Analysis of Soil Nail Excavation Considering Rayleigh Wave with Log-Spiral Failure Surface

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Abstract In this paper, an analysis has been carried out to evaluate the stability of soil nailed vertical or nearly vertical excavations assuming the logarithmic spiral failure surface and using total seismic wave. The internal failure mode of the nailed cut is considered either by pullout, rupture or excessive bending whichever is critical. Also, the three different failure modes of rupture surface such as rupture surface passing through toe, slope or base is considered for the analysis and critical one is taken for presentation of factor of safety. Expression for the factor of safety is derived using moment equilibrium method. Results have been arranged in non-dimensional tabular form considering ranges of the design parameters that usually occur in practice. Analytical results have been compared with the findings of model tests and reasonably good agreement has been observed.

Keywords Toe failure · Base failure · Slope failure · Soil nailing · Rayleigh wave analysis · Factor of safety · Parametric study · Comparison · Good agreement

Introduction

Soil nailing is a method of reinforcing the soil with steel bars or other materials. It has been an alternative technique to other conventional supporting system as it offers flexibility, rapid construction and competitive costs. The purpose is to increase the tensile and shear strength of soil and restrain its displacements. The nails are either placed in drilled boreholes and grouted along their total length to form grouted nails or simply driven into the ground as driven nails.

Many investigators [1–8] have proposed different methods for investigating the stability of vertical/nearly vertical excavations. In each method, the assumed geometry of the slip surface is based on observations in either small scale model tests or full scale structures. The methods vary with the geometry of the assumed failure surface, the definition of the factor of safety and the forces assumed to act on the active zone. Basudhar [9] presented an overview of experimental and theoretical studies leading to the development of various methods of analysis of soil nailed structures. The methods proposed by Raju [7] and Gupta [8] are based on moment equilibrium approach assuming the rupture surface as log-spiral meeting the ground at right angle. The formation of log-spiral rupture surface is supported by earlier investigators [6, 10–12] including the observations of Raju [7] in his small scale model tests and trench tests. The behavior of soil nailed walls under dynamic loads has been studied by very few investigators. Mizuno and Chen [13] performed seismic analysis of vertical slopes using plasticity models, and compared the results with finite element method and limit analysis methods. Saran et al. [14] analyzed the pseudo-static seismic stability of soil nail slope. Sabhahit et al. [15] presented a seismic design of nailed slopes based on pseudo-dynamic approach. Saha et al. [16] carried out dynamic analysis of soil nailed vertical cuts using an explicit finite difference tool fast Lagrangian analysis of continua (FLAC), where it is considered sinusoidal harmonic horizontal shear loading only. Katdare and Choudhury [17] and Choudhury and Katdare [18] determined the seismic active earth pressure considering the effect of Rayleigh wave, whereas, Choudhury et al. [19] introduced the Rayleigh wave in the analysis of retaining wall for the determination of seismic passive earth pressure. Choudhury et al. [20] also analyze the seismic behaviour of retaining structure and design issues for the stability analysis.

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Here, an attempt is made to evaluate the behavior of soil nail slope considering Rayleigh wave assuming log spiral failure surface. The present study also explores the effect of angle of friction, horizontal seismic acceleration, nail number, slope angle on the stability of slope.

Geometry of Rupture Surface

A homogeneous soil slope of height H and inclination i_s with horizontal is shown in Fig. 1. Let us consider the equilibrium of the potential sliding mass QIU formed by the assumed failure surface arc QU represented by a logarithmic spiral portion as shown in Fig. 1. Logarithmic spiral portion of the failure surface (QU) is governed by height of the soil slope (QG) and the location of centre of the logarithmic spiral arc (A). The soil is considered as a generalized c - ϕ soil, where c is the cohesion of soil and ϕ is the angle of internal friction of soil. It has been assumed that the occurrence of an earthquake forces does not affect the basic soil parameters; the unit cohesion of soil (c), the soil friction angle (ϕ) and the unit weight of the soil (γ). Let us consider, AU and AQ respectively the initial and final radius of the logarithmic spiral passing through the centre A .

The rupture surface having equation as

$$r_f = r_o \exp(\theta_1 \tan \phi). \quad (1)$$

Since $AQ = AL_0 + L_0Q$,

So, $r_o \exp(\theta_1 \tan \phi) = \frac{r_o \sin \theta_2}{\sin(\theta_2 + \theta_1)} + H \operatorname{cosec}(\theta_2 + \theta_1)$

$$r_o = \frac{H \operatorname{cosec}(\theta_2 + \theta_1)}{e^{\theta_1 \tan \phi} - \frac{\sin \theta_2}{\sin(\theta_2 + \theta_1)}} \quad (2)$$

$$r_o = Hx,$$

$$\text{where } x = \frac{\operatorname{cosec}(\theta_2 + \theta_1)}{e^{\theta_1 \tan \phi} - \frac{\sin \theta_2}{\sin(\theta_2 + \theta_1)}}.$$

Since $UI = EU - EG - GI$

$$UI = \frac{r_o \sin \theta_1}{\sin(\theta_1 + \theta_2)} - H \cot(\theta_1 + \theta_2) - H \cot i_s \quad (3)$$

$$UI = Hy = L_g(\text{Let}) \quad (4)$$

$$y = \frac{x \sin \theta_1}{\sin(\theta_1 + \theta_2)} - \cot(\theta_1 + \theta_2) - \cot i_s. \quad (5)$$

