Lagrange's Interpolating Polynomial Approach to Solve Multi-choice Transportation Problem

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Abstract This paper deals with the multi-choice transportation problem (TP) in which the parameters of the TP such as the cost coefficients of the objective function, supplies and demands of the constraints are all followed multi-choice parameters. The main aim of this paper is to select an appropriate choice from the multi-choices for the cost coefficients of the objective function, supplies and demands of the constraints in TP by introducing Lagrange's interpolating polynomial in such a way that the total cost is minimum. Finally, a non-linear mathematical model has been formulated. Using LINGO software, the optimal solution of the proposed problem is obtained. A real-life problem on multi-choice TP is considered to illustrate the proposed method in this paper.

Keywords Transportation problem · Multi-choice programming · Lagrange's interpolating polynomial

Mathematics Subject Classification 90B06 · 65K05

Introduction

Transportation problem deals with the distribution of goods from several points of supply, such as factories, often known as sources, to a number of points of demand, such as warehouses, often known as destinations. Each source is able to supply a fixed number of units of the product, usually called the capacity or availability, and each destination has a fixed demand, usually known as requirement. The main objective of TP is to schedule shipments from sources to destinations so that the total transportation cost is minimum. The transportation problem mainly contains three parameters namely cost coefficient, supply and demand

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parameters. Due to some uncountable factors, these parameters are not always stable or not known exactly. This imprecision occurs from the lack of exact information.

Most of the real world decision making problems of practical importance are modeled on multi-choice parameters. The methods of multi-choice optimization have become increasingly important in the field of economics, industry, transportation, military purpose and technology etc. The cost coefficients C_{ij} of the objective function may represent the transportation cost of unit product from source i to destination j . Due to increase of the fuel price and other considerable factors, let us introduce multi-choice type for the cost coefficients of the objective function for the transportation problem. Again due to the fluctuations of market price for any type of products, the supply and demand parameters should also follow multi-choice type.

During past few years, many researchers have showed great interest in the field of transportation problem and a number of methods have been proposed for solving it by several researchers [1–4]. The transportation model was first studied by [2], who gave an incomplete algorithm for obtaining the solution and [1] considered the problem of minimizing the cost of distribution of products from several factories to a number of customers. He also developed a procedure to solve the transportation problem.

Healy [5] first developed a procedure for linear programming problem with zero-one variables, which is called multiple choice programming. A method for modeling the multi-choice programming problem, using binary variables was presented by Chang [6]. He presented a mathematical model where the multiplicative terms of binary variables were replaced by continuous variables. Again Chang [7] proposed a revised method of multi-choice goal programming model which does not involve multiplicative terms of binary variables to satisfy the multiple aspiration levels. Biswal and Acharya [8] formulated the interpolating polynomials to handle the multi-choice parameters included in the linear programming problem in which the constraints were associated with some multi-choice parameters. Roy et al. [9], Mahapatra et al. [10] and Roy [11] presented the procedures to solve the multi-choice stochastic transportation problem where the authors converted the multi-choice transportation problem into a standard mathematical programming through the selection of binary variables, bounds for binary codes and restriction of binary codes using auxiliary constraints. Recently Maity and Roy [12, 13] presented the studies on multi-choice TP and their applications to real-life transportation problems.

Most of the multi choice programming problems are based on selection of binary variables, bounds for binary codes and restriction of binary codes using auxiliary constraints which are very difficult to transform and solve the mathematical programming involving more number of alternatives or choices. To avoid the computational burden and other difficulties, Lagrange's interpolating polynomial is incorporated to select a particular choice from the multi choice parameters. The multi choice parameters of TP are replaced by Lagrange's interpolating polynomial and using some standard numerical methods [14] we have formulated the nonlinear programming problem and then to solve it, which is the main motivation in this paper. Due to increase the fuel price, road tax, market fluctuation, and other considerable factors, this paper considers the cost coefficients of the objective function, supply and demand parameters based on multi-choices for the real-life transportation problem.

