Numerical Investigation of Pulsatile Blood Flow in Stenosed Artery

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Abstract In the present study, a two-layered model of pulsatile flow of blood through a stenosed elastic artery is numerically examined. The two-fluid model consists of a core layer of a suspension of erythrocytes and peripheral plasma layer. It is assumed that the core and peripheral plasma layer behave as micropolar and Newtonian fluids respectively. The resulting system of nonlinear partial differential equations is numerically solved using the finite difference scheme by exploiting the suitably prescribed conditions. Effects of the tapering angle, wall deformation and severity of the stenosis on flow characteristics are discussed. The present results are compared with literature and found to be in good agreement.

Keywords Finite difference scheme · Blood flow · Micropolar fluid · Newtonian fluid · Stenosed artery

Introduction

Cardiovascular diseases are the main causes of death in many countries [18]. One of the most common of these diseases is atherosclerosis, which is a constriction due to the multiple plaques formation and the accumulation of fatty materials such as cholesterol and triglyceride in the blood vessel lumen, and causes narrowing of the blood vessels. The effect of such constriction significantly reduces blood transport in the region beyond the narrowing. Atherosclerosis of the coronary arteries increase risk of the myocardial infarct and it can lead to stroke if it is in the vessels supplying blood to the brain. Several theories have been expressed about the existence of stenosis and the occurrence of cardiovascular disease at vascular stenosis, but the exact theory about the initiation of this phenomenon is not

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clearly known [16,29]. Many researchers emphasized that the initiation and development of atherosclerosis, is strongly related to the blood flow characteristics [5,10,25,28]. So, this matter shows the importance of the study of blood flow characteristics in stenosed regions.

Blood is a suspension of erythrocytes, white blood cells, leukocytes, and platelets in a liquid called plasma [17,24,31]. Erythrocytes in terms of the number in ratio of other blood suspended cells are in the majority, and their properties are predominant for other cells [22]. In most literatures, blood flow has been considered as single layered Newtonian [4,6,15] or non-Newtonian fluid [14,16,25,29,30]. Chakravarty and Mandal presented a mathematical model of non-linear two-dimensional blood flow in tapered arteries in the presence of the stenosis. In their investigation blood flow was assumed as an incompressible and Newtonian fluid and the axial and radial velocity profiles were expressed as power series [4]. The study of dispersion through non-Newtonian fluids has abundant applications in polymer and biochemical processing and in cardiovascular system. Mandal studied an unsteady and nonlinear blood flow through a single stenosed tapered artery. Non-Newtonian rheology of the blood flow was characterized by the generalized Power-law model. He solved the governing equations using a finite difference scheme and indicated that the non-Newtonian behavior of the blood flow affects the blood flow characteristics significantly and should not be neglected [16]. Siddiqui et. al. considered a fully developed and pulsatile non-Newtonian Casson fluid through a single stenosed and non-tapered artery and investigated the effect of the pulsatility and the non-Newtonian nature of the blood flow [30].

Blood behaves like a Newtonian fluid when it flows through larger arteries at high shear rates, whereas, it exhibits a non-linear shear stress vs shear rate characteristics when it flows through narrow arteries at low shear rates [14,16,19,25,32]. Also in some diseased conditions, blood exhibits remarkable non-Newtonian characteristics [11,13] and many researchers pointed out that for the blood flow through narrow arteries existence of a peripheral layer of plasma and a core region of suspension of all the erythrocytes was observed. [3,5,26] Thus, for a realistic description of the blood flow, it is appropriate to treat blood as a two-layered model.

In most literatures, blood vessel has been taken to be rigid [6,15,21,23], but due to the elastic properties of blood vessels, assumption of rigid nature of vessels isn't acceptable [16]. Moreover the blood vessels bifurcate at intervals and the cross section of the vessels changes with the distance [4] and mammalian vessels are tapered in nature [15]. Eringen presented the microfluids theory to investigate the behavior of fluids that Newtonian and non-Newtonian fluid flow theory is not able to describe them [7,8]. A subclass of these microfluids is known as micropolar fluids, which exhibit microscopic effects arising from the local structure and microrotation of fluid microelements are considered to be rigid [2]. The theory can be used to explain the flow of colloid fluids, liquid crystals, animals blood, etc. The classical Navier-Stokes equations are incapable of explain the microrotations or particles' spin of suspensions [1,12]. Micropolar fluid takes care of the rotation of fluid particles by means of spin vector, so using these fluids for blood stream is more appropriate than other non-Newtonian fluids [17].

Abdullah and Amin [1] presented a nonlinear two-dimensional mathematical model for the blood flow in a tapered artery with a single stenosis. They assumed that the blood flow as a single-layered micropolar fluid and concluded that the increase in the value of the taper angle leads to an increase in the axial velocity and the flow rate and a decrease in the radial velocity and the resistance to flow. Pralhad and Schultz studied a steady two-layered blood flow through a single stenosed artery by treating the blood in the core region as a micropolar fluid and the plasma in the peripheral layer as a Newtonian fluid. They studied the flow characteristics for normal blood and for the different blood diseases [23]. The motivation

Fig. 1 Geometry of the two-layered blood flow in the tapered artery with an overlapping stenosis

of this paper is to study the effects of an overlapping stenosis on the flow characteristics of two-layered flowing blood through an elastic tapered artery.

