Modeling of Blood Flow in a Constricted Porous Vessel Under Magnetic Environment: An Analytical Approach

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Abstract A theoretical investigation concerning the effect of arterial constriction on the flow of blood through a porous vessel in the presence of externally applied magnetic field has been carried out. In this study blood is assumed to be homogeneous Newtonian fluid flowing through a large blood vessel. In order to solve the problem, Adomian's decomposition method is employed for obtaining its analytical solution. The analytical expressions for stream function, axial velocity component, wall shear stress are obtained and the corresponding numerical solutions for a specific set of dimensionless parameters have shown graphically. The results obtained for different values of the parameters show that the flow is appreciably influenced by the presence of magnetic field as well as the porosity of fluid medium. Thus the method developed here can capture small changes in the complex flow domain.

Keywords Stenosed artery · Magnetohydrodynamics · Porous medium · Adomian's decomposition method

Introduction

Many cardiovascular diseases such as atherosclerosis (also called stenosis) are responsible for world wide deaths of people. The initiation and progression of many such diseases are closely related to the nature of blood movement and the dynamical behaviour of the artery. Medical survey reveals that more than 80 % of the total deaths of humans are due to the diseases of arterial walls. Among them atherosclerosis is a dangerous disease that is caused due to deposition of cholesterol and some other substances on the endothelium and by the proliferation of connective tissues in the arterial wall. Stenosis may develop at more than one location of the cardiovascular system. Many researchers [1–5] indicated that the rheology

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of blood and the fluid dynamical properties of blood flow can play a significant role in the basic understanding, diagnosis and treatment of many cardiovascular, cerebrovascular and arterial diseases. To understand the effects of stenosis on blood flow in arteries, several studies have been conducted by Misra and Shit [6] and Misra et al. [7,8]. Shit and Roy [9] examined numerically the pulsatile flow and heat transfer of blood in stenosed artery under the influence of body acceleration, wherein they considered microrotation of microparticles.

Flow of blood through arteries is an important physiological problem and is of considerable interest to biomedical researchers, physiologists and clinicians. One of the important aspects of this problem that requires special attention is the deviation in the flow characteristics of blood when the artery is exposed to an externally applied magnetic field and therefore it has been the subject of vigorous research [10,11] in recent years. It is known that blood is a suspension of various cells (the major component being the erythrocytes) in plasma. Since erythrocytes have small negative charge, an applied magnetic field bears the capacity to influence the movement of erythrocytes and thereby to affect the flow of blood. Information available in the literatures regarding the exact mechanism by which the rheological properties of blood are affected in the presence of a magnetic field is, however, inadequate.

Higashi et al. [12] and Yamagishi [13] investigated the orientation of erythrocytes in magnetic fields of strengths 8 and 4 T, respectively. They proclaimed that the erythrocytes orient themselves with their flat side in the direction of the magnetic field. A three-parameter rheological model was used by Kirkovskii et al. [14] in order to study the influence of a variable magnetic field on the rheological properties of blood of patients suffering from rheumatoid arthritis. These parameters of the model are related to the hydrodynamic properties of erythrocytes suspension. They observed that out of the three parameters, only one has the capacity to affect the rheological properties of blood to an appreciable extent.

Nomenclature

(u', v')	Dimensionless velocity components
p'	Pressure
ψ	Stream function
τ_w	Wall shear stress
B_0	Magnetic field strength
k'	Permeability constant
l'_0	Length of stenosis
l	Non-dimensional length of arterial segment
R_0	Radius of arterial segment in normal portion
$m(\geq 2)$	Shape parameter of stenosis
ϵ'	Maximum height of the stenosis
ν	Kinematic viscosity of blood
ρ	Density of blood
σ	Electrical conductivity
Re	Reynolds number
M	Hartmann number
K	Permeability parameter