Let $\angle QAB_1 = \beta$

$$p = \frac{\left(\sum x_j^2 y_j \sum x_j^2 - \sum x_j y_j \sum x_j^2 - \sum x_j^3 \sum y_j \sum x_j + \sum y_j \left(\sum x_j^2 \right)^2 - n \sum x_j^2 y_j \sum x_j^2 + n \sum x_j^3 \sum x_j y_j \right)}{\left(\sum x_j^4 \sum x_j^2 - 2 * \sum x_j \sum x_j^2 \sum x_j^3 + \left(\sum x_j^2 \right)^3 - n \sum x_j^4 \sum x_j^2 + n \left(\sum x_j^3 \right)^2 \right)} \quad (12)$$

$$q = \frac{- \left(n \sum x_j^4 \sum x_j y_j - n \sum x_j^3 \sum x_j^2 y_j - \left(\sum x_j^2 \right)^2 \sum x_j y_j + \sum x_j \sum x_j^2 \sum x_j^2 y_j - \sum x_j \sum x_j^4 \sum y_j + \sum x_j^2 \sum x_j^3 \sum y_j \right)}{\left(\sum x_j^4 \left(\sum x_j \right)^2 - 2 \sum x_j \sum x_j^2 \sum x_j^3 + \left(\sum x_j^2 \right)^3 - n \sum x_j^4 \sum x_j^2 + n \left(\sum x_j^3 \right)^2 \right)} \quad (13)$$

Fig. 1 Forces acting on soil nailing wedge

From Fig. 1,

$$\beta = 90^\circ - \theta_2 - \theta_1, \quad (6)$$

where

$$\angle QAU = \theta_1$$

$$\angle JAU = \theta_2$$

Equation of failure surface in cartesian coordinate system.

Let, n number of points taken on curve QB_2U such that the angle subtended by the two consecutive points at centre, $\theta_j = \theta_1/n$,

where

$$r_j = r_f \exp(-j\theta_j \tan \phi) \quad (7)$$

$$x_j = r_j \sin(j\theta_j + \beta) \quad (8)$$

$$y_j = r_j \cos(j\theta_j + \beta) \quad j = 1, 2, 3, \dots, n. \quad (9)$$

The equation of the curve QB_2U can be expressed as,

$$y = px^2 + qx + t \quad (10)$$

in which,

$$t = AB_1 = Y_0 \quad (11)$$

here,

$$\Sigma x_j = \sum_{j=0}^n r_j \sin(j\theta_j + \beta)$$

$$\Sigma y_j = \sum_{j=0}^n r_j \cos(j\theta_j + \beta)$$

$$\Sigma x_j y_j = \sum_{j=0}^n r_j \cos(j\theta_j + \beta) r_j \sin(j\theta_j + \beta)$$

$$\Sigma x_j^2 = \sum_{j=0}^n (r_j \sin(j\theta_j + \beta))^2$$

$$\Sigma x_j^3 = \sum_{j=0}^n (r_j \sin(j\theta_j + \beta))^3$$

$$\Sigma x_j^4 = \sum_{j=0}^n (r_j \sin(j\theta_j + \beta))^4$$

$$\Sigma x_j^2 y_j = \sum_{j=0}^n (r_j \sin(j\theta_j + \beta))^2 r_j \cos(j\theta_j + \beta)$$

$j = 1, 2, 3, \dots, n.$

Inertia Forces Acting on Soil Slope Considering the Effect of Rayleigh Wave

A homogeneous slope of height H and inclination i_s with horizontal is considered. All the possible earthquake waves like shear wave, primary wave and as well as Rayleigh wave are considered in this present study. In a pseudo-dynamic analysis, both shear wave velocity $V_s = \sqrt{\frac{G}{\rho}}$ and primary wave velocity $V_p = \sqrt{\frac{2G(1-\mu)}{\rho(1-2\mu)}}$ of the earthquake waves through backfill medium are assumed to act within the soil mass during earthquake. Where G , ρ and μ are shear modulus, density and Poisson's ratio of slope material respectively and the Rayleigh wave progressing horizontally in x -direction with displacement u and vertically in z direction with displacement w . Now, the displacement components, u and w are as follows:

Displacement potential:

$$d = \nabla \phi + \nabla \psi \quad (14)$$

Displacement components

a. Displacement of the particle in transverse direction

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z} \quad (15)$$

b. Displacement of the particle in normal direction

$$w = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x} \quad (16)$$

Potential function,

$$\phi = Ae^{-qz} e^{i(k_r x - \omega t)}, \quad \psi = Be^{-sz} e^{i(k_r x - \omega t)}, \quad (17)$$

where

$$q^2 = K_r^2 - \frac{\omega^2}{V_p^2} \text{ and } s^2 = K_r^2 - \frac{\omega^2}{V_s^2} \quad (18)$$

$$K_r^6 - 8K_r^4 + 8(3 - 2\xi^2)K_r^2 - 16(1 - \xi^2) = 0. \quad (19)$$

K_r is the Rayleigh wave no. and ξ is the ratio of shear wave velocity and primary wave velocity which depends on Poisson's ratio (μ) of soil are given by the following equation

$$\xi = \frac{V_s}{V_p} = \sqrt{\frac{1 - 2\mu}{2 - 2\mu}} \quad (20)$$

Viktorov [21] shows an approximate equation of K_r which depends on Poisson's ratio given by Bergmann's [22] following formula

$$K_r \cong \frac{0.87 + 1.12\mu}{1 + \mu}$$

From this equation $K_r = 0.93$ as $\mu = 0.3$.

