Nonlinear Feedback Control of Three-Phase Inverter Systems Using the Derivative-Free Nonlinear Kalman Filter

G. Rigatos¹ · P. Siano² · N. Zervos¹ · C. Cecati³

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Abstract The use of three-phase voltage inverters (DC to AC converters) is frequently met in the electric power system, such as in the connection of photovoltaics with the rest of the grid. The paper proposes a nonlinear feedback control method for three-phase inverters, which is based on differential flatness theory and a new nonlinear filtering method under the name Derivative-free nonlinear Kalman Filter. First, it is shown that the inverter's dynamic model is a differentially flat one. This means that all its state variables and the control inputs can be written as functions of a single algebraic variable which is the flat output. By exploiting differential flatness properties it is shown that the inverter's model can be transformed to the linear canonical (Brunovsky's) form. For the latter description the design of a state feedback controller becomes possible, e.g. using pole placement methods. Moreover, to estimate the non-measurable state variables of the linearized equivalent of the inverter, the Derivative-free nonlinear Kalman Filter is used. This consists of the Kalman Filter recursion applied on the linearized inverter's model and of an inverse transformation that is based on differential

flatness theory, which enables to compute estimates of the state variables of the initial nonlinear system. Furthermore, by redesigning the aforementioned filter as a disturbance observer it becomes also possible to estimate disturbance terms that affect the inverter and subsequently to compensate for them. The performance and disturbance rejection capability of the proposed nonlinear feedback control scheme is evaluated through simulation experiments.

Keywords Nonlinear feedback control · Differential flatness theory · Derivative-free nonlinear Kalman Filter · Three-phase voltage inverters · Photovoltaics · HVDC lines

Introduction

A nonlinear control scheme is developed for three-phase inverters (DC to AC converters) using differential flatness theory and the Derivative-free nonlinear Kalman Filter [1–3]. Efficient control of inverters is important for the reliable operation of the electric power grid and the connection to it of distributed DC power generation units (such as photovoltaic arrays providing power to the main grid or local photovoltaic cells connected to a microgrid infrastructure) [4–10]. Moreover, inverters find extensive use in HVDCs (High Voltage DC) lines and in their connection with the rest of the AC electric power network [11–13]. Indicative results on nonlinear control of inverters with the use of feedback linearization methods can be found in [14–16]. In the same direction, the present paper proposes a method for inverters control, based on differential flatness theory and on a new nonlinear filtering method under the name Derivative-free nonlinear Kalman Filter.

The dynamic model of the inverter is a nonlinear multi-variable one [7, 12]. The control inputs are the voltages generated by an H-bridge circuit while the state vector com-

✉ G. Rigatos
grigat@ieee.org
P. Siano
psiano@unisa.it
N. Zervos
nzervos@isi.gr
C. Cecati
c.cecati@ieee.org

¹ Unit of Industrial Automation, Industrial Systems Institute, 26504 Rion Patras, Greece

² Department of Industrial Engineering, University of Salerno, 84084 Fisciano, Italy

³ Department of Industrial Engineering, University of L'Aquila, 67100 L'Aquila, Italy

$$X_{ab} = X_a + jX_b \quad (3)$$

Next, the voltage and current variables are represented in the rotating dq reference frame [7]. It holds that $X_{dq} = x_d + jx_q$ and

$$X_{dq} = X_{ab}e^{-j\theta} \Rightarrow X_{ab} = X_{dq}e^{j\theta}$$

where $\theta(t) = \int_0^t \omega(t)dt + \theta_0$ (4)

By differentiating with respect to time one obtains the following description of the system's dynamics

$$\dot{X}_{ab} = \frac{d}{dt}X_{dq} + j\omega X_{dq} \quad (5)$$

Thus, one has for the current and voltage variables respectively,

$$\begin{aligned} \dot{i}_{I,ab} &= \frac{d}{dt}i_{I,dq} + (j\omega)i_{I,dq} \\ \dot{V}_{L,ab} &= \frac{d}{dt}V_{L,dq} + (j\omega)V_{L,dq} \end{aligned} \quad (6)$$

By substituting Eq. (6) into Eq. (1) one obtains

$$\begin{aligned} \frac{d}{dt}i_{I,dq} + j\omega i_{I,dq} &= \frac{1}{L_f}V_{I,dq} - \frac{1}{L_f}V_{L,dq} \\ \frac{d}{dt}V_{L,dq} + j\omega V_{L,dq} &= \frac{1}{C_f}i_{I,dq} - \frac{1}{C_f}i_{L,dq} \end{aligned} \quad (7)$$

Using Eq. (7) and by rearranging rows one finally obtains the inverter's dynamic model expressed in the dq reference frame:

The state vector of the system is taken to be $\tilde{X} = [V_{L,d}, V_{L,q}, i_{I,d}, i_{I,q}]^T$ while the control input is taken to be the vector $\tilde{U} = [V_{I,d}, V_{I,q}]$. The load currents $i_{L,d}$ and $i_{L,q}$ are taken to be unknown parameters which can be considered as perturbation terms. Alternatively, these currents can be expressed as functions of the inverter's active and reactive power. In the latter approach one has that the active power of the inverter is [10]

$$p_f = V_{L,d}i_{L,d} + V_{L,q}i_{L,q} \quad (9)$$

while the reactive power, consisting of reactive power at the load, reactive power at the capacitor and reactive power at the inductance is given by

$$\begin{aligned} q_f &= V_{L,q}i_{L,d} - V_{L,d}i_{L,q} - \omega C_f(V_{L,d}^2 + V_{L,q}^2) \\ &\quad + \omega L_f(i_{I,d}^2 + i_{I,q}^2) \end{aligned} \quad (10)$$

By solving Eqs. (9) and (10), with respect to the currents $i_{L,d}$ and $i_{L,q}$ one obtains

$$i_{L,d} = \frac{p_f V_{L,d} + q_f V_{L,q}}{V_{L,d}^2 + V_{L,q}^2} + \omega C_f V_{L,q} - \frac{\omega L_f V_{L,q}(i_{I,d}^2 + i_{I,q}^2)}{(V_{L,d}^2 + V_{L,q}^2)} \quad (11)$$

$$i_{L,q} = \frac{p_f V_{L,q} - q_f V_{L,d}}{V_{L,d}^2 + V_{L,q}^2} - \omega C_f V_{L,d} + \frac{\omega L_f V_{L,d}(i_{I,d}^2 + i_{I,q}^2)}{(V_{L,d}^2 + V_{L,q}^2)} \quad (12)$$

