A Model to Predict Strain Rate Dependency of Dry Silica Sand in Triaxial Compression

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Received: 20 September 2015 / Accepted: 28 October 2015 / Published online: 11 November 2015
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Abstract A new method is proposed and evaluated to model the effect of strain rate on sand during triaxial compression. Triaxial compression tests were conducted on dry silica sand with rates of 0.1–10 %/s at 100–400 kPa confining pressures. Rate effects become significant at the higher confining pressures. It is found that a modified strain–energy–density approach provides a useful means to interpolate and extrapolate constant strain rate triaxial data to other strain rates.

Keywords Strain energy density · Strain rate · Sand · Granular · Soft matter · Triaxial compression · Shear strength

Introduction

The mechanical behavior of granular media subjected to high strain rate (HSR) loading has been of great interest for many civilian and military engineering projects. The applications of HSR loading can be related to blasting [1], earthquakes [2], mine blasts [3], aircraft wheel loading [4], dynamic compaction [5]; in situ subsurface explorations [6–8], pile driving and rapid load testing of piles [9], and projectile penetration [10, 11]. These applications provoke transient loading characterized by large strains in the soil over very short periods of time.

Ideally predictions of constitutive response are provided by laboratory data obtained at the strain rates of interest.

However, although individual laboratory tests are normally conducted at a specific predetermined strain rate, event strain rates are seldom constant. Moreover, experimental characterization of the behavior of soils at strain rates higher than the quasi-static is difficult when conventional test setups are used. Acquiring high quality results in highly dynamic testing is challenging for many reasons, including equipment capabilities, uncertain boundary conditions, and complexities associated with non-uniform sample deformations. As a result, simple models that can be used for interpolation or extrapolation of individual test data are highly desirable.

A previous work by Suescun-Florez et al. [12] showed that the strain energy density model (SED) was capable of describing the rate-dependency of granular materials under uniaxial compression loading. The starting point of this theory is based on an experiential argument that stresses are proportional to a power of the ratio of strain rates, at points having the same strain energy density. The power is a material property. In addition to our application to granular materials, this method has been successfully utilized to predict creep of polymers such as high-density polyethylene (HDPE) [13, 14], low density polyethylene (LDPE) [15], pavements [16], and cracking evolution in concrete [17].

The stress–strain behavior of sand is typically nonlinear, irreversible and time-dependent. Strain rates are usefully divided into regimes, which can be loosely defined as creep, quasi-static, intermediate, and HSR. The HSR regime itself is typically divided into several zones depending on the strain rate of interest. This paper focuses on intermediate to low-HSR strain rate zones, namely 0.01–10 %/s.

The main goal of this paper is to develop an empirical method to predict the stress–strain response of granular media under triaxial compression when different strain rates are imposed. The method has been calibrated on silica

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sand loaded at strain rates varying from 0.01 to 10 %/s and subjected to different confining pressures in the range of 100–400 kPa.

Triaxial Compression Testing in Soils

Conventional triaxial compression is a testing condition, in which the soil is subjected to stresses along its three axes in order to describe shear strength and stress–strain characteristics of soils. These characteristics include: (a) soil stiffness, (b) yield, peak, and ultimate stresses, (c) volumetric changes, and (d) pore water pressure variations when undrained tests are conducted. Specimens are cylindrical in shape, and typically maintain a height-to-diameter ratio varying between 2 and 2.5. The specimen stands, along its vertical axis, on a pedestal containing a porous disc. The specimen is covered on top by another porous stone, and a stiff impermeable disc (Fig. 1a). The porous disks are connected to valves that permit controlling drainage into and out of the specimen. A flexible latex membrane, made from non-porous latex rubber, with negligible thickness wraps and seals the specimen from top to bottom. The test chamber contains a pressurized liquid that provides lateral confining pressure to the specimen.

In a triaxial compression test, a constant confining pressure is applied and maintained throughout the entire test duration. While the confining pressure remains constant, the vertical stress is increased until the ultimate stress state, also known as *critical state*, is reached (Fig. 1b). The critical state is identified when the soil experiences continuous shear strain at a sustained shear resistance while volume change remains constant [18]. Normal and shear stresses can also be plotted to illustrate stress states according to the Mohr–Coulomb failure criteria (Fig. 1c). Shear resistance in sand, prior to reaching the critical state, is governed by several variables including: confining pressure, initial density or void ratio, particle composition, and strain rate. Deformation at the critical state primarily occurs along a slip plane (Fig. 2). *Consolidated–drained triaxial (CD)* tests were performed where the specimen is first consolidated and the pore air pressure is allowed to dissipate. Drainage is also permitted during the shear phase of the test.