The remainder is organized as follows: "Mathematical Model" section derives the mathematical model of the proposed transportation problem. "Case Study" section contains a case study to illustrate the proposed methodology. "Conclusion" section discusses the conclusion regarding this article.

Mathematical Model

Considering the situation having m origins (sources) S_i ($i = 1, 2, \dots, m$) and n destinations (demands) D_j ($j = 1, 2, \dots, n$) for the classical transportation problem. The sources may be production facilities, warehouses, supply points and the destinations are consumption facilities, warehouses or demand points. Let the supplies $(a_1, a_2, \dots, a_m)$ be the quantity of homogeneous product, which are to transport from m origins S_i ($i = 1, 2, \dots, m$) to n destinations D_j ($j = 1, 2, \dots, n$) and satisfy the demands $(b_1, b_2, \dots, b_n)$ respectively. The cost coefficients C_{ij} of the objective function may represent the transportation cost, unfulfilled supply and demand, and others are provided with transporting a unit of product from source i to destination j . Here z represents the minimum value of the objective function and assuming that $a_i > 0$, $b_j > 0$, and $C_{ij} > 0$. Again assuming that $C_{ij} \in \{C_{ij}^1, C_{ij}^2, \dots, C_{ij}^{s_{ij}}\}$, $a_i \in \{a_i^1, a_i^2, \dots, a_i^{o_i}\}$ and $b_j \in \{b_j^1, b_j^2, \dots, b_j^{d_j}\}$, ($i = 1, 2, \dots, m$), ($j = 1, 2, \dots, n$) are multi-choice parameters. The mathematical model of the multi-choice transportation problem is presented as follows:

Model 1

$$\min : z = \sum_{i=1}^m \sum_{j=1}^n \{C_{ij}^1, C_{ij}^2, \dots, C_{ij}^{s_{ij}}\} x_{ij} \quad (1)$$

$$\text{subject to } \sum_{j=1}^n x_{ij} \leq \{a_i^1, a_i^2, \dots, a_i^{o_i}\} \quad i = 1, 2, \dots, m \quad (2)$$

$$\sum_{i=1}^m x_{ij} \geq \{b_j^1, b_j^2, \dots, b_j^{d_j}\} \quad j = 1, 2, \dots, n \quad (3)$$

$$x_{ij} \geq 0, \quad \forall i \text{ and } j \quad (4)$$

where

$$\sum_{i=1}^m \max [a_i^1, a_i^2, \dots, a_i^{o_i}] \geq \sum_{j=1}^n \max [b_j^1, b_j^2, \dots, b_j^{d_j}] \quad (\text{Feasibility Condition})$$

The following cases are to be considered:

1. When C_{ij} ($i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$) follows multi-choice parameter.
2. When a_i ($i = 1, 2, \dots, m$) follows multi-choice parameter.
3. When b_j ($j = 1, 2, \dots, n$) follows multi-choice parameter.

Model Under Lagrange's Interpolating Polynomial

Lagrange's interpolating polynomial is constructed for the multi-choice parameters taking some integral values for the nodal points. Each node corresponds to exactly one functional value of a multi-choice parameter. Exactly o_i or d_j or s_{ij} number of nodal points are needed if a parameter has o_i or d_j or s_{ij} number of choices. The multi-choice parameters are replaced by Lagrange's interpolating polynomial and using some standard numerical methods which selects a parameter from multi-choice parameters.

Table 1 Transportation costs at the node points for TP

Node points	z^{ij}	0	1	2	...	$s_{ij} - 1$
Transportation costs	$f(z^{ij}) = C_{ij}$	C_{ij}^1	C_{ij}^2	C_{ij}^3	...	$C_{ij}^{s_{ij}}$

When C_{ij} ($i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$) Follows Multi-choice Parameter

The objective function (1) of **Model 1** is defined as follows:

$$\min : z = \sum_{i=1}^m \sum_{j=1}^n \{C_{ij}^1, C_{ij}^2, \dots, C_{ij}^{s_{ij}}\} x_{ij}$$

Let $0, 1, \dots, (s_{ij} - 1)$ be s_{ij} number of node points, where $C_{ij}^1, C_{ij}^2, \dots, C_{ij}^{s_{ij}}$ are the associated functional values of the interpolating polynomial at s_{ij} different node points.