The surface wave solution is a combination of coupled partial (longitudinal and shear) waves. The B/A ratio is determined by the condition that the surface is traction free. Both partial waves are evanescent, i.e., $K_r > K_s > K_p$.

$$\text{Where } K_p^2 = \frac{\omega^2}{V_p^2}, \quad K_s^2 = \frac{\omega^2}{V_s^2}$$

$$\frac{B}{A} = i\sqrt{\frac{q}{s}} = -i\xi$$

Now, the normal and tangential displacement components for Rayleigh wave can be written as,

$$u = iA(K_r e^{-qz} - \xi e^{-sz}) e^{i(k_r x - \omega t)} \quad (21)$$

$$w = -A(q e^{-qz} - \xi K_r e^{-sz}) e^{i(k_r x - \omega t)} \quad (22)$$

u and w are displacements in the x and z directions respectively. Differentiating twice displacement Eqs. (21) and (22) we get respective accelerations

$$a_h(x, z, t) = \omega^2 A(K_r e^{-qz} - \xi s e^{-sz}) \sin(k_r x - \omega t) \quad (23)$$

$$a_v(x, z, t) = \omega^2 A(q e^{-qz} - \xi K_r e^{-sz}) \cos(k_r x - \omega t). \quad (24)$$

Calculation of $Q_h(t)$ and $Q_v(t)$

Equation of line QI : linear equation passing through the two points $Q(r_f \sin \beta, Y_0)$ and $I((H \cot i_s + r_f \sin \beta), (Y_0 - H))$ is given as,

$$\begin{aligned} y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \\ y &= -x \tan i_s + \{r_f \sin \beta + Y_0\} \\ y &= mx + c_0 \end{aligned} \quad (25)$$

where, $m = \tan i_s$

$$c_0 = r_f \sin \beta + Y_0$$

Equation of surface QB_2U : The equation of the failure surface is

$$px^2 + qx + t = 0. \quad (26)$$

In Fig. 1, mass of a thin element of thickness dy at depth y from the top of the surface and width dx at a distance x from the slope line is given by

$$m(x, z) = \frac{\gamma}{g} dx dy.$$

The total horizontal seismic inertia force acting on the failure wedge can be written as,

$$\begin{aligned} Q_h(t) &= \int_{Y_0-H}^{Y_0} \int_{\frac{1}{m}(y-c_0)}^{\frac{-q \pm \sqrt{q^2-4pt}}{2p}} m(x, z) a_h(x, z, t) dx dy \\ &= \frac{\omega^2 A \gamma}{g K_r} \int_{Y_0-H}^{Y_0} (K_r e^{-qz} - \xi s e^{-sz}) \\ &\quad - \left[\cos \left(K_r x - 2\pi \frac{t}{T} \right) \right]_{\frac{1}{m}(y-c_0)}^{\frac{-q \pm \sqrt{q^2-4pt}}{2p}} \\ &= \frac{\omega^2 A \gamma}{g K_r} I_1. \end{aligned} \quad (27) \quad (28)$$

The total vertical seismic inertia force acting on the failure wedge can be written as,

$$\begin{aligned} Q_v(t) &= \int_{Y_0-H}^{Y_0} \int_{\frac{1}{m}(y-c_0)}^{\frac{-q \pm \sqrt{q^2-4pt}}{2p}} m(x, z) a_v(x, z, t) dx dy \\ &= \int_{Y_0-H}^{Y_0} \frac{\omega^2 A \gamma}{g K_r} (K_r e^{-qz} - \xi s e^{-sz}) \\ &\quad \times \left[\sin \left(K_r x - 2\pi \frac{t}{T} \right) \right]_{\frac{1}{m}(y-c_0)}^{\frac{-q \pm \sqrt{q^2-4pt}}{2p}} \\ &= \frac{\omega^2 A \gamma}{g K_r} I_2 \end{aligned} \quad (29) \quad (30)$$

$Q_h(t)$ and $Q_v(t)$ can be expressed in the form of k_h and k_v by replacing A in Eqs. (28) and (30) from the Eqs. (23) and (24) respectively. Since, the variation in the x direction is not very much significant, so, all the values are computed for $x = 0$ and $z = (Y_0 - H)$ in this present study. Now, $Q_h(t)$ and $Q_v(t)$ can be written as,

$$Q_h(t) = \frac{\gamma k_h}{K_r (K_r e^{-q(Y_0-H)} - \xi s e^{-s(Y_0-H)}) \sin(2\pi \frac{t}{T})} I_1 \quad (31)$$

$$Q_v(t) = \frac{\gamma k_v}{K_r (q e^{-q(Y_0-H)} - \xi K_r e^{-s(Y_0-H)}) \cos(2\pi \frac{t}{T})} I_2. \quad (32)$$

For the evaluation of Eqs. (31) and (32), I_1 and I_2 is solved numerically.

