$$

$$\frac{dP_{N_2}}{dt} = \frac{RT}{V_c}[N_{2in} - N_{2out}] \tag{5}$$

$$\frac{dP_{H_2O_c}}{dt} = \frac{RT}{V_c}[H_2O_{cin} - H_2O_{c\,produced} - H_2O_{cout} + H_2O_{cmbr}] \tag{6}$$

In the above state equations $H2_{in}$, $O2_{in}$, $H2O_{Ain}$, $N2_{in}$, and H_2O_{cin} are the inlet flow rates of hydrogen, oxygen, nitrogen, anode-side water and cathode-side water. Moreover, $H2_{out}$, $O2_{out}$, $H2O_{Aout}$, $N2_{out}$ and and H_2O_{cout} are the outlet flow rates of each reactant. Additionally, $H2_{used}$, $O2_{used}$ and $H_2O_{c\,produced}$ are the usage and produced concentrations of the reactants. Furthermore, H_2O_{mbr} is the water concentration transferred through the membrane and is a function of the stack current and of the humidity (which is assumed to remain constant). It is also noted that V_a is the anode's volume and V_c is the cathode's volume (multiplied by the reactant's mass concentration in mole).

2.2 A Nonlinear State Equations Model of the PEM Fuel Cells

In continuation to the previous analysis a nonlinear model of the PEM fuel cells system is presented. Focusing on the cathode, the state vector of the model is defined as $x = [p_{O_2}, p_{N_2}, \omega_{cp}, p_{sin}]^T$, where p_{O_2} is the oxugen pressure at the cathode, p_{N_2} is the nitrogen pressure at the cathode, ω_{cp} is the compressor's rotational speed (r/min), and p_{sm} is the supply manifold pressure [10].

By applying the ideal gas law and by considering that the volume of the cathode is known one has

$$\begin{aligned}\frac{dp_{O_2}}{dt} &= \frac{RT}{M_{O_2}V_{ca}}(W_{O_{2,in}} - W_{O_2,out} - W_{O_{2,react}})\\ \frac{dp_{N_2}}{dt} &= \frac{RT}{M_{N_2}V_{ca}}(W_{N_{2,in}} - W_{N_2,out})\end{aligned} \tag{7}$$

where V is the volume of the cathode, R is the universal gas constant, and M_{O_2}, M_{N_2} are the mass concentrations (in mole) of oxygen and nitrogen. The incoming flow rates of oxygen and nitrogen are given by

$$\begin{aligned}W_{O_2,in} &= x_{O_2}W_{ca,in}\\ W_{N_2,in} &= (1 - x_{O_2})W_{ca,in}\end{aligned} \tag{8}$$

where x_{O_2} is the oxygen mass fraction of the inlet air, $1 - x_{O_2}$ is the nitrogen mass fraction of the inlet air, and $W_{ca,in}$ is the mass flow rate entering the cathode which is given by

$$W_{ca,in} = \frac{1}{1+\omega_{atm}}k_{ca,in}(p_{sm} - p_{in}) \tag{9}$$

where ω_{atm} is the humidity ratio

$$\omega_{atm} = \frac{M_v}{M_{a,ca,in}}\frac{\phi_{ca}p_{sat}(T_{atm})}{p_{atm} - \phi_{ca}p_{sat}(T_{atm})} \tag{10}$$

M_v is the mass of the vapor in mole, $M_{a,ca,in}$ is the mass of the air in mole, ϕ_{ca} is the relative humidity in ambient conditions, $p_{sat}(T_{atm})$ is the saturation pressure in ambient temperature, p_{atm} is the atmospheric pressure and $k_{ca,in}$ is the cathode inlet orifice constant.

The outlet flow rates of oxygen and nitrogen $W_{O_2,out}$ and $W_{N_2,out}$ are calculated from the mass fraction of oxygen and nitrogen in the stack after reaction

$$\begin{aligned}W_{O_2,out} &= \frac{M_{O_2}p_{O_2}}{M_{O_2}p_{O_2}+M_{N_2}p_{N_2}+M_v p_{sat}}W_{ca,out}\\ W_{N_2,out} &= \frac{M_{N_2}p_{N_2}}{M_{O_2}p_{O_2}+M_{N_2}p_{N_2}+M_v p_{sat}}W_{ca,out}\end{aligned} \tag{11}$$

The flow rate at the cathode's exit $W_{ca,out}$ is calculated by the nozzle flow equation

$$\begin{aligned}W_{ca,out} &= \frac{C_D A_T p_{ca}}{\sqrt{RT}}\left(\frac{p_{atm}}{p_{ca}}\right)^{\frac{1}{T}}\\ \text{if } \frac{p_{atm}}{p_{ca}} &> \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}\left\{\frac{2\gamma}{\gamma-1}\left[1 - \left(\frac{p_{atm}}{p_{ca}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}\end{aligned} \tag{12}$$

where γ is the ratio of the specific heat capacities of the air, $p_{ca} = p_{O_2} + p_{N_2} + P_{sat}$. The mass flow rate of oxygen is expressed as

$$W_{O_2,\text{react}} = \frac{nI_{st}}{4F}M_{O_2} \tag{13}$$

where n is the number of cells in the stack, F is the Faraday number and I_{st} is the stack current. The compressor's turn speed is related to the associated mechanical torque

$$\frac{d\omega_{cp}}{dt} = \frac{1}{J_{cp}}(\tau_{cm} - \tau_{cp}) \tag{14}$$

where τ_{cm} is the mechanical input torque, τ_{cp} is the load torque [10]

$$\tau_{cm} = \eta_{cm}\frac{K_v}{R_{cm}}(v_{cm})k_v\omega_{cp} \tag{15}$$

$$\tau_{cp} = \frac{C_p}{\omega_{cp}}\frac{T_{atm}}{\eta_{cp}}\left[\left(\frac{p_{atm}}{p_{ca}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]W_{cp} \tag{16}$$

where k_t, R_{cm} and k_v are motor constants, η_{cm} is a coefficient that denotes the motor's mechanical efficiency. C_p is the specific heat capacity of air and W_{cp} is the compressor mass flow rate. The dynamics of the air pressure in the supply manifold depend on the compressor flow into the supply manifold $W_{cp} = A\omega_{cp}$, on the flow out of the supply manifold into the cathode $W_{co,in}$ and on the compressor flow temperature T_{cp} [10]

$$\frac{dp_{sm}}{dt} = \frac{RT_{cp}}{M_a V_{sm}}[W_{cp} - k_{ca,in}(p_{sm} - p_{ca})] \tag{17}$$

where V_{sm} is the supply manifold volume and T_{cp} is the temperature of the air leaving the compressor

$$T_{cp} = T_{atm} + \frac{T_{atm}}{\eta_{cp}}\left[\left(\frac{p_{sm}}{p_{atm}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{18}$$