In drained tests, the shear resistance of sand under quasi-static loading occurs due to the particle's interaction. These forces are mainly driven by three mechanisms: kinetic friction, rearrangement and dilation, and particle crushing. The combination of the aforementioned mechanisms contributes to the internal friction angle (ϕ) of the sand, which is usually employed to describe the shear strength of the material. In natural soils with no cohesion, the internal friction angle is the tangent of the ratio of shear stress to normal stress on the

Fig. 1 Concept of triaxial compression test under static loading; **a** Triaxial compression setup, **b** stress–strain response compared to uniaxial test, and **c** Mohr–Coulomb failure criteria (Color figure online)

Fig. 2 Shear bands created from triaxial compression tests: **a** triaxial chamber with specimen exhibiting typical shear failure on very dense silica sand at 400 kPa of confining pressure inside the testing cell, and **b** well defined “x” shape shear band formed at $\theta = 45 + \phi/2$, where ϕ is the internal friction angle of the sand

failure plane. The ϕ angle can be graphically identified when the Mohr–Coulomb criteria is employed, and it is the slope of the failure envelope containing Mohr circles at failure stages. This is one of the most commonly used strength parameter in geotechnical engineering. Experimental values of the friction angle can be determined by post-test inspection, as shown in Fig. 2 and its caption.

Experimental Program

An experimental program was conducted to observe the effect of strain rate on the stress–strain behavior of silica sand loaded under triaxial compression. Results were then employed to calibrate a proposed SED based method to predict the stress–strain response of sand. Dry consolidated drained (via vacuum) compression tests were conducted following ASTM D7181 [19], with some variations during the test. The main variations being (1) that the tests were performed on dry rather than saturated specimens, and (2) that the specimen was confined using air rather than water. Both variations were performed to prevent compliance errors that may take place if water is used at the test rates. Triaxial tests were performed on one type of granular material as described in the following subsection.

Material Tested

A silica sand, named #1-Dry by its distributor *US Silica*, was utilized in this investigation. Particle size distribution

and aspect ratio were assessed using a particle size analyzer, where particles are fed through a hopper and dispersed using compressed air. Particle shapes are captured using a high-speed camera at a resolution of 4 Megapixels. Edges are precisely detected when the particle flow is imaged in transmission using a special objective lens transmitting light rays to the camera that are parallel to the optical axis. Further details about the dynamic image technique can be found in Yu and Hancock [20] and Nalluri et al. [21].

Examination of silica sand under the scanning electron microscope also demonstrates that particle edges are sharp with a well-defined cleavage (Fig. 3). Particles are sub-rounded to sub-angular with medium to high sphericity having an aspect ratio between 0.6 and 0.75. Aspect ratio is defined as the ratio of the smallest to the largest dimension that are orthogonal to each other. The particle sizes range from 10 to 800 μm , as shown in Fig. 4 which depicts the percentage of particles (by weight) that is smaller (i.e. passing) than the indicated size. The mass median particle size (D_{50}) was found to be 340 μm and the coefficient of uniformity was determined as 1.59. The material is poorly graded and classified as SP according to ASTM's Unified Soil Classification System (USCS). Maximum (γ_{max}) and minimum (γ_{min}) densities were determined following ASTM 4253 [22] and D4254 [23] standards, respectively. Maximum and minimum densities were found to be 1.84 and 1.43 g/cm^3 respectively, and the specific gravity was 2.66. A summary of the physical properties of this sand is shown in Table 1.

Fig. 3 SEM Images of the tested material. General views of **a** silica sand and **b** close-up view of silica sand showing identified cleavage

Fig. 4 Size and shape analysis of tested silica sand: **a** Particle size distribution; the *inset* depicts aspect ratio variation with respect to particle size. **b** Typical particle sizes and shapes at D_{50}

Specimen Preparation and Triaxial Cell

Dry triaxial specimens were prepared in a commercial triaxial mold approximately 76-mm in diameter with a 2:1 length-to-diameter ratio. Target relative density values of 80 and 55 % corresponding to dense and medium dense were chosen for sample preparation. For each density condition, a known mass was carefully poured into the mold using a spoon. To achieve the goal density, the mold was subjected to vibration and lateral tapping. A trial-and-error process permitted estimating vibration times of

Table 1 Physical properties for the tested sand

Property	Silica sand
Median diameter, D_{50} (μm)	340
Coefficient of uniformity, C_u	1.59
Coefficient of curvature, C_c	1.00
USCS classification	SP
Specific gravity, G_s	2.66
Maximum, γ_{max} (g/cm^3)	1.84
Minimum, γ_{min} (g/cm^3)	1.43
Tested, γ_{test} (g/cm^3)	1.75

15 min for 80 % of relative density, whereas tamping was the only compaction mechanism required for medium dense samples. Limited space within the mold, along with all the attachments employed for sample preparation make it impossible to continuously check density uniformity inside the specimen. Nonetheless, dimensions and final weight were carefully measured to ensure consistent densities and initial void ratios. There is no reason to believe that vibratory compaction resulted in particle segregation.

Triaxial samples were subjected to confining pressures of 100, 200, and 400 kPa. A commercial triaxial cell was employed to maintain a constant confining stress throughout the whole test time. During sample preparation, a negative vacuum of 100 kPa was applied to maintain stability of the sample. Once the external air confinement reached 100 kPa, the initial vacuum was released and positive air pressure was applied until the target confinement is reached.