The primary contributors [15–17] studied the MHD blood flow in lungs, where blood can be visualized as the fluid flowing between the opposing layers of capillary endothelium, held apart by endothelium-covered “posts” of special tissue. The capillary endothelium is, in turn, covered by a thin layer (interstitial space) lining the alveoli, which has been treated as a porous medium. The application of magneto-hydrodynamic principles is the method for affecting the flow field in a desired direction by altering the structure of the boundary layer. In fact, the application of MHD principles to biology and medicine has been receiving growing interest of physiologists, medical practitioners and fluid dynamicists, owing to its importance in biomedical engineering as well as in the treatment of various pathological conditions. While investigating the effect of a magnetic field on the apparent viscosity of human blood, Haik et al. [18, 19] observed that blood viscosity is significantly affected under the action of a strong magnetic field. This can be attributed to the variation in the orientation of the erythrocytes due to the action of an external magnetic field. In case of some arterial diseases, in the lumen of an artery a porous structure is formed by fatty substances, cholesterol as well as due to the presence of blood clots. Coronary arteries are more prone to such pathological states [20]. In such an arterial segment, blood has to flow through a porous structure [21]. Several researchers [22, 23] have studied some problems of free convection and mass transfer in the flow of a viscous fluid through a porous medium.

In the 1980's, George Adomian introduced a new method to solve nonlinear functional equations [24]. The advantage of decomposition method is to give analytical approximate solution of nonlinear ordinary or partial differential equation which is rapidly convergent [24, 25]. The speed of convergency depends upon the choice of operator which may be a highest-ordered differential operator or a combination of linear operators or a multidimensional operator. This method does not take the help of any simplification for handling the nonlinear terms. Since the decomposition parameter is used only for grouping the terms, therefore, the non-linearity can be handled easily in the operator equation and accurate approximate solution may be obtained for any physical problem. The ADM has been successfully applied to solve nonlinear equations in studying many interesting problems arising in applied sciences and engineering [26, 27], and is usually characterized by its higher degree of accuracy. Haldar [28] and Mamaloukas et al. [29] have also used Adomian decomposition method for solving a problem of blood flow through a constricted artery under the action of an applied magnetic field. However, the above studies did not considered porosity effect as it is an important biological phenomena in the blood flow.

In the present paper, the effect of an external magnetic field on blood flow through a porous medium having single stenosis with different shape has been analyzed by assuming the flow to be Newtonian as in the case of large blood vessels. The analysis is carried out by employing Adomian decomposition method and some important predictions have been made basing upon the present study. Since the study has been carried out for a situation when the human body is subjected to an external magnetic field, it bears the promise of significant application in magnetic/electromagnetic therapy for the treatment of tumor/cancer.

Mathematical Modeling

Let us consider steady and laminar flow of blood through a porous medium in an arterial segment having single stenosis under the influence of an externally applied magnetic field (cf. Fig. 1). Although, blood is a suspension of erythrocytes (red blood cells) in plasma, it is assumed to be uniformly dense throughout. The analysis that follows pertains to situations where the electromagnetic force produced is negligible, i.e., the magnetic Reynolds number

Fig. 1 Schematic diagram of the model geometry

is very small. This is true for blood flow, since the electrical conductivity is very small. We also assume that flow is axially symmetric and usual no-slip boundary condition is taken into account for the present problem. The set of equations governing the flow of blood in artery in cylindrical polar co-ordinates (cf. [28,30]) can be put as

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial r'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + \nu \left(\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'} \frac{\partial u'}{\partial r'} + \frac{\partial^2 u'}{\partial x'^2} \right) - \frac{\nu}{k'} u' - \frac{\sigma B_0^2 u'}{\rho}, \quad (1)$$

$$u' \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial r'} = -\frac{1}{\rho} \frac{\partial p'}{\partial r'} + \nu \left(\frac{\partial^2 v'}{\partial r'^2} + \frac{1}{r'} \frac{\partial v'}{\partial r'} - \frac{v'}{r'^2} + \frac{\partial^2 v'}{\partial x'^2} \right) - \frac{\nu}{k'} v', \quad (2)$$

and the continuity equation

$$\frac{\partial(r' u')}{\partial x'} + \frac{\partial(r' v')}{\partial r'} = 0. \quad (3)$$

where (u', v') are the component of blood velocity in the axial and radial directions respectively, B_0 is the strength of applied magnetic field, ρ is the density of blood, ν is the kinematic coefficient of viscosity, σ is the electrical conductivity and k' denotes the permeability of the porous medium.

