Magnetic Torque in Single Crystal Ni–Mn–Ga

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Abstract Magnetic shape memory alloys deform in an external magnetic field in two distinct ways: by axial straining—known as magnetic-field-induced strain—and by bending when exposed to torque. Here, we examine the magnetic torque that a magnetic field exerts on a long Ni–Mn–Ga rod. A single crystal specimen of Ni–Mn–Ga was constrained with respect to bending and subjected to an external magnetic field. The torque required to rotate the specimen in the field was measured as a function of the orientation of the sample with the external magnetic field, strain, and the magnitude of the external magnetic field. The torque was analyzed based on the changes in the free energy with the angle between the field and the sample. The contributions of magnetocrystalline anisotropy and shape anisotropy to the Zeeman energy determine the net torque. The torque is large when magnetocrystalline and shape anisotropies act synergistically and small when these anisotropies act antagonistically.

Keywords Magnetic shape memory · Magneto-mechanics · Twinning · Torque · Magnetic energy · Magnetic anisotropy

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Introduction

Magnetic shape memory alloys (MSMAs) change shape when exposed to a mechanical stress or a magnetic field [1]. The shape change results from a reorientation of the unit cell via motion of twin boundaries. The most widely studied magnetic shape memory alloy, monocrystalline Ni–Mn–Ga, has a monoclinic martensite structure with 10 M lattice modulation. The lattice is often approximated as tetragonal with $c < a$. The maximum strain is $1-c/a$, which is approximately 6%. At 0% strain, the material is a single domain with c parallel to the strain direction, and at 6% strain, the material is a single domain with c perpendicular to the strain direction.

Martensitic Ni–Mn–Ga has uniaxial magnetic anisotropy with the axis of easy magnetization parallel to c , the short axis of the unit cell. The easy axis of magnetization changes across a twin boundary. If an external magnetic field is not parallel to the easy axis of a domain, the magnetic field imposes a magnetostress in that domain [2]. If the magnetostress is larger than the twinning stress, the twin boundary will move. The twin domain with axis of easy magnetization most parallel to the external field grows.

The magnetic-field-induced shape change of a MSM element enables contact-free actuation without moving parts. Since the sample length determines stroke, a long element maximizes the stroke and work output of the material. Long samples tend to bend when a uniform magnetic field is applied perpendicular to the long dimension [3–5]. When bent, MSM elements may contact guide walls, which results in friction and a reduction of axial strain [3]. Although bending is a natural deformation mode of a long MSM element in a magnetic field, studies on the magnetomechanical properties of magnetic shape

memory alloys focus on magnetic-field-induced strain (MFIS) and often neglect magnetic-torque-induced bending (MTIB).

MTIB was reported in a nonuniform magnetic field [6]. The authors noted that, even in a uniform field, the magnetic and shape anisotropies would lead to a torque on the sample. MTIB was reported in a uniform field for wires with sub-millimeter diameter by Zheng et al. [4] who attributed it to the difference in Zeeman energies depending on the orientation of the sample. Kucza et al. studied bending in single crystal beams with 1 mm^2 cross-sectional area and various lengths, and found that the amount of bending depended on the beam's aspect ratio, while the axial strain did not depend on the sample length [5]. The authors attributed the bending strain to torque introduced by the Zeeman, shape anisotropy, and magnetocrystalline anisotropy energies [4, 5].

In both studies, sample bending was attributed to differences in the magnetic energy depending on the orientation of the sample in the external magnetic field. The relative contributions of the energies to torque was qualitatively discussed [5] based on the sample behavior in different magnetic field strengths. MSM alloys may be used in various shapes, sizes, and magnetic field strengths. A quantitative understanding of the contributors to this torque is necessary to understand the response of actuating elements to a variable magnetic field.

Several studies applied free energy functions to describe the driving force behind twin boundary motion [7–9] as well as the dependence of the critical field on the geometry of domains due to demagnetization [10]. The energy functions contain the contributions described above as well as external stress and magnetoelasticity. The studies quantitatively describe or analytically calculate the contribution of each energy term to the equilibrium of the twin domains, but do not consider the torque imposed on a sample by these energies.

O'Handley et al. [11] calculated the torque on an unfavorably oriented domain in an external magnetic field in order to describe the driving force for twin boundary motion. The contributions of the external field and magnetocrystalline anisotropy to the torque on the unfavorable twin domain were related to the shear stress at the twin boundary. In their theoretical approach, those authors considered torque as it relates to twin boundary motion in a specimen which is constrained on its ends such that it can deform only by straining. They did not discuss the total torque on a specimen without these constraints, such as in the experiments in Refs. 4 and 5. In addition, only one orientation of the applied field with the specimen was considered in contrast with the rotating field in Refs. 4 and 5.