The Geometry of the Stenosis

Consider a two-dimensional, laminar, unsteady and fully developed pulsatile flow of blood in a tapered elastic artery with axially symmetric overlapping stenosis. A two-layered model with a micropolar fluid in the core region and a peripheral plasma layer of the thickness $(R - R_1)$ is used to represent the flowing blood. Cylindrical polar coordinate system (r, θ, z) is used to analyze the flow, where r and z are taken along the radial and axial directions respectively, and θ is taken along the circumferential directions. The geometry of the time variant overlapping stenosis (cf. Fig. 1) in the peripheral region and core region are described mathematically as: [4, 9, 14].

$$R(z, t) = \begin{cases} \left[(mz + 1) - \frac{\tau_m \cos(\theta)(z-d)}{l_0} \right. \\ \quad \times \left. \left\{ 11 - \frac{94(z-d)}{3l_0} + \frac{32(z-d)^2}{l_0^2} - \frac{32(z-d)^3}{3l_0^3} \right\} \right] a_1(t), & d \leq z \leq d + \frac{3l_0}{2} \\ (mz + 1)a_1(t), & \text{in the normal artery region,} \end{cases}$$

$$R_1(z, t) = \begin{cases} \left[(\alpha)(mz + 1) - \frac{\tau_m \cos(\theta)(z-d)}{l_0} \right. \\ \quad \times \left. \left\{ 11 - \frac{94(z-d)}{3l_0} + \frac{32(z-d)^2}{l_0^2} - \frac{32(z-d)^3}{3l_0^3} \right\} \right] a_1(t), & d \leq z \leq d + \frac{3l_0}{2} \\ (\alpha)(mz + 1)a_1(t), & \text{in the normal artery region,} \end{cases}$$

in which $R(z, t)$ is the radius of the stenosed artery with peripheral layer, $R_1(z, t)$ is the radius of the stenosed core region of the artery, R_0 is the radius of the nonstenotic artery, $\frac{3l_0}{2}$ is the length of the stenosis, L is taken to be the finite length of the arterial segment, d is the upstream length of the artery, ψ is the angle of tapering, $a = \tan(\psi)$ is the slope of the tapered vessel, τ_m is the critical height of the overlapping stenosis and δ is the maximum size of the bulging of the interface. The time variant parameter $a_1(t)$ is given by $a_1(t) = 1 + k_r \cos(\omega t - \phi)$ [13, 25], where k_r represents the amplitude parameter and ϕ the phase angle.

The Mathematical Formulation

The governing equations for the axisymmetric micropolar flow in the cylindrical polar coordinate system may be written in nondimensional forms as follows [1, 12]:

$$\frac{\partial u_1}{\partial t} + v_1 \frac{\partial u_1}{\partial r} + u_1 \frac{\partial u_1}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re} \left(\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r} \frac{\partial u_1}{\partial r} + \frac{\partial^2 u_1}{\partial z^2} \right) + \frac{m}{Re} \left(\frac{\partial w}{\partial r} + \frac{w}{r} \right), \quad (1)$$

$$\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial r} + u_1 \frac{\partial v_1}{\partial z} = -\frac{\partial p}{\partial r} + \frac{1}{Re} \left(\frac{\partial^2 v_1}{\partial r^2} + \frac{1}{r} \frac{\partial v_1}{\partial r} + \frac{\partial^2 v_1}{\partial z^2} - \frac{v_1}{r^2} \right) + \frac{m}{Re} \left(\frac{\partial w}{\partial z} \right), \quad (2)$$

$$\frac{JM}{1-m} \left(\frac{\partial w}{\partial t} + v_1 \frac{\partial w}{\partial r} + u_1 \frac{\partial w}{\partial z} \right) = -\frac{2N}{Re} w + \frac{N}{Re} \left(\frac{\partial v_1}{\partial z} - \frac{\partial u_1}{\partial r} \right) + \frac{1}{Re} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} - \frac{w}{r^2} \right). \quad (3)$$

The Navier–Stokes equations for Newtonian flow together with the equation of continuity, may be written in nondimensional forms [4, 6, 9, 12, 15] as:

$$\frac{\partial u_2}{\partial t} + v_2 \frac{\partial u_2}{\partial r} + u_2 \frac{\partial u_2}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re} \left(\frac{\partial^2 u_2}{\partial r^2} + \frac{1}{r} \frac{\partial u_2}{\partial r} + \frac{\partial^2 u_2}{\partial z^2} \right), \quad (4)$$

$$\frac{\partial v_2}{\partial t} + v_2 \frac{\partial v_2}{\partial r} + u_2 \frac{\partial v_2}{\partial z} = -\frac{\partial p}{\partial r} + \frac{1}{Re} \left(\frac{\partial^2 v_2}{\partial r^2} + \frac{1}{r} \frac{\partial v_2}{\partial r} + \frac{\partial^2 v_2}{\partial z^2} - \frac{v_2}{r^2} \right), \quad (5)$$

$$\frac{\partial u_i}{\partial z} + \frac{\partial v_i}{\partial r} + \frac{v_i}{r} = 0. \quad (6)$$