We consider a generalized geometry of single stenosis [31] as

$$R'(x') = R_0 \left[1 - \frac{\epsilon'}{R_0 l_0^m} \frac{m^{m-1}}{m-1} \left(l_0^{m-1} (x' - d') - (x' - d')^m \right) \right], \quad d' \leq x' \leq l_0' + d' \quad (4)$$

in which $R'(x')$ is the radius of the arterial segment in the stenotic region at an axial distance x' , R_0 is the radius of a segment of a normal artery, l_0' is the length of the stenosis, m is a parameter that determines the shape of the stenosis and ϵ' is the maximum height of the stenosis, at the location

$$x' = d' + l_0' / (m^{\frac{1}{m-1}}). \quad (5)$$

The ratio of the height of the stenoses to the radius of the normal portion of the artery is considered to be much less than unity.

The boundary conditions (cf. [30]) for the present problem may be put mathematically in the form

$$u' = v' = 0 \quad \text{at } r' = R'(x') \quad (6)$$

$$\text{and } \frac{\partial u'}{\partial r'} = 0 \quad \text{at } r' = 0. \quad (7)$$

Let us introduce the following transformations

$$u = \frac{u'}{U_0}, \quad v = \frac{v'}{U_0}, \quad r = \frac{r'}{R_0}, \quad x = \frac{x'}{R_0}, \quad p = \frac{p'}{\rho U_0^2} \quad (8)$$

where U_0 is the characteristic velocity.

Use of these dimensionless variables the governing equations (1) and (2) take the form

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + \frac{1}{Re} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial x^2} \right) - \frac{1}{Re} \left(M^2 + \frac{1}{K} \right) u, \quad (9)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial r} = -\frac{\partial p}{\partial r} + \frac{1}{Re} \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial x^2} \right) - \frac{v}{ReK}, \quad (10)$$

where Re , M and K are the Reynolds number, Hartmann number and porosity permeability parameter respectively, defined by

$$Re = \frac{U_0 R_0}{\nu}, \quad M^2 = \frac{\sigma B_0^2 R_0^2}{\mu}, \quad K = \frac{k'}{R_0^2}. \quad (11)$$

The dimensionless continuity equation (3) is

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0. \quad (12)$$

The boundary conditions also transformed to

$$u = v = 0 \quad \text{at } r = R(x) \quad (13)$$

$$\text{and } \frac{\partial u}{\partial r} = 0 \quad \text{at } r = 0, \quad (14)$$

where

$$R(x) = 1 - \frac{\epsilon}{l_0^m} \frac{m^{m-1}}{m-1} \left(l_0^{m-1} (x-d) - (x-d)^m \right), \quad d \leq x \leq l_0 + d \quad (15)$$

in which $\epsilon = \frac{\epsilon'}{R_0}$, $l_0 = \frac{l'_0}{R_0}$ and $d = \frac{d'}{R_0}$.

Let Q' be the constant volumetric flux across any cross-section of the artery may be defined as

$$Q' = 2\pi \int_0^{R'(x')} r' u' dr'. \quad (16)$$

Now substituting (8) into the above equation, we get

$$\int_0^{R(x)} ru dr = \frac{Q}{2}, \quad (17)$$

where

$$Q = \frac{Q'}{Q_0} \quad (18)$$

in which $Q_0 = \pi U_0 R_0^2$.