This study examines experimentally the torque of a Ni–Mn–Ga specimen constrained with respect to bending

strain in a magnetic field and explores the causes for this torque. The torque as a function of angle with the magnetic field was measured and compared with the same test using a soft magnetic material and a permanent magnet as references. We discuss the contributions of the torque in terms of the external field energy, magnetocrystalline anisotropy energy, and internal field energy.

Experimental Procedure

Single crystal specimens of Ni–Mn–Ga were grown by a modified Bridgman–Stockbarger technique [12]. Specimens were grown with a nominal composition of $\text{Ni}_{50.5}\text{Mn}_{27.75}\text{Ga}_{21.75}$ (atomic percent); however, due to chemical segregation during growth, there was a compositional gradient across the crystal resulting in sections with different martensite structures at room temperature. Crystals were characterized on a Hitachi S3400 scanning electron microscope coupled with energy dispersive X-ray spectroscopy (EDS, Oxford Instruments Energy⁺ detector) and a Bruker D8 X-ray diffractometer with a Cu K_α source. Samples were cut using a Princeton Scientific WS-22 wire saw with a 50- μm -diameter tungsten wire and polished using successively smaller grit SiC paper and then diamond slurries down to 1- μm -diameter diamonds. The Ni–Mn–Ga sample tested had a 10 M structure with the composition $\text{Ni}_{51.1}\text{Mn}_{25.7}\text{Ga}_{23.2}$ (atomic percent, as determined with EDS with accuracy 0.5 atomic percent) and measured $20.02 \times 5.02 \times 3.62\text{ mm}^3$ fully extended in the longest dimension, and compressed in the intermediate dimension. The sample was subjected to constant magnetic fields between 25 and 150 milli-Tesla (mT) using a Varian Associates V3603 electromagnet. Once the samples had reached an equilibrium angle with the external magnetic field, a torque was applied as described below, and the angular deflection was measured.

A sample holder made from Teflon contained a groove to accommodate the sample (Fig. 1). The sample holder had a diameter of 25.4 mm on which the sample was laid and a shoulder with a smaller diameter of 9.53 mm which was press-fit into the inner bore of a ZrO_2 nonmagnetic bearing (Fig. 2a). The bearing was placed in Styrofoam between the pole pieces of an electromagnet (Varian Associates V3603). The angle of the sample with the magnetic field γ was measured by taking pictures using a Canon PowerShot A3000 digital camera and comparing the edge of the sample with a compass aligned in the field (Fig. 2a) using the angle tool in ImageJ. We define Cartesian coordinates such that the x direction coincides with the long edge of the sample, and the rotation axis constitutes the z direction. The angle γ describes the deviation of the magnetic field direction from the

Fig. 1 The magnetic samples (a) were placed in a round Teflon sample holder (b). The sample holder had a shoulder machined to press-fit into the inner bore of a nonmagnetic ZrO₂ ceramic bearing (c). A groove was machined at the circumference, and a string (d) was wrapped around the sample holder in this groove. The string was connected to a spring force gauge (not shown) to apply a tangential force (torque) to the sample holder and sample

x direction. When the magnetic field was parallel to the x direction, $\gamma = 0^\circ$. The angular error was less than 0.5° . The sample holder had a groove cut around the circumference where a string was wrapped around and extended to a spring force gauge (PCE Instruments) and pulley. The string transferred a displacement of the pulley to apply a tangential force to the sample holder and, thus, a torque to the sample.

Two spring force gauges from PCE Instruments were used alternatively, with maximum forces of 10 g (0.098 N) and 1 N and resolution of 0.001 N and 0.01 N, respectively. For experiments with a magnetic field of 25 mT and for tests with a magnetic field of 50 mT and strain equal to or larger than 5%, the maximum force to fully rotate the sample was less than 0.098 N. For these experiments, we

measured the torque with the 10 g force gauge. For all other experiments, we applied the 1 N force gauge.

The sample was tested with a constant twin structure throughout each experiment. The sample contained three twin domains with two twin boundaries, such that the center domain had the easy axis in the x direction. Figure 2b outlines the edges of the sample with light-colored lines, showing the kink between the domains. The approximate locations of the two twin boundaries between the domains are given by the lines running at a 45° angle to the sample edges. Twin boundaries intersect the location of the kinks on either side of the sample. The thick white lines in the domains give the directions of the c axis in each domain. The image in Fig. 2b depicts a sample at 4% strain, which would correspond to the domain with c parallel to x making up one-third of the sample. This is visually verified in Fig. 2b. The resolution of the angle measurement in ImageJ was approximated by measuring the angle between the edges of the two domains, and compared to the expected angle from lattice parameters.