Since the flow is axisymmetric, all the variables are independent of θ . The index $i = 1$ is for the core region variables and the index $i = 2$ is for the peripheral layer variables. The following nondimensional variables have been used:

$$u = \frac{u^*}{U}, \quad v = \frac{v^*}{U}, \quad w = \frac{R_0 w^*}{U}, \quad r = \frac{r^*}{R_0}, \quad z = \frac{z^*}{R_0}, \\ t = \frac{t^* U}{R_0}, \quad p = \frac{p^*}{\rho_i U^2}, \quad l_0 = \frac{l_0^*}{R_0}, \quad J = \frac{J^*}{R_0^2}.$$

The other non-dimensional parameters in the above equations are defined as:

$$Re = \frac{\rho_1 U R_0}{\mu_1 + \kappa}, \quad N = \frac{\kappa R_0^2}{\nu}, \quad M = \frac{\mu_1 R_0^2}{\nu}, \quad m = \frac{\kappa}{\mu_1 + \kappa},$$

where u_i , v_i and w are the axial, radial and microrotational dimensionless velocity components respectively, p is the pressure, ρ_i is the density, J is the micro-inertia constant, κ is the rotational viscosity, ν is the material constant and μ_1 is the viscosity of the micropolar fluid.

The dimensionless pressure gradient $\frac{\partial p}{\partial z}$ appearing in equation (1) is given by $-\frac{\partial p}{\partial z} = A_0 + A_1 \cos \omega t$ [4, 13, 25]. Where A_0 is the constant amplitude of the pressure gradient, A_1 is the amplitude of the pulsatile component giving rise to the systolic and diastolic pressures, $\omega = 2\pi f_p$, f_p is the pulse frequency.

The boundary conditions are:

$$\begin{aligned} v_1(r, z, t) = w(r, z, t) = \frac{\partial u_1(r, z, t)}{\partial r} &= 0 \quad \text{for } r = 0, \\ u_1 = u_2, v_1 = v_2, (\tau_{rz})_1 = (\tau_{rz})_2, w(r, z, t) &= -\lambda \frac{\partial u_1}{\partial r} \quad \text{for } r = R_1(z, t), \\ v_2(r, z, t) = \frac{\partial R}{\partial t}, u_2(r, z, t) &= 0 \quad \text{for } r = R(z, t). \end{aligned}$$

The initial conditions are as follows [12,20]:

$$\begin{cases} u_1(r, z, 0) = 2\bar{u}_1 \left[1 - \left(\frac{r}{R} \right)^2 + \frac{4\gamma I_0(\beta)}{\beta^2} \left\{ \frac{I_0\left(\frac{\beta r}{R}\right)}{I_0(\beta)} - 1 \right\} \right]; \\ u_2(r, z, 0) = 2\bar{u}_2 \left[1 - \left(\frac{r}{R} \right)^2 \right], v_i(r, z, 0) = w(r, z, 0) = 0, \end{cases}$$

where $\gamma = \frac{m\beta}{4I_0(\beta)}$, $\beta^2 = N(2 - m)$, $\bar{u}_i$ is the mean axial velocity at any given cross section. I_0 is the modified first kind Bessel function of zeroth order, I_1 is the modified first kind Bessel function of first order.

The Radial Coordinate Transformation

The radial coordinate transformation given by $\xi = \frac{r}{R}$ [4,16,20], is introduced to transform tapered and stenosed artery to straight artery. Using this transformation, equation of continuity (6) and prescribed boundary conditions take the forms:

$$\frac{1}{R} \frac{\partial v_i}{\partial \xi} + \frac{v_i}{\xi R} + \frac{\partial u_i}{\partial z} - \frac{\xi}{R} \frac{\partial R}{\partial z} \frac{\partial u_i}{\partial \xi} = 0, \quad (7)$$

$$v_1(\xi, z, t) = w(\xi, z, t) = \frac{\partial u_1(\xi, z, t)}{\partial \xi} = 0 \quad \text{for } \xi = 0, \quad (8)$$

$$u_1 = u_2, v_1 = v_2, (\tau_{\xi z})_1 = (\tau_{\xi z})_2 w(r, z, t) = -\lambda \frac{\partial u_1}{\partial \xi} \quad \text{for } \xi = \alpha, \quad (9)$$

$$u_2(\xi, z, t) = 0, v_2(\xi, z, t) = \frac{\partial R}{\partial t} \quad \text{for } \xi = 1. \quad (10)$$

The Velocity Profile

The Radial Velocity Component

Multiplying Eq. (7) by ξR and integrating with respect to ξ from 0 to ξ , one obtains,

$$\begin{aligned} \int_0^\xi \xi \frac{\partial v_1}{\partial \xi} d\xi + \int_0^\xi v_1 d\xi + \int_0^\xi \xi R \frac{\partial u_1}{\partial z} d\xi - \int_0^\xi \xi^2 \frac{\partial R}{\partial z} \frac{\partial u_1}{\partial \xi} d\xi &= 0, \\ \xi v_1(\xi, z, t) - \int_0^\xi v_1 d\xi + \int_0^\xi v_1 d\xi + R \int_0^\xi \xi \frac{\partial u_1}{\partial z} d\xi - \frac{\partial R}{\partial z} \left(\xi^2 u_1 \Big|_0^\xi - \int_0^\xi 2\xi u_1 d\xi \right) &= 0, \\ v_1(\xi, z, t) &= -\frac{R}{\xi} \int_0^\xi \xi \frac{\partial u_1}{\partial z} d\xi + \frac{\partial R}{\partial z} \left(\xi u_1 - \frac{2}{\xi} \int_0^\xi \xi u_1 d\xi \right). \end{aligned} \quad (11)$$