Using Eqs. (8), (11) and (12) one obtains the state-space description of the inverter's dynamics

$$\frac{d}{dt} \begin{pmatrix} V_{L,d} \\ V_{L,q} \\ i_{I,d} \\ i_{I,q} \end{pmatrix} = \begin{pmatrix} \omega V_{L,q} + \frac{1}{C_f}i_{I,d} - \frac{1}{C_f}\frac{p_f V_{L,d} + q_f V_{L,q}}{V_{L,d}^2 + V_{L,q}^2} + \omega C_f V_{L,q} - \frac{\omega L_f V_{L,q}(i_{I,d}^2 + i_{I,q}^2)}{(V_{L,d}^2 + V_{L,q}^2)} \\ -\omega V_{L,d} + \frac{1}{C_f}i_{I,q} - \frac{1}{C_f}\frac{p_f V_{L,q} - q_f V_{L,d}}{V_{L,d}^2 + V_{L,q}^2} - \omega C_f V_{L,d} + \frac{\omega L_f V_{L,d}(i_{I,d}^2 + i_{I,q}^2)}{(V_{L,d}^2 + V_{L,q}^2)} \\ \omega i_{I,q} - \frac{1}{L_f}V_{L,d} \\ -\omega i_{I,d} - \frac{1}{L_f}V_{L,q} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L_f} & 0 \\ 0 & \frac{1}{L_f} \end{pmatrix} \begin{pmatrix} V_{I,d} \\ V_{I,q} \end{pmatrix} \quad (13)$$

$$\begin{aligned} \frac{d}{dt}V_{L,d} &= \omega V_{L,q} + \frac{1}{C_f}i_{I,d} - \frac{1}{C_f}i_{L,d} \\ \frac{d}{dt}V_{L,q} &= -\omega V_{L,d} + \frac{1}{C_f}i_{I,q} - \frac{1}{C_f}i_{L,q} \\ \frac{d}{dt}i_{I,d} &= \omega i_{I,q} + \frac{1}{L_f}V_{I,d} - \frac{1}{L_f}V_{L,d} \\ \frac{d}{dt}i_{I,q} &= -\omega i_{I,d} + \frac{1}{L_f}V_{I,q} - \frac{1}{L_f}V_{L,q} \end{aligned} \quad (8)$$

while the measurement equation of the inverter's model is

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} V_{L,d} \\ V_{L,q} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} V_{L,d} \\ V_{L,q} \\ i_{I,d} \\ i_{I,q} \end{pmatrix} \quad (14)$$

and by using the state variables notation $x_1 = V_{L,d}$, $x_2 = V_{L,q}$, $x_3 = i_{L,d}$ and $x_4 = i_{L,q}$ one has

$$\frac{d}{dt} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \omega x_2 + \frac{1}{C_f} x_3 - \frac{1}{C_f} \frac{p_f x_1 + q_f x_2}{x_1^2 + x_2^2} + \omega C_f x_2 - \frac{\omega L_f x_2 (x_3^2 + x_4^2)}{(x_1^2 + x_2^2)} \\ -\omega x_1 + \frac{1}{C_f} x_4 - \frac{1}{C_f} \frac{p_f x_2 - q_f x_1}{x_1^2 + x_2^2} - \omega C_f x_1 + \frac{\omega L_f x_1 (x_3^2 + x_4^2)}{(x_1^2 + x_2^2)} \\ \omega x_4 - \frac{1}{L_f} x_1 \\ -\omega x_3 - \frac{1}{L_f} x_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L_f} & 0 \\ 0 & \frac{1}{L_f} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad (15)$$

while the measurement equation of the inverter's model is

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} V_{L_d} \\ V_{L_q} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \quad (16)$$

thus, the inverter's model is written in the nonlinear state-space form

$$\begin{aligned} \dot{x} &= f(x) + G(x)u \\ y &= h(x) \end{aligned} \quad (17)$$

where $f(x) \in \mathbb{R}^{4 \times 1}$, $G(x) \in \mathbb{R}^{4 \times 1}$ and $h(x) \in \mathbb{R}^{2 \times 4}$.

Lie Algebra-based Control of the Inverter's Model

The following functions are defined

$$\begin{aligned} y_1 &= h_1(x) = x_1 \\ y_2 &= h_2(x) = x_2 \end{aligned} \quad (18)$$

It holds that

$$\begin{aligned} z_1 &= h_1(x) = x_1 \\ z_2 &= L_f h_1(x) \\ \dot{z}_2 &= L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2 \\ z_3 &= h_2(x) = x_2 \\ z_4 &= L_f h_2(x) \\ \dot{z}_4 &= L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2 \end{aligned} \quad (19)$$

Using the above, one has

$$\begin{aligned} z_2 = L_f h_1(x) &\Rightarrow z_2 = \frac{\partial h_1}{\partial x_1} f_1 + \frac{\partial h_1}{\partial x_2} f_2 + \frac{\partial h_1}{\partial x_3} f_3 + \frac{\partial h_1}{\partial x_4} f_4 \Rightarrow \\ z_2 = 1 f_1 &\Rightarrow z_2 = \omega x_2 + \frac{1}{C_f} x_3 - \frac{1}{C_f} \left\{ \frac{p_f x_1 + q_f x_2}{x_1^2 + x_2^2} \right. \\ &\quad \left. + \omega C_f x_2 - \frac{\omega L_f x_2 (x_3^2 + x_4^2)}{(x_1^2 + x_2^2)} \right\} \end{aligned} \quad (20)$$

In a similar manner one computes

$$\begin{aligned} L_f^2 h_1(x) = L_f z_2 &\Rightarrow L_f^2 h_1(x) = \frac{\partial z_1}{\partial x_1} f_1 + \frac{\partial z_1}{\partial x_2} f_2 + \frac{\partial z_1}{\partial x_3} f_3 \\ &\quad + \frac{\partial z_1}{\partial x_4} f_4 \Rightarrow \end{aligned} \quad (21)$$