Hydraulic Loading System

Specimens were loaded at different strain rates using a two-column load frame equipped with an MTS hydraulic actuator having 110kip force capacity. The hydraulic actuator is operated by an Instron digital controller. Stress, strain, and time were recorded for every test performed.

The time dependent behavior of these sands was observed when samples were loaded at four strain rates including 0.01, 0.1, 1, and 10 %/s corresponding to test durations of 2500, 250, 25, and 2.5 s respectively. Engineering stress and strain are used throughout this paper with area correction based on elastic deformations to facilitate the analysis.

Results and Discussion

Repeatability of Results

Each triaxial compression test condition was replicated three to four times to assess repeatability. Specimens were loaded at four different strain rates: 0.01, 0.1, 1, and 10 %/s. Specimens were prepared at target densities of 55 and 80 % for loose and very dense packing condition respectively. However, final measurements of weights and dimensions resulted in densities fluctuating plus or minus 5 percent of the desired value. There was very little data scatter, which can be attributed to slight differences of test densities of each sample. Therefore, results for each test condition were deemed to be sufficiently repeatable to justify the use of their average for further analysis.

Effect of Strain Rate on the Mechanical Behavior of Sand

Average deviatoric stress from each set of experimental condition was computed as a function of global strain rate (Fig. 5). Figures were grouped according to intrinsic and external parameters in order to assess the stress–strain behavior of the material. These parameters include: a) confining pressure, b) density, and c) variation of the strain rate. Based on the results, the following observations can be gleaned. At all strain rates, the maximum deviatoric stresses vary with axial strain in a conventional manner. Stress increases up to a maximum value, then falls to the critical state. At high confining pressure and high relative density, the fall from peak is relatively abrupt. This fall is associated with high density and confining pressure that promote a steeper response of the sand until the maximum deviatoric stress is reached. Eventually, local strains and high concentration of stresses among particles are concentrated in the vicinity where a shear band is formed. Once the local maximum strength between particles located on the shear band is overcome, friction becomes less influencing, and higher displacement take places at the same stress. The opposite is observed when the specimen is either loose, low confined, or both. Higher porosity permits sufficient particle rearrangement to occur, and low confining pressures are not enough to constrain the sample to

induce a stiff reaction. Thus in these cases, the maximum stress is not easily identified as a peak, although ultimate stress levels are observed.

Generally, strain rate significantly affects strength at high packing densities, but hardly at all at the lowest relative densities. Specifically, at 80 % relative density and high confining pressure, the peak strength of silica sand increases by almost 40 % when the strain rate increases by 3 orders of magnitude. At a 55 % relative density, the rate effect is less, about 25 %. The reduction of confining pressure drastically reduces or eliminates rate effect on the stress–strain behavior of sand.

The effect of strain rate on secant modulus is much more pronounced at higher confining pressure (Fig. 6). The secant modulus was calculated taking measurements of stress and strain levels at the maximum deviatoric stress of each curve. From Fig. 6 it can be seen that strain rate has a consistent effect on the secant modulus of the material. Data from the figure are the pair values of the secant modulus and the strain rate at which the tests were carried out. Three different markers, triangles, squares, and circles, correspond to tests confined at 100, 200, and 400 kPa respectively. The secant modulus of silica sand remains constant when the confinement is very low (100 kPa) regardless of its initial density. At the highest confining pressure, it increases about 10 MPa per log cycle in strain rate.

Thus the behavior of the secant modulus is governed more by the confining pressure rather than initial packing. In fact, there is not a distinct trend when results of dense and medium dense samples are combined. Low values of confining pressure result in similar stiffness. When the confining pressure is raised up to 200 kPa, the stress–strain curves become steeper, and they become much steeper when the confining pressure reaches 400 kPa.

Modeling the Effect of Strain Rate on the Behavior of Sand

The mechanical behavior of granular media exposed to quasi-static and HSR loading was recently reviewed [24]. Shear strength of sand under drained quasi-static loading is a result of inter-particle forces. This mechanical response is governed mainly by three mechanisms: (a) sliding friction, (b) rearrangement, and (c) crushing of the particles, and they are described in the following:

- The initial portion of the stress–strain behavior represents an elastic compression due to the stiffness of the soil grains.
- Once the elastic limit has been overcome, deformation becomes irreversible, *i.e.* plastic. In this zone, applied shear stress causes slippage and rearrangement of

Fig. 5 Average stress–strain curves of dry silica sand loaded at 0.01, 0.1, 1, and 10 %/s and under the following testing condition: **a** Dr: 80 % and σ_3 : 100 kPa; **b** Dr: 55 % and σ_3 : 100 kPa; **c** Dr: 80 % and σ_3 : 200 kPa; **d** Dr: 55 % and σ_3 : 400 kPa; **e** Dr: 80 % and σ_3 : 400 kPa; and **f** Dr: 55 % and σ_3 : 400 kPa. Dr relative density, σ_3 confining pressure)

particles. Particle rearrangement can give rise to volume changes (*i.e.* expansion and/or contraction) of the specimen. As a result, a new packing condition evolves, at least locally. In triaxial compression, particle rearrangement is clearly manifested in the bulging of the test specimen (Fig. 2).