Lagrange's interpolating polynomial $P_{s_{ij}-1}(z^{ij})$ of degree $(s_{ij} - 1)$ which interpolates the data from Table 1 has been derived as follows:

$$P_{(s_{ij}-1)}(s) = C_{ij}^{s+1}, \quad s = 0, 1, \dots, (s_{ij} - 1), \quad i = 1, 2, \dots, m \quad \text{and} \quad j = 1, 2, \dots, n \quad (5)$$

Then Lagrange's interpolating polynomial for ij -th multi-choice parameters has been formulated as follows:

$$\begin{aligned} P_{s_{ij}-1}(z^{ij}) = & \frac{(z^{ij} - 1)(z^{ij} - 2) \cdots (z^{ij} - s_{ij} + 1)}{(-1)^{(s_{ij}-1)}(s_{ij} - 1)!} C_{ij}^1 \\ & + \frac{(z^{ij})(z^{ij} - 2) \cdots (z^{ij} - s_{ij} + 1)}{(-1)^{(s_{ij}-2)}(s_{ij} - 2)!} C_{ij}^2 \\ & + \frac{(z^{ij})(z^{ij} - 1)(z^{ij} - 3) \cdots (z^{ij} - s_{ij} + 1)}{(-1)^{(s_{ij}-3)}(s_{ij} - 3)!} C_{ij}^3 + \cdots \\ & + \frac{(z^{ij})(z^{ij} - 1)(z^{ij} - 2) \cdots (z^{ij} - s_{ij} + 2)}{(s_{ij} - 1)!} C_{ij}^{s_{ij}}, \quad \forall i \text{ and } j \quad (6) \end{aligned}$$

Hence, the objective function (1) is to redefine by considering the Eq. (6) as follows:

$$\begin{aligned} \min : z = & \sum_{i=1}^m \sum_{j=1}^n \left[\frac{(z^{ij} - 1)(z^{ij} - 2) \cdots (z^{ij} - s_{ij} + 1)}{(-1)^{(s_{ij}-1)}(s_{ij} - 1)!} C_{ij}^1 \right. \\ & + \frac{(z^{ij})(z^{ij} - 2) \cdots (z^{ij} - s_{ij} + 1)}{(-1)^{(s_{ij}-2)}(s_{ij} - 2)!} C_{ij}^2 \\ & + \frac{(z^{ij})(z^{ij} - 1)(z^{ij} - 3) \cdots (z^{ij} - s_{ij} + 1)}{(-1)^{(s_{ij}-3)}(s_{ij} - 3)!} C_{ij}^3 + \cdots \\ & \left. + \frac{(z^{ij})(z^{ij} - 1)(z^{ij} - 2) \cdots (z^{ij} - s_{ij} + 2)}{(s_{ij} - 1)!} C_{ij}^{s_{ij}} \right] x_{ij} \quad (7) \end{aligned}$$

Though, in this paper, I have used the Lagrange's interpolation formula to formulate the objective function into deterministic form, one can use a transformation technique using binary variables (see [13]) to formulate objective function in another form also.

Table 2 Supply values at the node points for TP

Node points	y^i	0	1	2	...	$o_i - 1$
Supply values	$f(y^i) = a_i$	a_i^1	a_i^2	a_i^3	...	$a_i^{o_i}$

When a_i ($i = 1, 2, \dots, m$) Follows Multi-choice Parameter

The constraint (2) whose right hand side i.e., supply follows multi-choice parameters of the TP of **Model 1** is recalled as follows:

$$\sum_{j=1}^n x_{ij} \leq \{a_i^1, a_i^2, \dots, a_i^{o_i}\}, \quad i = 1, 2, \dots, m$$

Let $0, 1, \dots, (o_i - 1)$ are o_i number of node points and $a_i^1, a_i^2, \dots, a_i^{o_i}$ are the functional values of Lagrange's interpolating polynomial at o_i distinct node points.

