

A mini-review on preference modeling and articulation in multi-objective optimization: current status and challenges

Handing Wang¹ · Markus Olhofer² · Yaochu Jin¹

Received: 12 May 2017 / Accepted: 25 July 2017
© The Author(s) 2017. This article is an open access publication

Abstract Evolutionary multi-objective optimization aims to provide a representative subset of the Pareto front to decision makers. In practice, however, decision makers are usually interested in only a particular part of the Pareto front of the multi-objective optimization problem. This is particularly true when the number of objectives becomes large. Over the past decade, preference-based multi-objective optimization has attracted increasing attention from both academia and industry due to its significance in both theory and practice. Significant progress has been made in evolutionary multi-objective optimization and multi-criteria decision communities, although many open issues still remain to be addressed. This paper provides a concise review on preference-based multi-objective optimization, including various preference modeling methods and existing preference-based optimization methods, as well as a brief discussion of the main future challenges.

Keywords Multi-objective optimization · Preference modeling · Preference learning

Introduction

Most real-world optimization problems in science, engineering and even daily life need to take into account multiple and often conflicting criteria [1,2]. Such problems are known as multi-objective optimization problems (MOPs), which can be formulated mathematically as follows:

$$\min_{x \in \Omega} F(\mathbf{x}) = \{f_1(\mathbf{x}), \dots, f_m(\mathbf{x})\}, \quad (1)$$

where m is the number of objectives, \mathbf{x} is an n -dimensional decision vector, and the feasible region is defined by Ω . For two arbitrary solutions $\mathbf{x}^1, \mathbf{x}^2 \in \Omega$, \mathbf{x}^1 is said to dominate \mathbf{x}^2 (notated as $\mathbf{x}^1 \leq \mathbf{x}^2$), if $f_i(\mathbf{x}^1) \leq f_i(\mathbf{x}^2)$, and $F(\mathbf{x}^1) \neq F(\mathbf{x}^2)$. A solution $\mathbf{x}^* \in \Omega$ that cannot be dominated by any other feasible solutions in Ω is then called a Pareto optimal solution [3]. Typically, a set of Pareto optimal solutions exists for the MOP is described in Eq. (1), the set of \mathbf{x}^* is called as Pareto set (PS), and their corresponding objective vectors $F(\mathbf{x}^*)$ are called as Pareto front (PF).

It is helpful for decision makers (DMs) to make their decisions if the whole Pareto optimal set is already known, because the whole set can provide an overall picture of the distribution of Pareto optimal solutions. To obtain the entire PF, or to be more exact, to obtain a representative subset of the PF, a huge number of algorithms and methodologies in both communities of traditional mathematical programming and evolutionary computation have been designed in recent decades. Traditional mathematical programming methods such as the weighted aggregation methods [4] cannot identify the whole PF in one single run. Evolutionary algorithms (EAs), as population-based search methods, are believed to be well suited for solving MOPs in that they can achieve a set of non-dominated solutions in one run. Multi-objective

✉ Handing Wang
wanghanding.patch@gmail.com;
handing.wang@surrey.ac.uk

Markus Olhofer
markus.olhofer@honda-ri.de

Yaochu Jin
yaochu.jin@surrey.ac.uk

¹ Department of Computer Science, University of Surrey,
Guildford, Surrey GU2 7XH, UK

² Honda Research Institute Europe, 63073 Offenbach, Germany

evolutionary algorithms (MOEAs) [5] have now become a mature tool to solve MOPs. Generally speaking, existing MOEAs can be divided into three categories according to their selection criteria, namely Pareto-, indicator-, and reference-based MOEAs [6–8], even though a number of MOEAs might fall into more than one category or employ additional selection criteria.

Pareto-based MOEAs employ the Pareto dominance as their main selection methodology for convergence. Different diversity maintenance strategies are adopted in different Pareto-based MOEAs, such as crowding distance in NSGA-II [9] and environment selection in SPEA2 [10]. However, it was shown that Pareto-based MOEAs fail to solve many-objective optimization problems (MaOPs) that are defined to be MOPs with more than three objectives [11], mainly due to the fact that the dominance comparison becomes less effective when the number of objectives increases for a limited population size [12].

Indicator-based MOEAs use a single indicator as the selection criterion to replace the Pareto dominance in Pareto-based MOEAs. $I_{\epsilon+}$ [13, 14], Hypervolume [15], and R2 [16] have been applied in IBEA [13], HypE [17], and MOMBI [18], respectively.

Reference-based MOEAs decompose an MOP into a set of sub-problems according to the pre-assigned references, such as weights [19], reference points [20], reference vectors [21], and direction vectors [22, 23]. Different aggregation functions have been suggested to convert an MOP into a set of single-objective optimization problems, including weighted sum [3], Tchebycheff approach [3], and penalty-based boundary intersection (PBI) approach [24].

Although a representative subset of the overall PF can be located using most MOEAs for two- or three-objective optimization problems, selecting a few solutions to be implemented is not trivial. The decision-making process will become much harder for many-objective optimization problem, because human beings are believed to be able to handle up to seven criteria [25–27]. Therefore, articulation of preferences is essential for solving MOPs [28], which can guide optimization algorithms to find the most preferred solutions rather than the whole PF. To incorporate preferences into multi-objective optimization algorithms, the modeling and articulation of preferences must be considered. Generally, preferences can be involved in different stages of multi-objective optimization algorithms, and preference-based optimization methods can be classified into three categories: a priori, interactive, and a posteriori methods [28]. However, it is unclear which preferences are able to effectively incorporated into MOEAs, and in many cases the user does not have a clear preference when little knowledge about the problem is available.

This paper offers a brief survey on preference modeling and articulation in multi-objective optimization. In

section “Preference modeling methods”, various preference modeling methods are summarized. Section “Preference-based optimization methods” gives an account of existing preference-based optimization methods. Future challenges in preference modeling and preference guided multi-objective optimization are discussed in section “Challenges”. Section “Conclusion” concludes this paper.

Preference modeling methods

Various preference models have been reported in the literature [29], which can be largely classified into goals, weights, reference vectors, preference relation, utility functions, out-ranking, and implicit preferences.

Goals

The most straightforward way to articulate preference is to provide goal information [30], as shown in Fig. 1. Usually, users have some targets for different objectives [31, 32]. Thus, goals act as the additional criteria in multi-objective optimization to provide ranking with the preference information [33, 34]. In interactive approaches [35], DMs need to provide a goal point for a tentative efficient solution in each iteration. However, when DMs have no priori knowledge about the problems, they might set unreasonable goals which may mislead the search process [36].

Weights

DMs can assign different levels of importance to different criteria by using weights $\mathbf{w} = \{w_1, \dots, w_m\}$, which are a vector in the weight space as Fig. 2 shows. With the weights, multiple objectives can be converted into a single-objective function using an aggregation function [37–39]. Two most popular

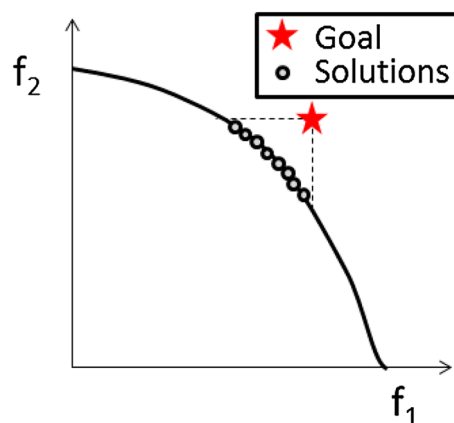


Fig. 1 Modeling preferences in terms of goals. In the figure, the *star* denotes a goal specified by the DM, whereas the *points* illustrate the optimal solutions that may be found by an optimization algorithm based on the goal