We set the strain on the sample and then placed it into the sample holder with the magnetic field turned off. Then we increased the field to the set point of interest. Once at the field set point, we measured the equilibrium angle of the sample with the magnetic field direction without applying a force to the string. Then, with the sample in a constant magnetic field, we increased the force on the spring gauge manually to apply torque to the sample holder. We took pictures to measure the resulting angle between the sample and the field at discrete force values. Upon exceeding the position of maximum torque, the mechanical loading system became unstable, and the sample holder spun uncontrolled. Thus, no data were collected at angles greater than the angle of maximum torque.

Fig. 2 **a** A digital image was captured using a Canon PowerShot A3000 digital camera. A compass was placed in between the poles of the electromagnet. The North–South pole was aligned parallel to the magnetic field, and the East–West pole was perpendicular to the magnetic field. Using the angle tool in ImageJ, the angle γ between the poles of the compass and the x axis of the sample was measured.

The magnetic field direction is the reference direction, and γ increases clockwise. **b** White lines highlight the kink between adjacent domains. Lines across the sample connect the kinks on the surfaces and indicate the location of twin boundaries. Thick white lines in each domain denote the direction of the easy axis of magnetization

We performed experiments first with the fully extended sample (i.e., at 6% strain) at magnetic fields of 25, 50, 75, 100, 125, and 150 mT. Between the experiments at different fields, we turned the field to zero, checked the sample strain and rewound the string around the sample holder. We then compressed the sample to 5% strain and conducted experiments at multiple magnetic fields. This process was repeated for multiple fields at integer values of strain (0–6%). At 100 mT, twin boundaries moved for strains lower than 3%, which changed the strain. Therefore, data below 3% strain at 100 mT and fields higher than 100 mT were not measured. We assume that the repeated straining of the sample required to properly establish each strain state caused a softening of the material which resulted in reducing the switching field [13] to below 100 mT. This is known as training and often applied intentionally to reduce the twinning stress [14]. In addition, the equilibrium angle as a function of strain at 50 mT external magnetic field was found in strain increments of 0.5%.

Results

The equilibrium angle γ_{eq} (i.e., at zero torque) was measured as a function of strain at different magnetic fields (Fig. 3). The amount of strain changed the equilibrium angle. At 0% strain (i.e., when the easy direction of magnetization was parallel to the longest axis of the sample), the equilibrium angle was 0°. With the increasing strain, the equilibrium angle increased nonlinearly and reached 90° when the sample was fully extended (i.e., at 6% strain). The equilibrium angle did not depend on the field strength except at 3% strain where the value for 100 mT deviated from those at 50 and 75 mT.

Fig. 3 Equilibrium angle γ_{eq} between the sample's long axis and magnetic field as a function of strain at different magnetic fields. At 100 mT, twin boundaries shifted for strains higher than 3%, and so data for strains higher than 3% were not obtained. The arrow shows the strain for the data point at 3% strain and 100 mT may have decreased, changing the equilibrium angle. Equation 7 is plotted as a dashed line

We derived the torque from the force measurements by multiplying them by the radius of the sample holder (Fig. 1). Figure 4a shows the torque at 6% strain as a function of γ and different magnetic fields. The tests started at or near -90° as this was the equilibrium angle for 6% strain independent of external field (Fig. 3). The torque increased until the sample spun in the magnetic field to another equilibrium angle 180° from the initial starting point. At low magnetic field (particularly at 25 mT) friction impeded the smooth motion of the sample holder and caused large jumps in angle. The torque required to rotate the sample increased with increasing magnetic field such that friction was negligible at fields of 50 mT and above. For example, the maximum torque measured at 25 mT was 0.19 mNm while the maximum torque at 150 mT was 5.9 mNm, an increase by a factor of 31. The torque as a function of external field at -75° , -60° , and -45° is given in Fig. 4b. The method used to interpolate torque values at these discrete angles is given in “Discussion” section. The torque needed to rotate the sample increases nonlinearly with external field for each angle.