The nonlinear state-space model of the PEM fuel-cells model is based om Eqs. (7), (14) and (17) [10]

$$\dot{x}_1 = c_1(x_4 - x_1 - x_2 - c_2) - \frac{c_3 x_1 W_{co,out}}{c_4 x_1 + c_5 x_2 + c_6} - c_7\zeta \quad (19)$$

$$\dot{x}_2 = c_8(x_4 - x_1 - x_2 - c_2) - \frac{c_3 x_2 W_{co,out}}{c_4 x_1 + c_5 x_2 + c_6} \quad (20)$$

$$\dot{x}_3 = -c_9 x_3 - c_{10}\left[\left(\frac{x_4}{c_{11}}\right)^{c_{12}} - 1\right] + c_{13}u \quad (21)$$

$$\dot{x}_4 = c_{14}\left\{1 + c_{15}\left[\left(\frac{x_4}{c_{11}}\right)^{c_{12}} - 1\right]\right\} \cdot [W_{cp} - c_{16}(x_4 - x_1 - x_2 - c_2)] \quad (22)$$

where the coefficients $c_1, c_2, \ldots, c_{16}$ are constants. The control input u depends the motor's current. The control input ζ is the stack current (which can be considered as an external perturbation to the model).

The previous set of state-space equations is also written in the form

$$\dot{x} = f(x) + g(x)u \Rightarrow$$

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{pmatrix} = \begin{pmatrix} c_1(x_4 - x_1 - x_2 - c_2) - \frac{c_3 x_1 W_{co,out}}{c_4 x_1 + c_5 x_2 + c_6} - c_7\zeta \\ c_8(x_4 - x_1 - x_2 - c_2) - \frac{c_3 x_2 W_{co,out}}{c_4 x_1 + c_5 x_2 + c_6} \\ -c_9 x_3 - c_{10}\left[\left(\frac{x_4}{c_{11}}\right)^{c_{12}} - 1\right] \\ c_{14}\left\{1 + c_{15}\left[\left(\frac{x_4}{c_{11}}\right)^{c_{12}} - 1\right]\right\} \cdot [W_{cp} - c_{16}(x_4 - x_1 - x_2 - c_2)] \end{pmatrix} + \begin{pmatrix}0\\0\\c_{13}\\0\end{pmatrix} u \quad (23)$$

By differentiating in time one obtains also

$$\dot{x}_4 = f_1(y, \dot{y}, \ddot{y}, x_2, \dot{x}_2) \quad (26)$$

Substituting Eqs. (25) into (20) one gets

$$\dot{x}_2 = f_3(y, \dot{y}, x_2) \quad (27)$$

By differentiating Eq. (19) with respect to time one has

$$\ddot{x}_1 = c_1(\dot{x}_4 - \dot{x}_1 - \dot{x}_2) - \frac{c_3 \dot{x}_1 W_{ca,out}(c_4 x_1 + c_5 x_2 + c_6) - c_3 x_1 W_{ca,out}(c_4 \dot{x}_1 + c_5 \dot{x}_2)}{(c_4 x_1 + c_5 x_2 + c_6)^2} \quad (28)$$

Using the flat output's notation $y = x_1$ the previous relation becomes

$$\ddot{y} = c_1(f_2(y, \dot{y}, \ddot{y}, x_2, f_3(y, \dot{y}, x_2)) - \dot{y} - f_3(y, \dot{y}, x_2)) - \frac{c_3 \dot{y} W_{ca,out}(c_4 y + c_5 x_2 + c_6) - c_3 y W_{ca,out}(c_4 \dot{y} + c_5 f_3(y, \dot{y}, x_2))}{(c_4 y + c_5 x_2 + c_6)^2} \quad (29)$$

3 Linearization of the Fuel Cells Dynamics Using Differential Flatness Theory

3.1 Differential Flatness of the PEM Fuel-Cells Model

It is proven that the dynamic model of the PEM fuel cells given in Eqs. (19), (22) is a differentially flat one, which means that all its state variables and its control inputs can be written as differential functions of the flat output and its derivatives.

The flat output of the model is taken to be $y = x_1$. Equation (19) is solved with respect to x_4. This gives

$$x_4 = \frac{1}{(c_4 y + c_5 x_2 + c_6)}\{(c_4 y\dot{y} + c_5 \dot{y} x_2 + c_6 \dot{y}) - (-c_1 y - c_1 x_2 - c_1 c_2)(c_4 y + c_5 x_2 + c_6) - c_3 y W_{ca,out}\} \Rightarrow$$

$$x_4 = f_1(y, \dot{y}, x_2) \quad (24)$$

or equivalently

$$x_4 = f_1(y, \dot{y}, x_2) \quad (25)$$

The above equation provides a relation between x_2 on the one side and y and its derivatives of the other side. Therefore, one obtains

$$x_2 = f_4(y, \dot{y}, \ddot{y}) \quad (30)$$

By substituting Eqs. (30) into (25) one arrives at

$$x_4 = f_2(y, \dot{y}, f_4(y, \dot{y}, \ddot{y})) \quad (31)$$

Next, by differentiating in time Eq. (31) one obtains

$$\dot{x}_4 = f_5(y, \dot{y}, \ddot{y}, y^{(3)}) \quad (32)$$

Thus, one has that state variables x_1, x_2 and x_3 are differential functions of the flat output y and its derivatives. By substituting Eqs. (30), (31) and $x_1 = y$ into Eq. (22), one obtains that x_3 is also a differential function of the flat output.

It holds that

$$x_3 = \frac{1}{A}\left[\frac{\dot{x}_4}{c_{14}\left\{1 + c_{15}\left[\left(\frac{x_4}{c_{11}}\right)^{c_{12}} - 1\right]\right\}} + c_{16}(x_4 - x_1 - x_2 - c_2)\right] \tag{33}$$

From Eq. (33) one also obtains that x_3 is a differential function of y, and also

$$\dot{x}_3 = f_6(y, \dot{y}, \ddot{y}, y^{(3)}, y^{(4)}) \tag{34}$$

Finally, by solving Eq. (21) with respect to the control input u one obtains that

$$u = \frac{1}{c_{13}}\left\{\dot{x}_3 + c_3 x_3 + c_{10}\left[\left(\frac{x_4}{c_{11}}\right)^{c_{12}} - 1\right]\right\} \tag{35}$$

or equivalently

$$u = f_7(y, \dot{y}, \ddot{y}, y^{(3)}, y^{(4)}) \tag{36}$$

Consequently, all state variables of the system and its control input can be expressed as differential functions of the flat output. This means that the PEM fuel-cells modelis a differentially flat one.