- When the load is further increased and/or sustained, particle crushing may take place. Particle crushing phenomena and its level depend on several factors such as confining pressure, initial soil density, and mineralogical composition of the grains. If particle crushing takes place, a new particle size distribution can be expected, which affects the response.

- As particles crush, strain softening can occur on a local level, which gives rise to strain localization in the form of shear bands.

When the rate of loading increases, shorter time scales begin to inhibit particle rearrangement, which can augment the relative role of particle crushing. Strain-rate hardening may arise because all strain mechanisms except grain elasticity require movement of particles, which may be truncated by insufficient load duration. This is a form of inertial resistance, because particle inertia prevents the movement that would take place at lower loading rates. Response to HSR loading of triaxial samples is also

Fig. 6 Effect of strain rate and confining pressure on the secant modulus of elasticity of silica sand. Secant moduli were measured at maximum deviatoric stresses

resisted by axial and radial inertial forces as the specimen accelerates along its boundaries. An interesting study of inertial effects due to boundary conditions on mechanical response of sand can be found in Schimming et al. [25]. They employed different manners to apply normal stresses: via dead weights and pneumatic system. Significant differences on the stress–strain response of sand were reported. However, these types of inertial forces are not intrinsic soil properties, but rather are test dependent, and should therefore be minimized. A discussion of inertial effects and other experimental issues in HSR testing is found in Suescun-Florez et al. [26].

Limited data from previous studies are available for sand at intermediate strain rates in the range of 0.1 to 100 %/s [27, 28]. Perhaps this is the range of strain rates where inertial forces begin to be important in the analysis. In any case, previous triaxial tests suggest an increase in shear strength between 10 and 20 % compared to static levels. This increase can be compared to a range of 25 up to 40 % herein observed (Fig. 5). This range difference can be attributed to several intrinsic parameters and different boundary conditions applied. Among the intrinsic properties of the sand that can affect this increase one can list (a) the mineralogical composition, (b) particle size and (c) particle shape. From external variables (d) the loading and confining system [29], (e) the sample size, and (f) sample's aspect ratio [30]. Nonetheless, these results are found to be consistent with our findings.

The rate-dependent behavior of a material can be studied based on the observed loading rate effects, which include: (a) Stress–strain curves at various constant strain rates; (b) transient responses of stress to step and gradual changes in the strain rate; (c) creep deformation; and (d) stress relaxation. The current work is mainly devoted to

the first of these approaches. The goal is to obtain realistic solutions for time-dependent engineering problems, in which the use of constitutive models accounting for time dependency of the stress–strain–strength properties of soils is crucial.

Liingaard et al. [31] classified existing rate dependent models in three main groups as follows:

- *Empirical models* mainly obtained by fitting experimental results from creep, stress relaxation, and constant rate of strain tests, and the constitutive relations are generally given by closed-form solutions or differential equations.
- *Rheological models* describe uniaxial conditions and are given in a differential form.
- *General stress–strain–time models* characterized for being, in principle, three-dimensional models, and often given in incremental form. Therefore, they can be adapted to numerical implementations. From this group, several classic formulations include (a) the Perzyna's theory [32], the most well-known formulation, where viscous behavior is modeled with a time-rate flow rule, and (b) the Duvant-Lions theory [33, 34], in which the viscous behavior is fabricated based on the difference between solutions for non-viscous and viscoplastic formulations.

Other constitutive models related to rate dependency have been developed based on the aforementioned formulations including the three-phase equation-of-state (EOS) described by Wang et al. [35], Laine and Sandvik [36] and Tong and Tuan [37]. EOS soil models take into account the different compressibilities of the solid, water and air phases of soil.

The proposed model falls within the empirical model category, and it represents a handy and useful tool for engineering practice. The constitutive relations are generally given by closed-form solutions, and the method is strictly limited to specific boundary and loading conditions. The model assumes strain energy density equivalency between a given stress–strain test and another one conducted at a higher strain rate. The equivalence is assumed to incorporate the effects of visco-elasticity and viscoplasticity through the use of an exponential relationship defined in the next section.

Method Based on the Equivalent Strain Energy Density (SED) for Linear Materials

The strain energy can be defined as the energy stored in a body subjected to deformation, and the strain energy per unit volume is called the strain energy density. The strain energy density can be identified as the area underneath the stress–strain curve up to the point of deformation of interest.

The concept of the strain energy density theory has been applied to study the instability of materials [38]. The theory is based on the hypothesis that the SED is a state variable that controls constitutive response.