Figure 5 shows the torque as a function of γ at a magnetic field of 75 mT and different strains. The equilibrium angle shifted to larger values with increasing strain according to the results shown in Fig. 3. For each strain, the torque increased with the increasing angle although with regressing slope. The slope of the torque versus angle curves decreased with increasing strain such that the maximum torque at 0% strain was 10.1 mNm and decreased to 1.72 mNm at 6% strain, a decrease by a factor of 5.87.

Discussion

The total magnetic energy of a specimen is the sum of the exchange energy (E_{ex}), the magnetocrystalline anisotropy energy (E_{anis}), the energy of the magnetized sample in an external field, also known as the Zeeman energy (E_Z), and the energy of the magnetized sample in its own internal field created by the net magnetization, often referred to as the shape anisotropy energy (E_{int}). The exchange energy causes the magnetic moments to align parallel (i.e., ferromagnetically). The magnetocrystalline anisotropy energy results from the symmetry of the crystal lattice and effectively keeps the magnetization vector parallel to the easy axis of magnetization. Ni–Mn–Ga with 10 M martensite is uniaxially anisotropic and the crystallographic c direction is the direction of easy magnetization. The magnetocrystalline anisotropy energy depends on the angle between the magnetization vector and the c direction. The Zeeman energy is the dot product of the magnetization vector and external magnetic field vector. The magnetic poles of a

Fig. 4 **a** Torque as a function of angle γ between the sample's long axis and magnetic field at 6% strain for different applied magnetic fields. **b** Interpolated torque values from empirically fit equations as a function of external field at -75° , -60° , and -45°

Fig. 5 Torque as a function of angle γ between the sample's long axis and magnetic field at 75 mT applied field for different values of strain. The maximum measured torque decreased by a factor of 6 from 0 to 6% strain

magnetized specimen create an internal magnetic field, which opposes the external field and is a function of sample geometry. The total energy is given by

$$E_{\text{total}} = E_{\text{ex}} + E_{\text{anis}} + E_Z + E_{\text{int}}. \quad (1)$$

All energy terms are given as energy densities referring to the sample volume. The magnetic moments orient such as to minimize E_{total} . The total magnetic energy is a function of the angle of the external magnetic field with the sample. The negative first derivative of the total magnetic energy with respect to the angle γ between the field and sample (Fig. 2a) gives the torque on the sample due to the magnetic energy:

$$\theta = -dE_{\text{total}}/d\gamma. \quad (2)$$

The equilibrium angle of the specimen in the external magnetic field is the angle at which the torque on the specimen is zero; it is the angle at which the magnetic energy is minimal.

As stated by Liang et al. [6] and Kucza et al. [5], the torque on a specimen is governed by the Zeeman, magnetocrystalline, and shape anisotropy energies. The specimen had three domains and two twin boundaries. The axis of easy magnetization changed across each twin boundary,

and each domain had a different shape anisotropy energy. As the strain of the sample changed, so did the relative sizes of each domain. The shape anisotropy for each domain was different at each strain value resulting in the variation of the equilibrium angle shown in Fig. 3. The equilibrium angle did not depend on the field strength except at 3% strain where the value for 100 mT deviated from those at 50 and 75 mT. We assume that this deviation resulted from the small movements of twin boundaries (as described in the experimental section). Thus, the actual strain value of that sample was lower (as indicated by the arrow in Fig. 3).

The torque as a function of angle for external fields ranging from 25 to 150 mT is plotted in Fig. 4a. A sine function for each external field was empirically fit to the torque values. The values of torque for -75° , -60° , and -45° at each external field were interpolated from the fitting functions. These torque values are plotted in Fig. 4b as a function of the magnetic field. The torque magnitude has a nonlinear dependence on the external field magnitude. In order to find this dependence, a linear equation was fitted to the logarithm of torque as a function of the logarithm of external field. The slope of the relationship for each angle was 1.88 ± 0.02 . The power dependence of the torque on magnetic field is slightly less than that of the quadratic.

The maximum torque measured on the sample changed by a factor of 5.87 between single variant states. The major difference in these two cases is the shape anisotropy of the domain and resulting internal field. We can define the internal field with a demagnetization factor ("Internal Field" section). The larger the demagnetization factor, the larger the internal field. At 0% strain, the single domain has easy axis of magnetization parallel to the long dimension, which has a small demagnetization factor of about 0.093 [15]. At 6% the easy axis is parallel to the sample dimension with a larger demagnetization factor of about 0.386 [15]. The ratio of large demagnetization to small demagnetization is 4.15.

Zeeman Energy

A magnetic material in an external magnetic field (Fig. 6) has a net magnetization, $\mathbf{M}$ with magnitude M . The angle γ is the angle between the longest axis of the sample (which is also the x direction) and the magnetic field, and α is the angle between the magnetization vector and the sample's longest axis.