3.2 Transformation of the PEM Fuel Cells Model into a Canonical Form

By proving that the PEM fuel-cells system is differentially flat it can be also assured that it can be transformed into an equivalent linearized form (which is the Brunovsky canonical form). From Eq. (19), and after omitting the disturbance term (unknown stack current) one has

$$\dot{x}_1 = c_1(x_4 - x_1 - x_2 - c_2) - \frac{c_3 x_1 W_{co,out}}{c_4 x_1 + c_5 x_2 + c_6} \tag{37}$$

By differentiating with respect to time one gets

$$\begin{aligned}\ddot{x}_1 = {} & c_1(\dot{x}_4 - \dot{x}_1 - \dot{x}_2) \\ & - \frac{(c_3\dot{x}_1 W_{co,out})(c_4x_1 + c_5x_2 + c_6) - (c_3x_1W_{ca,out})(c_4\dot{x}_1 + c_5\dot{x}_2)}{(c_4x_1 + c_5x_2 + c_6)^2}\end{aligned} \tag{38}$$

Bu substituting in Eq. (19) the derivatives $\dot{x}_1$ from Eq. (19), $\dot{x}_2$ from Eq. (20) and $\dot{x}_4$ from Eq. (22) one gets

$$\begin{aligned}\ddot{x}_1 = {} & c_1\left(c_{14}\left[1 + c_{15}\left[\left(\frac{x_4}{c_{11}}\right)^{c_{12}} - 1\right]\right]\right) \\ & \times [Ax_3 - c_{16}(x_4 - x_1 - x_2 - c_2)] \\ & - c_1(x_4 - x_1 - x_2 - c_2) - \frac{(c_3x_1W_{ca,out})}{(c_4x_1 + c_5x_2 + c_6)} \\ & - c_8(x_4 - x_1 - x_2 - c_2) - \frac{(c_3x_2W_{ca,out})}{(c_4x_1 + c_5x_2 + c_6)} \\ & \frac{- c_3(c_1(x_4 - x_1 - x_2 - c_2) - \frac{c_3x_1W_{ca,out}}{c_4x_1 + c_5x_2 + c_6})W_{ca,out}}{(c_4x_1 + c_5x_2 + c_6)} \\ & + \frac{(c_3x_1W_{ca,out})}{(c_4x_1 + c_5x_2 + c_6)^2} \\ & \times \Bigg[c_4(c_1(x_4 - x_1 - x_2 - c)) - \frac{c_3x_1W_{ca,out}}{c_4x_1 + c_5x_2 + c_6} \\ & + c_5(c_8(x_4 - x_1 - x_2 - c_2)) - \frac{c_3x_2W_{ca,out}}{c_4x_1 + c_5x_2 + c_6}\Bigg]\end{aligned} \tag{39}$$

By differentiating the previous relation once more with respect to time one gets

$$\begin{aligned}x_1^{(3)} = {} & c_1\Bigg\{c_{14}\Bigg[c_{15}\Bigg[\Bigg(\frac{x_4}{c_{11}}^{c_{12}-1)}\dot{x}_3\Bigg)\Bigg]\Bigg][Ax_3 - c_{16}(x_4 - x_1 - x_2 - c_2)] \\ & + c_{14}\left[1 + c_{15}\left[\left(\frac{x_4}{c_{11}}\right) - 1\right]\right][A\dot{x}_3 - c_{16}(\dot{x}_4 - \dot{x}_1 - \dot{x}_2)] \\ & - c_1(\dot{x}_4 - \dot{x}_1 - \dot{x}_2 \\ & - \frac{(c_3\dot{x}_1W_{ca,out})(c_4x_1 + c_5x_2 + c_6) - (c_3x_1W_{ca,out})(c_4\dot{x}_1 + c_5\dot{x}_2)}{(c_4x_1 + c_5x_2 + c_6)^2} \\ & - c_8(\dot{x}_4 - \dot{x}_1 - \dot{x}_2 \\ & - \frac{(c_3\dot{x}_2W_{ca,out})(c_4x_1 + c_5x_2 + c_6) - (c_3x_2W_{ca,out})(c_4\dot{x}_1 + c_5\dot{x}_2)}{(c_4x_1 + c_5x_2 + c_6)^2}\Bigg\} \\ & - \frac{W_{ca,out}}{(c_4x_1 + +c_5x_2 + c_6)^2}\{-c_3(c_1(\dot{x}_4 - \dot{x}_1 - \dot{x}_2) \\ & \frac{-(c_2\dot{x}_1)W_{ca,out}(c_4x_1 + c_5x_2 + c_6) - (c_3x_1W_{ca,out})(c_4\dot{x}_1 + c_5\dot{x}_2)}{(c_4x_1 + c_5x_2 + c_6)^2}\Bigg\} \\ & + \frac{(c_3\dot{x}_1W_{ca,out})}{(c_4x_1 + c_5x_2 + c_6)^2}\Bigg[c_4\Bigg(c_1(x_4 - x_1 - x_2 - c_2) \\ & - \frac{c_3x_1W_{ca,out}}{c_4x_1 + c_5x_2 + c_6}\Bigg) \\ & - c_5\left(c_8(x_4 - x_1 - x_2 - c_2) - \frac{c_3x_2W_{ca,out}}{c_4x_1 + c_5x_2 + c_6}\right)\Bigg] \\ & + \frac{(c_3x_1W_{ca,out})}{(c_4x_1 + c_5x_2 + c_6)^2}\Bigg[c_4\Bigg(c_1(\dot{x}_4 - \dot{x}_1 - \dot{x}_2) \\ & - \frac{(c_3\dot{x}_1W_{ca,out})(c_4x_1 + c_5x_2 + c_6) - (c_3x_1W_{ca,out})(c_4\dot{x}_1 + c_5\dot{x}_2)}{(c_4x_1 + c_5x_2 + c_6)^2} \\ & c_5(c_8(\dot{x}_4 - \dot{x}_1 - \dot{x}_2) \\ & - \frac{(c_3\dot{x}_2W_{ca,out})(c_4x_1 + c_5x_2 + c_6) - (c_3x_2W_{ca,out})(c_4\dot{x}_1 + c_5\dot{x}_2)}{(c_4x_1 + c_5x_2 + c_6)^2}\Bigg)\Bigg] \\ & - \frac{(c_3x_1W_{ca,out})}{c_4x_1 + c_5x_2 + c_6{}^2}\Bigg[c_4\Bigg(c_1(x_4 - x_1 - x_2 - c_2) \\ & - \frac{c_3x_1W_{ca,out}}{(c_4x_1 + c_5x_2 + c_6)}\Bigg] \\ & + c_5(c_8(x_4 - x_1 - x_2 - c_2) - \frac{c_3x_2W_{ca,out}}{c_4x_1 + c_5x_2 + c_6}\Bigg)\Bigg]2(c_4\dot{x}_1 + c_5\dot{x}_2)\end{aligned} \tag{40}$$