Weight of the log-spiral portion

$$W_{QB_2UI} = W_{AQB_2U} - W_{AL_0U} - W_{QL_0G} - W_{QIG}.$$

If the distance of centre of gravity of log-spiral portion from centre A is given by $\bar{x}$ in x -direction and $\bar{y}$ in y -direction then,

$$\bar{x} = \frac{M_{AQB_2U} - M_{AL_0U} - M_{QL_0G} - M_{QIG}}{W_{QB_2U} - W_{AL_0U} - W_{QL_0G} - W_{QIG}}$$

$$\bar{y} = \frac{M'_{AQB_2U} - M'_{AL_0U} - M'_{QL_0G} - M'_{QIG}}{W_{QB_2U} - W_{AL_0U} - W_{QL_0G} - W_{QIG}}$$

The expression for W_{AQB_2U} , W_{AL_0U} , W_{QL_0G} , W_{QIG} , M_{AQB_2U} , M_{AL_0U} , M_{QL_0G} , M_{QIG} , M'_{AQB_2U} , M'_{AL_0U} , M'_{QL_0G} , M'_{QIG} are given in Appendix.

Forces Acting on Sliding Wedge

The forces acting on the sliding wedge bounded by the logarithmic spiral arc are shown in Fig. 1, and defined as below:

1. W is the weight of the soil wedge QB_2UI along with vertical seismic force $Q_v(t)$. It acts at the center of gravity (c.g.) of the wedge in the vertical direction. So, the disturbing moment due to $W + Q_v(t)$,

$$\begin{aligned} W_{QB_2UI} &= W_{AQB_2U} - W_{AL_0U} - W_{QL_0G} - W_{QIG} \\ M_{wv} &= (W_{QB_2UI} \pm Q_v(t)) \bar{x}, \end{aligned} \quad (33)$$

where $\bar{x}$ is the distance of centre of gravity of failure wedge in x direction. (The detail calculation of which is given in Appendix.)

2. $Q_h(t)$ the horizontal seismic force acting at the c.g. of wedge. So, the Moment due to $Q_h(t)$.

$$M_{WH} = Q_h(t) \bar{y} \quad (34)$$

where $\bar{Y}$ is the distance of centre of gravity of failure wedge in Y direction. (The detail calculation of which is given in [Appendix](#).)

3. Surcharge Load q (kN/m²) is acting on the slope. So, the Moment due to q

$$M_{qh} = k_h \times q \times UI \times r_o \sin \theta_2$$

$$= qH^2 k_{hxy} \sin \theta_2 \quad (35)$$

$$M_{qv} = (1 + k_v)q \times UI \times (r_o \cos \theta_2 - UI/2). \quad (36)$$

4. C , the force due to cohesion acting along the surface of sliding wedge QB_2U . So, moment due to cohesion,

$$M_c = \frac{C}{2 \tan \phi} [r_i^2 - r_o^2] \quad (37)$$

5. T_i , the tensile or pull out capacity, (whichever is smaller) of the i th nail. So, M_T due to that for whole mass

$$M_T = \sum_{i=1}^{i=n} T_i L_i = \sum_{i=1}^{i=n} f_1 L_{ei} p / s_h \times r_o \exp(\theta \tan \phi) \times \sin(\theta_2 + \theta)$$

OR $\sum_{i=1}^{i=n} (\sigma_{ni} \tan \delta) L_{ei} p / s_h r_o \exp(\theta \tan \phi) \times \sin(\theta_2 + \theta) \quad (38)$

where, f_1 = limit bond stress of the soil–nail interface. It is obtained from pullout tests. $P = \pi d$, s_h = horizontal spacing between two nails, L_{ei} = pullout length of nail, $\sigma_{ni} = \frac{\sigma_y \cos^2 \theta - \sigma_x \sin^2 \theta}{\cos 2\theta + \sin 2\theta \tan \delta}$, δ = the angle of friction between nail and soil = ϕ (assumed), $\sigma_y = [\gamma(i - 0.5)s_v + q]$, θ = nail inclination with horizontal, s_v = vertical Spacing between two nails, $\sigma_x = k_a \sigma_y$, k_a = Active earth pressure coefficient = $\frac{1 - \sin \phi}{1 + \sin \phi}$

6. T_{ci} , the maximum shear resistance mobilized in the i th nail due to bending stiffness according to Jewell and Pedley [23]. So, moment due to T_{ci} ,

$$M_{TC} = \sum_{i=1}^{i=n} T_{ci} L_{ci} \quad (39)$$

where $T_{ci} = \frac{4M_p}{s_h l_{si}} \left[1 - \frac{T_i}{T_p} \right]$, $M_p = 0.166 d^3 f_y$, $T_p = f_y A_p$, f_y is Yield strength of steel nail, where T_i = Axial nail force = $f_1 L_{ei} p / s_h$ or $(\sigma_{ni} \tan \delta) L_{ei} p / s_h$, which one is less $L_{ci} = r_o \exp(\theta \tan \phi) \times \sin(\theta_2 + \theta) \cos \theta_2$

The procedure of computation of various forces acting on the sliding wedge (c , T_i , T_{ci}) resisting and driving moments (M_c , M_T , M_{TC}) is presented elsewhere (Gosavi [24]).

If grouted nails are used for stabilizing the cuts, then certain changes are required to be made in the analysis of

driven reinforcement. Let ‘ D ’ be the diameter of the grouted nail (grout hole diameter).