The Zeeman energy is the energy of a magnetized body in an external magnetic field and is given by the scalar product of the magnetization and magnetic field vectors:

$$E_Z = -\mu_0 \mathbf{M} \mathbf{H} \cos(\gamma - \alpha) \quad (3)$$

where μ_0 is the magnetic permeability of free space, and H is the external magnetic field. In general, α depends on γ . In the case of a permanent magnet, the net magnetization direction is constant (relative to the sample) independent of the field direction.

In a soft, magnetically anisotropic material, like Ni–Mn–Ga, at a low magnetic field, the magnetic anisotropy energy holds the direction of magnetization nearly parallel to the easy axis of magnetization. (Ni–Mn–Ga saturates at fields well below the external fields used in the experiment.) When saturated in the easy axis, the Zeeman energy is

$$E_Z = -\mu_0 M_s H |\cos(\gamma - \alpha)| \quad (4)$$

A soft magnet will magnetize in both directions in the easy axis. This means the Zeeman energy is always negative, hence the absolute value of the cosine function. Eq. 4 has two energy minima at $\gamma = 0^\circ$ and 180° .

Consider a Ni–Mn–Ga sample with two domains, A and B, where the easy axis of magnetization of domains A and B are parallel to the x and y directions (Fig. 7). The fraction of each domain, f_A and f_B , depends on the overall strain of the sample. Zero strain corresponds to only domain A, or $f_A = 1$ and $f_B = 1 - f_A = 0$. Full strain (6% for 10 M) corresponds to $f_B = 1$ and $f_A = 0$. For simplicity, we refer

Fig. 6 A magnetic material subjected to an external magnetic field at an angle γ to the sample edge experiences a magnetization M at an angle α to the sample edge

here to normalized strain $\epsilon = \epsilon/\epsilon_{\max}$, such that $\epsilon = f_B$. Since the orientation of the easy axis of magnetization is different in each domain, the energy must be calculated for each domain individually. The total Zeeman energy for a specimen is:

$$E_Z = -\mu_0 M_s H [f_B |\sin(\gamma - \alpha)| + f_A |\cos(\gamma - \alpha)|] \quad (5)$$

Assuming α is small, the torque due to Zeeman energy (for brevity Zeeman torque, θ_Z)

$$\theta_Z = -\frac{dE_Z}{d\gamma} = \mu_0 M_s H [\epsilon(\sin \gamma + \cos \gamma) - \sin \gamma] \quad (6)$$

For the equilibrium angle, Eq. 6 yields by setting $\theta_Z = 0$:

$$\gamma = \arctan(\epsilon/(1 - \epsilon)) \quad (7)$$

Equation 7 is plotted in Fig. 3. It accounts well for the equilibrium angle at small (i.e., compressed sample) and large (i.e., extended sample) strain but overestimates the equilibrium angle at intermediate strains.

The magnitude of the Zeeman torque depends on the amount of strain on the sample. The zero of the torque (equilibrium angle) in Eq. 7 also depends on strain. The Zeeman torque has a maximum of $\mu_0 M_s H$ when $\epsilon = 0$ or 1. For an external field of 75 mT, the maximum torque density is 36 kNm/m³.

Magnetocrystalline Anisotropy

Consider a material with uniaxial magnetic anisotropy, i.e., it has an easy axis of magnetization. Magnetizing the sample in a direction not parallel to the easy axis is unfavorable and has a positive energy. The energy density is represented by the product of the anisotropy energy constant K_u with the square of the sine of the angle α between the easy axis of magnetization and the direction of net magnetization (Fig. 6):

$$E_{\text{anis}} = K_u \sin^2 \alpha \quad (8)$$

For martensitic Ni–Mn–Ga with 10 M structure, the anisotropy energy constant K_u is 2.45×10^5 J/m³ [16].

Fig. 7 Specimen containing two domains, A and B, with easy axis of magnetization parallel to the x and y directions, respectively. As the strain is increased, the fraction of domain B increases and domain A shrinks

When a component of the magnetic field is perpendicular to the easy axis of magnetization, the magnetic moments rotate to reduce the angle to the magnetic field direction (Fig. 6). The angle α varies with the magnitude and direction of the external magnetic field.