Thus, it holds that

$$\begin{aligned} L_f^2 h_1(x) &= -\frac{1}{C_f} \left\{ \frac{p_f (x_1^2 + x_2^2) - (p_f x_1 + q_f x_2)(2x_1)}{(x_1^2 + x_2^2)^2} \right. \\ &\quad \left. + \frac{\omega L_f x_2 (x_3^2 + x_4^2)(2x_1)}{(x_1^2 + x_2^2)^2} \right\} \dot{x}_1 \\ &\quad + \left[\omega - \frac{1}{C_f} \left\{ \frac{q_f (x_1^2 + x_2^2) - (p_f x_1 + q_f x_2)2x_2}{(x_1^2 + x_2^2)^2} + \omega C_f \right. \right. \\ &\quad \left. \left. - \frac{\omega L_f (x_3^2 + x_4^2)(x_1^2 + x_2^2) + \omega L_f x_2 (x_3^2 + x_4^2)2x_2}{(x_1^2 + x_2^2)^2} \right\} \right] \dot{x}_2 \\ &\quad \times \left[\frac{1}{C_f} - \frac{1}{C_f} \left\{ -\frac{\omega L_f x_2 2x_3}{(x_1^2 + x_2^2)^2} \right\} \right] \left(\omega x_4 - \frac{1}{L_f} x_1 \right) \\ &\quad + \left[-\frac{1}{C_f} \left\{ \frac{-\omega L_f x_2 2x_4}{(x_1^2 + x_2^2)} \right\} \right] \left(-\omega x_3 - \frac{1}{L_f} x_2 \right) \end{aligned} \quad (22)$$

and after intermediate operations one arrives at the formulation

$$\begin{aligned} L_f^2 h_1(x) &= \omega \dot{x}_2 + \frac{1}{C_f} \left(\omega x_4 - \frac{1}{L_f} x_1 \right) \\ &\quad - \frac{1}{C_f} \left\{ \frac{(p_f \dot{x}_1 + q_f \dot{x}_2)(x_1^2 + x_2^2) - (p_f x_1 + q_f x_2)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} \\ &\quad - \omega \dot{x}_2 + \frac{1}{C_f} \left\{ \frac{\omega L_f \dot{x}_2 (x_3^2 + x_4^2)(x_1^2 + x_2^2) - \omega L_f x_2 (x_3^2 + x_4^2)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} \\ &\quad + \frac{\omega L_f x_2}{C_f} \cdot \frac{2x_3 \left(\omega x_4 - \frac{1}{L_f} x_1 \right)}{(x_1^2 + x_2^2)} + \frac{\omega L_f x_2}{C_f} \cdot \frac{2x_1 \left(-\omega x_3 - \frac{1}{L_f} x_2 \right)}{(x_1^2 + x_2^2)} \end{aligned} \quad (23)$$

In a similar manner one computes

$$\begin{aligned} L_{g_a} L_f h_1(x) &= L_{g_a} z_1 \Rightarrow L_{g_a} L_f h_1(x) \\ &= \frac{\partial z_2}{\partial x_1} g_{a1} + \frac{\partial z_2}{\partial x_2} g_{a2} + \frac{\partial z_2}{\partial x_3} g_{a3} + \frac{\partial z_2}{\partial x_4} g_{a4} \Rightarrow \end{aligned}$$

$$\begin{aligned}
L_{g_a} L_f h_1(x) &= \frac{1}{L_f} \frac{\partial z_2}{\partial x_3} \Rightarrow \\
L_{g_a} L_f h_1(x) &= \frac{1}{C_f} \left\{ \frac{\omega L_f x_2 (2x_3)}{(x_1^2 + x_2^2) L_f} + \frac{1}{L_f} \right\}
\end{aligned} \quad (24)$$

and also

$$\begin{aligned}
L_{g_b} L_f h_1(x) &= L_{g_a} z_2 \Rightarrow L_{g_b} L_f h_2(x) \\
&= \frac{\partial z_2}{\partial x_1} g_{b1} + \frac{\partial z_2}{\partial x_2} g_{b2} + \frac{\partial z_2}{\partial x_3} g_{b3} + \frac{\partial z_2}{\partial x_4} g_{b4} \Rightarrow \\
L_{g_b} L_f h_1(x) &= \frac{1}{L_f} \frac{\partial z_2}{\partial x_4} \Rightarrow \\
L_{g_b} L_f h_1(x) &= \frac{\omega L_f x_2}{C_f} \frac{2x_4}{(x_1^2 + x_2^2)} \frac{1}{L_f}
\end{aligned} \quad (25)$$

Following a similar procedure one obtains

$$\begin{aligned}
z_4 &= L_f h_2(x) \Rightarrow z_4 = \frac{\partial h_2}{\partial x_1} f_1 + \frac{\partial h_2}{\partial x_2} f_2 + \frac{\partial h_2}{\partial x_3} f_3 + \frac{\partial h_2}{\partial x_4} f_4 \Rightarrow \\
z_4 &= f_2 \Rightarrow z_4 = -\omega x_1 + \frac{1}{C_f} x_4 - \frac{1}{C_f} \left\{ \frac{p_f x_2 - q_f x_1}{(x_1^2 + x_2^2)} \right. \\
&\quad \left. - \omega C_f + \frac{\omega L_f x_1 (x_3^2 + x_4^2)}{(x_1^2 + x_2^2)} \right\}
\end{aligned} \quad (26)$$

Moreover, one computes

$$\begin{aligned}
L_f^2 h_2(x) &= L_f z_4 \Rightarrow L_f^2 h_2(x) = \frac{\partial z_4}{\partial x_1} f_1 + \frac{\partial z_4}{\partial x_2} f_2 \\
&\quad + \frac{\partial z_4}{\partial x_3} f_3 + \frac{\partial z_4}{\partial x_4} f_4 \Rightarrow \\
L_f^2 h_2(x) &= \frac{\partial z_4}{\partial x_1} \dot{x}_1 + \frac{\partial z_4}{\partial x_2} \dot{x}_2 + \frac{\partial z_4}{\partial x_3} \omega x_4 + \frac{\partial z_4}{\partial x_4} - \omega x_3
\end{aligned} \quad (27)$$

which is also written as

$$\begin{aligned}
L_f^2 h_2(x) &= \left[-\omega - \frac{1}{C_f} \right. \\
&\quad \times \left\{ \frac{-g_f(x_1^2 + x_2^2) - (p_f x_2 - q_f x_1)(2x_1)}{(x_1^2 + x_2^2)^2} - \omega C_f \right\} \\
&\quad \left. + \frac{\omega L_f(x_3^2 + x_4^2)(x_1^2 + x_2^2) - \omega L_f x_1(x_3^2 + x_4^2)2x_1}{(x_1^2 + x_2^2)^2} \right] \dot{x}_1 \\
&\quad \left[-\frac{1}{C_f} \left\{ \frac{p_f(x_1^2 + x_2^2) - (p_f x_2 - q_f x_1)2x_2}{x_1^2 + x_2^2} \right. \right. \\
&\quad \left. \left. - \frac{\omega L_f x_1(x_3^2 + x_4^2)2x_2}{(x_1^2 + x_2^2)^2} \right\} \right] \dot{x}_2 \\
&\quad \left[-\frac{1}{C_f} \left\{ \frac{\omega L_f x_1 2x_3}{(x_1^2 + x_2^2)} \right\} \right] \left(\omega x_4 - \frac{1}{L_f} x_1 \right)
\end{aligned}$$