If the flow is steady and the system is closed (that is, no outward/inward flow takes place through the arterial wall), then $Q = 1$. In this case, Eq. (17) becomes

$$\int_0^{R(x)} ru \, dr = \frac{1}{2}. \quad (19)$$

Let us now introduce the stream function ψ defined by

$$u = -\frac{1}{r} \frac{\partial \psi}{\partial r}, \quad v = \frac{1}{r} \frac{\partial \psi}{\partial x}. \quad (20)$$

Then the continuity equation (12) is automatically satisfied and using (20), eliminating p from (9) and (10), we obtain the following governing equation

$$Re \left[\frac{1}{r} J - \frac{2}{r^2} \frac{\partial \psi}{\partial x} \nabla^2 \psi \right] = \nabla^4 \psi - \left(M^2 + \frac{1}{K} \right) r \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) - \frac{1}{K} \frac{\partial^2 \psi}{\partial x^2}, \quad (21)$$

where J is the Jacobian defined as

$$J = \frac{\partial(\nabla^2 \psi, \psi)}{\partial(r, x)} \quad (22)$$

and the operator ∇^2 is given by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial x^2}. \quad (23)$$

The boundary conditions for ψ can be written as

$$\psi = -\frac{1}{2}, \quad \frac{\partial \psi}{\partial r} = 0, \quad \text{at } r = R \quad (24)$$

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) = \psi = 0, \quad \text{at } r = 0. \quad (25)$$

Analytical Solution

In this section we are analyzing the solution of the Eq. (21) subjected to the boundary conditions (24) and (25) by using Adomian decomposition method (ADM).

Let us introduce the operator F , define as

$$F = \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r}.$$

Thus Eq. (21) becomes

$$F^2 \psi = Re T \psi - \frac{\partial^4 \psi}{\partial x^4} - 2 \frac{\partial^2}{\partial x^2} (F \psi) + \left(M^2 + \frac{1}{K} \right) F \psi + \frac{1}{K} \frac{\partial^2 \psi}{\partial x^2}, \quad (26)$$

where

$$T \psi = \frac{1}{r} J - \frac{2}{r^2} \frac{\partial \psi}{\partial x} \nabla^2 \psi. \quad (27)$$

Operating with the inverse F^{-2} on both sides of (26) we have [24, 25]

$$\psi = \psi_0 + F^{-2} \left[ReT\psi - \frac{\partial^4 \psi}{\partial x^4} - 2 \frac{\partial^2}{\partial x^2} (F\psi) + \left(M^2 + \frac{1}{K} \right) F\psi + \frac{1}{K} \frac{\partial^2 \psi}{\partial x^2} \right], \quad (28)$$

where ψ_0 is the solution of the homogeneous equation $F^2\psi_0 = 0$ and is given by

$$\psi_0 = \frac{1}{16} \alpha(x) r^4 + \beta(x) F_1^{-1} r \log r + \frac{1}{2} \gamma(x) r^2 + \delta(x). \quad (29)$$

The integration constants α , β , γ and δ can be obtained by using the boundary conditions (24) and (25). The inverse operator F_1^{-1} involved in (29) is defined by

$$F_1^{-1} = \int (\cdot) dr \quad (30)$$

has to be determined first.

Let us consider the equation

$$F\psi = G \quad (31)$$

and its solution yields,

$$\psi = [F_1^{-1} r (F_1^{-1} r^{-1})] G. \quad (32)$$

The above expression shows that F^{-1} has a relation of the form

$$F^{-1} = F_1^{-1} r (F_1^{-1} r^{-1}) \quad (33)$$

and also

$$F^{-2} = F_1^{-1} \left[r F_1^{-1} \left\{ r^{-1} F_1^{-1} r \left(r F_1^{-1} r^{-1} \right) \right\} \right]. \quad (34)$$

Now the stream function ψ and the nonlinear term $T\psi$ can be decomposed into the following forms

$$\psi = \sum_{n=0}^{\infty} \lambda^n \psi_n \quad (35)$$

and

$$T\psi = \sum_{n=0}^{\infty} \lambda^n P_n, \quad (36)$$

where the parameter λ used in (35) and (36) is not a perturbation parameter; it is used only for grouping the terms of different orders.