By substituting in the previous relation $\dot{x}_3$ from Eq. (21) an input–output linearized description of the system's dynamics is obtained in the form

$$x_1^{(3)} = \tilde{f}(x) + \tilde{g}(x)u \tag{41}$$

where function $\tilde{f}(x)$ is given by

$$\begin{aligned}
\tilde{f}(x) = c_1 \bigg\{ & c_{14}\bigg[c_{15}\bigg[\bigg(\frac{x_4}{c_{11}}^{c_{12}-1)}\dot{x}_4\bigg)\bigg]\bigg][Ax_3 - c_{16}(x_4 - x_1 - x_2 - c_2)] \\
& + c_{14}\bigg[1 + c_{15}\bigg[\bigg(\frac{x_4}{c_{11}}\bigg) - 1\bigg]\bigg]\bigg[A\bigg(-c_9x_3 - c_{10}\bigg[\bigg(\frac{x_4}{c_{11}}\bigg)^{c_{12}} - 1\bigg]\bigg) \\
& - c_{16}(\dot{x}_4 - \dot{x}_1 - \dot{x}_2)\bigg] \\
& - c_1(\dot{x}_4 - \dot{x}_1 - \dot{x}_2 \\
& - \frac{(c_3\dot{x}_1W_{ca,out})(c_4x_1 + c_5x_2 + c_6) - (c_3x_1W_{ca,out})(c_4\dot{x}_1 + c_5\dot{x}_2)}{(c_4x_1 + c_5x_2 + c_6)^2} \\
& - c_8(\dot{x}_4 - \dot{x}_1 - \dot{x}_2 \\
& - \frac{(c_3\dot{x}_2W_{ca,out})(c_4x_1 + c_5x_2 + c_6) - (c_3x_2W_{ca,out})(c_4\dot{x}_1 + c_5\dot{x}_2)}{(c_4x_1 + c_5x_2 + c_6)^2}\bigg\} \\
& - \frac{W_{ca,out}}{(c_4x_1 + +c_5x_2 + c_6)^2}\bigg\{-c_3\bigg(c_1(\dot{x}_4 - \dot{x}_1 - \dot{x}_2) \\
& - \frac{(c_2\dot{x}_1)W_{ca,out}(c_4x_1 + c_5x_2 + c_6) - (c_3x_1W_{ca,out})(c_4\dot{x}_1 + c_5\dot{x}_2)}{(c_4x_1 + c_5x_2 + c_6)^2}\bigg\} \\
& + \frac{(c_3\dot{x}_1W_{ca,out})}{(c_4x_1 + c_5x_2 + c_6)^2}\bigg[c_4\bigg(c_1(x_4 - x_1 - x_2 - c_2) \\
& - \frac{c_3x_1W_{ca,out}}{c_4x_1 + c_5x_2 + c_6}\bigg) \\
& - c_5\bigg(c_8(x_4 - x_1 - x_2 - c_2) - \frac{c_3x_2W_{ca,out}}{c_4x_1 + c_5x_2 + c_6}\bigg)\bigg] \\
& + \frac{(c_3x_1W_{ca,out})}{(c_4x_1 + c_5x_2 + c_6)^2}\bigg[c_4\bigg(c_1(\dot{x}_4 - \dot{x}_1 - \dot{x}_2) \\
& - \frac{(c_3\dot{x}_1W_{ca,out})(c_4x_1 + c_5x_2 + c_6) - (c_3x_1W_{ca,out})(c_4\dot{x}_1 + c_5\dot{x}_2)}{(c_4x_1 + c_5x_2 + c_6)^2} \\
& - c_5(c_8(\dot{x}_4 - \dot{x}_1 - \dot{x}_2) \\
& - \frac{(c_3\dot{x}_2W_{ca,out})(c_4x_1+c_5x_2+c_6)-(c_3x_2W_{ca,out})(c_4\dot{x}_1 + c_5\dot{x}_2)}{(c_4x_1 + c_5x_2 + c_6)^2}\bigg)\bigg] \\
& - \frac{(c_3x_1W_{ca,out})}{c_4x_1 + c_5x_2 + c_6{}^2}\bigg[c_4\bigg(c_1(x_4 - x_1 - x_2 - c_2) \\
& - \frac{c_3x_1W_{ca,out}}{(c_4x_1 + c_5x_2 + c_6)}\bigg] \\
& + c_5(c_8(x_4 - x_1 - x_2 - c_2) - \frac{c_3x_2W_{ca,out}}{c_4x_1 + c_5x_2 + c_6}\bigg)\bigg]2(c_4\dot{x}_1 + c_5\dot{x}_2)
\end{aligned} \tag{42}$$

and function $\tilde{g}(x)$ is given by

$$\tilde{g}(x) = c_1\left(c_{14}\left[1 + c_{15}\left[\left(\frac{x_4}{c_{11}}\right)^{c_{12}} - 1\right]\right]Ac_{13}\right) \tag{43}$$

4 Linearization of the Fuel Cells Dynamics Using Lie Algebra

One can attempt linearization of the PEM fuel cells dynamics using also Lie algebra. The linearizing output $z_1 = x_1$ is defined. It holds that

$$\begin{aligned}
& z_2 = L_f z_1 \Rightarrow z_2 = \frac{\partial z_1}{\partial x_1}f_1 + \frac{\partial z_1}{\partial x_2}f_2 + \frac{\partial z_1}{\partial x_3}f_3 + \frac{\partial z_1}{\partial x_4}f_4 \Rightarrow \\
& z_2 = f_1 \Rightarrow z_2 = c_1(x_4 - x_1 - x_2 - c_2) \\
& \quad - \frac{c_3x_1W_{ca,out}}{c_4x_1 + c_5x_2 + c_6} \Rightarrow \\
& z_2 = \dot{z}_1
\end{aligned} \tag{44}$$

Similarly

$$z_3 = L_f z_2 \Rightarrow z_3 = \frac{\partial z_2}{\partial x_1}f_1 + \frac{\partial z_2}{\partial x_2}f_2 + \frac{\partial z_2}{\partial x_3}f_3 + \frac{\partial z_2}{\partial x_4}f_4 \tag{45}$$

which after intermediate operations gives

$$z_3 = L_f^2 z_1 = \dot{z}_2 = \ddot{x}_1 \tag{46}$$

Equivalently

$$L_f z_3 = \frac{\partial z_3}{\partial x_1}f_1 + \frac{\partial z_3}{\partial x_2}f_2 + \frac{\partial z_3}{\partial x_3}f_3 + \frac{\partial z_3}{\partial x_4}f_4 \tag{47}$$