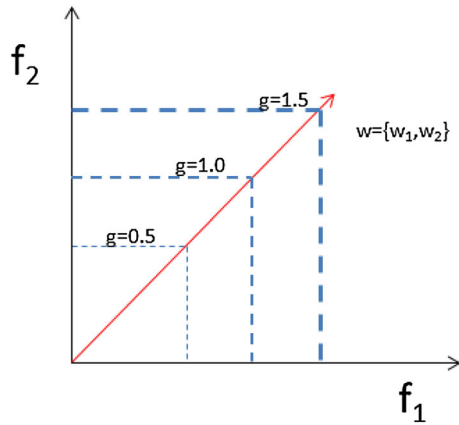


Fig. 2 Modeling preferences using weights, where w is a weight vector and the dotted lines are contour lines of the aggregation function g . The contour lines show the convergence tendency of an optimization algorithm on the aggregation function g

aggregation functions are the weighted sum [40] as shown in Eq. (2) and the Tchebycheff approach [3], as described in Eq. (3), where f_i is the i -th objective and w_i is the i -th weight. In Fig. 2, the dotted line is the contour of the aggregation function g , which indicates the convergence tendency of the search of g on the specific weight w . Authors in [41, 42] have modified the dominance by a weight vector. However, similar to goals, it is hard for DMs to provide accurate weights without a full understanding of the characteristics of the problem.

$$g^{ws} = \sum_{i=1}^m w_i f_i \tag{2}$$

$$g^{te} = \max_{1 \leq i \leq m} \{w_i f_i\} \tag{3}$$

Reference vectors

Reference vectors or points provide the expectation to or importance of the objectives. Reference vectors and weights are similar in their aggregation functionality, although they do have different physical meanings, and consequently, different influences on the search process. Usually, reference vectors represent the directions of the solution vector, whereas weights indicate the importance of different objectives. Reference vectors are in the objective space, whilst weights are in the weight space. Because of the inherent connection between reference vectors and weights, they can be converted into each other. The reference vectors in RVEA [21] and reference points in NSGA-III [20] are converted from uniformly distributed weights.

Taking the PBI approach [24] in Fig. 3 as an example, which is the fitness function in NSGA-III [20], the relationship between a solution and a reference vector v is described by two distances, where d_1 is the projection distance and d_2

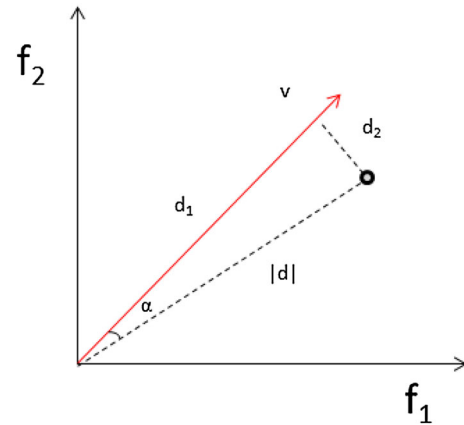


Fig. 3 The PBI approach decomposes distance $|d|$ to two orthogonal distance d_1 and d_2 , while the APD approach penalizes replaces the distance with an angle α

is the perpendicular distance to a reference vector v . With d_1 accounting for convergence and d_2 promoting diversity, PBI selects solutions based on Eq. (4), where θ is the penalty factor. The recently proposed angle penalized distance (APD) in RVEA [21] adopts the acute angle between the reference vector and solution vectors to replace the Euclidean distance as shown in Eq. (5), where $p(\alpha)$ is a penalty function related to the angle α . It has been shown that angles provide a more scalable measure for diversity in high-dimensional spaces.

$$g^{PBI} = d_1 + \theta d_2 \tag{4}$$

$$g^{APD} = (1 + p(\alpha))|d| \tag{5}$$

Neither the Tchebycheff nor PBI method is suited to the PFs in different shapes [43]. Recently, different aggregation functions are proposed for both preferences in reference vectors and weights. For example, adaptive scalarizing methods in [44–46] change the aggregation function during the MOEA, the Tchebycheff method is used in a reversed form for a convex PF [47], and the PBI method is inverted based on a nadir point [48].

Preference relation

DMs have different preferences on different objectives; thus, some objectives might be not equally important during the process of decision making [49–51]. Table 1 lists the symbol representation of the importance of objectives, and as a result, objectives can be sorted with a preferred order as $f_1 \geq f_2 \geq f_3 \approx f_4$. With that preference relation [52], the search can be narrowed down by converting into weights, the method in [2] is one example with the binary preference. The main disadvantage is that the preference relation cannot handle non-transitivity. During the process of decision making, DMs

Table 1 Preference relation

Relation	Meaning	Relation	Meaning
\prec	Less important	\succ	More important
\ll	Much less important	\gg	Much more important
\approx	Equally important	$\#$	Do not care
\dashv	Not important	$!$	Important

gradually learn their preferences. Analytic hierarchy process (AHP) [53] is a measurement using pairwise comparisons to calculate priority scales based on the judgements from DMs, which might be inconsistent. Various examples have employed AHP for decision making.

Utility functions

Preferences can be characterized by utility functions [54–56], where the preference information is implicitly involved in the fitness function to rank solutions [57,58]. Unlike preference relations, the utility function sorts solutions rather than objectives in an order. For example, there are N solutions \mathbf{x}_1 to \mathbf{x}_N , DMs are required to input their preferences for those solutions, $\mathbf{x}_1 \prec_{\text{pref}} \mathbf{x}_2 \prec_{\text{pref}} \dots \prec_{\text{pref}} \mathbf{x}_N$, for instance. Then, an imprecisely specified multi-attribute value theory (ISMAUT) formulation is employed to infer the relative importance of objectives to modify the fitness function. However, utility functions are based on a strong assumption that all attributes of the preferences are independent, thereby being unable to handle non-transitivity [59,60].

Outranking

Outranking [61] is a different ranking for objective preferences allowing non-transitivity, which is different from the preference relation [62]. To construct an outranking [63], the preference and indifference thresholds for each objective are input by a preference ranking organization method for enrichment evaluations (PROMETHEE) [64]. Every two solutions are compared according to those thresholds. Then, a preference ranking is obtained for outranking-based methods to search the preferred solutions [65]. However, the outranking-based methods require too many parameter settings, which is hard for DMs when the number of objectives increases [64].

Implicit preferences

In some cases, DMs have little knowledge to articulate any sensible preferences. Nevertheless, there are some solutions on the PF that are naturally preferred, even if no problem specific preference can be proposed. Those solutions can be detected based on the curvature of PF [66]. For example, a knee point, around which a small improvement of any objec-

tive causes a large degeneration of others, is always of interest to DMs as an implicit preferred solution [67–69]. Examples include model selection in machine learning [70,71] and sparse reconstruction [72].

There is no widely accepted definition for knee points, and specifying knee points are notoriously difficult in high-dimensional objective spaces. Existing approaches to identifying knee points can be divided into two categories: angle- and distance-based approaches [68]. Angle-based approaches measure the angle between a solution and its two neighbors and search the knee point according to the obtained angle [72]. Although angle-based approaches are straightforward, they can be applied to bi-objective optimization problems only. Distance-based approaches can handle problems with more than two objectives, which search the knee point according to the distance to a pre-defined hyperplane [73].

In addition to knee points, extreme points or the nadir point can work as a special form of preferences [74]. Extreme points are the solutions with the worst objective values on the PF. A nadir point is a combination of extreme points. With extreme points or the nadir point, DMs can acquire the knowledge on the range of the PF to input their preferences more accurately [75–77].

Discussions

The above formulations of preferences share several similarities. For example, although weights and reference vectors are different concepts, weights are sometimes used as references, and vice versa. All the existing preference formulations are scalable to many objectives, but their complexity significantly increases. Although different preference models may have very different properties, they all describe the objective importance or priority in their own ways, except that utility functions sort the importance of solutions rather than objectives.

DMs might articulate preferences with uncertainty. To model uncertainty in preferences, small perturbations can be introduced into goals, weights, or reference vector-based methods. Thus, fuzzy logic can be used as a natural means for handling uncertainty in preferences [78,79], such as reference points [35], weights [80], preference relation [81,82], and outranking [63]. Preference relation, utility function, and

outranking are not strictly based on objective importance in values, which allow uncertainty to a certain degree. DMs might have inconsistent preferences during the search. In these such cases, goal-, weight-, and reference vector based methods might fail, because they focus on the previous preferences too much and may lose diversity. Also, preference relation and utility function based methods cannot handle preference inconsistency. Only outranking allows inconsistency in preferences to some degree. Furthermore, DMs can introduce inappropriate preferences, which might lead to infeasible solutions. There has not been any specific research dedicated to handling inappropriate preferences, and fuzzy preferences might provide a solution to this problem.