The magnetocrystalline anisotropy keeps the magnetic moment parallel to the easy axis. In an external field, the magnetization vector has an equilibrium position which is the angle α at which the total magnetic energy is minimal. The total magnetic energy is

$$E_{\text{total}} = -\mu_0 M_s H \cos(\gamma - \alpha) + K_u \sin^2 \alpha \quad (9)$$

In order to find the equilibrium angle of the magnetization with the x axis α , we set the first derivative of Eq. 9 with respect to α equal to zero and solve for α . With this angle, we find the total magnetic energy density as a function of γ for domains A ($c \parallel x$) and B ($c \parallel y$) [17]:

$$E_{\text{MA}} = -\mu_0 M_s H \cos \gamma + \frac{1}{4K_u} (\mu_0 M_s H_a)^2 \sin^2 \gamma \quad (10A)$$

$$E_{\text{MB}} = -\mu_0 M_s H \sin \gamma + \frac{1}{4K_u} (\mu_0 M_s H)^2 \cos^2 \gamma \quad (10B)$$

We find the torque density due to both domains A and B with Eqs. 2 and 10A, B:

$$\theta_{\text{MA}} = -\mu_0 M_s H \sin \gamma - \frac{1}{4K_u} (\mu_0 M_s H)^2 \sin 2\gamma \quad (11A)$$

$$\theta_{\text{MB}} = \mu_0 M_s H \cos \gamma + \frac{1}{4K_u} (\mu_0 M_s H)^2 \sin 2\gamma \quad (11B)$$

Equation 11A, B give the torque density normalized by the sample volume for each domain. The relative size of each domain changes as the strain changes. In order to find the total torque density as a function of strain, the torque density in domain A is multiplied by the relative strain $1 - \epsilon = f_A$ and the torque density in domain B is multiplied by $\epsilon = f_B$. The sum of the two torque densities results in the total torque due to the Zeeman and magnetocrystalline anisotropy energies as a function of the relative strain and γ :

$$\theta_{\text{MT}} = (\mu_0 M_s H) [\epsilon (\sin \gamma + \cos \gamma) - \sin \gamma] + \frac{1}{4K_u} (\mu_0 M_s H)^2 \sin 2\gamma (2\epsilon - 1) \quad (12)$$

The first term in Eq. 12 is equivalent to the right side of Eq. 6. The second term is the torque density due to magnetocrystalline anisotropy energy. The terms both contain a constant multiplied by a function of γ and ϵ . For an external field of 75 mT and saturation magnetization 0.6 T, the constant in the Zeeman term has a value of 36 kNm/m³, and the magnetocrystalline term has a value of 1.3 kNm/m³. The Zeeman term is more than an order of magnitude larger than the contribution of the magnetocrystalline anisotropy energy.

The functions of strain and angle in each term have different characteristics. The angle γ at which the Zeeman torque is zero and the energy is minimized changes continuously as the strain changes, varying from 0° to 90° as the normalized strain varies from 0 to 1. The zero of the magnetocrystalline torque corresponding to the energy minimum is only either 0° or 90°. If $\epsilon < 0.5$, the equilibrium angle is 0° and if $\epsilon > 0.5$, the equilibrium angle is 90°. If the strain $\epsilon = 0.5$, the magnetocrystalline torque is zero for all angles of γ . The torque contribution of the magnetocrystalline anisotropy energy does not have the same symmetry as the Zeeman energy, but is small and does not significantly contribute to the torque at the low fields in the experiments. The equilibrium angle is almost identical to Eq. 7 and cannot be distinguished in Fig. 3 from the solution which neglects the magnetocrystalline anisotropy energy (i.e., Eq. 7). These two energies do not predict accurately the equilibrium angles for intermediate strain values.

Internal Field

The equations in “Zeeman Energy” section for the Zeeman energy assume that the magnetic field inside the material is the same as the external magnetic field. In fact, a magnetized body creates its own demagnetizing internal magnetic field in addition to the external field. The demagnetizing field (internal field) is a function of the specimen geometry and the orientation in the external field.

The effective magnetic field in the body is

$$H_{\text{eff}} = H - N_d M \quad (13)$$

where N_d is the geometric demagnetization factor, and M the average magnetization of the sample. The sum of the Zeeman and internal field energies is

$$E_{\text{net}} = -\mu_0 \left(H - \frac{1}{2} N_d M \right) \cdot M \quad (14)$$

The summation of the interaction between magnetic moments counts each interaction twice; thus, a factor of $\frac{1}{2}$ is included in the internal field energy term. The energy E_{net} refers to the total magnetic field energy. In non-spherical samples, such as a bar, N_d is different for each direction.