Lagrange's interpolating polynomial $P_{o_i-1}(y^i)$ of degree $(o_i - 1)$ which interpolates the data from Table 2 has been derived as follows:

$$P_{(o_i-1)}(r) = a_i^{r+1}, \quad r = 0, 1, \dots, (o_i - 1), \quad i = 1, 2, \dots, m \quad (8)$$

Lagrange's interpolating polynomial for i -th multi-choice parameters has been constructed as follows:

$$\begin{aligned} P_{o_i-1}(y^i) = & \frac{(y^i - 1)(y^i - 2) \cdots (y^i - o_i + 1)}{(-1)^{(o_i-1)}(o_i - 1)!} a_i^1 \\ & + \frac{(y^i)(y^i - 2) \cdots (y^i - o_i + 1)}{(-1)^{(o_i-2)}(o_i - 2)!} a_i^2 \\ & + \frac{(y^i)(y^i - 1)(y^i - 3) \cdots (y^i - o_i + 1)}{(-1)^{(o_i-3)}(o_i - 3)!} a_i^3 + \cdots \\ & + \frac{(y^i)(y^i - 1)(y^i - 2) \cdots (y^i - o_i + 2)}{(o_i - 1)!} a_i^{o_i}, \quad i = 1, 2, \dots, m. \end{aligned} \quad (9)$$

Using the Eq. (9), the constraint (2) can be transformed as follows:

$$\begin{aligned} \sum_{j=1}^n x_{ij} \leq & \frac{(y^i - 1)(y^i - 2) \cdots (y^i - o_i + 1)}{(-1)^{(o_i-1)}(o_i - 1)!} a_i^1 \\ & + \frac{(y^i)(y^i - 2) \cdots (y^i - o_i + 1)}{(-1)^{(o_i-2)}(o_i - 2)!} a_i^2 \\ & + \frac{(y^i)(y^i - 1)(y^i - 3) \cdots (y^i - o_i + 1)}{(-1)^{(o_i-3)}(o_i - 3)!} a_i^3 + \cdots \\ & + \frac{(y^i)(y^i - 1)(y^i - 2) \cdots (y^i - o_i + 2)}{(o_i - 1)!} a_i^{o_i}, \quad i = 1, 2, \dots, m. \end{aligned} \quad (10)$$

When b_j ($j = 1, 2, \dots, n$) Follows Multi-choice Parameter

The constraint (3) whose right hand side i.e., demand follows multi-choice parameters of the TP of **Model 1** is rewritten as follows:

$$\sum_{i=1}^m x_{ij} \geq \{b_j^1, b_j^2, \dots, b_j^{d_j}\}, \quad j = 1, 2, \dots, n$$

Table 3 Demand values at the node points for TP

Node points	z^j	0	1	2	...	$d_j - 1$
Demand values	$f(z^j) = b_j$	b_j^1	b_j^2	b_j^3	...	$b_j^{d_j}$

Let $0, 1, \dots, (d_j - 1)$ are d_j number of node points and $b_j^1, b_j^2, \dots, b_j^{d_j}$ are the functional values of Lagrange's interpolating polynomial at d_j distinct node points.