Preference-based optimization methods

The existing preference-based optimization methods can be classified into three categories according to the time when preferences are incorporated, i.e. a priori, interactive, and a posteriori methods [28]:

- *A priori methods* In these methods, DMs need to input their preferences before optimization starts. In such methods, the main difficulty lies in the fact that DMs may have limited knowledge about the problem and their preferences may be inaccurate or even misleading.
- *A posteriori methods* In a posteriori methods, a set of representative Pareto optimal is obtained using an optimization algorithm, from which DMs choose a small number of solutions according to their preferences. In comparison with the a priori methods, DMs are able to better understand the trade-off relationships between the objectives in the a posteriori methods. Most existing multi-objective evolutionary algorithms (MOEAs) [83] belong to this category. It should be noted, however, that it becomes increasingly hard to obtain a representative solution set as the number of objectives increases [84].
- *Interactive methods* Interactive methods [85, 86] enable DMs to articulate their preferences in the course of optimization. In interactive methods, DMs are allowed to modify their preferences, typically based on the domain knowledge acquired during the optimization [32, 38, 87]. With the increasing understanding of the problem as the optimization proceeds, DMs are able to fine tune their preferences according to the obtained solutions in each iteration. With the revised preference, the interactive methods search for new preferred solutions, which usually needs less computational cost compared with the a posteriori methods. In the existing interactive methods, only one single preference model is adopted, such as

reference vectors [88–91], weights [92–95], preference relation [96–98], and utility functions [99].

Non-evolutionary preference-based optimization methods

Traditional multiple criteria decision making (MCDM) methodologies are non-evolutionary and usually involve in a certain type of preference information. During the MCDM processes, the following assumptions hold [4, 100–102]:

- Parts of non-dominated solutions are expected to be found.
- DMs are expected to understand the problem and are able to provide reasonable preferences.
- Satisfactory optimal solutions are expected to be output finally.

According to [103], classical MCDM approaches can be divided into two types: aggregation procedures and synthesizing criteria.

The aggregation-based MCDM approaches are based on weights [104]. Thus, decision making is mathematically defined by Eq. (2), where m is the number of criteria, w is the weight. For those approaches, DMs need to have a clear idea about how to set the weights. However, it is very hard for human beings to provide precise quantitative importance levels for different objectives. In some cases, good solutions cannot be easily distinguished from the poor solutions by Eq. (2).

Unlike the aggregation-based MCDM approaches which are based on explicit mathematical formula as a fixed preference, the synthesizing criterion-based approaches are based on implicit rules. For example, outranking and utility function are two implicit and flexible preference models. Outranking sorts the objective preferences, and utility function sorts the solution preferences. So far, the elimination and choice expressing reality (ELECTRE) [105] and preference ranking organization method for enrichment evaluations methods (PROMETHEE) [106] are two main outranking approaches; utilities additives (UTA) methods [107] are utility function-based approaches [108].

In addition to the above mentioned aggregation procedures and synthesizing criteria, fuzzy logic [109], decision rules [110], multi-objective mathematical programming [111], and objective classification [112] have been employed to improve the performance of the MCDM approaches.

Evolutionary preference-based optimization methods

While non-evolutionary methods pay much attention to preference handling, most MOEAs focus on obtaining the whole solution set as the a posteriori methods. In this section, we

discuss the a priori methods in MOEAs, which embed preferences into their fitness functions for narrowing down the selection [113]. So far, goals, weights reference and utility functions have been used to integrate preferences in MOEAs [29].

As mentioned in section “Preference modeling methods”, goals are straightforward preferences for MOEAs. Different formulations have been used to incorporate preferences in existing MOEAs. For example, the algorithm in [114] considers goal preferences as constraints by Eq. (6), where g_i is the goal for the i -th objective. One issue with this approach is that no solution can be achieved if the goals are set unreasonably by DMs for MOPs with a discontinuous PF. To address this issue, an algorithm is proposed in [115] that divides the constraints into hard and soft constraints according to the priority of objectives.

$$f_i(\mathbf{x}) < g_i \quad (6)$$

In fact, existing reference-based MOEAs can naturally be seen as preference-based MOEAs, which assign preferences uniformly distributed in the whole objective space and decompose one MOP into a number of single-objective optimization problems. Preferences in those algorithms are presented in different models, such as weights in MOEA/D [19], direction vectors in DVCMOA [22] and MOEA/D-M2M [23], reference vectors in RVEA [21], and reference points in NSGA-III [20]. So far, a majority of preference-based MOEAs are based on weights [37,38,116–120]. The second most widely-used preference model in MOEAs is reference vectors. The algorithms reported in [20,21,76] model preferences by reference vectors or points. Most recently, preference articulation methods based on reference points, reference vectors, and weights have been examined and compared on a hybrid electric vehicle control problem [121].

Another popular type of preferences adopted in MOEAs is the achievement scalarizing function (ASF) [28,122–124]. The formulation of ASF is shown in Eq. (7), where \mathbf{w} is a weight and \mathbf{z} is a reference point. The light beam search [125,126] projects a beam of light from a reference point onto the PF, resulting in a small neighborhood on the PF. To increase the robustness of the preference-based MOEAs to DMs, the light beam is used [127] to replace the reference point in the achievement scalarizing function [128]. Moreover, the achievement scalarizing function has been employed to approximate hypervolume [34,129,130]. Based on ASF, an interactive MOEA termed I-SIBEA [131] is proposed by selecting new solutions according to a weighted hypervolume.

$$g^{\text{ASF}} = \max_{1 \leq i \leq m} (w_i f_i - z_i) \quad (7)$$

Several utility function-based MOEAs have also been developed. For example, an algorithm was presented in [58], which might be the first MOEA that implicitly involves the preferences in the fitness function using a utility function. To guide MOEAs toward preferred solutions, robust ordinal regression is employed to approximate the utility in [97].

Challenges

Even though preferences have recently gained increasing attention and have been studied for decades, many issues remain to be addressed in the future.

Preference adaptation for various formulations

As mentioned before, different preference models have been developed and existing preference-based MOEAs are designed according to a specific preference model. However, preferences provided by DMs might be in different forms, thus no single MOEA is able to deal with various types of preferences, making them less flexible to be used in practice. Thus, it would be very desirable if various preference models can be converted into a single preference so that they can be incorporated into a preference-based MOEA. So far, not much work has been reported on converting one preference model into another, with a few exceptions, e.g., preference relations are converted into weights in [2] and fuzzy preferences are turned into weights in [82]. Thus, it is necessary to develop a general framework for converting different preference models so that the advantages and disadvantages of the existing methods can be properly compared in terms of their ability to handle uncertainty, conflicts, as well as the robustness in obtain preferred solutions.

Preference learning

Learning user preferences

Preferences play a very important role in MCDM. Preferences given by DMs are consistent to a certain degree, notwithstanding that the fact that DMs might change their preferences in interacting with the optimizer. Thus, the system should be able to learn the preferences of DMs based on history data. Although there are many mature techniques in machine learning [132] and data mining [133] that can help learn preferences of DMs, little attention has been paid to this research topic with a few exceptions [134], where preferences of DMs are learned by training a single or multiple surrogate models [135] using a semi-supervised learning algorithm. As the work in [134] indicated, a proper learning algorithm should be chosen and attention should be paid to

the fact that the learned preferences are able to be incorporated in MOEAs.

Handling preference violation

Without sufficient information about the problem, it is likely for DMs to provide less reasonable or even misleading preferences. In some cases, no solutions can be found for some preferences, for example when the Pareto front is discontinuous.

In case there are a group of DMs, it should be taken into account that the preferences given by different group members might be conflicting to each other [136]. As pointed out in [36], priority, independence, and unanimity of individual preferences need to be taken into account in using preferences from multiple DMs.

Psychological study

Decision making can be seen as a psychological construct in the selection of several alternative actions [137]. In some cases, the processing capacity of DMs is limited due to the overwhelmed results from decision making systems [138]. To ensure that decision making systems are compatible with the psychology of DMs, attention should be paid to theory of decision making in the psychological level [139]. Experiments reported in [140] indicate that the improvement of the forecasting performance can be achieved with the help of a psychological model. Therefore, we believe that a further understanding of the psychology of DMs would build a proper bridge between decision making systems and DMs, which can further improve the efficiency of the preference-based methods [25, 26, 57, 112].