The shape anisotropy and internal field significantly affect the torque in the Ni–Mn–Ga specimen. Figure 5 shows that the torque required to rotate the specimen decreases with increasing strain. As the strain increases, the domain with easy axis of magnetization parallel to the short dimension of the sample increases. The geometry of the domain with easy axis parallel to the long dimension changes such that the demagnetization factor in the easy

axis increases as well. At 6% strain, the internal field is large due to large demagnetization factor, while at 0% strain, the demagnetization factor and internal field are small. The maximum torque decreased by a factor of 5.87 between 0 and 6% strain, while the relative demagnetization factor decreased by a factor of 4.15 [15].

Ni–Mn–Ga has uniaxial anisotropy along the crystallographic c direction, which causes the magnetization vectors to align in that direction. We know the direction of magnetization; however, for a twinned specimen (Fig. 7), the c direction varies throughout the sample, and the domain pattern changes with varying strains. The geometry of each domain changes with strain. The demagnetizing field is a function of the magnetization as well as the demagnetization factor; this means that N_d must be calculated for each twin domain at each strain value. In addition, the susceptibility along the easy axis is very high, and the magnetization saturates at a low field (~ 15 mT).

Let us assume a single domain with an easy axis of magnetization parallel to x (Fig. 6), saturates at an effective field of 15 mT. We represent the average magnetization as a function of the effective field with a piecewise function (Eq. 15) instead of a linear one.

$$M_{av} = \begin{cases} \chi H_{eff} & 0 \leq H_{eff} < 15 \text{ mT} \\ M_s & H_{eff} \geq 15 \text{ mT} \end{cases} \quad (15)$$

The internal field energy does not significantly contribute to α at low external fields. We analyze the external and internal field energies separately. The energies in Eq. 10A, B can be combined with the internal energy calculated from Eqs. 14 and 15 to find the total energies of domains A and B:

$$E_{TA} = -\mu_0 M_s H \cos \gamma + \frac{1}{4K_u} (\mu_0 M_s H)^2 \sin^2 \gamma + \frac{1}{2} H_d^x M_{av} \quad (16A)$$

$$E_{TB} = -\mu_0 M_s H \sin \gamma + \frac{1}{4K_u} (\mu_0 M_s H)^2 \cos^2 \gamma + \frac{1}{2} H_d^y M_{av} \quad (16B)$$

where H_d^x and H_d^y are the demagnetizing fields for the A domain in the x direction and the B domain in the y direction, respectively. The internal field energy in Eq. 16A, B) is not a continuous function of γ if the magnetization saturates. The torque due to the internal field must be calculated for specific values of strain, γ , and H . Some values for the magnetotorque are given in Table 1 and discussed in “Comparison of different energy contributions to magnetic torque” section..

Comparison of Different Energy Contributions to Magnetic Torque

Table 1 contains the torque contributions from the Zeeman energy, magnetocrystalline anisotropy energy, and shape anisotropy energy, as well as the measured torque in a Ni–Mn–Ga specimen. For relative strains of $\epsilon = 0, 0.5$, and 1, the symbolic equation for each contribution is given along with the calculated maximum value, angle of maximum torque, and the angle for which the torque vanishes. The angle refers to γ , which is the angle between the long axis of the sample and the magnetic field. The calculated values are given for an external field $H = 75$ mT, saturation magnetization $M_s = 0.6$ T, and magnetocrystalline anisotropy energy of $K_u = 2.45 \times 10^5$ J/m³. The torque values

Table 1 Torque contributions from the Zeeman energy, magnetocrystalline anisotropy energy, and shape anisotropy energy, as well as the measured torque

ϵ	Zeeman energy (kJm/m ³)			Magnetocrystalline anisotropy energy (kJm/m ³)			Shape anisotropy energy (kJm/m ³)			Experimental (kJm/m ³)		
0	$-\mu_0 M_s H_a \sin \gamma$			$\frac{-1}{4K_u} (\mu_0 M_s H_a)^2 \sin 2\gamma$			$\frac{1}{2} H_d^x M_s$			−80	57°	4°
	−36	90°	0°	−1.3	45°	0°	11	90°	0°			
0.5	$\frac{1}{2} \mu_0 M_s H_a (\cos \gamma - \sin \gamma)$			0			$\frac{1}{4} M_s (H_d^x + H_d^y)$			−23	67°	22°
	±18	0°, 90°	45°	0	none	All angles	±8	90°, 0°	45°			
1	$\mu_0 M_s H_a \cos \gamma$			$\frac{1}{4K_u} (\mu_0 M_s H_a)^2 \sin 2\gamma$			$\frac{1}{2} H_d^y M_s$			−13	133°	89°
	36	0°	90°	1.3	45°	90°	−17	0°	90°			