In general, the SED, W , of a material is defined as the strain energy, U , per volume, V . It is equal to the area under the stress–strain diagram of a material, measured from $\varepsilon_x = 0$ to $\varepsilon_x = \varepsilon_1$ as follows

$$W = \frac{U}{V} = \int_0^{\varepsilon_1} \sigma_x * d\varepsilon_x \quad (1)$$

where σ_x and ε_x are the stress and strain components at a given point.

The SED concept hypothesizes that the stress is proportional to strain with a modulus E that depends of strain rate to an exponential power, m . For two loading paths, 1 and 2, which are conducted at two different constant strain rates but have the same SED,

$$\left(\frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1}\right)^m = \frac{\sigma_2}{\sigma_1} = \frac{\varepsilon_1}{\varepsilon_2} \quad (2)$$

where subscripts 1 and 2 are any two states, representing tests conducted at different strain rates as visualized in Fig. 7. Thus any point on a stress–strain curve has a corresponding point on a different stress–strain curve conducted at a different strain rate so that the two points have the same energy density and satisfy Eq. (2) [39]. It can be observed from the Eq. (2) that small values of m imply a low time-dependent behavior.

When a particular material exhibits this behavior; in other words, the stress–strain relation is linear for a given strain rate, the SED hypothesis can be graphically understood as shown in Fig. 7. By equaling areas A_1 and A_2 the following relationship can be expressed as:

$$\frac{E_{r1} \times \varepsilon_{r1}^2}{2} = \frac{E_{r2} \times \varepsilon_{r2}^2}{2} \quad (3)$$

Fig. 7 Linear stress–strain behavior of visco-elastic materials with equal strain energy density ($A_1 = A_2$; $\dot{\varepsilon}_1 > \dot{\varepsilon}_2$)

$$\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 = \frac{E_{r1}}{E_{r2}} \quad (4)$$

$$\frac{\varepsilon_2}{\varepsilon_1} = \sqrt{\frac{E_{r1}}{E_{r2}}} \quad (5)$$

The availability of two stress–strain tests on the same material at different constant strain rates allows the prediction of a third stress–strain curve at any other strain rate. The first two tests are used to define m . Once m is known, the third stress strain curve can be obtained by using one of the other two stress strain tests as a reference. Rearranging Eqs. (2) and (5), the m value can be found as,

$$\left(\frac{\dot{\varepsilon}_1}{\dot{\varepsilon}_2}\right)^m = \sqrt{\frac{E_{r1}}{E_{r2}}} \cdot m = \frac{\log(E_{r1}/E_{r2})}{2 \log(\dot{\varepsilon}_1/\dot{\varepsilon}_2)} \quad (6)$$

Assuming that E changes with the rate of loading, but remains a constant for any particular strain rate, the term m would also be a constant number that no longer depends on the strain energy density. The value of m converges to a constant value of 0.048 and 0.027, for virgin and recycled HDPE, respectively, when tests with at least a difference of 2 or more orders of magnitude in their strain rates are used [40, 41].

SED Applied to Modeling Stress–Strain Behavior of Granular Media with Non-linear Behavior

A non-linear model inspired by the SED concept is proposed in this paper to predict the stress–strain response of dry silica sand. Unlike the viscoelastic linear behavior exhibited by HDPE, the mechanical response of sand is considerably nonlinear, and time and material composition dependent. Yielding stresses occur gradually with a steady transition from linear to nonlinear response under triaxial compression as shown in Figs. 5 and 8. Granular materials exhibit rate-dependent and strain-dependent moduli even at small loading ranges, which makes it difficult to identify where yielding starts. However, viscoelasticity and rate-dependency are the fundamental assumption employed in SED. For this reason, the conventional strain energy density method for viscoelastic materials, which employs a linear relationship and therefore a constant value of m , is unsuitable for characterizing rate effects in sand.

In the conventional SED method, a linear fit is applied to the strain–stress curves to find the modulus of elasticity and calculate the strain energy density as the area of a triangle under the stress–strain curve [40, 41]. The proposed model, on the other hand, accounts for the viscoplasticity of sand by applying a stepped visco-elastic model piece-meal, such that each portion of the stress–strain curve has a different value of m . In keeping with the fundamental SED hypothesis, the value of m changes as SED increases along

Fig. 8 Modified strain energy density—based concept for prediction of stress–strain in triaxial compression

a loading curve. As a result, values of m are incremental and may depend on confining pressure and the strain level at any specific point of analysis (Fig. 8). Thus a spline function was employed, instead of the linear fit, to represent the stress–strain response. A spline is a numerical function that is piecewise-defined by polynomial functions, which possess a sufficiently high degree of smoothness at the places where the polynomial pieces connect. The proposed approach is aimed at accounting for both viscoelastic and viscoplastic components of the behavior of sand.