By performing intermediate operations and by using Eq. (42) one finds that

$$L_f z_3 = L_f^2 z_1 = \tilde{f}(x) \tag{48}$$

Moreover, one finds that

$$L_g z_1 = \frac{\partial z_1}{\partial x_1}g_1 + \frac{\partial z_1}{\partial x_2}g_2 + \frac{\partial z_1}{\partial x_3}g_3 + \frac{\partial z_1}{\partial x_4}g_4 = 0 \tag{49}$$

Similarly

$$L_g L_f z_1 = \frac{\partial z_2}{\partial x_1}g_1 + \frac{\partial z_2}{\partial x_2}g_2 + \frac{\partial z_2}{\partial x_3}g_3 + \frac{\partial z_2}{\partial x_4}g_4 = 0 \tag{50}$$

Equivalently

$$\begin{aligned}
& L_g L_f^2 z_1 = \frac{\partial z_3}{\partial x_1}g_1 + \frac{\partial z_3}{\partial x_2}g_2 + \frac{\partial z_3}{\partial x_3}g_3 + \frac{\partial z_3}{\partial x_4}g_4 \Rightarrow \\
& \quad L_g L_f^2 z_1 = \frac{\partial z_3}{\partial x_3}c_{13} \Rightarrow L_g L_f^2 z_1 = \frac{\partial \ddot{x}_1}{\partial x_3}c_{13}
\end{aligned} \tag{51}$$

and using the previously computed relation about $\ddot{x}_1$ that was given in Eq. (39) one has the result of Eq. (43), that is

$$\begin{aligned}
& L_g L_f^2 z_1 = c_1\left(c_{14}\left[1 + c_{15}\left[\left(\frac{x_4}{c_{11}}\right)^{c_{12}} - 1\right]\right]Ac_{13}\right) \Rightarrow \\
& \quad L_g L_f^2 z_1 = \tilde{g}(x)
\end{aligned} \tag{52}$$

According to the above, the relative degree of the system is $r - 1 = 2 \Rightarrow r = 3$. Consequently, by applying Lie algebra one arrives again at the input–output linearized description of the system

$$z_1^3 = (L_f^3 z_1) + (L_g L_f^2 z_1)u$$
$$z_1^3 = \tilde{f}(x) + \tilde{g}(x)u. \quad (53)$$

5 Flatness-Based Control of the Nonlinear Fuel Cells Dynamics

Using the input–output linearized description of the system, that is

$$x_1^{(3)} = \tilde{f}(x) + \tilde{g}(x)u \quad (54)$$

and using also that $x_1 = y$, and by defining $v = \tilde{f}(x)+\tilde{g}(x)u$ one gets

$$y^{(3)} = v \quad (55)$$

The control input that is actually applied to the PEM fuel-cells system is

$$u = \frac{1}{\tilde{g}(x)}[v - \tilde{f}(x)] \quad (56)$$

Then, a stabilizing feedback controller for the system is defined as

$$v = y_d^{(3)} - k_1(\ddot{y} - \ddot{y}_d) - k_2(\dot{y} - \dot{y}_d) - k_3(y - y_d) \quad (57)$$

and one gets the closed-loop dynamics

$$(y^{(3)} - y_d^{(3)}) + k_1(\ddot{y} - \ddot{y}_d) + k_2(\dot{y} - \dot{y}_d) + k_3(y - y_d) \Rightarrow$$
$$e^{(3)} + k_1\ddot{e} + k_2\dot{e} + k_3 e = 0 \quad (58)$$

and by choosing the feedback gains $k_i,\ i = 1, 2, 3$ so as the characteristic polynomial associated with the tracking error's differential equation to be a Hurwitz one, gives

$$lim_{t\to\infty} e(t) = 0 \Rightarrow lim_{t\to\infty} y(t) = y_d(t) \Rightarrow$$
$$lim_{t\to\infty} x_1(t) = x_{1,d}(t) \quad (59)$$

Next, the problem of compensation of model uncertainties and external perturbations [such as the unknown stack current $c_7\zeta$ given in Eq. (19)] has to be treated. The linearized dynamics of the system is written as

$$y^{(3)} = v + \tilde{d} \quad (60)$$

where $\tilde{d}$ represents the cumulative disturbance terms. Without loss of generality it is assumed that the disturbance is modelled by the associated 3rd order derivative, plus initial conditions. Since estimation is going to be performed with the use of the Kalman filter and the filter's convergence is not dependent on knowledge of initial conditions, the latter can be omitted from the problem's formulation. The following state variables are defined

$$z_1 = y \quad z_2 = \dot{y} \quad z_3 = \ddot{y}$$
$$z_4 = \tilde{d} \quad z_5 = \dot{\tilde{d}} \quad z_6 = \ddot{\tilde{d}} \quad (61)$$

The extended state vector of the system is $Z = [z_1, z_2, \ldots, z_6]^T$. It holds that

$$\begin{pmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\\ \dot{z}_4\\ \dot{z}_5\\ \dot{z}_6\end{pmatrix} = \begin{pmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&0&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{pmatrix}\begin{pmatrix}z_1\\ z_2\\ z_3\\ z_4\\ z_5\\ z_6\end{pmatrix} + \begin{pmatrix}0&0\\0&0\\1&0\\0&0\\0&0\\0&1\end{pmatrix}\begin{pmatrix}v\\ d^{(3)}\end{pmatrix} \quad (62)$$

$$z_1 = \begin{pmatrix}1\ 0\ 0\ 0\ 0\ 0\end{pmatrix} Z \quad (63)$$

Defining the extended control input as $\tilde{v} = [v\ \tilde{d}^{(3)}]^T$, the above state-space description of the extended system is also written as

$$\dot{Z} = AZ + B\tilde{v}$$
$$z_1 = CZ \quad (64)$$

Next, a disturbance estimator is designed for the extended state-space model of the system. This has the form

$$\dot{\hat{z}} = A_o\hat{z} + B_o v + K_f(z_1 - \hat{z}_1)$$
$$\hat{z}_1 = C_o\hat{Z} \quad (65)$$

where $A_o = A$, $C_o = C$ and

$$B_o = \begin{pmatrix}0\ 0\ 1\ 0\ 0\ 0\end{pmatrix}^T \quad (66)$$

By applying common discretization methods, the discrete-time equivalents of matrices A_o, B_o and C_o are obtained. These are written as A_d, B_d and C_d respectively. In this estimation problem the process and measurement noise covariance matrices are denoted as $Q(k)$ and $R(k)$ respectively, while the estimation error's covariance matrix is denoted as $P(k)$. The disturbance estimator's gain is computed with the use of the Kalman Filter recursion [26–28].