Lagrange's interpolating polynomial $P_{d_j-1}(z^j)$ of degree $(d_j - 1)$ which interpolates the data from Table 3 has been obtained as follows:

$$P_{(d_j-1)}(t) = b_j^{t+1}, \quad t = 0, 1, \dots, (d_j - 1), \quad j = 1, 2, \dots, n \quad (11)$$

Lagrange's interpolating polynomial for j -th multi-choice parameters has been formulated as follows:

$$\begin{aligned} P_{d_j-1}(z^j) = & \frac{(z^j - 1)(z^j - 2) \cdots (z^j - d_j + 1)}{(-1)^{(d_j-1)}(d_j - 1)!} b_j^1 \\ & + \frac{(z^j)(z^j - 2) \cdots (z^j - d_j + 1)}{(-1)^{(d_j-2)}(d_j - 2)!} b_j^2 \\ & + \frac{(z^j)(z^j - 1)(z^j - 3) \cdots (z^j - d_j + 1)}{(-1)^{(d_j-3)}(d_j - 3)!} b_j^3 + \cdots \\ & + \frac{(z^j)(z^j - 1)(z^j - 2) \cdots (z^j - d_j + 2)}{(d_j - 1)!} b_j^{d_j}, \quad j = 1, 2, \dots, n. \end{aligned} \quad (12)$$

Using the Eq. (12), the constraint (3) can be transformed as follows:

$$\begin{aligned} \sum_{i=1}^m x_{ij} \geq & \frac{(z^j - 1)(z^j - 2) \cdots (z^j - d_j + 1)}{(-1)^{(d_j-1)}(d_j - 1)!} b_j^1 \\ & + \frac{(z^j)(z^j - 2) \cdots (z^j - d_j + 1)}{(-1)^{(d_j-2)}(d_j - 2)!} b_j^2 \\ & + \frac{(z^j)(z^j - 1)(z^j - 3) \cdots (z^j - d_j + 1)}{(-1)^{(d_j-3)}(d_j - 3)!} b_j^3 + \cdots \\ & + \frac{(z^j)(z^j - 1)(z^j - 2) \cdots (z^j - d_j + 2)}{(d_j - 1)!} b_j^{d_j}, \quad j = 1, 2, \dots, n. \end{aligned} \quad (13)$$

Finally, using the Eqs. (7), (10) and (13), the equivalent mathematical programming (**Model 2**) from the multi-choice transportation problem of **Model 1** has been derived as follows:

Model 2

$$\begin{aligned} \min : z = & \sum_{i=1}^m \sum_{j=1}^n \left[\frac{(z^{ij} - 1)(z^{ij} - 2) \cdots (z^{ij} - s_{ij} + 1)}{(-1)^{(s_{ij}-1)}(s_{ij} - 1)!} C_{ij}^1 \right. \\ & + \frac{(z^{ij})(z^{ij} - 2) \cdots (z^{ij} - s_{ij} + 1)}{(-1)^{(s_{ij}-2)}(s_{ij} - 2)!} C_{ij}^2 \\ & + \frac{(z^{ij})(z^{ij} - 1)(z^{ij} - 3) \cdots (z^{ij} - s_{ij} + 1)}{(-1)^{(s_{ij}-3)}(s_{ij} - 3)!} C_{ij}^3 + \cdots \\ & \left. + \frac{(z^{ij})(z^{ij} - 1)(z^{ij} - 2) \cdots (z^{ij} - s_{ij} + 2)}{(s_{ij} - 1)!} C_{ij}^{s_{ij}} \right] x_{ij} \end{aligned}$$