The following steps describe the use of the Modified SED method in the granular soils at low to intermediate strain rates:

Log (m) Calculation for Different Reference Strain Rate Pairs

An initial attempt to calculate m as a rate dependent parameter for silica based sand was performed by modifying Eq. (6), when ε_1 and ε_2 are equal, as follows

$$\text{Log}(m) = \text{Log} \left(\frac{\log \left(\frac{\sigma_1}{\sigma_2} \right)}{\log \left(\frac{\dot{\varepsilon}_{r1}}{\dot{\varepsilon}_{r2}} \right)} \right) \quad (7)$$

The Log of m , computed from Eq. 7, allows smoothing and linearizing the relationship between m and its corresponding strains. In this equation, m is a power law exponent expressing how the secant modulus varies with strain rate; i is strain energy density at any desired strain; $\dot{\varepsilon}_{r1}$ and $\dot{\varepsilon}_{r2}$ are the strain rates for the two initial reference tests (1 and 2); and σ_1 and σ_2 are the axial stress values corresponding to the reference stress–strain curves 1 and 2 respectively. Since four different strain rates were used here in (0.01, 0.1, 1, and 10 %/s), the number of possible combinations to calculate the value of m is six. Thus, six different Log m – strain results were calculated following Eq. 7 in order to explore the repeatability of this trend, and examine the sensitivity of Log m to strain. Each confining pressure state requires finding its values of Log m – strain.

Fig. 9 Relationship of Log m and axial strain for silica sand **a** $\sigma_3 = 100$ kPa, **b** $\sigma_3 = 200$ kPa, and **c** $\sigma_3 = 400$ kPa. Best linear fit is shown as a dashed red line (Color figure online)

Finding the m Function

The relationship between Log m and strain is shown in Fig. 9 for 100, 200, and 400 kPa of confining pressure. The best linear fit is calculated based on the m -values obtained from all six pairs of strain rate combinations. It can be seen that the data scatter becomes greater as the confining pressure is less (Fig. 9a). This scatter effect is the result of double logarithmic of fractions that are extremely small or negative considering there is a negligible variation of the stress strain curves at low confinement. It can also be seen that data scatter is much less when the confining value is

Table 2 Best fit power regression constants for m values in silica sand

Confining pressure (kPa)	a	b
100	2.5413	−2.3421
200	−2.4707	−1.3065
400	−1.7554	−1.1312

400 kPa (Fig. 9c). It is also observed that specimens confined at at 400 kPa are more sensitive to strain rate variations. The developed theory is most applicable for cases of high strain sensitivity; in this case it applies best to dense sand confined at 400 kPa.

Although Eq. 7 is not explicitly a function of strain, each reference and predicted stress values correspond to its strain pair. As a result, the best linear fit from Fig. 9, in all three confining stresses condition, can be expressed as follows:

$$\text{Log } m(i) = a(\varepsilon) + b \quad (8)$$

where a and b can be identified as material and confinement constants (Table 2). The variation of a and b could also be attributed to other intrinsic variables of the material such as particle size distribution, angularity and roundness, particle flaws, among others. However, the practical current approach assumes a general variation that is only material and confining pressure dependent. Thus, m becomes a power function that depends on the axial strain and on two constants shown as follows,

$$m = 10^{(a\varepsilon+b)} \quad (9)$$

As a result a relationship between m and axial strain is established so m is no longer constant as previously employed for materials with linear behavior. It may be possible to relate a and b to confining pressure; but further testing is required for calibration at higher confining pressures, especially since the low confining pressures did not exhibit sensitivity to strain rates.

Predicted Stresses at Higher Strain Rates

By rearranging Eqs. (2) and (9) the predicted stress σ_p at any desired strain rate can be calculated as the following

$$\sigma_p = \sigma_1 \left(\frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1} \right)^{10^{(a\varepsilon+b)}} \quad (10)$$

Prediction of Experimental Observations Using SED

Predictions of stress–strain response of silica sand loaded at different strain rates can be done as follows:

- (1) For each test condition (density, and confinement), it is suggested that at least three triaxial compression tests are performed in order to obtain their mathematical average. Averaging smooths out variations due to minor density changes during test setup.
- (2) A different battery of tests maintaining the same condition but conducted at a different strain rate needs to be performed. A constant confining pressure and target density should be used across various strain rates.
- (3) The two average stress–strain curves of different strain rates are used to define the double Logarithmic function of m using Eq. (7). Equation 7 is applied—repeatedly for various strain levels in order to define m as a function of strain.
- (4) A best fit for the Log m versus strain is obtained, in which fixed parameters a and b are found.
- (5) The predicted stress for the imaginary test at a higher strain rate (σ_p) is obtained from the reference stress strain test conducted at a slower strain rate using Eq. (10), and substituting the values of a and b as well the strain rates for each point on the reference test.
- (6) An example of finding predicted stresses at 100 %/s of strain rate, based on two references stress–strain tests at 1 and 10 %/s under 400 kPa of confinement, is presented in Table 3.