Therefore, the Eq. (28) can be written as

$$\psi = \psi_0 + \lambda F^{-2} \left[ReT\psi - \frac{\partial^4 \psi}{\partial x^4} - 2 \frac{\partial^2}{\partial x^2} (F\psi) + \left(M^2 + \frac{1}{K} \right) F\psi + \frac{1}{K} \frac{\partial^2 \psi}{\partial x^2} \right] \quad (37)$$

Now we substitute (35) and (36) into (37) and then comparing the like powers of λ on both sides of the resulting expression, we get

$$\psi_{n+1} = F^{-2} \left[ReP_n - \frac{\partial^4 \psi_n}{\partial x^4} - 2 \frac{\partial^2}{\partial x^2} (F\psi_n) + \left(M^2 + \frac{1}{K} \right) F\psi_n + \frac{1}{K} \frac{\partial^2 \psi_n}{\partial x^2} \right] \quad (38)$$

where $n = 0, 1, 2, \dots$. Once the component ψ_0 is determined, the other components of ψ can be easily determined from (38).

We further consider the parameterized decomposition of ψ_0 as

$$\psi_0 = \sum_{n=0}^{\infty} \lambda^n \psi_{0,n}. \quad (39)$$

Substituting (35), (36) and (39) into (37), we get

$$\psi_{n+1} = \psi_{0,n+1} + F^{-2} \left[Re P_n - \frac{\partial^4 \psi_n}{\partial x^4} - 2 \frac{\partial^2}{\partial x^2} (F \psi_n) + \left(M^2 + \frac{1}{K} \right) F \psi_n + \frac{1}{K} \frac{\partial^2 \psi_n}{\partial x^2} \right] \quad (40)$$

Since the integration constants involved in the solution of ψ_0 are $\alpha(x)$, $\beta(x)$, $\gamma(x)$ and $\delta(x)$, the parameterized decomposition of these constants are taken to be

$$\alpha = \sum_{n=0}^{\infty} \lambda^n \alpha_n, \quad \beta = \sum_{n=0}^{\infty} \lambda^n \beta_n, \quad \gamma = \sum_{n=0}^{\infty} \lambda^n \gamma_n, \quad \delta = \sum_{n=0}^{\infty} \lambda^n \delta_n. \quad (41)$$

We now substitute (39) and (41) into (29) and then compare the like powers of λ , we obtain

$$\psi_{0,n+1} = \frac{1}{16} \alpha_{n+1} r^4 + \beta_{n+1} F_1^{-1} r \log r + \frac{1}{2} \gamma_{n+1} r^2 + \delta_{n+1}. \quad (42)$$

The polynomials $P_0, P_1, \dots, P_n$ involved in (36) are known as Adomian's polynomials. They are defined in such a way that $P_0 \equiv P_0(\psi_0)$, $P_1 \equiv P_1(\psi_0, \psi_1)$, $P_2 \equiv P_2(\psi_0, \psi_1, \psi_2)$, $\dots$, $P_n \equiv P_n(\psi_0, \dots, \psi_n)$.

In order to determine these polynomials, we substitute (35) and (36) into (27) and then comparison of the terms of like powers of λ on the both sides of the resulting equation gives the following set of Adomian's polynomials

$$P_0 = \frac{1}{r} \frac{\partial(\nabla^2 \psi_0, \psi_0)}{\partial(r, x)} - \frac{2}{r^2} \frac{\partial \psi_0}{\partial x} \nabla^2 \psi_0 \quad (43)$$

$$P_1 = \frac{1}{r} \left[\frac{\partial(\nabla^2 \psi_0, \psi_1)}{\partial(r, x)} + \frac{\partial(\nabla^2 \psi_1, \psi_0)}{\partial(r, x)} \right] - \frac{2}{r^2} \left[\frac{\partial \psi_0}{\partial x} \nabla^2 \psi_1 + \frac{\partial \psi_1}{\partial x} \nabla^2 \psi_0 \right] \quad (44)$$

and so on.