1. For the calculation of the nail axial force with the pull-out resistance criteria, perimeter (P) of the grouted nails calculated as: πD
2. The value of apparent coefficient of friction ($\tan \delta$) for grouted nail is to be considered in between grout and soil interface.
3. Shear width l_{si} , is calculated for the grouted nail as:

$$l_{si} = \sqrt{\frac{8M_p}{\sigma_b} \times \frac{1}{D} \times \left(1 - \frac{T_i}{T_p} \right)}$$

where T_i = Axial nail force = $f_1 L_{ei} p / s_h$ or $(\sigma_{ni} \tan \delta) L_{ei} p / s_h$, which one is less, θ = nail inclination with horizontal, $\delta = \phi$, $M_p = 0.166 d^3 f_y$, $\sigma_b = \sigma_v \left[\frac{1+k_a}{2} \right] \tan(\phi/2 + \pi/2) e^{(\pi/2 + \phi) \tan \phi}$ = soil bearing Pressure (Jewell and Pedley [23]), f_1 = limit bond stress of the soil–nail interface. It is obtained from pullout tests.

Equation of Factor of safety

The equation of factor of safety (FOS) is defined as:

$$\text{FOS}_{ro} = \frac{\sum \text{Resisting moments}}{\sum \text{Disturbing moments}} \quad (40)$$

$$\text{FOS}_{ro} = \frac{M_T + M_C + M_{TC} + M_R}{M_{WV} + M_{WH} + M_{qv} + M_{qh}}$$

here M_R = resisting moment due to reaction force about centre $A = 0$,

M_{qh} , M_{qv} are given by Eqs. (35) and (36) respectively.

$$M_{WV} = (W_{QU} + Q_v) \bar{x} \quad (41)$$

$$M_{WH} = Q_h(t) \bar{Y} \quad (42)$$

M_c = moment of cohesion force c about the point centre A ,

$$M_c = \frac{C}{2 \tan \phi} [r_i^2 - r_o^2] \quad (43)$$

$$M_T = \sum_{i=1}^{i=n} T_i L_i = \sum_{i=1}^{i=n} f_1 L_{ei} p / s_h \times r_o \exp(\theta \tan \phi) \times \sin(\theta_2 + \theta)$$

OR $\sum_{i=1}^{i=n} (\sigma_n \tan \delta) L_{ei} p / s_h r_o \exp(\theta \tan \phi) \times \sin(\theta_2 + \theta) \quad (44)$

$$M_{TC} = \sum_{i=1}^{i=n} T_{ci} L_{ci} \quad (45)$$

Table 1 Factor of safety for soil friction angle (ϕ) = 30°, 35°, 40°, angle of slope (i_s) = 75°, 90° horizontal seismic acceleration coefficient, k_h = 0, 0.1, 0.2, $c/\gamma H$ = 0, $c/\gamma H$ = 0.3, $c/\gamma H$ = 0.06, surcharge coefficient ($q/\gamma H$) = 0, 0.3, 0.5, γ = 16.5 kN/m³, k_v = 0, N = 14, 15, 16, $S_v = S_h = H/(N + 1)$, and H = 10 m