In order to assess the model's capabilities, a set of stress–strain predictions up to 10 %/s was computed. The m function was found based on the average values of Fig. 5. m values were computed for the entire strain range of 0–20 %. Following the step-by-step process, the Log of m was calculated as a function of the strain (Fig. 9). It is clear that Log of m converges to a linear trend. However, the trend appears more linear as the confining pressure increases, and the scattering seen in Fig. 9a, b, and c is reduced when the confining pressure is higher. In addition, the slope of the linear trend of the 100 kPa of confinement is reversed from those of the 200 and 400 kPa confinement, and can be attributed to negligible strain rate effects. In other words, at very low confinement, the stress–strain curves do not provide trends of strength gain due to strain rate increase as shown in Fig. 5a compared to Fig. 5e. The Log of m of silica sand is somewhat sensitive to the axial strain, although its values exhibit low scattering along the strain domain. As a result, the best-fit logarithmic functions can be identified using Eq. (8).

The variation of m with axial strain is illustrated in Fig. 10, with the error bars representing the standard error. m exhibits a decreasing trend with increasing strain at 200–400 kPa, but the trend is smaller and reversed for the 100 kPa tests. m describes the strain rate sensitivity of the

Table 3 Representative calculation of stress–strain response at different strain rate using SED method

Material			Silica sand			
Strain rate			1 and 10 %/s (reference tests)			
			100 %/s (predicted)			
Confining pressure			400 kPa			
Strain	$\sigma_1 \dot{\epsilon}_{r1} = 1 \text{ %/s}$ (MPa)	$\sigma_2 \dot{\epsilon}_{r1} = 10 \text{ %/s}$ (MPa)	Log m (i) Eq. (7)	Log m Best fit Equation (8)	m Equation (9)	σ_p (100 %/s predicted) (MPa) Equation (10)
0.00	0.00	0.00	0.000	−1.131	0.074	0.95
0.01	793.81	1032.07	−0.943	−1.149	0.071	1012.76
0.02	1287.81	1488.69	−1.201	−1.166	0.068	1784.30
0.03	1521.31	1699.17	−1.319	−1.184	0.065	2107.35
0.04	1642.53	1816.59	−1.359	−1.201	0.063	2233.37
0.05	1700.95	1874.11	−1.376	−1.219	0.060	2268.08
0.06	1717.19	1890.04	−1.380	−1.237	0.058	2246.45
0.07	1703.48	1866.42	−1.402	−1.254	0.056	2175.83
0.08	1667.20	1811.98	−1.442	−1.272	0.054	2062.90
0.09	1616.74	1724.90	−1.551	−1.289	0.051	1917.02
0.10	1546.62	1604.60	−1.796	−1.307	0.049	1754.07
0.11	1375.66	1466.05	−1.558	−1.324	0.047	1609.76
0.12	1202.17	1412.43	−1.155	−1.342	0.046	1515.89
0.13	1181.88	1407.44	−1.120	−1.359	0.044	1444.93
0.14	1180.55	1403.95	−1.123	−1.377	0.042	1381.79
0.15	1180.57	1399.06	−1.132	−1.395	0.040	1324.51
0.16	1184.14	1393.94	−1.150	−1.412	0.039	1273.17
0.17	1187.05	1388.47	−1.167	−1.430	0.037	1227.85
0.18	1181.50	1386.71	−1.158	−1.447	0.036	1188.58
0.19	1177.67	1378.06	−1.166	−1.465	0.034	1155.36
0.20	1176.61	1377.14	−1.165	−1.482	0.033	1130.62

Fig. 10 Values of m as function of axial strain for different confining pressures, and their standard error bars

test materials and the data (Fig. 10) depicts a decreasing strain rate effect with strain in the test conditions where strain rate effects are most evident (200–400 kPa). It can be seen that m is higher as the confinement increases. The reversal of trends can be understood as that samples confined at 100 kPa have experienced significant grain movement and rearrangement during loading, which is not explicitly accounted for in the model. Ideally, the red dashed value, corresponding to 100 kPa of confinement should move horizontally across the x axis since rate effects are negligible. However this method is based on and calibrated using existing data so the flipped trends at low confinement can be observed, perhaps due to mechanisms not explicitly considered herein.

The m function for silica sand is then substituted in Eq. (10) to predict the stress, using another test as a reference test. Since the stress levels, and the exponential function of m differ from one confining stress to another, the predicted stresses are also different as expected. The

results of the actual experiments are plotted together with the stress–strain curves predicted using the SED method in Figs. 11, 12, and 13 for silica sand confined at 100, 200, and 400 kPa respectively. The shown predictions were performed with tests conducted at strains rates that are one order of magnitude less than the predicted values. The stress strain curves predicted by the SED technique agree with measured curves generally within the repeatability of experimental data. Experimental results of silica sand tested under 100 kPa of confinement depict that the strain rate does not affect the response, and the modeled results corroborate the observation (Fig. 11). A confining pressure of

Fig. 11 Experimental (dashed lines) versus predicted (solid lines) stress–strain curves for silica sand using the SED inspired method at **a** 0.1 %/s, **b** 1 %/s, and **c** 10 %/s under $\sigma_3 = 100$ kPa

Fig. 12 Experimental (dashed lines) versus predicted (solid lines) stress–strain curves for silica sand using the SED inspired method at **a** 0.1 %/s, **b** 1 %/s, and **c** 10 %/s under $\sigma_3 = 200$ kPa

200 kPa provides negligible to very low strain rate dependency, especially when low strain rates are applied (Fig. 12). Rate dependency is more prone when higher confinement is acting on the sample. Figure 13 shows that there is an accurate agreement between the experimental stress–strain curves and predictions of 0.1, 1, and 10 %/s. In fact, the model results provide better results when the confining pressure increases. Thus, the method provides a very useful engineering approach for interpolation or modest extrapolation of available data.