For $\xi = \alpha$, using the boundary conditions (9), Eq. (11) becomes,

$$\int_0^\alpha \frac{\partial u_1}{\partial z} d\xi = -\frac{1}{R} \int_0^\alpha 2 \frac{\partial R}{\partial z} \xi u_1 d\xi + \frac{\alpha}{R} \left(\alpha \frac{\partial R}{\partial z} u_{1\alpha} - v_{1\alpha} \right) \quad (12)$$

Suppose function $f(\xi)$ represents an arbitrary function satisfying $\int_0^1 \xi f(\xi) d\xi = 1$. Equation (12) can be written as:

$$\begin{aligned} \int_0^\alpha \frac{\partial u_1}{\partial z} d\xi &= -\frac{1}{R} \int_0^\alpha 2 \frac{\partial R}{\partial z} \xi u_1 d\xi + \frac{\alpha}{R} \left(\alpha \frac{\partial R}{\partial z} u_{1\alpha} - v_{1\alpha} \right) \left(\int_0^1 \xi f(\xi) d\xi \right) \\ \int_0^\alpha \xi \frac{\partial u_1}{\partial z} d\xi &= \int_0^\alpha \xi \left[\frac{-2}{R} \frac{\partial R}{\partial z} u_1 + \frac{\alpha}{R} \left(\frac{\partial R}{\partial z} \alpha u_{1\alpha} - v_{1\alpha} \right) f(\xi) \right] d\xi, \end{aligned} \quad (13)$$

Taking the approximation of treating the equality between the integrals to integrands, Eq. (13) can be written as:

$$\frac{\partial u_1}{\partial z} = -\frac{2}{R} \frac{\partial R}{\partial z} u_1 + \frac{\alpha}{R} \left(\frac{\partial R}{\partial z} \alpha u_{1\alpha} - v_{1\alpha} \right) \frac{4(\xi^2 - 1)}{\alpha^2(\alpha^2 - 1)}. \quad (14)$$

The radial velocity component for the core region is achieved by substituting Eq. (14) into Eq. (11) as follows

$$\begin{aligned} v_1(\xi, z, t) &= -\frac{R}{\xi} \int_0^\xi \xi \left\{ -\frac{2}{R} \frac{\partial R}{\partial z} u_1 + \frac{\alpha}{R} \left(\frac{\partial R}{\partial z} \alpha u_{1\alpha} - v_{1\alpha} \right) \frac{4(\xi^2 - 1)}{\alpha^2(\alpha^2 - 1)} \right\} d\xi \\ &\quad + \frac{\partial R}{\partial z} \left(\xi u_1 - \frac{2}{\xi} \int_0^\xi \xi u_1 d\xi \right) \\ v_1(\xi, z, t) &= \xi \left[\frac{\partial R}{\partial z} u_1 - \frac{(\alpha \frac{\partial R}{\partial z} u_{1\alpha} - v_{1\alpha})(\xi^2 - 2)}{\alpha(\alpha^2 - 2)} \right]. \end{aligned} \quad (15)$$

To obtain the radial velocity for the peripheral layer, multiplying Eq. (7) by ξR and integrating with respect to ξ from α to ξ , and exploiting the boundary condition at the wall (10) one obtains:

$$v_2(\xi, z, t) = \frac{\xi^2 + 1}{2\xi} \left[\frac{\partial R}{\partial z} u_2 - \frac{R}{2} \left(\frac{\xi^2 - 1}{\xi^2 + 1} \right) \frac{\partial u_2}{\partial z} + \frac{2}{\xi^2 + 1} \frac{\partial R}{\partial t} \right]. \quad (16)$$

The Axial and Microrotational Velocity Components

Although many computational methods are available for solving the system of nonlinear partial differential equations, the finite difference scheme is easier, cheaper and more efficient to employ [25, 28]. The Central difference formula is used to express the spatial derivatives and the forward difference formula is applied to the time derivative and these are given by:

$$\begin{aligned} (u_i)_{f\xi} &= \frac{\partial u_i}{\partial \xi} = \frac{(u_i)_{i,j+1}^k - (u_i)_{i,j-1}^k}{2\Delta\xi}, \quad (u_i)_{fz} = \frac{\partial u_i}{\partial z} = \frac{(u_i)_{i+1,j}^k - (u_i)_{i-1,j}^k}{2\Delta z}, \\ (u_i)_{s\xi} &= \frac{\partial^2 u_i}{\partial \xi^2} = \frac{(u_i)_{i,j+1}^k - 2(u_i)_{i,j}^k + (u_i)_{i,j-1}^k}{\Delta\xi^2}, \\ (u_i)_{sz} &= \frac{\partial^2 u_i}{\partial z^2} = \frac{(u_i)_{i+1,j}^k - 2(u_i)_{i,j}^k + (u_i)_{i-1,j}^k}{\Delta z^2}, \end{aligned}$$