Analysis of relationships between decision variables and objectives

Relationship between objectives

The conflict between two objectives means that the improvement on one objective would deteriorate the other. The conflict might be global or local [141–143]. For locally conflicting objectives, they are conflicting with each other in some regions but not in other regions. However, the existing research on objective reduction focuses on global redundancy between objectives [144–146], but little work has been conducted on locally conflicting objectives. The search on locally redundant objectives wastes computational cost, and the results in [141] indicate that objective reduction approaches for some problems with globally conflicting objectives can still improve the performance of MOEAs on the problems with locally redundant objectives. Therefore, detecting locally conflicting objectives, reducing locally

redundant objectives, and analyzing the effects of locally conflicting objectives on the PFs are of great interest. Moreover, analysis of the correlation between objectives can help group objectives into a number of groups to simplify the representation to DMs, because human beings can only handle around seven objectives.

Several approaches can be used to help DMs to understand the relationship between objectives. In [147], objectives are divided into five classes to help DMs understand the trade-off. Self-organizing maps (SOMs) [148] have been shown to be promising in revealing the tradeoff relationships between objectives [149, 150]. Correlation is another effective tool for analyzing the relationship between objectives. Different metrics have been proposed to measure the degree of correlation (both linear and non-linear), covariance, mutual information entropy [151], and non-linear correlation information entropy (NCIE) [152, 153], for instance. Based on these relations, many mature data mining techniques can be employed to choose a subset of conflicting objectives to simplify the original problem, such as feature selection [146], principal component analysis (PCA) [154], and maximum variance unfolding (MVU) [155]. The Pareto corner search evolutionary algorithm (PCSEA) [145] is a newly proposed objective reduction approach. It only searches the corners on PFs. Then, it uses the obtained solutions to analyze the relationship between objectives and identify a subset of non-correlated objectives.

Knee points show the conflicting degree and are interesting to DMs if they do not have specific preferences [68]. Knee point detection is based on the different definition for 2 or 3-objective problems [73]. The definition of knee point in MaOPs is not yet well-established, because the conflicting degree might vary with different pairs of objectives. The sensitivity to changes in individual objectives may exist in some particular regions on the PF, which can be considered as partial knee points that are of interested to DMs.

Functional maps from decision variables to objectives

For the real-world applications, noises or uncertainties are inevitable. In such situations, DMs prefer solutions that are robust against small changes in decision variables [156–158]. There have been some discussions on robust multi-objective optimization [159–164], but little research has studied the robustness in decision making, except for measuring attractiveness by a categorical based evaluation technique (MACBETH) [165]. The analysis of the mapping relationship from decision variables to objectives [166] helps searching robust solutions in the preference-based methods.

To analyze the mapping relationship from decision variables to objectives, artificial neural network (ANN) [167, 168], Bayesian learning [169, 170], and the estimation of distribution algorithm (EDA) [171, 172] have been employed.

Benchmark design

So far, several MOP test suites have been proposed, such as ZDT [173], DTLZ [174], and WFG [175] problems. However, no benchmark problems have been proposed to test preference-based optimization methods. Thus, it is very desirable to design MOP test suites tailored for evaluating the performance of preference based MOEAs. To design preference-based MOP benchmarks, following aspects need to be considered.

- *Preference simulation* It is necessary to simulate the preferences with artificial functions [176], where uncertainty and the response to the algorithm should also be taken into account.
- *Objective correlation* Both global and local conflicts should be designed in the benchmark.
- *Ground truth* The true optimal solutions should be provided for assessing the performance.

Performance assessment

Several performance indicators for measuring the performance of MOEAs have been proposed, such as generational distance (GD) [177], inverted generational distance (IGD) [173], and hypervolume [15]. However, not many performance indicators exist that are dedicated to evaluation of preference-based methods with few exceptions [178], which considers both dominance and the distance to the preferences. In addition, an ideal metric for preference-based methods should evaluate whether the obtained solutions truly reflect the preferences, regardless of their preference modeling types.

Visualization

Visualization plays an important role in interactions between DMs and preference-based optimization methods. When the number of objectives equals or larger than four, visualization becomes a challenge. Existing approaches can be divided into three classes, namely parallel coordinate, mapping, and aggregation tree [179].

The approaches based on parallel coordinates provide the visualization of individual solutions by a parallel coordinate system. In that system, there are parallel axes that can describe values of all objectives. Parallel coordinates [1] use a polyline with vertices on the parallel axes, while heatmap [180] uses color to present the values on the parallel axes. Those approaches can only show the trade-off between two adjacent objectives.

Other approaches include those adopt dimension reduction techniques that can preserve the Pareto dominance relationship among individuals in both global and local areas,

such as Sammon mapping [181], neuroscale [182], radial coordinate visualization (RadViz) [183], SOM [149,184], and Isomap [185]. These approaches are not as straightforward as the parallel coordinate-based approaches in analyzing the tradeoff relationships between the objectives and are time-consuming.

Approaches based on aggregation tree [186,187] measure the harmony between objectives to visualize the relation between objectives. However, this kind of approaches cannot show individual solutions.

Most existing visualization tools are not straightforward for DMs to understand. Ideally, both dominance and preference relationship should be presented in the visualization. Moreover, DMs should be able to zoom in interesting regions to get more detailed information.

Scalable multi-objective optimization test problems. In: Proceedings of the 2002 Congress on Evolutionary Computation, CEC 2002, vol 1. IEEE Computer Society, pp 825–830

Conclusion

Since preference-based multi-objective optimization is strongly motivated from the real-world applications, research interests in this area have increased in recent years. Indeed, preference modelling is also a common need in many areas of artificial intelligence in which decision making is involved [188–190]. It becomes thus clear that preference modelling and learning are important not only for decision making and evolutionary optimization, but also for artificial intelligence research.

In this paper, we provide a concise review of research on preference modelling and preference-based optimization methods. We discuss the open issues in preference modelling and preference based optimization. It is emphasized that the importance of preference-based multi-objective optimization is of paramount practical significance and preferences must be incorporated in many-objective optimization, where obtaining a representative subset of the entire Pareto front is less likely.

Acknowledgements This work was supported in part by an EPSRC Grant (No. EP/M017869/1) on “Data-driven surrogate-assisted evolutionary fluid dynamic optimisation”, in part by the Joint Research Fund for Overseas Chinese, Hong Kong and Macao Scholars of the National Natural Science Foundation of China (No. 61428302), and in part by the Honda Research Institute Europe.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made.