ϕ		30°			35°			40°		
N	k_h	$c/\gamma H = 0$	$c/\gamma H = 0.03$	$c/\gamma H = 0.06$	$c/\gamma H = 0$	$c/\gamma H = 0.03$	$c/\gamma H = 0.06$	$c/\gamma H = 0$	$c/\gamma H = 0.03$	$c/\gamma H = 0.06$
$i_s = 90^\circ$ $q/\gamma H = 0$										
14	0	1.489	1.9315	2.3737	2.2051	2.7411	3.2772	3.2678	3.926	4.5842
15	0	1.6287	2.104	2.5792	2.4077	2.9871	3.5665	3.5644	4.2799	4.9953
16	0	1.7723	2.2825	2.7927	2.6156	3.2413	3.8669	3.8685	4.6451	5.4216
14	0.1	1.0448	1.3649	1.6849	1.4874	1.849	2.2106	1.9706	2.368	2.7653
15	0.1	1.1452	1.4864	1.8276	1.6242	2.0149	2.4056	2.1493	2.5813	3.0134
16	0.1	1.2486	1.6123	1.976	1.7647	2.1864	2.6081	2.3325	2.8016	3.2707
14	0.2	0.7765	1.0143	1.2521	1.1216	1.3943	1.667	1.4104	1.6948	1.9791
15	0.2	0.8511	1.1046	1.3582	1.2248	1.5194	1.814	1.5383	1.8475	2.1567
16	0.2	0.9279	1.1982	1.4685	1.3307	1.6487	1.9667	1.6694	2.0051	2.3408
$i_s = 90^\circ$ $q/\gamma H = 0.3$										
14	0	0.9927	1.2906	1.5886	1.473	1.8309	2.1915	2.1786	2.621	3.0633
15	0	1.0856	1.4054	1.7252	1.6049	1.9945	2.3841	2.3755	2.8562	3.3369
16	0	1.181	1.5251	1.2172	1.743	2.1635	2.5841	2.5773	3.0988	3.6204
14	0.1	0.7525	0.9848	1.3198	1.0834	1.3491	1.6149	1.4757	1.7759	2.0762
15	0.1	0.8245	1.0722	1.4267	1.1826	1.4697	1.7568	1.6087	1.9352	2.2617
16	0.1	0.8986	1.1626	0.9529	1.2843	1.5942	1.9041	1.7451	2.0996	2.4541
14	0.2	0.5892	0.7711	1.033	0.8577	1.0681	1.2784	1.1145	1.3412	1.568
15	0.2	0.6455	0.8394	1.117	0.9632	1.1635	1.3908	1.215	1.4616	1.7081
16	0.2	0.7036	0.9103	1.294	1.0168	1.2621	1.5074	1.3179	1.5857	1.8534
$i_s = 90^\circ$ $q/\gamma H = 0.5$										
14	0	0.7469	0.9715	1.1959	1.1071	1.3787	1.6502	1.6402	1.9733	2.3063
15	0	0.8169	1.0579	1.2987	1.2085	1.5019	1.7952	1.7885	2.1504	2.5123
16	0	0.8889	1.1473	1.4056	1.3125	1.6291	1.9456	1.9404	2.3331	2.7257
14	0.1	0.5906	1.0579	0.9553	0.8563	1.0663	1.2763	1.1859	1.4273	1.6686
15	0.1	0.6471	1.1473	1.0359	0.9347	1.1616	1.3885	1.2929	1.5553	1.8177
16	0.1	0.7053	0.773	1.1197	1.0151	1.26	1.5049	1.4024	1.6874	1.9723
14	0.2	0.477	0.8415	0.7714	0.6981	0.8694	1.0406	0.9267	1.1153	1.3039
15	0.2	0.5226	0.9125	0.8365	0.7621	0.947	1.132	1.0103	1.2154	1.4204
16	0.2	0.5696	0.6242	0.9042	0.8276	1.0273	1.227	1.0959	1.3186	1.5412
$i_s = 75^\circ$ $q/\gamma H = 0$										
14	0	1.8301	2.4061	2.9808	2.9679	3.7111	4.4543	3.7521	5.8102	6.7927
15	0	2.0078	2.6193	3.23	3.2465	4.0432	4.8398	4.1982	6.3338	7.395
16	0	2.1908	2.8402	3.4894	3.5328	4.3865	5.2401	5.3347	6.8741	8.0194
14	0.1	1.1492	1.504	1.8588	1.8561	2.3246	2.7931	2.4928	2.9994	3.506
15	0.1	1.2592	1.6374	2.0156	2.0314	2.5327	3.0339	2.7219	3.2696	3.8172
16	0.1	1.3723	1.7756	2.1788	2.2117	2.7478	3.284	2.9569	3.5483	4.1397
14	0.2	0.8319	1.0888	1.3456	1.3464	1.6863	2.0261	1.6281	1.9594	2.2906
15	0.2	0.9115	1.1853	1.4591	1.4737	1.8372	2.2008	1.7749	2.1351	2.4954
16	0.2	0.9934	1.2853	1.5772	1.6044	1.9933	2.3823	1.9253	2.3164	2.7076
$i_s = 75^\circ$ $q/\gamma H = 0.3$										
14	0	1.1955	1.5817	1.9657	1.9638	2.4617	2.9589	3.2205	3.8796	4.1534
15	0	1.3131	1.7216	2.1283	2.1499	2.6821	3.2141	3.5194	4.2297	4.4497
16	0	1.4344	1.8666	2.2975	2.3413	2.91	3.4788	3.8262	4.5912	5.1023
14	0.1	0.8731	1.1426	1.4121	1.37	1.7158	2.0616	1.9736	2.3747	2.4553
15	0.1	0.9566	1.2439	1.5313	1.4995	1.8694	2.2394	2.155	2.5885	2.6785

Table 1 continued

ϕ		30°			35°			40°		
N	k_h	$c/\gamma H = 0$	$c/\gamma H = 0.03$	$c/\gamma H = 0.06$	$c/\gamma H = 0$	$c/\gamma H = 0.03$	$c/\gamma H = 0.06$	$c/\gamma H = 0$	$c/\gamma H = 0.03$	$c/\gamma H = 0.06$
16	0.1	1.0426	1.3489	1.6552	1.6325	2.0282	2.424	2.341	2.8092	2.9318
14	0.2	0.6626	0.8741	1.0804	1.0512	1.3166	1.5819	1.4056	1.6912	1.9769
15	0.2	0.7319	0.9517	1.1715	1.1505	1.4344	1.7183	1.5348	1.8436	2.1523
16	0.2	0.7976	1.032	1.2663	1.2526	1.5563	1.8599	1.6673	2.007	2.3342
$i_s = 75^\circ$ $q/\gamma H = 0.5$										
14	0	0.8759	1.1648	1.2678	1.4514	1.8233	1.932	2.3902	2.8805	2.9672
15	0	0.9635	1.2677	1.3122	1.59	1.9856	2.0938	2.6126	3.1407	3.2345
16	0	1.0534	1.3744	1.432	1.7325	2.1543	2.2764	2.841	3.4092	3.4765
14	0.1	0.6858	0.9094	0.9546	1.0823	1.3567	1.4327	1.6333	1.9653	2.1094
15	0.1	0.7536	0.9897	1.0632	1.1849	1.4782	1.5467	1.7835	2.1423	2.2366
16	0.1	0.8236	1.0731	1.1599	1.2903	1.6038	1.6643	1.9374	2.3249	2.4123
14	0.2	0.558	0.7302	0.7754	0.8608	1.0791	1.1855	1.2091	1.4548	1.5329
15	0.2	0.6114	0.795	0.8532	0.9424	1.1757	1.2891	1.3202	1.5858	1.6564
16	0.2	0.6663	0.8621	0.932	1.0263	1.2756	1.4564	1.4342	1.721	1.791