Measurement update:

$$K_f(k) = P^-(k)C_d^-(k)[C_d(k)P^-(k)C_d^T(k) + R(k)]^{-1}$$
$$\hat{x}(k) = \hat{x}^-(k) + K_f(k)[z_1(k) - C_d(k)\hat{Z}(k)]$$
$$P(k) = P^-(k) - K_f(k)C_d(k)P^-(k) \quad (67)$$

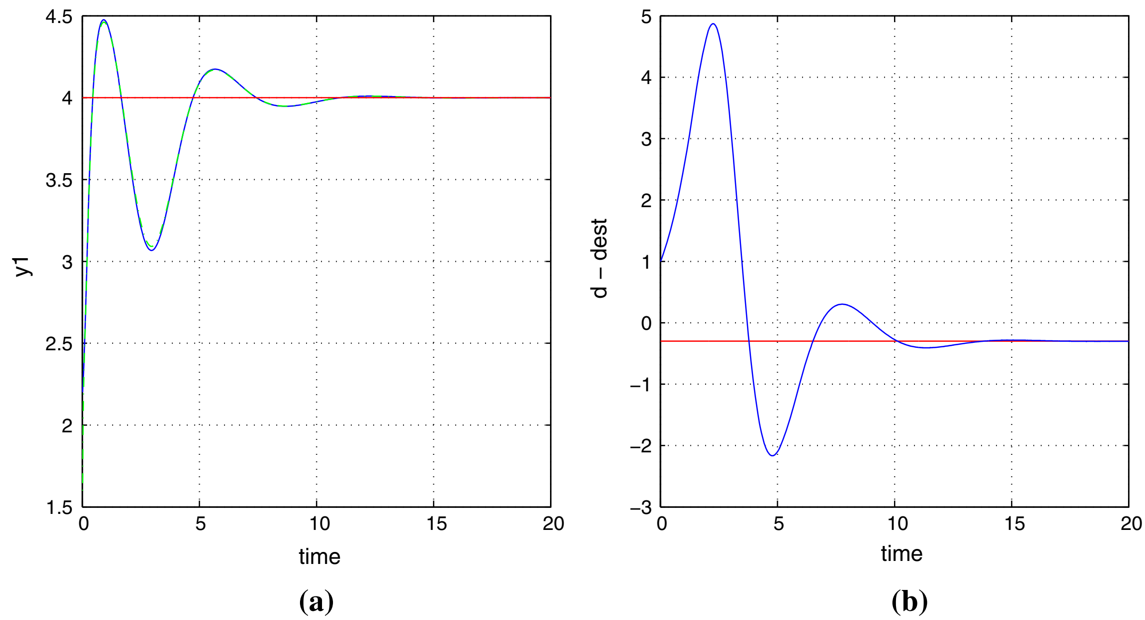


Fig. 2 **a** Convergence of state variable $x_1 = P_{O_2}$ (*green line*) to setpoint 1 (*red line*) **b** Kalman filter-based estimation (*blue line*) of the aggregate disturbance $\tilde{d}$ (*red line*) that affects the PEM fuel cells model

Time update:

$$\begin{aligned} P^-(k+1) &= A_d(k)P(k)A_d(k)^T + Q(k) \\ \hat{x}^-(k+1) &= A_d\hat{x}(k) + B_d v(k) \end{aligned} \tag{68}$$

To compensate for the disturbance's effects, the control input that is actually exerted on the system is

$$v^*(k) = v(k) - \hat{d}(k) \tag{69}$$

It is noted that the feedback control input is actually computed with the use of the estimated state vector

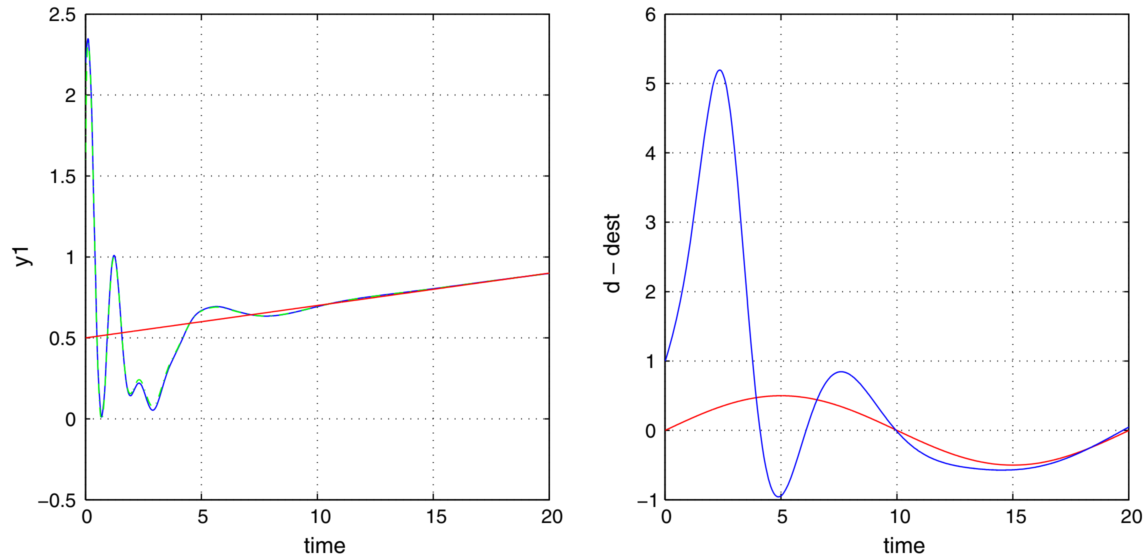
$$\begin{aligned} v^*(k) = y_d(k) &- k_1(\hat{\ddot{y}} - \ddot{y}_d) \\ &- k_2(\hat{\dot{y}} - \dot{y}_d) - k_3(\hat{y} - y_d) - \hat{d}(k). \end{aligned} \tag{70}$$

6 Simulation Tests

The performance of the proposed differential flatness theory-based control scheme has been confirmed through simulation experiments. In the results which are presented in Figs. 2, 3, 4 and 5 it can noticed that the developed control scheme achieves fast and accurate tracking of the reference setpoints. Besides, it can be noticed that the proposed Kalman Filter-based disturbance observer enables to identify fast the aggregate term of model uncertainties and external perturbations that affect the control loop. This permits finally to compensate for the disturbance's effects.