References

- Fleming P, Purshouse R, Lygoe R (2005) Many-objective optimization: An engineering design perspective. In: Evolutionary multi-criterion optimization. Springer, New York, pp 14–32
- Parmee IC, Cvetković D, Watson AH, Bonham CR (2000) Multiobjective satisfaction within an interactive evolutionary design environment. *Evol Comput* 8(2):197–222
- Miettinen K (1999) Nonlinear multiobjective optimization. Springer, New York
- Steuer RE (1986) Multiple criteria optimization: theory, computation, and applications. Wiley, New York
- Zhou A, Bo-Yang Q, Li H, Zhao S-Z, Suganthan PN, Zhang Q (2011) Multiobjective evolutionary algorithms: a survey of the state of the art. *Swarm Evol Comput* 1(1):32–49
- Li B, Li J, Tang K, Yao X (2015) Many-objective evolutionary algorithms: a survey. *ACM Comput Surv* 48(1):13
- Wang H, Jin Y, Yao X (2017) Diversity assessment in many-objective optimization. *IEEE Trans Cybern* 47(6):1510–1522
- Wagner T, Beume N, Naujoks B (2007) Pareto-, aggregation-, and indicator-based methods in many-objective optimization. In: Evolutionary multi-criterion optimization. Springer, New York, pp 742–756
- Deb K, Pratap A, Agarwal S, Meyarivan TAMT (2002) A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Trans Evol Comput* 6(2):182–197
- Zitzler E, Laumanns M, Thiele L (2001) SPEA2: Improving the strength Pareto evolutionary algorithm. In: Proceedings of EUROGEN 2001. Evolutionary methods for design, optimization and control with applications to industrial problems. Citeseer, pp 1–21
- Praditwong K, Yao X (2007) How well do multi-objective evolutionary algorithms scale to large problems. In: Proceedings of the Congress on Evolutionary Computation. IEEE, pp 3959–3966
- Ishibuchi H, Tsukamoto N, Nojima Y (2008) Evolutionary many-objective optimization: a short review. In: Proceedings of the Congress on Evolutionary Computation. IEEE, pp 2419–2426
- Zitzler E, Künzli S (2004) Indicator-based selection in multiobjective search. In: International Conference on Parallel Problem Solving from Nature. Springer, New York, pp 832–842
- Wang H, Jiao L, Yao X (2015) Two_Arch2: an improved two-archive algorithm for many-objective optimization. *IEEE Trans Evol Comput* 19(4):524–541
- Zitzler E, Thiele L (1999) Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach. *IEEE Trans Evol Comput* 3(4):257–271
- Brockhoff D, Wagner T, Trautmann H (2012) On the properties of the R2 indicator. In: The genetic and evolutionary computation conference. ACM, New York, pp 465–472
- Bader J, Zitzler E (2011) HypE: an algorithm for fast hypervolume-based many-objective optimization. *Evol Comput* 19(1):45–76
- Gómez RH, Coello CAC (2013) MOMBI: a new metaheuristic for many-objective optimization based on the R2 indicator. In: Proceedings of the Congress on Evolutionary Computation. IEEE, pp 2488–2495
- Zhang Q, Li H (2007) MOEA/D: a multiobjective evolutionary algorithm based on decomposition. *IEEE Trans Evol Comput* 11(6):712–731
- Deb K, Jain H (2014) An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part i: solving problems with box constraints. *IEEE Trans Evol Comput* 18(4):577–601
- Cheng R, Jin Y, Olhofer M, Sendhoff B (2016) A reference vector guided evolutionary algorithm for many-objective optimization. *IEEE Trans Evol Comput* 20(5):773–791. doi:10.1109/TEVC.2016.2519378
- Jiao L, Wang H, Shang R, Liu F (2013) A co-evolutionary multi-objective optimization algorithm based on direction vectors. *Inf Sci* 228:90–112
- Liu H-L, Fangqing G, Zhang Q (2014) Decomposition of a multiobjective optimization problem into a number of simple multiobjective subproblems. *IEEE Trans Evol Comput* 18(3):450–455
- Dennis J, Das I (1998) Normal-boundary intersection: a new method for generating Pareto optimal points in nonlinear multicriteria optimization problems. *SIAM J Optim* 8(3):631–657
- Miller GA (1956) The magical number seven, plus or minus two: some limits on our capacity for processing information. *Psychol Rev* 63(2):81
- Nisbett RE, Wilson TD (1977) Telling more than we can know: verbal reports on mental processes. *Psychol Rev* 84(3):231
- Slovic P, Lichtenstein S (1971) Comparison of Bayesian and regression approaches to the study of information processing in judgment. *Organ Behav Human Perform* 6(6):649–744
- Thiele L, Miettinen K, Korhonen PJ, Molina J (2009) A preference-based evolutionary algorithm for multi-objective optimization. *Evol Comput* 17(3):411–436
- Hirsch C, Shukla PK, Schmeck H (2011) Variable preference modeling using multi-objective evolutionary algorithms. In: Evolutionary multi-criterion optimization. Springer, New York, pp 91–105
- Gembicki FW (1974) Vector optimization for control with performance and parameter sensitivity indices. Ph.D. thesis, Ph.D. Thesis, Case Western Reserve Univ., Cleveland, Ohio
- Wang R, Purshouse RC, Fleming PJ (2013) Preference-inspired coevolutionary algorithms for many-objective optimization. *IEEE Trans Evol Comput* 17(4):474–494
- Fonseca CM, Fleming PJ (1993) Genetic algorithms for multiobjective optimization: Formulation discussion and generalization. In: Proceedings of the International Conference on Genetic Algorithms, vol 93. Citeseer, pp 416–423
- Deb K (1999) Solving goal programming problems using multi-objective genetic algorithms. In: Proceedings of the 1999 congress on evolutionary computation, CEC 99, vol 1. IEEE, pp 77–84
- Wagner T, Trautmann H (2010) Integration of preferences in hypervolume-based multiobjective evolutionary algorithms by means of desirability functions. *IEEE Trans Evol Comput* 14(5):688–701
- Sakawa M, Kato K (2002) An interactive fuzzy satisficing method for general multiobjective 0–1 programming problems through genetic algorithms with double strings based on a reference solution. *Fuzzy Sets Syst* 125(3):289–300
- Coello CAC (2000) Handling preferences in evolutionary multi-objective optimization: a survey. In: Proceedings of the Congress on Evolutionary Computation, vol 1. IEEE, pp 30–37
- Phelps S, Köksalan M (2003) An interactive evolutionary metaheuristic for multiobjective combinatorial optimization. *Manage Sci* 49(12):1726–1738
- Köksalan M, Karahan I (2010) An interactive territory defining evolutionary algorithm: iTDEA. *IEEE Trans Evol Comput* 14(5):702–722
- Wang R, Purshouse RC, Fleming PJ (2015) Preference-inspired co-evolutionary algorithms using weight vectors. *Eur J Oper Res* 243(2):423–441
- Wang R, Zhou Z, Ishibuchi H, Liao T, Zhang T (2016) Localized weighted sum method for many-objective optimization. *IEEE Trans Evol Comput*. doi:10.1109/TEVC.2016.2611642