$$\left[\frac{1}{C_f} - \frac{1}{C_f} \left\{ \frac{\omega L_f x_1 2x_4}{(x_1^2 + x_2^2)} \right\} \right] \left(-\omega x_3 - \frac{1}{L_f} x_2 \right) \quad (28)$$

which after intermediate operations gives

$$\begin{aligned}
L_f^2 h_2(x) &= -\omega \dot{x}_1 + \frac{1}{C_f} (-\omega x_3) \\
&\quad - \frac{1}{C_f} \left\{ \frac{(p_f \dot{x}_2 - g_f \dot{x}_1)(x_1^2 + x_2^2) - (p_f x_2 - q_f x_1)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} \\
&\quad + \omega \dot{x}_1 - \frac{1}{C_f} \left\{ \frac{\omega L_f \dot{x}_1(x_3^2 + x_4^2)(x_1^2 + x_2^2) - \omega L_f x_1(x_3^2 + x_4^2)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} \\
&\quad - \frac{1}{C_f} \frac{\omega L_f x_1 2x_3}{(x_1^2 + x_2^2)} \left(\omega x_4 - \frac{1}{L_f} x_1 \right) - \frac{1}{C_f} \frac{\omega L_f x_1 2x_4}{(x_1^2 + x_2^2)} \left(-\omega x_3 - \frac{1}{L_f} x_2 \right)
\end{aligned} \quad (29)$$

Following a similar procedure one finds

$$\begin{aligned}
L_{g_a} L_f h_2(x) &= L_{g_a} z_4 \Rightarrow L_{g_a} L_f h_2(x) \\
&= \frac{\partial z_4}{\partial x_1} g_{a1} + \frac{\partial z_4}{\partial x_2} g_{a2} + \frac{\partial z_4}{\partial x_3} g_{a3} + \frac{\partial z_4}{\partial x_4} g_{a4} \Rightarrow \\
L_{g_b} L_f h_2(x) &= \frac{1}{L_f} \frac{\partial z_4}{\partial x_3} \Rightarrow \\
L_{g_a} L_f h_2(x) &= -\frac{1}{C_f} \frac{\omega L_f x_1 2x_3}{(x_1^2 + x_2^2)} \frac{1}{L_f}
\end{aligned} \quad (30)$$

and equivalently

$$\begin{aligned}
L_{g_b} L_f h_2(x) &= L_{g_b} z_4 \Rightarrow L_{g_b} L_f h_2(x) \\
&= \frac{\partial z_4}{\partial x_1} g_{b1} + \frac{\partial z_4}{\partial x_2} g_{b2} + \frac{\partial z_4}{\partial x_3} g_{b3} + \frac{\partial z_4}{\partial x_4} g_{b4} \Rightarrow \\
L_{g_b} L_f h_2(x) &= \frac{-1}{L_f} \frac{\partial z_4}{\partial x_4} \Rightarrow \\
L_{g_b} L_f h_2(x) &= -\frac{1}{C_f} \left\{ \frac{\omega L_f x_1 2x_4}{(x_1^2 + x_2^2)} \frac{1}{L_f} + \frac{1}{L_f} \right\}
\end{aligned} \quad (31)$$

After the Lie algebra-based linearization procedure, the dynamics of the inverter is written in the form

$$\begin{aligned}
\dot{x}_1 &= L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2 \\
\dot{x}_2 &= L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2
\end{aligned} \quad (32)$$

The following new control inputs are defined

$$\begin{aligned}
v_1 &= L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2 \\
v_2 &= L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2
\end{aligned} \quad (33)$$

This results in the linearized system's dynamics

$$\begin{aligned}
\dot{x}_1 &= v_1 \\
\dot{x}_2 &= v_2
\end{aligned} \quad (34)$$

which is also written in a state-space form, after defining the state variables $z_1 = x_1$, $z_2 = \dot{x}_1$, $z_3 = x_2$ and $z_4 = \dot{x}_2$

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (35)$$

$$\begin{pmatrix} z_1^m \\ z_2^m \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} \quad (36)$$

Differential Flatness of the Inverter's Model

The flat output of the inverter is taken to be the vector

$$y = [y_1, y_2] = [V_{L_d}, V_{L_q}] \quad (37)$$

It will be shown that the system is a differentially flat one, which means that all its state variables and the control inputs can be written as functions of the flat output and their derivatives. It holds that

$$\begin{aligned} x_1 &= V_{L_d} \Rightarrow x_1 = y_1 \\ x_2 &= V_{L_q} \Rightarrow x_2 = y_2 \end{aligned} \quad (38)$$

The first row of the state-space equations is

$$\begin{aligned} \dot{x}_1 &= \omega x_2 + \frac{1}{C_f} x_3 - \frac{1}{C_f} \frac{p_f x_1 + q_f x_2}{x_1^2 + x_2^2} \\ &\quad - \omega x_2 + \frac{1}{C_f} \frac{\omega L_f x_1 (x_3^2 + x_4^2)}{x_1^2 + x_2^2} \end{aligned} \quad (39)$$

The second row of the state-space equations is

$$\begin{aligned} \dot{x}_2 &= \omega x_1 + \frac{1}{C_f} x_4 - \frac{1}{C_f} \frac{p_f x_2 + q_f x_1}{x_1^2 + x_2^2} \\ &\quad + \omega x_1 + \frac{1}{C_f} \frac{\omega L_f x_2 (x_3^2 + x_4^2)}{x_1^2 + x_2^2} \end{aligned} \quad (40)$$

Both equations are solved with respect to their first term

$$\begin{aligned} \frac{1}{C_f} \frac{\omega L_f x_2 (x_3^2 + x_4^2)}{x_1^2 + x_2^2} &= \dot{x}_1 - \omega x_2 - \frac{1}{C_f} x_3 \\ &\quad + \frac{1}{C_f} \frac{p_f x_1 + q_f x_2}{(x_1^2 + x_2^2)} + \omega x_2 \end{aligned} \quad (41)$$

$$\begin{aligned} \frac{1}{C_f} \frac{\omega L_f x_1 (x_3^2 + x_4^2)}{x_1^2 + x_2^2} &= \dot{x}_2 + \omega x_1 - \frac{1}{C_f} x_4 \\ &\quad + \frac{1}{C_f} \frac{p_f x_2 + q_f x_1}{(x_1^2 + x_2^2)} - \omega x_1 \end{aligned} \quad (42)$$