$$\frac{\partial u_i}{\partial t} = \frac{(u_i)_{i,j}^{k+1} - (u_i)_{i,j}^k}{\Delta t}, \quad \begin{cases} i = 1 \Rightarrow \Delta = \Delta\xi \\ i = 2 \Rightarrow \Delta = \Delta\xi' \end{cases}$$

where $\Delta\xi$ and $\Delta\xi'$ are the increments in the radial direction of the core layer and the peripheral layer, Δz and Δt are respectively, axial direction and time increments. Similar necessary expressions can also be obtained for the remaining parameters. We define:

$$\begin{cases} \xi_j = (j-1)\Delta\xi, & (j = 1, 2, \dots, N_c + 1); \quad \xi_{(N_c+1)} = \alpha \\ \xi'_j = \alpha + (j - (N_c + 1))\Delta\xi', & (j = N_c + 1, N_c + 2, \dots, N + 1); \quad \xi_{(N+1)} = 1 \\ z_i = (i-1)\Delta z, & (i = 1, 2, \dots, M + 1) \\ t_k = (k-1)\Delta t, & (k = 1, 2, \dots). \end{cases}$$

Using the presented transformation and the finite difference scheme, Eqs. (1), (3) and (4) and the prescribed boundary and initial conditions (8)–(10), take the following forms:

$$\begin{aligned} (u_1)_{i,j}^{k+1} &= (u_1)_{i,j}^k + \Delta t \left[- \left(\frac{\partial p}{\partial z} \right)_i^k + (T_1)_{i,j}^k (u_1)_{f\xi} + (T_2)_{i,j}^k (u_1)_{s\xi} - (u_1)_{i,j}^k (u_1)_{fz} \right. \\ &\quad \left. + \frac{1}{Re} (u_1)_{sz} + \frac{m}{Re\xi_j R_i^k} \left((w)_{i,j}^k + \xi_j (w)_{f\xi} \right) \right], \\ (u_2)_{i,j}^{k+1} &= (u_2)_{i,j}^k + \Delta t \left[- \left(\frac{\partial p}{\partial z} \right)_i^k + (T_3)_{i,j}^k (u_2)_{f\xi} + (T_2)_{i,j}^k (u_2)_{f\xi} \right. \\ &\quad \left. - (U_2)_{i,j}^k (u_2)_{fz} + \frac{1}{Re} (u_2)_{sz} \right], \\ (w)_{i,j}^{k+1} &= (w)_{i,j}^k + \Delta t \left[(T_4)_{i,j}^k (w)_{f\xi} + (T_5)_{i,j}^k (w)_{s\xi} - (u_1)_{i,j}^k (w)_{fz} \right. \\ &\quad \left. + \frac{1-m}{MReJ} \left\{ (w)_{sz} - \frac{(w)_{i,j}^k}{(\xi_j R_i^k)^2} \right\} + \frac{(1-m)N}{MReJR_i^k} \right. \\ &\quad \left. \left\{ -2R_i^k (w)_{i,j}^k + R_i^k (v_1)_{fz} - \xi_j \left(\frac{\partial R}{\partial z} \right)_i^k (v_1)_{f\xi} - (u_1)_{f\xi} \right\} \right], \\ (v_1)_{i,1}^k &= (w)_{i,1}^k = 0, \quad (u_1)_{i,1}^k = (u_1)_{i,2}^k, \quad (u_1)_{i,N_c+1}^k = (u_2)_{i,N_c+1}^k, \\ (v_1)_{i,N_c+1}^k &= (v_2)_{i,N_c+1}^k, \quad (w)_{(i,N+1)}^k = -\frac{\lambda}{R_i^k} (u_{f\xi})_{i,N+1}^k, \\ (u_1)_{i,N_c}^k &= (u_1)_{i,N_c+1}^k - \frac{\Delta\xi}{C} \left[(u_2)_{f\xi} + (1-a)R_i^k (v_2)_{fz} - \left(\frac{\partial R}{\partial z} \right)_i^k \right. \\ &\quad \left. \{ a\xi_j R_i^k (u_2)_{fz} + \xi_j (v_2)_{f\xi} + a(v_2)_{i,j}^k \} - cR_i^k (w)_{i,j}^k \right], \\ (u_2)_{i,N+1}^k &= 0, \quad (v_2)_{i,N+1}^k = \left(\frac{\partial R}{\partial t} \right)_i^k, \quad (u_2)_{i,j}^1 = 2\overline{u_2}(1 - \xi_j^2); \quad (j = N_c + 1, \dots, N + 1) \\ (u_1)_{i,j}^1 &= 2\overline{u_1} \left[1 - \xi_j^2 + \frac{4\gamma}{\beta^2} I_0(\beta) \left\{ \frac{I_0(\beta\xi_j)}{I_0(\beta)} - 1 \right\} \right]; \quad (j = 1, 2, \dots, N_c + 1) \\ (v_i)_{i,j}^1 &= (w)_{i,j}^1 = 0, \quad \text{where } a = \frac{\mu_1}{\mu_2}, b = \frac{\mu_1 + \kappa}{\mu_2}, c = \frac{\kappa}{\mu_2}, C = b - a \left(\xi \frac{\partial R}{\partial t} \right)^2. \end{aligned}$$