41. Branke J, Kaußler T, Schmeck H (2001) Guidance in evolutionary multi-objective optimization. *Adv Eng Softw* 32(6):499–507
42. Branke J, Deb K (2005) Integrating user preferences into evolutionary multi-objective optimization. In: *Knowledge Incorporation in Evolutionary Computation*. Springer, New York, pp 461–477
43. Ma X, Zhang Q, Yang J, Zhu Z (2017) On Tchebycheff decomposition approaches for multi-objective evolutionary optimization. *IEEE Trans Evol Comput*. doi:10.1109/TEVC.2017.2704118
44. Ishibuchi H, Sakane Y, Tsukamoto N, Nojima Y (2009) Adaptation of scalarizing functions in MOEA/D: an adaptive scalarizing function-based multiobjective evolutionary algorithm. In: *Evolutionary multi-criterion optimization*. Springer, New York, pp 438–452
45. Ishibuchi H, Sakane Y, Tsukamoto N, Nojima Y (2010) Simultaneous use of different scalarizing functions in MOEA/D. In: *The genetic and evolutionary computation conference*. ACM, New York, pp 519–526
46. Wang R, Zhang Q, Zhang T (2016) Decomposition-based algorithms using Pareto adaptive scalarizing methods. *IEEE Trans Evol Comput* 20(6):821–837
47. Liu HL, Gu FQ, Cheung YM (2010) T-MOEA/D: MOEA/D with objective transform in multi-objective problems. In: *Information Science and Management Engineering (ISME), International Conference of*, vol 2. IEEE, pp 282–285
48. Sato H (2014) Inverted PBI in MOEA/D and its impact on the search performance on multi and many-objective optimization. In: *The Genetic and Evolutionary Computation Conference*. ACM, New York, pp 645–652
49. Haimes YY, Hall WA (1974) Multiobjectives in water resource systems analysis: the surrogate worth trade off method. *Water Resour Res* 10(4):615–624
50. Brafman RI (2011) Relational preference rules for control. *Artif Intell* 175(7):1180–1193
51. Zitzler E, Thiele L, Bader J (2008) SPAM: set preference algorithm for multiobjective optimization. In: *International Conference on Parallel Problem Solving from Nature*, vol 5199. Springer, New York, pp 847–858
52. Jaimes AL, Coello CAC (2009) Study of preference relations in many-objective optimization. In: *The genetic and evolutionary computation conference*. ACM, New York, pp 611–618
53. Thomas L (2008) Saaty. Decision making with the analytic hierarchy process. *Int J Serv Sci* 1(1):83–98
54. Feldman AM (1989) Preferences and utility. In: *Welfare economics and social choice theory*. Springer, New York, pp 9–22
55. Jeantet G, Spanjaard O (2011) Computing rank dependent utility in graphical models for sequential decision problems. *Artif Intell* 175(7):1366–1389
56. Pedro LR, Takahashi R (2011) Modeling decision-maker preferences through utility function level sets. In: *Evolutionary multi-criterion optimization*. Springer, New York, pp 550–563
57. Costa CAC (2012) *Readings in multiple criteria decision aid*. Springer Science & Business Media, New York
58. Greenwood GW, Hu X, D'Ambrosio JG (1996) Fitness functions for multiple objective optimization problems: Combining preferences with Pareto rankings. In: *FOGA*, vol 96, pp 437–455
59. White CC III, Sage AP, Dozono S (1984) A model of multiattribute decisionmaking and trade-off weight determination under uncertainty. *IEEE Trans Syst Man Cybern* 14(2):223–229
60. Cvetković D, Parmee IC (2002) Preferences and their application in evolutionary multiobjective optimization. *IEEE Trans Evol Comput* 6(1):42–57
61. Rekiek B, De Lit P, Pellichero F, L'Eglise T, Falkenauer E, Delchambre A (2000) Dealing with user's preferences in hybrid assembly lines design. *IFAC Proc Vol* 33(17):989–994
62. Waegeman W, De Baets B (2011) On the ERA ranking representability of pairwise bipartite ranking functions. *Artif Intell* 175(7):1223–1250
63. Siskos J, Lombard J, Oudiz A (1986) The use of multicriteria outranking methods in the comparison of control options against a chemical pollutant. *J Oper Res Soc* 37(4):357–371
64. Brans JP, Vincke P, Mareschal B (1986) How to select and how to rank projects: the PROMETHEE method. *Eur J Oper Res* 24(2):228–238
65. Massebeuf S, Fonteix C, Kiss LN, Marc I, Pla F, Zaras K (1999) Multicriteria optimization and decision engineering of an extrusion process aided by a diploid genetic algorithm. In: *Proceedings of the 1999 congress on evolutionary computation, CEC 99*, vol 1. IEEE, pp 14–21
66. Shukla PK, Emmerich M, Deutz A (2013) A theoretical analysis of curvature based preference models. In: *International Conference on Evolutionary Multi-Criterion Optimization*. Springer, New York, pp 367–382
67. Rachmawati L, Srinivasan D (2009) Multiobjective evolutionary algorithm with controllable focus on the knees of the Pareto front. *IEEE Trans Evol Comput* 13(4):810–824
68. Branke J, Deb K, Dierolf H, Osswald M (2004) Finding knees in multi-objective optimization. In: *International Conference on Parallel Problem Solving from Nature*. Springer, New York, pp 722–731
69. Deb K, Gupta S (2011) Understanding knee points in bicriteria problems and their implications as preferred solution principles. *Eng Optim* 43(11):1175–1204
70. Jin Y, Bernhard S (2008) Pareto-based multi-objective machine learning: An overview and case studies. *IEEE Trans Syst Man Cybern Part C Appl Rev* 38(3):397–415
71. Smith C, Jin Y (2014) Evolutionary multi-objective generation of recurrent neural network ensembles for time series prediction. *Neurocomputing* 143:302–311
72. Li L, Yao X, Stolkin R, Gong M, He S (2014) An evolutionary multiobjective approach to sparse reconstruction. *IEEE Trans Evol Comput* 18(6):827–845
73. Zhang X, Tian Y, Jin Y (2015) A knee point-driven evolutionary algorithm for many-objective optimization. *IEEE Trans Evol Comput* 19(6):761–776
74. Wang H, He S, Yao X (2017) Nadir point estimation for many-objective optimization problems based on emphasized critical regions. *Soft Comput* 21(9):2283–2295
75. Branke J, Deb K, Miettinen K, Slowiński R (2008) *Multiobjective optimization. Interactive and evolutionary approaches*, vol 5252. Springer, New York
76. Deb K, Kumar A (2007) Interactive evolutionary multi-objective optimization and decision-making using reference direction method. In: *The genetic and evolutionary computation conference*. ACM, New York, pp 781–788
77. Amiri JC, Ekhtiari M, Yazdani M (2011) Nadir compromise programming: a model for optimization of multi-objective portfolio problem. *Expert Syst Appl* 38(6):7222–7226
78. Voget S, Kolonko M (1998) Multidimensional optimization with a fuzzy genetic algorithm. *J Heuristics* 4(3):221–244
79. Hadjali A, Mokhtari A, Pivert O (2012) Expressing and processing complex preferences in route planning queries: towards a fuzzy-set-based approach. *Fuzzy Sets Syst* 196:82–104
80. Pirjanian P (1998) *Multiple objective action selection and behavior FPCsion using voting*. Ph.D. thesis, Department of Medical Informatics and Image Analysis, Institute of Electronic Systems, Aalborg University, Aalborg, Denmark
81. Fodor JC, Roubens MR (1994) *Fuzzy preference modelling and multicriteria decision support*, vol 14. Springer Science & Business Media, New York