$$\begin{aligned}
\text{subject to } \sum_{j=1}^n x_{ij} &\leq \frac{(y^i - 1)(y^i - 2) \cdots (y^i - o_i + 1)}{(-1)^{(o_i-1)}(o_i - 1)!} a_i^1 \\
&+ \frac{(y^i)(y^i - 2) \cdots (y^i - o_i + 1)}{(-1)^{(o_i-2)}(o_i - 2)!} a_i^2 \\
&+ \frac{(y^i)(y^i - 1)(y^i - 3) \cdots (y^i - o_i + 1)}{(-1)^{(o_i-3)}(o_i - 3)!2!} a_i^3 + \cdots \\
&+ \frac{(y^i)(y^i - 1)(y^i - 2) \cdots (y^i - o_i + 2)}{o_i - 1!} a_i^{o_i}, \quad i = 1, 2, \dots, m \\
\sum_{i=1}^m x_{ij} &\geq \frac{(z^j - 1)(z^j - 2) \cdots (z^j - d_j + 1)}{(-1)^{(d_j-1)}(d_j - 1)!} b_j^1 \\
&+ \frac{(z^j)(z^j - 2) \cdots (z^j - d_j + 1)}{(-1)^{(d_j-2)}(d_j - 2)!} b_j^2 \\
&+ \frac{(z^j)(z^j - 1)(z^j - 3) \cdots (z^j - d_j + 1)}{(-1)^{(d_j-3)}(d_j - 3)!2!} b_j^3 + \cdots \\
&+ \frac{(z^j)(z^j - 1)(z^j - 2) \cdots (z^j - d_j + 2)}{(d_j - 1)!} b_j^{d_j}, \quad j = 1, 2, \dots, n \\
x_{ij} &\geq 0, \quad i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n \\
y^i &= 0, 1, 2, \dots, o_i - 1, \quad i = 1, 2, \dots, m \\
z^j &= 0, 1, 2, \dots, d_j - 1, \quad j = 1, 2, \dots, n \\
z^{ij} &= 0, 1, 2, \dots, s_{ij} - 1, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n
\end{aligned}$$

The following case study on petroleum refinery industry is considered to demonstrate the use of proposed methodology in this paper.

Case Study

A petroleum refinery company acts as a leader in the petroleum refinery and marketing sector. The company processes crude oil to produce petroleum products such as petrol, diesel, LPG and kerosene. The company has three storage terminals (here three supplies in terms of TP) and four depots (here four destinations in terms of TP) in a particular region. The company transports refined oil from the storage terminals to depots via tankers over the highways and railways. The transportation cost (in Rupees) for one unit (one gallon) of refined oil, each treated as of multi-choice parameter. This happens for different uncountable factors such as toll tax, fluctuation of fuel price etc. Due to these factors, assuming that the cost coefficients of TP follow multi-choice parameters (i.e., there are various routes are available from sources to destinations) and the cost matrix of the TP is defined as follows:

$$\begin{array}{cccc}
& D_1 & D_2 & D_3 & D_4 \\
\begin{array}{l} S_1 \\ S_2 \\ S_3 \end{array} & \left(\begin{array}{cccc} \{16, 19, 20\} & \{18, 19, 21\} & \{12, 16, 18, 19\} & \{14, 16\} \\ \{20, 21, 23, 26, 29\} & \{13, 16, 18\} & \{13, 17\} & \{14, 17, 19, 22\} \\ \{15, 16\} & \{20, 21, 22, 25\} & \{14, 16, 17\} & \{10, 12, 13, 15, 17\} \end{array} \right)
\end{array}$$

where the first element {16, 19, 20} of the above cost matrix indicates that x_{11} route either 16 or 19 or 20 required admissible cost/unit in Rupees.

The other elements of the above cost matrix of TP are defined in the similar way.

Due to manpower, electricity and other factors, assuming that supply amounts follow multi-choice parameters at S_1 , S_2 , and S_3 which are given as $a_1 \in \{7, 9, 10, 11, 14\}$, $a_2 \in \{10, 12, 15\}$ and $a_3 \in \{4, 5, 7, 8\}$ respectively. Due to market condition and other unpredictable factors, demand amounts also follow multi-choice type at each destination D_1 , D_2 , D_3 and D_4 which are given as $b_1 \in \{9, 11\}$, $b_2 \in \{3, 4, 6\}$, $b_3 \in \{2, 3, 5, 6\}$ and $b_4 \in \{5, 6, 7, 9\}$ respectively. Then the mathematical model of the case study can be expressed as follows:

Model 3:

$$\begin{aligned}
 \min : z = & \{16, 19, 20\}x_{11} + \{18, 19, 21\}x_{12} + \{12, 16, 18, 19\}x_{13} + \{14, 16\}x_{14} \\
 & + \{20, 21, 23, 26, 29\}x_{21} + \{13, 16, 18\}x_{22} + \{13, 17\}x_{23} + \{14, 17, 19, 22\}x_{24} \\
 & + \{15, 16\}x_{31} + \{20, 21, 22, 25\}x_{32} + \{14, 16, 17\}x_{33} + \{10, 12, 13, 15, 17\}x_{34} \\
 \text{subject to } & \sum_{j=1}^4 x_{1j} \leq a_1, \quad a_1 \in \{7, 9, 10, 11, 14\} \\
 & \sum_{j=1}^4 x_{2j} \leq a_2, \quad a_2 \in \{10, 12, 15\} \\
 & \sum_{j=1}^4 x_{3j} \leq a_3, \quad a_3 \in \{4, 5, 7, 8\} \\
 & \sum_{i=1}^3 x_{i1} \geq b_1, \quad b_1 \in \{9, 11\} \\
 & \sum_{i=1}^3 x_{i2} \geq b_2, \quad b_2 \in \{3, 4, 6\} \\
 & \sum_{i=1}^3 x_{i3} \geq b_3, \quad b_3 \in \{2, 3, 5, 6\} \\
 & \sum_{i=1}^3 x_{i4} \geq b_4, \quad b_4 \in \{5, 6, 7, 9\} \\
 & x_{ij} \geq 0, \quad \forall i \text{ and } j
 \end{aligned}$$

Using the methodology presented in see “Model Under Lagrange’s Interpolating Polynomial” section, the following equivalent mathematical model has been formulated (**Model 4**) of the above **Model 3** as:

Model 4

$$\begin{aligned}
 \min : z = & [-z_{11}^2 + 4z_{11} + 16]x_{11} + \left[\frac{1}{2}z_{12}^2 + \frac{1}{2}z_{12} + 18\right]x_{12} \\
 & + \left[\frac{1}{6}z_{13}^3 - \frac{3}{2}z_{13}^2 + \frac{16}{3}z_{13} + 12\right]x_{13} + [2z_{14} + 14]x_{14} \\
 & + \left[-\frac{1}{24}z_{21}^4 + \frac{1}{4}z_{21}^3 + \frac{841}{24}z_{21}^2 + \frac{3}{4}z_{21} + 20\right]x_{21} + \left[\frac{1}{2}z_{22}^2 + \frac{7}{2}z_{22} + 13\right]x_{22}
 \end{aligned}$$

$$\begin{aligned}
& + [4z_{23} + 13]x_{23} + \left[\frac{1}{3}z_{24}^3 - \frac{3}{2}z_{24}^2 + \frac{25}{6}z_{24} + 14 \right]x_{24} + [z_{31} + 15]x_{31} \\
& + \left[\frac{1}{3}z_{32}^3 - 1z_{32}^2 + \frac{5}{3}z_{32} + 20 \right]x_{32} + \left[-\frac{1}{2}z_{33}^2 + \frac{5}{2}z_{33} + 14 \right]x_{33} \\
& + \left[-\frac{1}{8}z_{34}^4 + \frac{13}{12}z_{34}^3 + \frac{137}{8}z_{34}^2 + \frac{7}{2}z_{34} + 10 \right]x_{34} \\
\text{subject to } & \sum_{j=1}^4 x_{1j} \leq -\frac{7}{8}y_1^4 + \frac{19}{3}y_1^3 + \frac{13}{12}y_1^2 + \frac{119}{12}y_1 + 7 \\
& \sum_{j=1}^4 x_{2j} \leq \frac{1}{2}y_2^2 + \frac{3}{2}y_2 + 10 \\
& \sum_{j=1}^4 x_{3j} \leq -\frac{1}{3}y_3^3 + \frac{3}{2}y_3^2 - \frac{1}{6}y_3 + 4 \\
& \sum_{i=1}^3 x_{i1} \geq 2z_1 + 9 \\
& \sum_{i=1}^3 x_{i2} \geq \frac{1}{2}z_2^2 + \frac{1}{2}z_2 + 3 \\
& \sum_{i=1}^3 x_{i3} \geq -\frac{1}{3}z_3^3 + \frac{3}{2}z_3^2 - \frac{1}{6}z_3 + 2 \\
& \sum_{i=1}^3 x_{i4} \geq \frac{11}{12}z_4^3 - \frac{11}{4}z_4^2 + \frac{17}{6}z_4 + 5 \\
& 0 \leq z_{11} \leq 2, \quad 0 \leq z_{12} \leq 2, \quad 0 \leq z_{13} \leq 3, \quad 0 \leq z_{14} \leq 1 \\
& 0 \leq z_{21} \leq 4, \quad 0 \leq z_{22} \leq 2, \quad 0 \leq z_{23} \leq 1, \quad 0 \leq z_{24} \leq 3 \\
& 0 \leq z_{31} \leq 1, \quad 0 \leq z_{32} \leq 3, \quad 0 \leq z_{33} \leq 2, \quad 0 \leq z_{34} \leq 4 \\
& 0 \leq y_1 \leq 4, \quad 0 \leq y_2 \leq 2, \quad 0 \leq y_3 \leq 3 \\
& 0 \leq z_1 \leq 1, \quad 0 \leq z_2 \leq 2, \quad 0 \leq z_3 \leq 3, \quad 0 \leq z_4 \leq 3 \\
& x_{ij} \geq 0, \quad \forall i \text{ and } j
\end{aligned}$$