For relative strains of $\epsilon = 0, 0.5$, and 1, the symbolic equation for each contribution is given along with the calculated maximum value, angle of maximum torque, and angle for which the torque vanishes. The angle refers to γ , which is the angle between the long axis of the sample and the magnetic field. The calculated values are given for an external field $H_a = 75$ mT, saturation magnetization $M_s = 0.6$ T, and magnetocrystalline anisotropy energy of $K_u = 2.45 \times 10^5$ J/m³. The torque values given are normalized by the volume, and have the units kNm/m³ = kPa

given are normalized by the volume, and have the units $\text{kNm/m}^3 = \text{kPa}$.

The Zeeman energy from the external field contributes the most to the torque. At low fields such as the ones used in this experiment, the magnetocrystalline anisotropy energy is negligible. The magnitude of the shape anisotropy energy is much greater in the case for $\epsilon = 1$ because domain A has its easy axis parallel to the direction with the largest demagnetization factor. Figure 5 shows that with the increasing fraction of domain B, the torque required to rotate the specimen decreases. This is due to the larger demagnetizing field in the direction of easy magnetization in domain B.

The sum of the magnitudes of the calculated torques does not match the measured torque. The case for $\epsilon = 0$ has a measured torque larger than the combined magnitudes of the torque calculated. The sum of the calculated torques in the case where $\epsilon = 1$ results in a torque of the opposite sign of the measured torque.

The analysis used to calculate torque internal field energy is insufficient to use for a twinned sample of Ni–Mn–Ga because (i) Eq. 14 assumes a homogeneous magnetization and (ii) the domains are not rectangular. Twin domains magnetize differently due to the difference in easy axis of magnetization. In addition, material interfaces including twin boundaries, affect the magnetization. Also, no published values of demagnetization factor for the shape of the domains exist. We can apply demagnetization factors to single-domain states, at 0% and 6% strain, but not at intermediate strains. Finally, we cannot assume that neighboring domains do not influence each other [10]. Thus, a quantitative analysis of the torque of twinned specimen requires numerical, micromagnetic calculations which are beyond the scope of this study.

The sample had two twin boundaries and three domains (Fig. 2b). The twin structure in the sample was B–A–B (using the domain nomenclature in Fig. 6). The center domain (domain A) has $c \parallel x$ while the two outside domains have $c \parallel y$. We also tested the sample with an opposite twin structure, A–B–A. At an equal magnetic field of 75 mT and equal strain value of 2%, the B–A–B structure required 1.7 times more torque to rotate than the A–B–A structure. The only difference was the geometry of each domain, which resulted in different internal field magnitudes in the domains.

The demagnetization factor does not provide sufficient information to describe the internal field in the sample. The above analysis assumes saturation of the magnetization and independent stray fields in each domain. Also, a sample with the same number of domains and overall strain can experience very different torques, even with relatively simple domain structures containing only two twin boundaries. We cannot account for the internal field with

an experimentally derived parameter using these values. The evaluation of the internal field requires a micromagnetic analysis of specific twin microstructures which is beyond the scope of this study.

Conclusion

The magnetocrystalline anisotropy energy caused the magnetic moments to remain parallel to the axis of easy magnetization; the direction of magnetic moments was invariant. The Zeeman energy was a function of the angle between the sample and field, resulting in torque on the sample. While the change in Zeeman energy contributed the most to the torque, the internal field energy drastically reduced the torque at large strain values. The change in the maximum torque as shown in Fig. 5 as well as the deviation of equilibrium angle from that calculated in Eq. 7 are due to the differences in the internal field energy due to shape anisotropy.

As the strain in the sample increased, the geometries of each domain changed such that the demagnetization factor in the direction of the easy axis increased. The increase in the internal field effectively decreased the magnitude of the external field, which resulted in smaller total energy and lower torque. When the magnetocrystalline and shape anisotropies worked synergistically (i.e., the easy axis was parallel to the long domain dimension), the sample experienced a large torque. When the magnetocrystalline and shape anisotropies worked antagonistically (i.e., the easy axis was parallel to the short sample dimension), the sample experienced smaller torque.

Not only the sample geometry, but the twin-domain microstructure also changes the contribution of the internal field to the magnetization and energetics of the sample. This in turn affects the magnetomechanical properties. Sample geometry and twin-domain structure can be manipulated in order to optimize the desired response of an MSMA sample.

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