By dividing the above two equations one obtains

$$-\frac{x_2}{x_1} = \frac{\dot{x}_1 - \omega x_2 - \frac{1}{C_f} x_3 + \frac{1}{C_f} \frac{p_f x_1 + q_f x_2}{(x_1^2 + x_2^2)} + \omega x_2}{\dot{x}_2 + \omega x_1 - \frac{1}{C_f} x_4 + \frac{1}{C_f} \frac{p_f x_2 + q_f x_1}{(x_1^2 + x_2^2)} - \omega x_1} \quad (43)$$

or using in the notation the elements of the flat output vector

$$-\frac{y_2}{y_1} = \frac{\dot{y}_1 - \omega y_2 - \frac{1}{C_f} x_3 + \frac{1}{C_f} \frac{p_f y_1 + q_f y_2}{(y_1^2 + y_2^2)} + \omega y_2}{\dot{y}_2 + \omega y_1 - \frac{1}{C_f} x_4 + \frac{1}{C_f} \frac{p_f y_2 + q_f y_1}{(y_1^2 + y_2^2)} - \omega y_1} \quad (44)$$

which equivalently gives

$$\begin{aligned} -\frac{y_2}{y_1} \dot{y}_2 - \omega y_2 + \frac{1}{C_f} \left(\frac{y_2}{y_1} \right) \frac{p_f y_2 + q_f y_1}{(y_1^2 + y_2^2)} + \omega y_2 \\ = \dot{y}_1 - \omega y_2 - \frac{1}{C_f} x_3 + \frac{1}{C_f} \frac{p_f y_1 + q_f y_2}{(y_1^2 + y_2^2)} + \omega y_2 \end{aligned} \quad (45)$$

Eq. (45) is solved with respect to x_3 . It holds that

$$\begin{aligned} x_3 &= C_f \left\{ \left(\frac{y_2}{y_1} \right) \dot{y}_2 + \omega y_2 - \frac{1}{C_f} \left(\frac{y_2}{y_1} \right) x_4 \right. \\ &\quad + \frac{1}{C_f} \frac{(y_2 p_f y_2 + q_f y_1)}{y_1^2 + y_2^2} - \omega y_2 \\ &\quad \left. + \dot{y}_1 - \omega \dot{y}_2 + \frac{1}{C_f} \frac{p_f y_1 + q_f y_2}{y_1^2 + y_2^2} + \omega y_2 \right\} \end{aligned} \quad (46)$$

which equivalently gives

$$\begin{aligned} x_3 &= -\frac{y_2}{y_1} x_4 + C_f \left\{ \frac{y_2}{y_1} \dot{y}_2 + \omega y_2 + \frac{1}{C_f} \frac{(y_2 p_f y_2 + q_f y_1)}{(y_1^2 + y_2^2)} \right. \\ &\quad \left. - \omega y_2 + \dot{y}_1 - \omega y_2 + \frac{1}{C_f} \frac{p_f y_1 + q_f y_2}{(y_1^2 + y_2^2)} + \omega y_2 \right\} \end{aligned} \quad (47)$$

which is also written as

$$x_3 = -\left(\frac{y_2}{y_1} \right) x_4 + f_a(y_1, \dot{y}_1, y_2, \dot{y}_2) \quad (48)$$

Next, Eq. (48) is substituted into Eq. (39). Thus, one obtains

$$\begin{aligned} \dot{x}_2 &= -\omega x_1 + \frac{1}{C_f} x_4 - \frac{1}{C_f} \frac{p_f x_2 + q_f x_1}{(x_1^2 + x_2^2)} + \omega x_1 \\ &\quad - \frac{1}{C_f} \frac{\omega L_f x_1 \left\{ \left[-\left(\frac{y_2}{y_1} \right) x_4 + f_a(y_1, \dot{y}_1, y_2, \dot{y}_2) \right]^2 + x_4^2 \right\}}{(x_1^2 + x_2^2)} \end{aligned} \quad (49)$$

or equivalently

$$\dot{y}_2 = -\omega y_1 + \frac{1}{C_f} x_4 - \frac{1}{C_f} \frac{p_f y_2 - q_f y_1}{(y_1^2 + y_2^2)} + \omega y_1 + \frac{1}{C_f} \frac{\omega L_f y_1 \left\{ \left[-\left(\frac{y_2}{y_1}\right) x_4 + f_a(y_1, \dot{y}_1, y_2, \dot{y}_2) \right]^2 + x_4^2 \right\}}{(y_1^2 + y_2^2)} \quad (50)$$

Eq. (50) is a binomial which can be solved with respect to x_4 . Thus, one obtains that

$$x_4 = f_b(y_1, \dot{y}_1, y_2, \dot{y}_2) \quad (51)$$

Substituting Eq. (52) into Eq. (48) one gets

$$x_3 = -\left(\frac{y_2}{y_1}\right) f_b(y_1, \dot{y}_1, y_2, \dot{y}_2) + f_4(y_1, \dot{y}_1, y_2, \dot{y}_2) \quad (52)$$

From Eqs. (51) and (52) one has that state variables x_3 and x_4 can be written as functions of the flat outputs and their derivatives. Moreover, from the last two rows of the state-space equations one has that

$$\dot{x}_3 = \omega x_4 - \frac{1}{L_f} x_1 + \frac{1}{L_f} u_1 \quad (53)$$

$$\dot{x}_4 = -\omega x_3 - \frac{1}{L_f} x_2 + \frac{1}{L_f} u_2 \quad (54)$$

From Eq. (53) one gets

$$u_1 = L_f \left\{ \dot{x}_3 - \omega x_4 + \frac{1}{L_f} x_1 \right\} \Rightarrow u_1 = f_c(y_1, \dot{y}_1, y_2, \dot{y}_2) \quad (55)$$

From Eq. (54) one gets

$$u_2 = L_f \left\{ \dot{x}_4 - \omega x_3 + \frac{1}{L_f} x_2 \right\} \Rightarrow u_2 = f_d(y_1, \dot{y}_1, y_2, \dot{y}_2) \quad (56)$$

Consequently, the control inputs are also functions of the flat outputs and their derivatives. Thus, the inverter's model is a differentially flat one.