Result and Discussion

Using Lingo10 package, the above **Model 4** is solved and listed the optimal solution. The optimal solution of the **Model 4** is obtained as: $x_{11} = 6.00$, $x_{13} = 2.00$, $x_{22} = 3.00$, $x_{31} = 3.00$, $x_{34} = 5.00$, and rest of the decision variables are zero. The minimum cost of the objective function is (in Rupees) 254.00. The selected parameters for cost coefficients of objective function, supply and demand that provide optimal solution to the problem are as follows:

$$\begin{aligned}
C_{11} &= 16, \quad C_{12} = 18, \quad C_{13} = 12, \quad C_{14} = 14, \\
C_{21} &= 20, \quad C_{22} = 13, \quad C_{23} = 13, \quad C_{24} = 17, \\
C_{31} &= 15, \quad C_{32} = 21, \quad C_{33} = 14, \quad C_{34} = 10,
\end{aligned}$$

$$a_1 = 9, \quad a_2 = 10, \quad a_3 = 8,$$

$$b_1 = 9, \quad b_2 = 3, \quad b_3 = 2, \quad b_4 = 5$$

After obtaining the optimal solution, the problem stated in the case study, the decision maker has decided for selecting the cost of each route, the necessary supply at each origin and the particular demand at each destination. The problem cannot be solved directly without using the multi-choice programming technique.

Conclusion

The aim of this paper is to discuss the solution procedure for multi-choice transportation problem with the consideration of all the parameters which are multi-choice types. Most of the real-life transportation problems, the cost coefficients of the objective function, supply and demand parameters may not be known previously due to uncountable factors. Due to this reason, the cost coefficients of the objective function, supply and demand parameters are multi-choices rather than single choice and keeping in view of all these factors, multi-choice transportation model has been formulated. In mathematical programming, there is no direct methodology to handle the multi-choice parameters. For the first time, and in this paper, Lagrange's interpolating polynomial is used to handle the multi-choice parameters of TP. Here, the following are the main advantages when Lagrange's interpolating polynomial is used to solve multi-choice transportation problem.

- Computation burden is less.
- The size of the problem is not increased due to absent of auxiliary constraints.
- Any number of choices for goal can be accommodated.

Finally, the proposed model is highly applicable for solving the real-life transportation problem and by following this model, the decision maker will be more benefitted to take right decision for giving more information.

In future study, the presented methodology of this paper can be extended to multi-objective interval valued multi-choice transportation problem and this methodology may be useful to extract better solution of decision making problems in supply chain management.

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