The finite difference formulas for the flow rate Q and the resistive impedance Λ are obtained as:

$$Q_i^k = 2\pi(R_i^k)^2 \int_0^1 \xi_j(u)_{i,j}^k d\xi_j, \quad (17)$$

$$\Lambda_i^k = \frac{|L(\frac{\partial p}{\partial z})_i^k|}{Q_i^k}. \quad (18)$$

Numerical Results and Discussion

Numerical computations have been carried out using the following parameter values [12, 21, 27]:

$$\begin{aligned} R_0 &= 1.52, L = 30, d = 7, l_0 = 15, \phi = 0, k_r = 0.05, f_p = 1.2, \alpha = 0.95, \\ Re &= 300, N = 1, M = 1, m = 0.85, J = 0.0001, \Delta t = 0.0001, \Delta \xi = 0.019, \\ \Delta' \xi &= 0.005, \Delta z = 0.1, A_0 = 0.1, A_1 = 0.2A_0. \end{aligned}$$

A few simulations were run with a finer and coarse grid size for the axial velocity profiles at the point $z = 9.6$ are presented in Fig. 2. One may notice from this figure that the profiles concerning three different grid sizes almost get overlapped one with another. Thus, the grid independence study in the present context of numerical simulation has its own importance to establish the accuracy of the results obtained.

The dimensionless axial velocity profiles in a non-tapered artery ($\psi = 0$) are plotted in Fig. 3. As shown the present results are in good agreement with R.N. Pralhad and D.H. Schultz [23] in the critical regions at $t = 4$.

Figure 4 illustrates the dimensionless axial velocity profiles of the flowing blood at a specific location of $z = 19$ in the stenotic region for different taper angles and different

Fig. 2 Dimensionless axial velocity profiles at $z = 9.6$ for different grid sizes (flexible wall) ($t = 3$, $\tau_m = 0.2R_0$, $\psi = 0$)

Fig. 3 Dimensionless axial velocity profile in non-tapered artery ($t = 4$, $\tau_m = 0.2R_0$, $\psi = 0$)

Fig. 4 Dimensionless axial velocity profiles with dimensionless radial distance at $z = 19$

stenosis size. According to Fig. 4, the axial velocity decreases by increasing the size of stenosis and increases by increasing the taper angle. It may be noted that the skin-friction velocity increases with the size of the stenosis and therefore by increasing the stenosis size, velocity decreases. The present figure also includes the axial velocity for the stenosis ($\tau_m = 0.2R_0$) at the time $t = 2$ through an elastic and rigid artery. It is clear that the axial velocity in rigid artery is more than the axial velocity in the elastic artery and this shows the importance of assumption of elastic nature of blood arteries. According to the continuity equation this behavior happened because of this fact that the radial velocity in rigid arteries is less than the radial velocity in elastic arteries.

Fig. 5 Dimensionless radial velocity profiles with dimensionless radial distance at $z = 19$

Fig. 6 Dimensionless mirorotational velocity with dimensionless radial distance at $z = 10$ for the time $t = 3$

The radial velocity profiles of the two-layered blood flow through a stenosed artery at specific location $z = 14.5$ at the times $t = 1$ and $t = 2$ are indicated in Fig. 5. The radial velocity at these times gives negative values. All the curves for blood flow in the elastic artery appear to decline from zero on the axis as one moves from away from it and finally to increase towards the wall to attend some finite value on the wall surface which clearly reflects the presence of wall motion encountered in the present model. However, the curves of the blood flow through the rigid artery, due to the no-slip boundary condition have a zero values in the wall of the artery. In Fig. 5 to better comparison, since the value of radial velocity of the blood flow in the rigid artery is much less than radial velocity of the blood flow in the elastic artery, the value of radial velocity in the rigid artery multiplied by 10. A thorough

Fig. 7 Variation of the flow rate versus the time at $z = 10$

analysis of all these curves helps estimate the effects of arterial tapering and the presence of stenosis, artery elasticity properties and quantitatively on the radial velocity component of the flowing blood. As shown, by increasing the stenosis severity the magnitude of the radial velocity decreases. Also, it is observed that the magnitude of the radial velocity through the diverging tapered artery ($\psi > 0$) is more than from the non-tapered artery ($\psi = 0$).

Figure 6 shows the effect of the tapering angle and the stenosis size on microrotational velocity at the same time ($t = 3$) in the critical location $z = 10$. The microrotational velocity decreases with the increase of the tapering angle and increases with the increase of the stenosis size. Also it is found that the microrotational velocity through the rigid artery is more than those in the elastic artery.