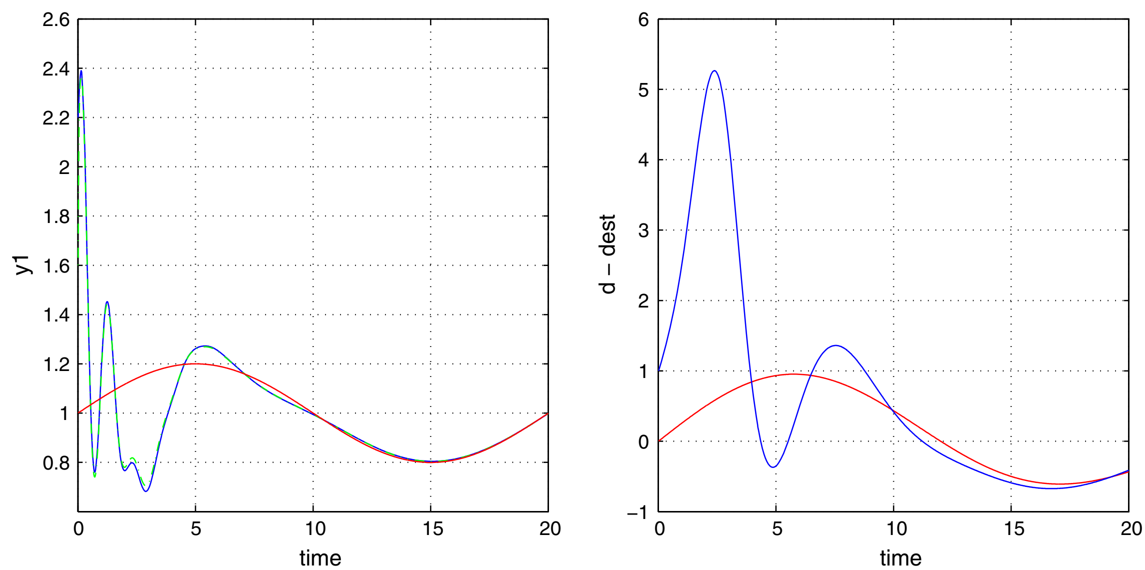
Remark 1 The article has proposed a new nonlinear filtering and control method that is based on differential flatness theory and that can be applied to the control problem of PEM fuel cells. By demonstrating that PEM fuel cells model satisfies differential flatness properties its transformation to the canonical Brunovsky form becomes possible and this enables to solve both the state estimation and the control problem. Moreover, by transforming the system into the linear canonical form, the separation principle holds and this allows to confirm separately stability and convergence conditions for the controller and the observer. The feedback gains of the flatness-based controller are chosen such that the poles of the closed-loop system are strictly found in the left complex semiplane. To solve the associated state estimation problem a new nonlinear filtering method, under the name Derivative-free nonlinear Kalman Filter, has been developed. The filter consists of the Kalman Filter recursion on the linearized equivalent model of the system that is obtained after application of the differential flatness diffeomorphism. Moreover, it comprises an inverse transformation, based again on differential flatness theory, which enables to compute estimates for the state variables of the initial nonlinear model. By redesigning the Derivative-free nonlinear Kalman Filter as a disturbance observer it becomes also possible to estimate in real-time modelling uncertainty and external perturbation terms that affect the PEM fuel cells model.

Remark 2 The proposed flatness-based control method exhibits specific advantages against other nonlinear control approaches such as sliding mode control or backstepping control. First about sliding mode control it is noted that



(a) **(b)**

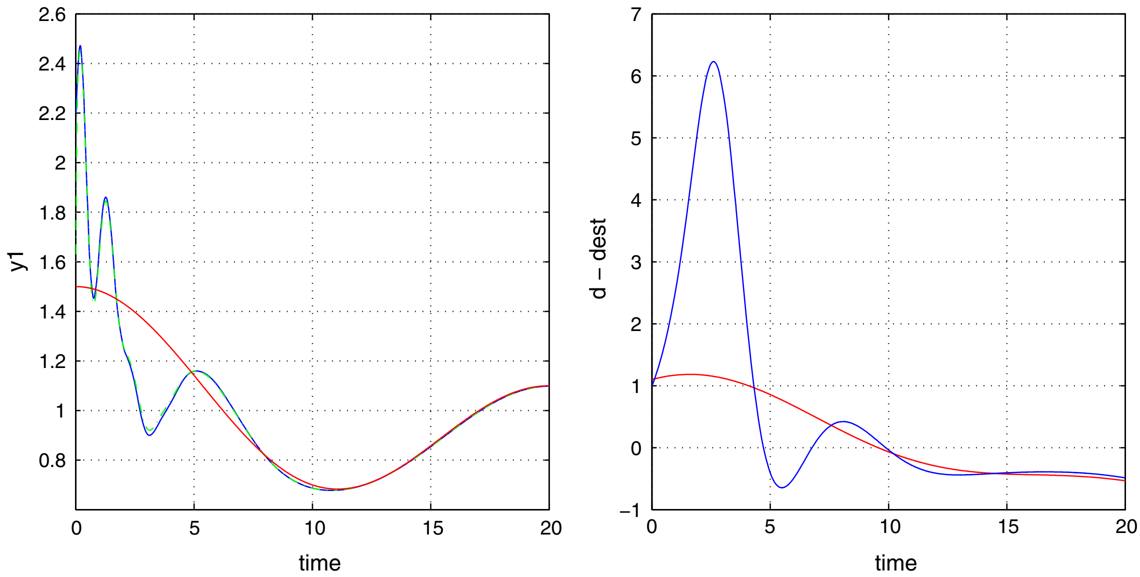
Fig. 3 **a** Convergence of state variable $x_1 = P_{O_2}$ (*green line*) to setpoint 2 (*red line*) **b** Kalman filter-based estimation (*blue line*) of the aggregate disturbance $\tilde{d}$ (*red line*) that affects the PEM fuel cells model



(a) **(b)**

Fig. 4 **a** Convergence of state variable $x_1 = P_{O_2}$ (*green line*) to setpoint 3 (*red line*) **b** Kalman Filter-based estimation (*blue line*) of the aggregate disturbance $\tilde{d}$ (*red line*) that affects the PEM fuel cells model

its application to the PEM fuel cells models is not recommended for the following reasons (i) the system is not in an input–output linearized form and therefore the selection of the sliding surface is not a straightforward procedure, (ii) uncertainty ranges for the model of PEM fuel cells and for external perturbations are not known, (iii) the switching control term of sliding mode control can cause undesirable oscillations and unacceptable transients for the PEM fuel cells' state vector, (iv) the solution of the state estimation problem with the use of a sliding mode observer will be also of inferior performance comparing to Kalman filtering due to chattering phenomena, (v) there is no direct and easy to implement stability proof for the joint sliding-mode controller and sliding-mode observer scheme. Second, about backstepping control it is noted that its application to the model of the PEM fuel cells model is not possible because this model is not found in the backstepping integral (triangular) form. For the PEM fuel cells model to be brought to such



(a) **(b)**

Fig. 5 **a** Convergence of state variable $x_3 = \omega_{c_p}$ (*green line*) to setpoint 1 (*red line*) **b** Convergence of state variable $x_4 = p_{s_m}$ (green line) to setpoint 1 (*red line*)

a form a prior transformation is needed, but this falls again to the problem of writing the PEM fuel cells' model in the canonical (Brunovsky) form through differential flatness diffeomorphisms. Similarly, in the backstepping approach there is no direct solution to the state and disturbances estimation problem for the PEM fuel cells model.