Fig. 13 Experimental (*dashed lines*) versus predicted (*solid lines*) stress–strain curves for silica sand using the SED inspired method at **a** 0.1 %/s, **b** 1 %/s, and **c** 10 %/s under $\sigma_3 = 400$ kPa. **d** Experimental results at HSR using a drop-weight tower

High Strain Rate Predictions

Stress–strain curves of HSR loading were also extrapolated from the available data. A strain rate of 1000 %/s was calculated using the SED inspired method, and based on a test conducted at 10 %/s strain rate. In order to corroborate the accuracy of the results, an experiment was carried out using a different loading frame. Mechanical limitations of the original hydraulic frame prevented strain rates higher than 10 %/s. A drop-weight tower was employed to deliver a transient loading in a shorter time. The device was used to drop weights of about 13.5 kg at heights about 0.10 m in order to achieve strain rates about 1000 %/s. The target specimen was prepared and loaded in the same triaxial cell. Thus, the sample had similar dimensions of previous tests. A similar relative density of about 85 % was also employed.

The results of HSR (1000 %/s) experimental versus predicted curves are plotted in Fig. 13d. It can be seen that at HSR the two curves begin to deviate after peak load. These discrepancies are believed to be due to changes in deformation processes that occur at 1000 %/s that were less significant at lower rates: (1) impact loading, causing some bouncing and load reversals, and (2) higher transient stresses caused by the impact probably resulting in particle crushing, which introduces new mechanisms not accounted for by the SED. Nevertheless, the measured and predicted responses are reasonably close, particularly for engineering purposes.

The additional HSR test allowed several observations as follows: first, the current method provides a very useful engineering approach for interpolation or modest extrapolation of available data. Second, this approach was more accurate when loading did not introduce new phenomena

that are not captured by the experimental data collected at lower strain rates.

Verification of Strain Rate Pairs Calculation

The availability of having results from 4 strain rates allows for investigating the effect of the difference in strain rates on the predicted stress–strain behavior. This provides 6 pairs of tests to compute m at each test condition. The variation of the stress–strain response with the order of

magnitude for strain rate pairs is illustrated in Fig. 14. Each plot, corresponding to strain rates of 0.1, 1, and 10 %/s includes six thin solid black lines that are equivalent to strain rate pairs. The solid lines shows good consistency, since all of them are close to each other and to the experimental results.

The results with more distance from the dashed experimental curve correspond to strain rate pairs differing by one to two orders of magnitude, whilst pairs differing by two to three orders of magnitude better approximate the actual data. These observations are also found in Suescun-Florez et al. [12] for granular media loaded in uniaxial compression. In addition, these results are consistent with Bozorg-Haddad et al. [40, 41] for virgin and recycled plastics respectively.

Conclusions

A series of triaxial compression tests was conducted to evaluate the dependency of the mechanical behavior of dry silica sand on strain rate. Laboratory test results show distinct strain rate effects for silica sand, depending on the confining pressures. Samples confined at 100 kPa did not exhibit strain rate dependency, while, samples confined at 400 kPa exhibited increase in strength with increasing rate of loading. The response varies depending on several factors that have not been fully explored in this study, but include density, with denser samples exhibiting higher strain rate effects.

An empirical approach, based on the equivalence of strain energy density (SED) between tests conducted at various strain rates was developed. The model predicts stress–strain response of sand at higher strain loading rates, using a reference stress–strain test conducted at a lower strain rate. The effects of both visco-elastic and visco-plastic mechanisms were lumped together by curve fitting a function m , which is material, confining pressure, and strain dependent. Comparison between conventional triaxial compression tests conducted at strain rates in the range of 0.1–1000 %/s, and prediction performed using the developed SED approach indicates that SED is an appropriate methodology for extrapolating stress strain behavior to scenarios involving higher strain rates. Thus, the proposed methodology provides a powerful approach for estimating higher strain rates, using test conducted at slower strain rates.

Acknowledgments The authors gratefully acknowledge the support of the Defense Threat Reduction Agency Grant No: HDTRA1-10-1-0049 and The United States National Science Foundation Grant No: DGE 0741714.

Fig. 14 Effect of variation of strain rate pairs in the analysis. Experimental data (*dashed lines*) versus predicted data (*solid lines*)

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