If we consider two-term approximation of the solution ψ , then from (35) we get

$$\psi = \psi_0 + \psi_1 \quad (45)$$

by taking $\lambda = 1$.

Substituting ψ from (35) into (24) and (25), we obtain the boundary conditions for ψ_0 and ψ_1 as

$$\frac{\partial \psi_0}{\partial r} = 0, \quad \psi_0 = -\frac{1}{2} \quad \text{at } r = R \quad (46)$$

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi_0}{\partial r} \right) = \psi_0 = 0 \quad \text{at } r = 0 \quad (47)$$

$$\frac{\partial \psi_1}{\partial r} = \psi_1 = 0 \quad \text{at } r = R \quad (48)$$

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi_1}{\partial r} \right) = \psi_1 = 0 \quad \text{at } r = 0 \quad (49)$$

Using the boundary conditions (46) and (47) in (29), we obtain the expression for ψ_0 as

$$\psi_0 = \frac{1}{2R^4} (r^4 - 2r^2 R^2). \quad (50)$$

From (40) and (42), we can obtain the expression for ψ_1 by putting $n = 0$ as

$$\psi_1 = A(x)r^{10} + B(x)r^9 + C(x)r^8 + D(x)r^6 + \frac{1}{16}\alpha_1 r^4 + \beta_1 F_1^{-1} r \log r + \frac{1}{2}\gamma_1 r^2 + \delta, \quad (51)$$

where α_1 , β_1 , γ_1 and δ_1 are integrating constants to be determined by using the boundary conditions (48) and (49) as

$$\begin{aligned} \alpha_1 &= -16R^2(4AR^4 + 3CR^2 + 2D) \\ \beta_1 &= \delta_1 = 0 \\ \gamma_1 &= 2R^4(3AR^4 + 2CR^2 + D). \end{aligned} \quad (52)$$

The expressions for A , B , C and D are given by

$$A = \frac{Re}{960R^{11}}(20R'^3 - 13RR'R'' + R^2R''') \quad (53)$$

$$B = \frac{1}{2205KR^4}(16R'^2 - 4R'') \quad (54)$$

$$C = \frac{Re}{144R^9}(4R' - R^2R''' + 11RR'R'' - 16R'^3) - \frac{1}{576R^8} \\ \left(15R^2R''^2 + 20R^2R'R'' - 180RR'^2R'' + 210R'^4 - R^3R^{iv}\right) \quad (55)$$

$$D = \frac{Re}{48R^7}(12R'^3 - 9RR'R'' + R^2R''' - 8R') - \frac{1}{96R^6} \left(R^3R^{iv} - 12R^2R'R''' - 9R^2R''^2 \right. \\ \left. + 72RR'^2R'' - 60R'^4 + 80R'^2 - 6RR''\right) + \frac{1}{48R^4} \left(M^2 + \frac{1}{K}\right) + \frac{1}{96KR^2}(R'' - 2R'^2) \quad (56)$$

where R' , R'' , R''' and R^{iv} are the derivatives of R with respect to x indicating the orders according to their suffices.

The resulting expression for ψ_1 is given by

$$\begin{aligned} \psi_1 &= Ar^{10} + Br^9 + Cr^8 + Dr^6 - R^2(4AR^4 + 3CR^2 + 2D) \\ &\quad + R^4(3AR^4 + 2CR^2 + D). \end{aligned} \quad (57)$$

The axial velocity component can be found as

$$u = -\left[\frac{1}{r}\frac{\partial\psi_0}{\partial r} + \frac{1}{r}\frac{\partial\psi_1}{\partial r}\right]. \quad (58)$$

