Preface for the special feature

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This interesting collection of papers shows how certain analytical methods can help us to understand some important problems in economics. The first part concerns the problem of social choice, to which Donald Saari and Amendola and his co-authors contribute. The second is more unusual in that it focuses on tropical geometry in economics, a topic with which few economists are familiar. An introduction to this topic is provided by Feichtner and it is followed by its application to international trade by Shiozawa and one on Perron rankings by Tran. The last part shows how these techniques can be applied to ranking and to production theory.

To Saari and his co-author Simon, we already owe a very important result which underlines the propositions of Sonnenschein, Mantel, and Debreu, who showed that with our standard assumptions on individual preferences it is impossible to show that the equilibria of an exchange economy are stable. Saari and Simon showed that, worse, even if we could find an adjustment process which would lead an economy from any out-of-equilibrium prices to an equilibrium it would require an infinite amount of information. In his contribution here, Saari shows that there is a common difficulty underlying these and many other issues in collective choice. At the heart of this is the problem posed by reductionism. By reducing a problem to its components and then solving each component individually, one runs into insurmountable difficulties. What is surprising, and came as a surprise to Saari himself, is that the reductionist dilemma underlies a whole class of collective choice problems. His illustration of this with a number of examples is extremely insightful.

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Amendola and al.'s contribution seems paradoxically to take an almost orthogonal point of view. They look at choice over a multidimensional space of possibilities, where importantly, the characteristics are interrelated. The idea is that one can reduce the complexity of the problem by looking at subsets or "objects" within the space which group together some of the individual possibilities. If one requires a less stringent criterion for an optimum, for example their u-local optima and local optima, these are much more likely to exist than a generalised Condorcet winner. However, there is a problem, which is that there may be many such optima and thus a central authority might be able to manipulate the result. They also mention the possibility which they intend to investigate in future work, that the individuals whose preferences are involved could also try to manipulate the outcome by misrepresenting their preferences. The merit of the approach proposed in this paper is that it cleverly combines an algebraic and geometric approach and shows how the use of both can make the problem more tractable. Its focus on the probability of finding a solution rather than on the possibility or impossibility recalls an old literature in which the probability of finding an intransitive order in Arrow's problem was discussed.

The second part of this issue deals with tropical geometry, a recent development in mathematics on which I am poorly qualified to comment. Nevertheless, despite its exotic name it might be thought of as a piecewise linear approach to algebraic geometry and seems to have found a number of useful applications in optimisation problems. In economics Klemperer has used it to design auctions and, in particular, auctions for the Bank of England at the beginning of the current crisis. Here, Shiozawa reviews Ricardian trade theory using these mathematical techniques. The geometry of this particularly simple model, where labour is the only input into commodities and it enters in a linear way, has been explored thoroughly in the past by McKenzie, Jones, and Kuhn. Many of the details of Shiozawa's presentation will have a familiar feel for those who have read Kuhn's notes on international trade theory. However, Shiozawa focuses particularly on the most interesting case, in which the number of commodities is greater than the number of goods. This produces a certain number of questions as to the efficient allocation of world resources. Shiozawa also re-examines an old and now overlooked theorem of McKenzie which, with the contribution of Minabe, can be corrected. It is fascinating that the obsession with geometry in international trade, which reached its height with the work of Meade, should now be reappearing on the scene, and those of us who were brought up on such results as Sperner's lemma can be forgiven for feeling a little nostalgia.

In the article by Tran, she discusses problems closely related to those raised by Saari and, in particular, the difficulties that arise when making pairwise comparisons over set alternatives. She discusses three cases for the latter: Perron Rank, Tropical Rank, and Hodge Rank. It is well known that these different criteria can lead to quite different outcomes for pairwise comparisons even though they may be related. She points out that an individual or group of individuals with cardinal preferences over alternatives can generate an n by n matrix where the ij th component indicates the preference for i over j . However, the problem then arises as to how to generate a score vector which will reflect the preferences over the items. The first theorem

given by Tran is discouraging and suggests that if one has two different rank score vectors then there will always be a pairwise comparison matrix and two of the three approaches that she considers, such that one will generate the first rank score vector and the other will generate the second. Nevertheless, she indicates that there are mathematical relations between the three and this might suggest some hope in situations with large numbers of alternatives.

The final article, by Dosi et al., also deals with the aggregation problem and uses geometric techniques. The idea here follows on from a much-neglected article by Hildenbrand in which he showed, with an empirical example using data from Norwegian oil tankers, that the data could not have been generated by a standard production function with the standard properties. The geometric technique introduced by Hildenbrand consisted of using Zonotopes which capture the productive activity of an industry, for example. The underlying idea is based on Koopman's activity analysis. If all firms operated with the same activity or technique all the observations from the firms would lie on a line and the associated Zonotope would have zero volume. As Hildenbrand showed, this is far from being the case and the authors develop the notion of the volume of the Zonotope as a measure of the heterogeneity of the productive units. Their approach can also be used to study the evolution of heterogeneity over time and should allow us to confirm the empirical finding that heterogeneity does not diminish as one adopts a more and more microeconomic point of view.

All in all these contributions cast new light on various aspects of aggregation and the use of geometrical approaches which are rather different from the mathematical techniques which have become standard in economics. In a sense they have quite early origins, but it is good to see this revival to shed light on some important and not fully answered economic questions.