Flatness-based Control of the Inverter

Next, a flatness-based controller will be designed. To this end Eqs. 39 and (40) are used again. By considering p_f, q_f as piecewise constant and by deriving Eq. (39) with respect to time, one has

$$\ddot{x}_1 = \omega \dot{x}_2 + \frac{1}{C_f} \dot{x}_3 - \frac{1}{C_f} \left\{ \frac{(p_f \dot{x}_1 + q_f \dot{x}_2)(x_1^2 + x_2^2) - (p_f x_1 + q_f x_2)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} - \omega \dot{x}_2 + \frac{1}{C_f} \left\{ \frac{\omega L_f \dot{x}_2 (x_3^2 + x_4^2)(x_1^2 + x_2^2) + \omega L_f x_2 (2x_3 \dot{x}_3 + 2x_4 \dot{x}_4)(x_1^2 + x_2^2)}{(x_1^2 + x_2^2)^2} \right\} - \frac{\omega L_f x_2 (x_3^2 + x_4^2)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \quad (57)$$

The time derivatives are substituted from the associated rows of the state-space equations. Next, substitutions of the terms $\dot{x}_3$ and $\dot{x}_4$ are performed. It holds that

$$\ddot{x}_1 = \omega \dot{x}_2 + \frac{1}{C_f} \left(\omega x_4 - \frac{1}{L_f} x_1 + \frac{1}{L_f} u_1 \right) - \frac{1}{C_f} \left\{ \frac{(p_f \dot{x}_1 + q_f \dot{x}_2)(x_1^2 + x_2^2) - (p_f x_1 + q_f x_2)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} - \omega \dot{x}_2 + \frac{1}{C_f} \left\{ \frac{\omega L_f \dot{x}_2 (x_3^2 + x_4^2)(x_1^2 + x_2^2) + \omega L_f x_2 (x_3^2 + x_4^2)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} + \frac{\omega L_f x_2}{C_f} \frac{2x_3 (x_1^2 + x_2^2)}{(x_1^2 + x_2^2)^2} \left(\omega x_4 - \frac{1}{L_f} x_1 + \frac{1}{L_f} u_1 \right) + \frac{\omega L_f x_2}{C_f} \frac{2x_4 (x_1^2 + x_2^2)}{(x_1^2 + x_2^2)^2} \left(-\omega x_3 - \frac{1}{L_f} x_2 + \frac{1}{L_f} u_2 \right) \quad (58)$$

The previous relation is rewritten by regrouping terms which multiply the control inputs u_1 and u_2 . Thus one obtains

$$\ddot{x}_1 = \omega \dot{x}_2 + \frac{1}{C_f} \left(\omega x_4 - \frac{1}{L_f} x_1 \right) - \frac{1}{C_f} \left\{ \frac{(p_f \dot{x}_1 + q_f \dot{x}_2)(x_1^2 + x_2^2) - (p_f x_1 + q_f x_2)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} - \omega \dot{x}_2 + \frac{1}{C_f} \left\{ \frac{\omega L_f \dot{x}_2 (x_3^2 + x_4^2)(x_1^2 + x_2^2) - \omega L_f x_2 (x_3^2 + x_4^2)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} + \frac{\omega L_f x_2}{C_f} \cdot \frac{2x_3 (\omega x_4 - \frac{1}{L_f} x_1)}{(x_1^2 + x_2^2)} + \frac{\omega L_f x_2}{C_f} \cdot \frac{2x_4 (-\omega x_3 - \frac{1}{L_f} x_2)}{(x_1^2 + x_2^2)} + \frac{1}{C_f} \left\{ \frac{\omega L_f x_2 (2x_3)}{(x_1^2 + x_2^2) L_f} + \frac{1}{L_f} \right\} u_1 + \frac{\omega L_f x_2}{C_f} \frac{2x_4}{(x_1^2 + x_2^2) L_f} u_2 \quad (59)$$

The previous relation can be also written in the form

$$\ddot{x}_1 = L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2 \quad (60)$$

In a similar manner, by differentiating the second row of the state-space equations with respect to time one has

$$\ddot{x}_2 = -\omega \dot{x}_1 + \frac{1}{C_f} \dot{x}_4 - \frac{1}{C_f} \left\{ \frac{(p_f \dot{x}_2 - g_f \dot{x}_1)(x_1^2 + x_2^2) - (p_f x_2 - g_f x_1)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} + \omega \dot{x}_1 - \frac{1}{C_f} \left\{ \frac{\omega L_f \dot{x}_1 (x_3^2 + x_4^2)(x_1^2 + x_2^2) + \omega L_f x_1 (2x_3 \dot{x}_3 + 2x_4 \dot{x}_4)(x_1^2 + x_2^2)}{(x_1^2 + x_2^2)^2} \right\} - \frac{\omega L_f x_1 (x_3^2 + x_4^2)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \quad (61)$$

By substituting the derivative terms for x_3 and x_4 , according to the third and fourth rows of the state-space equations one has

$$\begin{aligned}\ddot{x}_2 = & -\omega\dot{x}_1 + \frac{1}{C_f} \left(-\omega x_3 - \frac{1}{L_f} x_2 + \frac{1}{L_f} u_2 \right) \\ & - \frac{1}{C_f} \left\{ \frac{(p_f \dot{x}_2 - g_f \dot{x}_1)(x_1^2 + x_2^2) - (p_f x_2 - g_f x_1)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} \\ & + \omega\dot{x}_1 - \frac{1}{C_f} \left\{ \frac{\omega L_f \dot{x}_1 (x_3^2 + x_4^2)(x_1^2 + x_2^2) - \omega L_f x_1 (x_3^2 + x_4^2)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} \\ & - \frac{1}{C_f} \frac{\omega L_f x_1 2x_3 (x_1^2 + x_2^2)}{(x_1^2 + x_2^2)^2} \left(\omega x_4 - \frac{1}{L_f} x_1 + \frac{1}{L_f} u_1 \right) \\ & - \frac{1}{C_f} \frac{\omega L_f x_1 2x_4 (x_1^2 + x_2^2)}{(x_1^2 + x_2^2)^2} \left(-\omega x_3 - \frac{1}{L_f} x_2 + \frac{1}{L_f} u_2 \right) \quad (62)\end{aligned}$$