82. Jin Y, Sendhoff B (2002) Fuzzy preference incorporation into evolutionary multi-objective optimization. In: Proceedings of the 4th Asia-Pacific Conference on Simulated Evolution and Learning, vol 1, pp 26–30
83. Abraham A, Jain L (2005) Evolutionary multiobjective optimization. Springer, New York
84. Khare V, Yao X, Deb K (2003) Performance scaling of multi-objective evolutionary algorithms. In: Evolutionary Multi-Criterion Optimization. Springer, New York, pp 376–390
85. Miettinen K, Hakanen J, Podkopaev D (2016) Interactive non-linear multiobjective optimization methods. In: Multiple criteria decision analysis. Springer, New York, pp 927–976
86. Said LB, Bechikh S, Ghédira K (2010) The r -dominance: a new dominance relation for interactive evolutionary multicriteria decision making. *IEEE Trans Evol Comput* 14(5):801–818
87. Deb K, Chaudhuri S (2005) I-EMO: an interactive evolutionary multi-objective optimization tool. In: Pattern Recognition and Machine Intelligence. Springer, New York, pp 690–695
88. Miettinen K, Podkopaev D, Ruiz F, Luque M (2015) A new preference handling technique for interactive multiobjective optimization without trading-off. *J Global Optim* 63(4):633–652
89. Miettinen K, Ruiz F (2016) NAUTILUS framework: towards trade-off-free interaction in multiobjective optimization. *J Bus Econ* 86(1–2):5–21
90. Deb K, Chaudhuri S (2007) I-MODE: an interactive multi-objective optimization and decision-making using evolutionary methods. In: Evolutionary Multi-Criterion Optimization. Springer, New York, pp 788–802
91. Sindhya K, Ruiz AB, Miettinen K (2011) A preference based interactive evolutionary algorithm for multi-objective optimization: PIE. In: Evolutionary Multi-Criterion Optimization. Springer, New York, pp 212–225
92. Gong M, Liu F, Zhang W, Jiao L, Zhang Q (2011) Interactive MOEA/D for multi-objective decision making. In: The genetic and evolutionary computation conference. ACM, New York, pp 721–728
93. Ruiz AB, Luque M, Miettinen K, Saborido R (2015) An interactive evolutionary multiobjective optimization method: Interactive WASF-GA. In: Evolutionary multi-criterion optimization. Springer, New York, pp 249–263
94. Ruiz AB, Luque M, Ruiz F, Saborido R (2015) A combined interactive procedure using preference-based evolutionary multi-objective optimization. application to the efficiency improvement of the auxiliary services of power plants. *Expert Syst Appl* 42(21):7466–7482
95. Liu R, Wang R, Feng W, Huang J, Jiao L (2016) Interactive reference region based multi-objective evolutionary algorithm through decomposition. *IEEE Access* 4:7331–7346
96. Battiti R, Passerini A (2010) Brain-computer evolutionary multi-objective optimization: a genetic algorithm adapting to the decision maker. *IEEE Trans Evol Comput* 14(5):671–687
97. Branke J, Greco S, Słowiński R, Zielniewicz P (2009) Interactive evolutionary multiobjective optimization using robust ordinal regression. In: Evolutionary multi-criterion optimization. Springer, New York, pp 554–568
98. Deb K, Sinha A, Korhonen PJ, Wallenius J (2010) An interactive evolutionary multiobjective optimization method based on progressively approximated value functions. *IEEE Trans Evol Comput* 14(5):723–739
99. Sinha A, Korhonen P, Wallenius J, Deb K (2014) An interactive evolutionary multi-objective optimization algorithm with a limited number of decision maker calls. *Eur J Oper Res* 233(3):674–688
100. Chankong V, Haimes YY (1983) Multiobjective decision making: theory and methodology. North Holland, New York
101. Hwang CL, Masud ASM (1979) Multiple objective decision making-methods and applications. Springer, New York
102. Torokhti A, Howlett P (1985) Theory of multiobjective optimization, vol 176. Elsevier, Amsterdam
103. Roy B (2005) Paradigms and challenges. In: Multiple criteria decision analysis: state of the art surveys. Springer, New York, pp 3–24
104. Bouyssou D (1986) Some remarks on the notion of compensation in MCDM. *Eur J Oper Res* 26(1):150–160
105. Figueira J, Mousseau V, Roy B (2005) ELECTRE methods. In: Multiple Criteria Decision Analysis: State of the Art Surveys. Springer, New York, pp 133–153
106. Brans JP, Mareschal B (2005) PROMETHEE methods. In: Multiple criteria decision analysis: state of the art surveys. Springer, New York, pp 163–186
107. Siskos Y, Grigoroudis E, Matsatsinis NF (2005) UTA methods. In: Multiple criteria decision analysis: state of the art surveys. Springer, New York, pp 297–334
108. Dyer JS (2005) MAUT-multiattribute utility theory. In: Multiple criteria decision analysis: state of the art surveys. Springer, New York, pp 265–292
109. Meyer P, Roubens M (2005) Choice, ranking and sorting in fuzzy multiple criteria decision aid. In: Multiple criteria decision analysis: state of the art surveys. Springer, New York, pp 471–503
110. Greco S, Matarazzo B, Słowiński R (2005) Decision rule approach. In: Multiple criteria decision analysis: state of the art surveys. Springer, New York, pp 507–555
111. Ehrgott M, Wiecek MM (2005) Multiobjective programming. In: Multiple criteria decision analysis: state of the art surveys. Springer, New York, pp 667–708
112. Larichev OI (1992) Cognitive validity in design of decision-aiding techniques. *J Multi Criteria Decis Anal* 1(3):127–138
113. Rachmawati L, Srinivasan D (2006) Preference incorporation in multi-objective evolutionary algorithms: a survey. In: Proceedings of the Congress on Evolutionary Computation. IEEE, pp 962–968
114. Fonseca CM, Fleming PJ (1998) Multiobjective optimization and multiple constraint handling with evolutionary algorithms. I. a unified formulation. *IEEE Trans Syst Man Cybern Part A Syst Hum* 28(1):26–37
115. Tan KC, Khor EF, Lee TH, Sathikannan R (2003) An evolutionary algorithm with advanced goal and priority specification for multi-objective optimization. *J Artif Intell Res* 18:183–215
116. Yang X, Gen M (1994) Evolution program for bicriteria transportation problem. *Comput Ind Eng* 27(1):481–484
117. Wilson PB, Macleod MD (1993) Low implementation cost IIR digital filter design using genetic algorithms. *IEE/IEEE Workshop Nat Algorithms Signal Process* 1:1–4
118. Wienke D, Lucasius C, Kateman G (1992) Multicriteria target vector optimization of analytical procedures using a genetic algorithm: part I. theory, numerical simulations and application to atomic emission spectroscopy. *Anal Chim Acta* 265(2):211–225
119. Quagliarella D, Vicini A (1997) Coupling genetic algorithms and gradient based optimization techniques. In: Genetic algorithms and evolution strategy in engineering and computer science: recent advances and industrial applications. Wiley, Hoboken, pp 289–309
120. Wang R, Purshouse RC, Giagkiozis I, Fleming PJ (2015) The iPICEA-g: a new hybrid evolutionary multi-criteria decision making approach using the brushing technique. *Eur J Oper Res* 243(2):442–453
121. Cheng R, Rodemann T, Fischer M, Olhofer M, Jin Y (2017) Evolutionary many-objective optimization of hybrid electric vehicle control: from general optimization to preference articulation. *IEEE Trans Emerg Top Comput Intell* 1(2):97–111
122. Wierzbicki AP (1980) The use of reference objectives in multiobjective optimization. In: Multiple criteria decision making theory and application. Springer, New York, pp 468–486

123. Korhonen PJ, Laakso J (1986) A visual interactive method for solving the multiple criteria problem. *Eur J Oper Res* 24(2):277–287
124. Nikulin Y, Miettinen K, Mäkelä MM (2012) A new achievement scalarizing function based on parameterization in multiobjective optimization. *OR Spectr* 34(1):69–87
125. Jaskiewicz A, Słowiński R (1999) The ‘light beam search’ approach—an overview of methodology applications. *Eur J Oper Res* 113(2):300–314
126. Liu R, Wang X, Liu J, Fang L, Jiao L (2013) A preference multi-objective optimization based on adaptive rank clone and differential evolution. *Nat Comput* 12(1):109–132
127. Deb K, Kumar A (2007) Light beam search based multi-objective optimization using evolutionary algorithms. In: *Proceedings of the Congress on Evolutionary Computation*. IEEE, pp 2125–2132
128. Molina J, Santana LV, Hernández-Díaz AG, Coello CAC, Caballero R (2009) g-dominance: Reference point based dominance for multiobjective metaheuristics. *Eur J Oper Res* 197(2):685–692
129. Ishibuchi H, Tsukamoto N, Sakane Y, Nojima Y (2009) Hypervolume approximation using achievement scalarizing functions for evolutionary many-objective optimization. In: *Proceedings of the Congress on Evolutionary Computation*. IEEE, pp 530–537
130. Ishibuchi H, Tsukamoto N, Sakane Y, Nojima Y (2010) Indicator-based evolutionary algorithm with hypervolume approximation by achievement scalarizing functions. In: *The Genetic and Evolutionary Computation Conference*. ACM, New York, pp 527–534
131. Chugh T, Sindhya K, Hakanen J, Miettinen K (2015) An interactive simple indicator-based evolutionary algorithm (I-SIBEA) for multiobjective optimization problems. In: *Evolutionary multi-criterion optimization*. Springer, New York, pp 277–291
132. Bishop CM (2006) *Pattern recognition and machine learning*. Springer, New York
133. Han J, Kamber M, Pei J (2011) *Data mining: concepts and techniques: concepts and techniques*. Elsevier, Amsterdam
134. Sun X, Gong D, Jin Y, Chen S (2013) A new surrogate-assisted interactive genetic algorithm with weighted semi-supervised learning. *IEEE Trans Cybern* 43(2):85–698
135. Jin Y (2011) Surrogate-assisted evolutionary computation: Recent advances and future challenges. *Swarm Evol Comput* 1(2):61–70
136. Arrow KJ (2012) *Social choice and individual values*, vol 12. Yale University Press, New Haven
137. Janis IL, Mann L (1977) *Decision making: a psychological analysis of conflict, choice, and commitment*. Free Press, New York
138. Ackoff RL (1967) Management misinformation system. *Manag Syst* 14(4):B-147
139. Edwards W (1954) The theory of decision making. *Psychol Bull* 51(4):380
140. Hoch SJ, Schkade DA (1996) A psychological approach to decision support systems. *Manag Sci* 42(1):51–64
141. Wang H, Yao X (2016) Objective reduction based on nonlinear correlation information entropy. *Soft Comput* 20(6):2393–2407
142. Freitas AR, Fleming PJ, Guimaraes F (2013) A non-parametric harmony-based objective reduction method for many-objective optimization. In: *IEEE International Conference on Systems, Man, and Cybernetics*. IEEE, pp 651–656
143. de Freitas ARR, Fleming PJ, Guimarães FG (2015) Aggregation trees for visualization and dimension reduction in many-objective optimization. *Inf Sci* 298:288–314
144. Brockhoff D, Zitzler E (2009) Objective reduction in evolutionary multiobjective optimization: theory and applications. *Evol Comput* 17(2):135–166
145. Singh HK, Isaacs A, Ray T (2011) A Pareto corner search evolutionary algorithm and dimensionality reduction in many-objective optimization problems. *IEEE Transactions on Evol Comput* 15(4):539–556
146. Jaimes AL, Coello CAC, Chakraborty D (2008) Objective reduction using a feature selection technique. In: *The Genetic and Evolutionary Computation Conference*. ACM, New York, pp 673–680
147. Hämäläinen JP, Miettinen K, Tarvainen P, Toivanen J (2003) Interactive solution approach to a multiobjective optimization problem in a paper machine headbox design. *J Optim Theory Appl* 116(2):265–281
148. Kangas JA, Kohonen Tk, Laaksonen JT (1990) Variants of self-organizing maps. *IEEE Trans Neural Netw* 1(1):93–99
149. Obayashi S, Sasaki D (2003) Visualization and data mining of Pareto solutions using self-organizing map. In: *Evolutionary multi-criterion optimization*. Springer, New York, pp 796–809
150. Nekolny B (2010) *Contextual self-organizing maps for visual design space exploration*. Master’s thesis, Iowa State University
151. Maes F, Collignon A, Vandermeulen D, Marchal G, Suetens P (1997) Multimodality image registration by maximization of mutual information. *IEEE Trans Med Imaging* 16(2):187–198
152. Wang Q, Shen Y, Zhang JQ (2005) A nonlinear correlation measure for multivariable data set. *Phys D* 200(3):287–295
153. Wang H, Jin Y (2017) Efficient nonlinear correlation detection for decomposed search in evolutionary multi-objective optimization. In: *Proceedings of the congress on evolutionary computation*, pp 649–656
154. Deb K, Saxena DK (2005) On finding Pareto-optimal solutions through dimensionality reduction for certain large-dimensional multi-objective optimization problems. Technical report, Indian Institute of Technology Kanpur
155. Saxena DK, Duro JA, Tiwari A, Deb K, Zhang Q (2013) Objective reduction in many-objective optimization: linear and nonlinear algorithms. *IEEE Trans Evol Comput* 17(1):77–99
156. Deb K, Gupta H (2006) Introducing robustness in multi-objective optimization. *Evol Comput* 14(4):463–494
157. Jin Y, Sendhoff B (2003) Trade-off between performance and robustness: an evolutionary multiobjective approach. In: Fonseca CM, Fleming PJ, Zitzler E, Thiele L, Deb K (eds) *Evolutionary multi-criterion optimization*, EMO 2003. *Lecture Notes in Computer Science*, vol 2632. Springer, Berlin, pp 237–251
158. Jin Y, Tang K, Xin Y, Sendhoff B, Yao X (2013) A framework for finding robust optimal solutions over time. *Memet Comput* 5(1):3–18
159. Deb K, Gupta H (2005) Searching for robust Pareto-optimal solutions in multi-objective optimization. *Lect Notes Comput Sci* 3410:150–164
160. Sülflow A, Drechsler N, Drechsler R (2007) Robust multi-objective optimization in high dimensional spaces. In: *Evolutionary multi-criterion optimization*. Springer, New York, pp 715–726
161. Gunawan S, Azarm S (2005) Multi-objective robust optimization using a sensitivity region concept. *Struct Multidiscip Optim* 29(1):50–60
162. Li M, Azarm S, Aute V (2005) A multi-objective genetic algorithm for robust design optimization. In: *The Genetic and Evolutionary Computation Conference*. ACM, New York, pp 771–778
163. Lim D, Ong Y-S, Jin Y, Sendhoff B, Lee BS (2006) Inverse multi-objective robust evolutionary optimization. *Genet Progr Evol Mach* 7(4):383–404
164. Greco S, Słowiński R, Figueira JR, Mousseau V (2010) Robust ordinal regression. In: *Trends in multiple criteria decision analysis*. Springer, New York, pp 241–283
165. Bana CA, De Corte JM, Vansnick JC et al (2005) *On the mathematical foundation of MACBETH*. Springer, New York
166. Wang H, Jiao L, Shang R, He S, Liu F (2015) A memetic optimization strategy based on dimension reduction in decision space. *Evol Comput* 23(1):69–100