Fig. 2 Influence of Φ on FOS_{ro} for various k_h computed for $L/H = 0.8$, $H = 9$ m, $\delta/\Phi = 1$, $k_h = 0.2$, $k_v = 0.5 k_h$, $c = 10$ kPa, $f_y = 4.15 \times 105$ kPa, $q/\gamma H = 0.3$, $i_s = 75^\circ$ **Fig. 3** Influence of c on FOS_{ro} for various k_h computed for $L/H = 0.8$, $H = 9$ m, $\Phi = 30^\circ$, $\delta/\Phi = 1$, $k_h = 0.2$, $k_v = 0.5 k_h$, $f_y = 4.15 \times 105$ kPa, $q/\gamma H = 0.3$, $i_s = 75^\circ$

where

$$T_{ci} = \frac{4M_p}{s_h l_{si}} \left[1 - \frac{T_i}{T_p} \right] \quad (46)$$

$$L_{ci} = r_o \exp(\theta \tan \phi) \times \sin(\theta_2 + \theta) \cos \theta_2 \quad (47)$$

Result

The objective is to locate the critical log-spiral failure surface to evaluate the minimum of factor of safety at equilibrium. MATLAB 2010 has been used to minimize the factor of safety (FOS_{ro}) subjected to bound

constraints such as $0^\circ < \theta_1 < 90^\circ$, $0^\circ < \theta_2 < 90^\circ$ and. This optimization gives the angles of critical failure surface (θ_{1cri} , θ_{2cri}), critical t/T ratio. The corresponding minimum factors of safety (FOS_{ro}) are obtained and are given in tabular form. The optimization considers all the modes of failure such as slope, base or toe failure. Table 1 shows the Factor of safety for soil friction angle, (ϕ) = 30° , 35° , 40° Angle of slope, (i_s) = 75° , 90° horizontal seismic acceleration coefficient, $k_h = 0, 0.1, 0.2$ $c/\gamma H = 0$ $c/\gamma H = 0.03$, $c/\gamma H = 0.06$, surcharge coefficient, $q/\gamma H = 0, 0.3, 0.5$, $\gamma = 16.5$ kN/m³ and $N = 14, 15, 16$.

Parametric Study

Effect of Angle of Friction on Rotational Factor of Safety

Figure 2 shows the variation of factor of safety with k_h for different values of ϕ at $c = 10$ kPa, $q/\gamma H = 0.3$, $i_s = 75^\circ$, $k_v = 0.5 k_h$, $\gamma = 16.5$ kN/m³. From the plot it has been seen that the magnitude of Factor of safety appreciably increases with the increase in ϕ . For example for $k_h = 0.2$

Fig. 4 Influence of $q/\gamma H$ on FOS_{ro} for various k_h computed for, $L/H = 0.8$, $H = 9$ m, $\Phi = 30^\circ$, $\delta/\Phi = 1$, $k_h = 0.2$, $k_v = 0.5 k_h$, $f_y = 4.15 \times 105$ kPa, $c = 10$, $i_s = 75^\circ$

Table 2 Comparison for various condition when $\Phi = 30^\circ$, $L/H = 0.8$, $f_y = 415,000$ kPa, $d = 25$ mm, $\theta = 0^\circ$

k_h	c	q	N	Meenal et al. [25]	Present study
0.1	0	0	16	1.82	1.2486
0.2	0	0	16	1.48	0.9279
0.1	10	0	16	2.18	1.976
0.2	10	0	16	1.99	1.478
0.1	10	80	16	1.87	1.2134
0.2	10	80	16	1.72	1.0785

Table 3 A comparison of width of top surface of the slope up to failure surface to the height of the slope ratio (L_g/H) for $k_h = 0.1, 0.2$ and 0.25 , $k_v = 0.0$ and $\Phi = 20^\circ, 30^\circ$ and 40° , $i_s = 90^\circ$

k_h	$\Phi = 20^\circ$			$\Phi = 30^\circ$			$\Phi = 40^\circ$		
	Ling and Leshchinsky [26]	Basha and Basudhar [27]	Present study	Ling and Leshchinsky [26]	Basha and Basudhar [27]	Present study	Ling and Leshchinsky [26]	Basha and Basudhar [27]	Present study
0.1	0.875	0.879	0.888	0.650	0.653	0.667	0.550	0.544	0.557
0.2	1.125	1.130	1.213	0.800	0.808	0.813	0.625	0.630	0.639
0.25	1.450	1.420	1.470	0.900	0.905	0.909	0.730	0.733	0.737

at $\phi = 30^\circ, 35^\circ, 40^\circ$, the magnitude of factor of safety is 1.2633, 1.8599, 2.3342 respectively. Due to increase in ϕ , the resistance capacity of the backfill increases which resembles for the increase in factor of safety.