By grouping the terms that multiply the control inputs u_1 and u_2 one obtains

$$\begin{aligned}\ddot{x}_2 = & -\omega\dot{x}_1 + \frac{1}{C_f} (-\omega x_3) \\ & - \frac{1}{C_f} \left\{ \frac{(p_f \dot{x}_2 - g_f \dot{x}_1)(x_1^2 + x_2^2) - (p_f x_2 - g_f x_1)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} \\ & + \omega\dot{x}_1 - \frac{1}{C_f} \left\{ \frac{\omega L_f \dot{x}_1 (x_3^2 + x_4^2)(x_1^2 + x_2^2) - \omega L_f x_1 (x_3^2 + x_4^2)(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2)}{(x_1^2 + x_2^2)^2} \right\} \\ & - \frac{1}{C_f} \frac{\omega L_f x_1 2x_3 (\omega x_4 - \frac{1}{L_f} x_1)}{(x_1^2 + x_2^2)} - \frac{1}{C_f} \frac{\omega L_f x_1 2x_4 (-\omega x_3 - \frac{1}{L_f} x_2)}{(x_1^2 + x_2^2)} \\ & - \frac{1}{C_f} \frac{\omega L_f x_1 2x_3}{(x_1^2 + x_2^2)} \frac{1}{L_f} u_1 - \frac{1}{C_f} \left\{ \frac{\omega L_f x_1 2x_4}{(x_1^2 + x_2^2)} \frac{1}{L_f} + \frac{1}{L_f} \right\} u_2 \quad (63)\end{aligned}$$

The previous equation can be also written in the form

$$\ddot{x}_2 = L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2 \quad (64)$$

Thus, one obtains an input-output linearized description of the inverter in the form

$$\begin{aligned}\ddot{x}_1 = & L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2 \\ \ddot{x}_2 = & L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2 \quad (65)\end{aligned}$$

The following control inputs are defined

$$\begin{aligned}v_1 = & L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2 \\ v_2 = & L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2 \quad (66)\end{aligned}$$

Thus, the inverter's dynamics becomes

$$\begin{aligned}\ddot{x}_1 = & v_1 \\ \ddot{x}_2 = & v_2 \quad (67)\end{aligned}$$

For this form of the system's dynamics the design of a state feedback controller is easy. This takes the form:

$$\begin{aligned}v_1 = & \ddot{x}_1 - k_d^1 (\dot{x}_1 - \dot{x}_1^d) - k_p^1 (x_1 - x_1^d) \\ v_2 = & \ddot{x}_2 - k_d^2 (\dot{x}_2 - \dot{x}_2^d) - k_p^2 (x_2 - x_2^d) \quad (68)\end{aligned}$$

The control input that is actually applied to the system is computed as:

$$\tilde{v} = \tilde{f} + \tilde{M}u \quad (69)$$

or equivalently

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} L_f^2 h_1(x) \\ L_f^2 h_2(x) \end{pmatrix} + \begin{pmatrix} L_{g_a} L_f h_1(x) & L_{g_b} L_f h_1(x) \\ L_{g_a} L_f h_2(x) & L_{g_b} L_f h_2(x) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad (70)$$

which means that the control input that is finally applied to the system is

$$\tilde{u} = \tilde{M}^{-1}(\tilde{v} - \tilde{f}) \quad (71)$$

To estimate the non-measurable state variables of the input-output linearized system the Derivative-free nonlinear Kalman Filter can be used. By defining the new state variables $z_1 = x_1$, $z_2 = \dot{x}_1$, $z_3 = x_2$ and $z_4 = \dot{x}_2$, the following state-space description of the system can be used

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (72)$$

$$\begin{pmatrix} z_1^m \\ z_2^m \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} \quad (73)$$

The associated state estimator has the form

$$\hat{z} = A\hat{z} + Bv + K_f(z^m - C\hat{z}) \quad (74)$$

For the linearized equivalent of the system it is possible to perform state estimation using the Derivative-free nonlinear Kalman Filter. The previously defined matrices A , B and C are substituted by their discrete-time equivalents A_d , B_d and C_d . This is done through common discretization methods. The recursion of the Kalman Filter is [30–33]:

measurement update:

$$\begin{aligned}K_f(k) = & P^-(k)C_d^T[C_dP^-(k)C_d^T + R(k)]^{-1} \\ \hat{x}(k) = & \hat{x}^-(k) + K_f(k)[C_d z(k) - C_d \hat{z}(k)] \\ P(k) = & P^-(k) - K(k)C_dP^-(k) \quad (75)\end{aligned}$$

time update:

$$\begin{aligned} P^-(k+1) &= A_d P(k) A_d^T + Q(k) \\ \hat{x}^-(k+1) &= A_d \hat{x}(k) + B_d v(k) \end{aligned} \quad (76)$$

The linearized equivalent model of the inverter can thus provide the state estimates $\hat{x}_1$, $\hat{\dot{x}}_1$, $\hat{x}_2$ and $\hat{\dot{x}}_2$. Finally using Eq. (51) and Eq. (52) one can obtain estimates for all state variables of the initial nonlinear model.

State and Disturbances Estimation with Nonlinear Kalman Filtering

The estimator can be also redesigned in the form of a disturbance observer. It is considered that the linearized model of the inverter is affected by additive input disturbances. The disturbance terms can describe both modelling uncertainties and external perturbations.

$$\begin{aligned} \ddot{x}_1 &= v_1 + \tilde{d}_1 \\ \ddot{x}_2 &= v_2 + \tilde{d}_2 \end{aligned} \quad (77)$$

These take the form $\tilde{d}_1^{(n)} = f_{d_1}$ and $\tilde{d}_2^{(n)} = f_{d_2}$. The disturbances' dynamics is represented by the n -th order derivative of the disturbances variables together with the associated initial conditions (however, since the disturbances are going to be estimated by the Kalman Filter, the knowledge of the initial conditions finally becomes obsolete).