167. Adra SF, Dodd TJ, Griffin IA, Fleming PJ (2009) Convergence acceleration operator for multiobjective optimization. *IEEE Trans Evol Comput* 13(4):825–847
168. Gaspar-Cunha A, Vieira A (2004) A hybrid multi-objective evolutionary algorithm using an inverse neural network. In: *Hybrid Metaheuristics*, pp 25–30
169. Laumanns M, Ocenasek J (2002) Bayesian optimization algorithms for multi-objective optimization. In: *International Conference on Parallel Problem Solving from Nature*. Springer, New York, pp 298–307
170. Khan N, Goldberg DE, Pelikan M (2002) Multi-objective Bayesian optimization algorithm. *Urbana* 51:684–684
171. Larranaga P, Lozano JA (2002) Estimation of distribution algorithms: a new tool for evolutionary computation, vol 2. Springer Science & Business Media, New York
172. Hauschild M, Pelikan M (2011) An introduction and survey of estimation of distribution algorithms. *Swarm Evol Comput* 1(3):111–128
173. Zitzler E, Deb K, Thiele L (2000) Comparison of multiobjective evolutionary algorithms: Empirical results. *Evol Comput* 8(2):173–195
174. Deb K, Thiele L, Laumanns M, Zitzler E (2002) Scalable multi-objective optimization test problems. In: *Proceedings of the 2002 congress on evolutionary computation, CEC 2002*, vol 1. IEEE Computer Society, pp 825–830
175. Huband S, Hingston P, Barone L, While L (2006) A review of multiobjective test problems and a scalable test problem toolkit. *IEEE Trans Evol Comput* 10(5):477–506
176. López-Ibáñez M, Knowles J (2015) Machine decision makers as a laboratory for interactive EMO. In: *Evolutionary Multi-Criterion Optimization*. Springer, New York, pp 295–309
177. Van Veldhuizen DA (1999) Multiobjective evolutionary algorithms: classifications, analyses, and new innovations. Technical report, DTIC Document
178. Yu G, Zheng J, Li X (2015) An improved performance metric for multiobjective evolutionary algorithms with user preferences. In: *Proceedings of the Congress on Evolutionary Computation*. IEEE, pp 908–915
179. He Z, Yen GG (2016) Visualization and performance metric in many-objective optimization. *IEEE Trans Evol Comput* 20(3):386–402
180. Pryke A, Mostaghim S, Nazemi A (2007) Heatmap visualization of population based multi objective algorithms. In: *Evolutionary Multi-Criterion Optimization*. Springer, New York, pp 361–375
181. Valdés JJ, Barton AJ (2007) Visualizing high dimensional objective spaces for multi-objective optimization: a virtual reality approach. In: *Proceedings of the Congress on Evolutionary Computation*. IEEE, pp 4199–4206
182. Lowe D, Tipping M (1996) Feed-forward neural networks and topographic mappings for exploratory data analysis. *Neural Comput Appl* 4(2):83–95
183. Hoffman P, Grinstein G, Marx K, Grosse I, Stanley E (1997) DNA visual and analytic data mining. In: *Proceedings of Visualization*. IEEE, pp 437–441
184. Chen S, Amid D, Shir OM, Limonad L, Boaz D, Anaby-Tavor A, Schreck T (2013) Self-organizing maps for multi-objective Pareto frontiers. In: *Visualization Symposium (PacificVis)*, IEEE Pacific. IEEE, pp 153–160
185. Tenenbaum JB, De Silva V, Langford JC (2000) A global geometric framework for nonlinear dimensionality reduction. *Science* 290(5500):2319–2323
186. Silva R, Salimi A, Li M, Freitas ARR, Guimarães FG, Lowther DA (2016) Visualization and analysis of tradeoffs in many-objective optimization: a case study on the interior permanent magnet motor design. *IEEE Trans Magn* 52(3):1–4
187. Freitas ARR, Silva RCP, Guimarães FG (2014) On the visualization of trade-offs and reducibility in many-objective optimization. In: *The Genetic and Evolutionary Computation Conference*. ACM, New York, pp 1091–1098
188. Pigozzi G, Tsoukiàs A, Viappiani P (2016) Preferences in artificial intelligence. *Ann Math Artif Intell* 77(3–4):361–401
189. Goldsmith J, Junker U (2009) Preference handling for artificial intelligence. *AI Mag* 29(4):9
190. Domshlak C, Hüllermeier E, Kaci S, Prade H (2011) Preferences in AI: an overview. *Artif Intell* 175(7):1037–1052