The expression for axial velocity is given by

$$u = -\left[\frac{2}{R^4}(r^2 - R^2) + 10Ar^8 + 9Br^7 + 8Cr^6 + 6Dr^4\right]. \quad (59)$$

The wall shear stress is defined by

$$\tau_w = -\frac{1}{4}\left(\frac{\partial u}{\partial r}\right)_{r=R}(1 + R'^2). \quad (60)$$

Substituting u from Eq. (59) into Eq. (60), one can further obtain

$$\tau_w = \frac{1}{R^3}\left(1 + 10AR^{10} + \frac{63}{4}BR^9 + 12CR^8 + 6DR^6\right)(1 + R'^2). \quad (61)$$

Results and Discussion

With a view to illustrating the applicability of the mathematical model developed and analyzed in the preceding sections, the analytical expressions for the stream function, axial velocity profile and wall shear stress are presented by taking into account the condition of no-slip velocity at the arterial wall. In order to have a proper insight into the flow behaviour of blood through a porous arterial segment with single stenosis, the variations of u and $\bar{\tau}$ have been estimated and the computed results have been presented in graphical form. The values (/ranges of values) of the physical parameters involved in the present model conform to experimental data for blood flow, existing in scientific literatures.

In the present model, the location positioned at $x = 0.25$, $x = 0.75$ and $x = 1.25$ correspond to the onset, throat and outset of the stenosis respectively (cf. Fig. 1).

Fig. 2 Velocity distribution at different axial positions when $m = 2$, $M^2 = 2.0$, $Re = 5.0$, $K = 0.1$ and $\epsilon = 0.4$

Fig. 3 Axial velocity distribution in the stenotic region for different values of m at $r = 0$ when $M^2 = 2.0$, $Re = 5.0$, $K = 0.1$ and $\epsilon = 0.4$

Fig. 4 Velocity variation at $x = 0.75$ for different values of M , when $Re = 5.0$, $K = 0.1$ and $m = 2$

Fig. 5 Velocity variation at $x = 1.25$ for different values of Re , when $M^2 = 2.0$, $K = 0.1$ and $m = 2$

Figures 2, 3, 4, 5, 6 and 7 give an idea of the axial velocity distribution in the case of blood flow in the vicinity of the stenotic portion of the arterial segment under the purview of the present study. Figure 2 gives the variation of the axial velocity at three different points (onset, throat and outlet) of the stenosis. It is seen from this figure that the axial velocity increases at the onset and outlet of the stenosis in the direction of increasing r (that is, from the axis of the artery to the wall), whereas the reverse trend is observed at the throat of the stenosis. It has been also observed from the figure that, axial velocity at the outlet, is greater than the axial velocity at onset of the stenosis. Figure 3 depicts of the distribution of the axial velocity along the axis of symmetry, in the stenotic region has been made for different values of shape parameter m . It is important to see that for $m = 2$, the axis of symmetry coincides with the axis of the stenosis, however, when $m > 2$, the axis of the symmetry shifts away from the stenosis-axis. This figure reveals that axial velocity increases upto a certain distance and after attaining its maximum, it decreases monotonically. Figure 4 illustrates the axial velocity distribution at $x = 0.75$ for different values of magnetic parameter (Hartmann

Fig. 6 Axial velocity distribution in the radial direction for different height of the stenosis with $M^2 = 2.0$, $Re = 5.0$, $K = 0.1$ and $m = 2$

Fig. 7 Variation of axial velocity with ϵ at the downstream of the stenosis for different values of K when $M^2 = 2.0$, $Re = 5.0$ and $m = 2$

number). It is seen that the axial velocity decreases with increasing Hartmann number. This observation agrees with the theory because with the increase in M , the Lorentz force increases and thereby flow velocity decreases. This implies that if we increase the strength of the magnetic field, the flow of blood will be impeded. As a result, the circulation diminishes indicating the favourable physiological condition. Therefore, it can be conclude that the effects of an external transverse magnetic field applied uniformly favours the condition of blood flow. Figure 5 gives the variation of axial velocity at the down stream of the stenosis i.e., at $x = 1.25$ with ϵ (maximum height of the stenosis) for different values of Reynolds number (Re). We observe from this figure that velocity is constant upto a certain height of the stenosis and beyond which it increases with the increase in stenosis height. Similarly, the axial velocity also increases with the increase of Reynolds number. Figure 6 represents the axial distribution at the throat of the stenosis for different height of the stenosis. This figure