Without loss of generality it is assumed that $n = 2$. The state vector is extended by including as additional state variables the disturbances and their derivatives. Thus, one has $z_1 = x_1$, $z_2 = \dot{x}_1$, $z_3 = x_2$, $z_4 = \dot{x}_2$, $z_5 = \tilde{d}_1$, $z_6 = \dot{\tilde{d}}_1$, $z_7 = \tilde{d}_2$ and $z_8 = \dot{\tilde{d}}_2$. Therefore, one has the extended state estimator of the form

$$\dot{\hat{z}}_e = A_e \hat{z}_e + B_e v + K_{f_e}(z_e^m - C_e \hat{z}_e) \quad (78)$$

where

$$A_e = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} B_e = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} C_e^T = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (79)$$

For the extended state-space description of the system it is possible again to perform state estimation using the Derivative-free nonlinear Kalman Filter. The previously defined matrices A_e , B_e and C_e are substituted by their

discrete-time equivalents A_{ed} , B_{ed} and C_{ed} . This is done again through common discretization methods. The recursion of the Kalman Filter in this case is:

measurement update:

$$\begin{aligned} K_f(k) &= P^-(k) C_{ed}^T [C_{ed} P^-(k) C_{ed}^T + R(k)]^{-1} \\ \hat{x}(k) &= \hat{x}^-(k) + K_f(k) [C_{ed} z(k) - C_{ed} \hat{z}^-(k)] \\ P(k) &= P^-(k) - K(k) C_{ed} P^-(k) \end{aligned} \quad (80)$$

time update:

$$\begin{aligned} P^-(k+1) &= A_{ed} P(k) A_{ed}^T + Q(k) \\ \hat{x}^-(k+1) &= A_{ed} \hat{x}(k) + B_{ed} v(k) \end{aligned} \quad (81)$$

After identifying the disturbance terms, the control input of the inverter is modified as follows:

$$\begin{aligned} v_1 &= \ddot{x}_1 - k_d^1(\dot{x}_1 - \dot{x}_1^d) - k_p^1(x_1 - x_1^d) - \hat{z}_5 \\ v_2 &= \ddot{x}_2 - k_d^2(\dot{x}_2 - \dot{x}_2^d) - k_p^2(x_2 - x_2^d) - \hat{z}_7 \end{aligned} \quad (82)$$

Simulation Tests

The efficiency of the inverter's nonlinear control scheme was tested in the tracking of several time-varying setpoints. The associated results are presented in Figs. 2–4. It can be noticed that through the proposed control approach, fast and accurate tracking of the output voltage reference setpoints can be succeeded. This is shown in Figs. 2a–4a. Moreover, it can be noticed that the Derivative-free nonlinear Kalman Filter enables precise estimation and compensation of the modelling uncertainties and the external perturbation terms that affect the inverter's model. This is shown in Figs. 2b–4b.

Using the state-space model of Eq. (13) as well as the linearized equivalent of Eq. (77), the disturbance terms affecting the inverter's model can as follows: (i) parametric uncertainty associated with the values of the capacitance C_f and of the inductance L_f , (ii) variations of the grid's frequency ω , (iii) uncertainty or variation of the active and reactive power p_f and q_f respectively, (iv) additive input disturbance terms affecting the control inputs V_{id} and V_{iq} (these can be due to parametric changes in the inverter's circuit). All these perturbations are denoted by the cumulative disturbance terms $\tilde{d}_1$ and $\tilde{d}_2$ which appear in Eq. (77). Therefore, by estimating these disturbance inputs with the use of the Derivative-free nonlinear Kalman Filter, it becomes possible to compensate for them and to assure the quality of the AC power provided by the inverter either to the stand-alone load or to the grid.

In the simulation experiments the disturbance inputs $\tilde{d}_1$ and $\tilde{d}_2$ of Eq. (77) were due to (i),(iii) and (iv) but it is also possible to obtain similar results in case that disturbance in

Fig. 2 **a** Convergence of the state vector elements $V_{L,d}$ and $V_{L,q}$ of the three-phase inverter to the associated reference setpoints 1, **b** Estimation of disturbance inputs affecting the inverter's model, with the use of the Derivative-free nonlinear Kalman Filter

Fig. 3 **a** Convergence of the state vector elements $V_{L,d}$ and $V_{L,q}$ of the three-phase inverter to the associated reference setpoints 2, **b** Estimation of disturbance inputs affecting the inverter's model, with the use of the Derivative-free nonlinear Kalman Filter

the grid's frequency ω is introduced. In all cases the proposed control scheme is proven to be robust and assures convergence of the AC voltage to the desirable setpoints. Regarding

the reference setpoints presented in the simulation tests these were constant signals expressed in the dq reference frame. The disturbance inputs contained also sinusoidal terms of

Fig. 4 **a** Three-phase voltage variables at the output of the inverter, **b** active and reactive power of the inverter

variable amplitude and frequency. It was confirmed that in all cases the control scheme succeeded elimination of the setpoints' tracking error.

Remark 1 From the stability and convergence analysis of the proposed state estimation-based control scheme in “Lie Algebra-based Control of the Inverter’s Model”—“Flatness-based Control of the Inverter” sections, it can be assured that both the steady-state and the transient response of the inverter’s control loop are satisfactory. The selection of the feedback control gains given in Eq. (82) are the ones that determine the closed-loops transient characteristics. Moreover, by choosing the Kalman Filter for estimating the state vector elements which are used by the state-feedback controller, one can assure that undesirable transients will be avoided (unlike for instance the transients generated by sliding-mode observer) [34,35]. Actually, the Kalman Filter assures fast and smooth elimination of the estimation error (thus also avoiding chattering that usually appears when sliding-mode observers are used in the control loop).

Remark 2 The proposed differential flatness theory-based control and filtering method for three-phase inverters can be generalized towards synchronizing control of parallel inverters connecting distributed DC power units to the grid. The latter problem is particularly significant for integrating into the electricity network renewable energy sources of the DC type such as photovoltaics, while also assuring that frequency and amplitude characteristics of the grid’s voltage will not be distorted and that the grid’s stability will be also assured [15,36–39].

Conclusions

The paper has analyzed a new nonlinear control and state estimation method for three-phase inverters. It has been shown that the MIMO nonlinear inverter’s model is a differentially flat one. The flat output of the model was shown to be a vector comprising as elements the voltage variables expressed in the dq reference frame and measured at the load’s side. It has been confirmed that all state variables of the inverter and all control inputs can be expressed as functions of the flat output and of its derivatives. By exploiting this property it has become possible to transform the inverter’s model to the linear canonical (Brunovsky) form. For the latter description, the design of a state feedback controller became possible, after applying pole placement methods.

Other problems that had to be dealt with in the development of the nonlinear feedback control scheme were: (i) there was need to estimate nonmeasurable variables of the inverter’s state vector, (ii) there was need to compensate for modeling uncertainties and perturbation terms affecting the inverter’s model. To solve (i) a new nonlinear filtering method, under the name Derivative-free nonlinear Kalman Filter, has been introduced. The filtering method consisted of the Kalman Filter recursion applied on the linearized equivalent of the inverter and of an inverse transformation that was based on differential flatness theory and which enabled to obtain estimates for the state variables of the initial nonlinear model. To solve (ii) the Derivative-free nonlinear Kalman Filter has been redesigned as a disturbance observer. The efficiency of the proposed control scheme was evaluated through simulation experiments.

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