7 Conclusions

A new nonlinear control method, that is based on differential flatness theory, has been proposed for the PEM fuel cells model. First it has been proven that differential flatness properties hold for the PEM fuel cells dynamics. This means that all state variables and the control input of the system can be expressed as differential functions of a particular state variable which is the so-called flat output. Next, by exploiting differential flatness properties it has been shown the system can be transformed into an equivalent input–output linear form, for which the design of a state feedback controller becomes possible.

Another problem that had to be dealt with in the design of this nonlinear feedback controller was robustness against to model uncertainties and external perturbations. To this end a Kalman Filter-based disturbance observer has been used in the control loop. The linearized state-space description of the system has been extended by considering as additional state variables the aggregate control input and its derivatives. Next, by processing the sequence of measurements of the system's output the Kalman Filter-based estimator enabled to identify the perturbation term and to compensate for them by including an additional element in the feedback control law.

References

1. Na, W., Gou, B., Diong, B.: Nonlinear control of PEM fuel cells by exact linearization. IEEE Trans. Ind. Appl. **43**(6), 1426–1433 (2007)
2. Na, W., Gou, B.: Feedback-linearization-based nonlinear control for PEM fuel cells. IEEE Trans. Energy Convers. **21**(1), 179–190 (2008)
3. Schultz, M., Horn, J.: PEM fuel cell system power control based on a feedback-linearization approach, IEEE MED 2012, mediterranean conference on control and automation, Barcelona, Spain (2012)
4. Talj, R., Hissel, D., Ortega, R., Becherif, M., Hilairet, M.: Experimental validation of a PEM fuel-cell reduced-order model and a moto-compressor higher order sliding-mode control. IEEE Trans. Ind. Electron. **57**(6), 1906–1913 (2010)
5. Ramos-Paja, C., Giral, R., Martinez-Salamero, L., Romano, J., Romero, A., Spagnuolo, G.: A PEM fuel-cell model featuring oxygen-excess-ratio estimation and power-electronics interaction. IEEE Trans. Ind. Electron. **57**(6), 1914–1924 (2010)
6. da Costa Lopes, F., Watanabe, E.H., Rolim, L.: A control-oriented model of a PEM fuel cell stack based on NARX and NOE neural networks. IEEE Trans. Ind. Electron. **62**(8), 5155–5163 (2015)
7. Valverde, L., Bordons, C., Rosa, F.: Integration of fuel cell technologies in renewable energy-based microgrids optimizing operational cost and durability. IEEE Trans. Ind. Electron. **63**(1), 167–177 (2015)
8. Laghrouche, S., Liu, J., Ahmed, F., Harmouche, M., Wack, M.: Adaptive second-order sliding mode observer-based fault reconstruction for PEM fuel cell air-feed system. IEEE Trans. Control Syst. Technol. **23**(3), 1098–1109 (2015)

9. Matraji, I., Laghrouche, S., Jemei, S., Wack, M.: Robust control of the PEM fuel cell air-feed system via sub-optimal second order sliding mode. Appl. Energy **104**, 945–957 (2013)
10. Liu, J., Laghrouche, S., Wack, M.: Differential flatness-based observer desing for a PEM fuel cell using adaptive-gain sliding-mode differentiators, ECC 2013, 2013 European Control Conference. Zurich, Switzerland (2013)
11. Rigatos, G.: Modelling and Control for Intelligent Industrial Systems: Adaptive Algorithms in Robotics and Industrial Engineering. Springer, New York (2011)
12. Rigatos, G.: Advanced Models of Neural Networks: Nonlinear Dynamics and Stochasticity of Biological Neurons. Springer, New York (2013)
13. Rigatos, G.: Nonlinear Control and Filtering Approaches Using Differential Flatness Theory: Applications to Electromechanical Systems. Springer, New York (2015)
14. Rudolph, J.: Flatness Based Control of Distributed Parameter Systems. Shaker Verlag, Aachen, Examples and Computer Exercises from Various Technological Domains. Shaker Verlag, Aachen (2003)
15. Sira-Ramirez, H., Agrawal, S.: Differentially Flat Systems. Marcel Dekker, New York (2004)
16. Lévine, J.: On necessary and sufficient conditions for differential flatness. Appl. Algebra Eng. Commun. Comput. **22**(1), 47–90 (2011)
17. Lévine, J.: Analysis and Control of Nonlinear Systems: A Flatness-Based Approach. Springer, New York (2009)
18. Fliess, M., Mounier, H.: Tracking control and π-freeness of infinite dimensional linear systems. In: Picci, G., Gilliam, D.S. (eds.) Dynamical Systems, Control, Coding and Computer Vision, vol. 258, pp. 41–68. Birkhaüser, Boston (1999)
19. Menhour, L., d'Andréa-Novel, B., Fliess, M., Mounier, H.: Coupled nonlinear vehicle control: flatness-based setting with algebraic estimation techniques. Control Eng. Pract. **2**, 135–146 (2014)
20. Rouchon, P.: Flatness-based control of oscillators. ZAMM Z. Angew. Math. Mech. **85**(6), 411–421 (2005)
21. Zhu, G., Lévine, J., Praly, L., Peter, Y.A.: Flatness-based control of electrostatically actuated MEMS with application to adaptive optics: a simulation study. IEEE J. Microelectromech. Syst. **15**(5), 1165–1174 (2006)
22. Bououden, S., Boutat, D., Zheng, G., Barbot, J.P., Kratz, F.: A triangular canonical form for a class of 0-flat nonlinear systems. Int. J. Control **84**(2), 261–269 (2011)
23. Laroche, B., Martin, P., Petit, N.: Commande par platitude: Equations différentielles ordinaires et aux derivées partielles. Ecole Nationale Supérieure des Techniques Avancées, Paris (2007)
24. Rigatos, G., Siano, P., Zervos, N.: Sensorless control of distributed power generators with the derivative-free nonlinear Kalman filter. IEEE Trans. Ind. Electron. **61**, 6369–6382 (2014)
25. Rigatos, G.G.: A derivative-free Kalman filtering approach to state estimation-based control of nonlinear dynamical systems. IEEE Trans. Ind. Electron. **59**(10), 3987–3997 (2012)
26. Rigatos, G.G., Tzafestas, S.G.: Extended Kalman filtering for Fuzzy modelling and multi-sensor fusion. Math. Comput. Model. Dyn. Syst. **13**, 251–266 (2007)
27. Bassevile, M., Nikiforov, I.: Detection of abrupt changes: Theory and Applications. Prentice-Hall, Englewood Cliffs (1993)
28. Rigatos, G., Zhang, Q.: Fuzzy model validation using the local statistical approach. Fuzzy Sets Syst. **60**(7), 882–904 (2009)