Fig. 8 Variation of the resistive impedance versus the time at $z = 10$

The variation of the flow rate with time spanned over a period of severity cardiac cycles at a critical location of $z = 10$ in the constricted region of the tapered artery, for different values of the tapering angle and the stenosis size is illustrated in Fig. 7. The pulsatile nature of the flow rate is shown in this figure. The flow rate for a positively tapered (diverging) artery $\psi = 0.01$ and $\psi = 0.02$ has a relatively higher magnitude than that of a non-tapered artery $\psi = 0$. Furthermore, It is observed that the flow rate decreases by increasing the stenosis size from $\tau_m = 0.1R_0$ to $\tau_m = 0.2R_0$. Studying all the results referred to the present figure, one may conclude that the presence of stenosis and the taper angle certainly bear the potential to influence the flow rate to a considerable extent in the realm of the physiological flow phenomena.

Figure 8 shows how the resistances to the flow is influenced by the unsteady behavior of the flowing blood by the tapering angle, the presence of stenosis, and elasticity properties of artery. Comparison between Figs. 7 and 8 shows the resistive impedance and volumetric flow rate are inversely related, such that the resistive impedance in a diverging tapering appear to be smaller than those in a non-tapering because the flow rate is higher in the former than that in the latter, as anticipated. Figure 8 also indicates that the resistive impedance increases with the increase of the stenosis size. From Figs. 7 and 8, we can conclude that more blood can flow freely through a diverging vessel, which has less effect of resistance.

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Appendix

$$\begin{aligned}
 (T_1)_{i,j}^k &= \frac{1}{(R_i^k)^2} \left[\xi_j R_i^k \left(\frac{\partial R}{\partial t} \right)_i^k + \xi_j R_i^k \left(\frac{\partial R}{\partial z} \right)_i^k (u_1)_{i,j}^k - (v_1)_{i,j}^k R_i^k \right. \\
 &\quad \left. + \frac{1}{Re} \left\{ \frac{1}{\xi_j} + 2\xi_j \left(\left(\frac{\partial R}{\partial z} \right)_i^k \right)^2 - \xi_j R_i^k \left(\frac{\partial^2 R}{\partial z^2} \right)_i^k \right\} \right] \\
 (T_2)_{i,j}^k &= \frac{1}{Re(R_i^k)^2} \left\{ 1 + \left(\xi_j \left(\frac{\partial R}{\partial z} \right)_i^k \right)^2 \right\} \\
 (T_3)_{i,j}^k &= \frac{1}{(R_i^k)^2} \left[\xi_j R_i^k \left(\frac{\partial R}{\partial t} \right)_i^k + \xi_j R_i^k \left(\frac{\partial R}{\partial z} \right)_i^k (u_2)_{i,j}^k - (v_2)_{i,j}^k R_i^k \right. \\
 &\quad \left. + \frac{1}{Re} \left\{ \frac{1}{\xi_j} + 2\xi_j \left(\left(\frac{\partial R}{\partial z} \right)_i^k \right)^2 - \xi_j R_i^k \left(\frac{\partial^2 R}{\partial z^2} \right)_i^k \right\} \right] \\
 (T_4)_{i,j}^k &= \frac{1}{(R_i^k)^2} \left[\xi_j R_i^k \left(\frac{\partial R}{\partial t} \right)_i^k + \xi_j R_i^k \left(\frac{\partial R}{\partial z} \right)_i^k (u_1)_{i,j}^k - (v_1)_{i,j}^k R_i^k \right. \\
 &\quad \left. + \frac{1-m}{MReJ} \left\{ \frac{1}{\xi_j} + 2\xi_j \left(\left(\frac{\partial R}{\partial z} \right)_i^k \right)^2 - \xi_j R_i^k \left(\frac{\partial^2 R}{\partial z^2} \right)_i^k \right\} \right] \\
 (T_5)_{i,j}^k &= \frac{1-m}{MReJ(R_i^k)^2} \left\{ 1 + \left(\xi_j \left(\frac{\partial R}{\partial z} \right)_i^k \right)^2 \right\}.
 \end{aligned}$$