Fig. 8 Variation of the wall shear stress (τ_w) at $x = 0.75$ for different values of m when $M^2 = 2.0$, $Re = 5.0$ and $K = 0.1$

indicates that for any value of ϵ , the velocity decreases in the direction of increasing r (that is, from the axis of the artery to the wall). Also it can be noted that as height of the stenosis increases, axial velocity increases upto a certain region from the axis of symmetry, beyond which a reverse trend is observed. For a given porous segment of an artery, Fig. 7 gives the variation in velocity with ϵ (maximum height of the second stenosis) for different values of permeability. This figure reveals that for the same permeability, when the value of ϵ is less than 0.3, the axial velocity decreases with the height of the stenosis, after that it increases monotonically. One can note from this figure that the velocity increases with the increase in permeability of porous medium. The computational results for the wall shear stress computed on the basis of the present study are presented in Figs. 8, 9, 10, 11 and 12 for different values of the parameters m , M^2 , Re , K and ϵ respectively. Figure 8 shows that the wall shear stress increases as the shape parameter m increases. The figure also shows that the shear stress increases for increasing values of ϵ (maximum height of stenosis). It is worthwhile to note from Fig. 9 the wall shear stress increases as the magnetic field strength increases. However, the wall shear stress reduces with the increase in Reynolds number and permeability of the porous medium (cf. Figs. 10, 11). Figure 12 depicts the longitudinal variation of the wall shear stress for the stenotic region. It is seen that for any values of ϵ , gradually wall shear stress increases and after attaining its maximum value, it decreases.

Conclusions

The present study deals with a theoretical investigation of blood flow through a porous artery having single stenosis under the action of an applied magnetic field. The study is quite suitable for the application to the hydrodynamical flow of blood in a coronary artery when it is subjected to the influence of an externally applied magnetic field. The reason is that the porosity of the coronary arteries is quite prominent. The porous matrix in the lumen of the artery is supposed to be formed on account of some particular type of pathology of the artery. The solution to the nonlinear equations that govern the flow is obtained by using Adomian's decomposition method. The main findings of the present problem are listed as follows:

Fig. 9 Variation of the wall shear stress (τ_w) at $x = 0.75$ for different values of M^2 when $m = 2$, $Re = 5.0$ and $K = 0.1$

Fig. 10 Variation of the wall shear stress (τ_w) at $x = 1.25$ for different values of Re when $m = 2$, $M^2 = 2.0$ and $K = 0.1$

- The investigation reveals very clearly that blood velocity can be controlled by suitably adjusting (increasing/decreasing) the magnetic field strength.
- The axial velocity of blood reduced with a considerable extend for sufficient strength of magnetic field.
- The blood velocity completely depends upon the formation of arterial plaques i.e., the shape of the arterial geometry.
- The axial velocity has an enhancing effect on permeability of the porous vessel.
- The wall shear stress has increasing effect on the depth of the stenosis.
- The low shear stress has occur at the downstream of the stenosis, which indicates for further deposition of cholesterol and other substances.

Therefore, the results presented here should be of sufficient interest to pathologists and medical scientists who usually want to understand the formation of plaques and to control blood flow during the entire surgical procedure.

Fig. 11 Variation of the wall shear stress (τ_w) at $x = 0.25$ for different values of K when $m = 2$, $M^2 = 2.0$ and $Re = 5.0$

Fig. 12 Longitudinal variation of the wall shear stress for different values of ϵ with $M^2 = 2.0$, $Re = 5.0$, $K = 0.1$ and $m = 2$

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