Effect of Cohesion

The present analysis shows that the cohesion of the soil appreciably increases the magnitude of factor of safety. Here, Fig. 3 represents one such variation of factor of safety with k_h for different value of c at $\phi = 30^\circ$, $k_v = 0.5 k_h$, $q/\gamma H = 0.3$, $\gamma = 16.5$ kN/m³, $i_s = 75^\circ$. In this plot it is seen that for $k_h = 0.2$, $FOS_{ro} = 0.7976$ at $c = 0$ increases to $FOS_{ro} = 1.0332$ at $c = 5$ kPa. Due to increases in c , intermolecular attraction of the soil particles increases which increases the resistances capacity of the backfill soil mass and thus increases factor of safety.

Effect of Surcharge Coefficient, $q/\gamma H$

Figure 4 shows the variation of factor of safety with k_h for different value $q/\gamma H$ at $\phi = 30^\circ$, $k_v = 0.5 k_h$, $c = 10$ kPa, $\gamma = 16.5$ kN/m³, $i_s = 75^\circ$. From the plot it is seen that FOS_{ro} decreases marginally due to increase in $q/\gamma H$. As the surcharge increases, disturbing moment increases and thus factor of safety decreases.

Limitation

1. The deformation of the soil in the active zone is sufficient to fully mobilize the shear strength of soil over the entire failure surface.
2. The normal stress on the nail surface in the horizontal direction is assumed to be k_a times the normal stress in the vertical direction, where k_a is the co-efficient of active earth pressure.
3. Mobilization of tension in the nail requires relative displacement of soil and reinforcement. Hence, in urban sites where ground movement must be avoided, technique may not feasible.

$$W_{AL_0U} = 0.5 \times AL_0 \times L_0U$$

$$= \frac{1}{4} \gamma \frac{H^2 x^2 \sin^2 \theta}{\sin^2(\theta_2 + \theta_1)} \frac{\sin(\theta_2 + \delta_1)}{\sin \delta} \sin \theta_2 \quad (51)$$

$$M_{AL_0U} = \frac{1}{12} \gamma H^3 x^3 \frac{\sin^3 \theta_1}{\sin^3(\theta_2 + \theta_1)}$$

$$\times \frac{\sin(\theta_2 + \delta_1) \sin^2 \theta_2 \cos(\theta_2 + \delta_1)}{\sin^2 \delta_1} \quad (52)$$

$$M'_{AL_0U} = \frac{1}{12} \gamma H^3 x^3 \frac{\sin^3 \theta_1}{\sin(\theta_2 + \theta_1)} \frac{\sin^2(\theta_2 + \delta_1) \sin^2 \theta_2}{\sin^2 \delta_1} \quad (53)$$

3. Weight and moment of QL_0G portion (Fig. 1)

$$W_{QL_0G} = 0.5 \gamma H^2 \cot(\theta + \theta_2) \quad (54)$$

$$M_{QL_0G} = 0.5 \gamma H^3 \cot(\theta + \theta_2)$$

$$\times \left[x \cos \theta_2 - y - \frac{\cot(\theta_2 + \theta)}{3} \right] \quad (55)$$

$$M'_{QL_0G} = 0.5 \gamma H^3 \cot(\theta + \theta_2) \left[x \sin \theta_2 + \frac{1}{3} \right] \quad (56)$$

4. Weight of QIG portion (Fig. 1)

$$W_{QIG} = 0.5 \gamma H^2 \cot \theta \quad (57)$$

$$M_{QIG} = 0.5 \gamma H^3 \cot \theta \left[x \cos \theta - y - \frac{2 \tan \theta}{3} \right] \quad (58)$$

$$M'_{QIG} = 0.5 \gamma H^3 \cot \theta \left[x \sin \theta + \frac{1}{3} \right] \quad (59)$$

Notations

A_s	Area of nail (mm ²)
C	Cohesion acting along the surface of sliding wedge (kPa)
C'	Constant and considered as $c' = 4$ for the plastic analysis (Dimensionless)
D	Diameter of driven nail (mm)
f_y	Yield strength of steel nail (kPa)
i	Number of nail varies from $i = 1$ to N (Dimensionless)
L_{ei}	Length of the i th nail behind the failure wedge (m)
M_T	Moment of nail axial force T_i of the i th nail about the point 'o' (kN m)
M_{TC}	Moment of the mobilised shear force T_{ci} acting normal to the axis of nail (kN m)
M_p	Fully Plastic bending Moment of nail (kN m)
T_{ci}	Mobilised shear in the i th nail which acts normal to the axis of nail at its intersection with slip surface (kN m)
T_p	Fully plastic axial force (kN)
ϕ	Angle of friction of soil (°)

r_o	Initial radius of log-spiral failure surface (m)
r_f	Final radius of log-spiral failure surface (m)
θ_1	Log spiral angle (°)
M_d	Disturbing moment (kN m)
M_R	Resisting moment (kN m)
k_h	Horizontal ground acceleration coefficient (Dimension less)
k_v	Vertical ground acceleration coefficient (Dimension less)
K_r	Rayleigh wave number (Dimension less)
ξ	Ratio of primary wave and shear wave velocity (Dimension less)
Q_h	Seismic inertia force in horizontal direction (kN)
Q_v	Seismic inertia force in vertical direction (kN)
q	Surcharge intensity (kN/m)
H	Height of slope (m)

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