References

1. Abdullaha, I., Amin, N.: A micropolar fluid model of blood flow through a tapered artery with a stenosis. *Math. Meth. Appl. Sci.* **33**, 1910–1923 (2010)
2. Bhargava, R., Agarwal, R., Kumar, L., Takhar, H.: Finite element study of mixed convection micropolar flow in a vertical circular pipe with variable surface conditions. *Int. J. Eng. Sci.* **42**, 1327 (2004)
3. Biswas, D., Chakraborty, U.: Two-layered pulsatile blood flow in a stenosed artery with body acceleration and slip at wall. *AAM: Int. J.* **5**(2), 303–320 (2010)
4. Chakravarty, S., Mandal, P.: Two-dimensional blood flow through tapered arteries under stenotic conditions. *Int. J. Non-Linear Mech.* **35**, 779–793 (2000)
5. Chakravarty, S., Mandal, P.: Unsteady flow of a two-layer blood stream past a tapered flexible artery under stenotic conditions. *Comput. Methods Appl. Math.* **4**(4), 391–409 (2004)
6. Deshpande, M., Giddens, D., Mabon, F.: Steady laminar flow through modelled vascular stenoses. *J. Biomech* **9**, 165–174 (1976)
7. Eringen, A.: Simple microfluids. *Int. J. Eng. Sci.* **2**, 205–217 (1964)
8. Eringen, A.: Theory of microfluids. *J. Math. Mech.* **16**, 1–18 (1966)
9. Haghighi, A.R., Asl, M.S., Kiyasatfar, M.: Mathematical modeling of unsteady blood flow through elastic tapered artery with overlapping stenosis. *J. Braz. Soc. Mech. Sci. Eng.* (2014). doi:[10.1007/s40430-014-0206-3](https://doi.org/10.1007/s40430-014-0206-3)
10. Haghighi, A.R.: Mathematical model of the impact of pressure drop on human body. *Selcuk J. Appl. Math.* **13**(1), 35–40 (2012)
11. Haghighi, A.R., Pralhad, R.N.: Mathematical modelling of blood flows under the effects of body forces and magnetism on human body. *Int. J. Biomed. Eng. Technol.* **2**(4), 295–303 (2009)
12. Ikbal, M., Chakravarty, S., Mandal, P.: Two-layered micropolar fluid flow through stenosed artery: Effect of peripheral layer thickness. *Comput. Math. Appl.* **58**, 1328–1339 (2009)
13. Ikbal, M., Chakravarty, S., Wong, K., Mazumdar, J., Mandal, P.: Unsteady response of non-Newtonian blood flow through a stenosed artery in magnetic field. *Comput. Appl. Math.* **230**, 243–259 (2009)
14. Ismail, Z., Abdullah, I., Mustapha, N., Amin, A.: A power-law model of blood flow through a tapered overlapping stenosed artery. *Appl. Math. Comput.* **195**, 669–680 (2008)
15. Liu, G., Wang, X., Ai, B., Liu, L.: Numerical study of pulsating flow through a tapered artery with stenosis. *Chin. J. Phys* **42**(4–I), 401–409 (2004)
16. Mandal, P.: An unsteady analysis of non-Newtonian blood flow through tapered arteries with a stenosis. *Int. J. Non-Linear Mech.* **40**, 151–164 (2005)
17. Mekheimer, K., El Kot, M.: The micropolar fluid model for blood flow through a tapered artery with a stenosis. *Acta. Mech. Sin.* **24**, 637–644 (2008)
18. Mortazavinia, Z., Zare, A., Mehdizadeh, A.: Effects of renal artery stenosis on realistic model of abdominal aorta and renal arteries incorporating fluid-structure interaction and pulsatile non-Newtonian blood flow. *Appl. Math. Mech. -Engl. Ed.* **33**(2), 165–176 (2012)
19. Mukhopadhyay, S., Layek, G.: Numerical modeling of a stenosed artery using mathematical model of variable shape. *AAM: Int.* **3**(6), 308–328 (2008)
20. Mustapha, N., Amin, N., Chakravarty, S., Mandal, P.: Unsteady magnetohydrodynamic blood flow through irregular multi-stenosed arteries. *Comput. Biol. Med.* **39**(896), 906 (2009)
21. Philip, D., Chandra, P.: Flow of Eringen fluid (Simple Microfluid) through an artery with mild stenosis. *Int. J. Eng. Sci.* **34**, 87–99 (1996)
22. Pontrelli, P.: Pulsatile blood flow in a pipe. *Comput. Fluids.* **27**(3), 367–380 (1998)
23. Pralhad, R., Schultz, D.: Two-layered blood flow through stenosed tubes for different diseases. *Biorheol.* **25**, 715–726 (1988)
24. Sankar, D.: A two-fluid model for pulsatile flow in catheterized blood vessels. *Int. J. Non-Linear Mech.* **44**(3), 37–351 (2009)
25. Sankar, D., Lee, U.: FDM analysis for MHD flow of a non-Newtonian fluid for blood flow in stenosed arteries. *J. Mech. Sci. Technol.* **25**(10), 2573–2581 (2001)
26. Sankar, D., Lee, U.: Nonlinear mathematical analysis for blood flow in a constricted artery under periodic body acceleration. *Commun. Nonlinear Sci. Numer. Simul.* **16**, 4390–4402 (2011)
27. Sankar, D., Goh, J., Ahmad Izani, M.: FDM analysis for blood flow through stenosed tapered arteries. *Bound. Value Prob.*, Article ID 917067, 16 p. (2010)
28. Sankar, D.: Two-phase non-linear model for blood flow in asymmetric and axisymmetric stenosed arteries. *Int. J. Non-Linear Mech.* **46**, 296–305 (2011)
29. Shaw, S., Murthy, P., Pradhan, S.: The effect of body acceleration on two dimensional flow of Casson fluid through an artery with asymmetric stenosis. *Open Transp. Phenom. J.* **2**, 55–68 (2010)

30. Siddiqui, S., Verma, N., Mishra, S., Gupta, R.: Mathematical modelling of pulsatile flow of Cassons fluid in arterial stenosis. *Appl. Math. Comput.* **210**, 1–10 (2009)
31. Srivastava, V., Vishnoi, R., Mishra, S., Sinha, P.: A two-layered non-Newtonian arterial blood flow through an overlapping constriction. *AAM: Int. J.* **6**(1), 41–57 (2001)
32. Varshney, G., Katiyar, V., Kumar, S.: Effect of magnetic field on the blood flow in artery having multiple stenosis: a numerical study. *Int. J. Eng. Sci. Technol.* **2**(2), 67–82 (2010)