Complex Intell. Syst. (2017) 3:17–32
DOI 10.1007/s40747-016-0032-6

ORIGINAL ARTICLE

Consensus-based local information coordination for the networked control of the autonomous intersection management

Chairit Wuthishuwong[1] · **Ansgar Traechtler**[2]

Received: 11 August 2016 / Accepted: 31 October 2016 / Published online: 14 November 2016

Abstract Autonomous intersection management (AIM) will be a future method for improving traffic efficiency in the urban area. Instead of using the traffic signal control like nowadays, it uses wireless communication with autonomous vehicles to support the management of road traffic more safely and efficiently. A single AIM shows an exceptional performance in managing traffics at an intersection. However, it could not be represented a traffic in the real world, which is composed of multiple intersections. We show that coordination of traffic information among vehicles and infrastructures is an essential part of macroscopic traffic management. Coordination of traffic information among the network of AIMs is the key to improve the overall traffic flow throughout the network not only has an optimal flow in some intersections and very heavy traffic in others. In this paper, we introduce the distributed control to a graph-based intersection network to control traffic in a macroscopic level. Vehicle to infrastructure and infrastructure to infrastructure communication are used to exchange the traffic information between a single autonomous vehicle to the network of autonomous intersections. We implement a discrete time consensus algorithm to coordinate the traffic density of an intersection with its neighborhoods and determine the control policy to maximize a traffic throughput of each intersection as well as stabilizing the overall traffic in the network. We use the Greenshields traffic model to define the boundary condition of various traffic flows to the corresponded traffic density and velocity. Our proposed method represents the ability to maintain traffic flow rate of each intersection without having a back up traffic. As well, every intersection operates under the uncongested flow condition. The simulation results of the graph-based networked control of a multiple autonomous intersection showed that the overall traffic flow in the network achieves up to 20% higher than using traffic signal system.

Keywords Autonomous intersection management · Autonomous vehicle · Vehicle to infrastructure communication · Infrastructure to infrastructure communication · Discrete time consensus algorithm · Traffic model

✉ Chairit Wuthishuwong
chairit.wuthishuwong@neclab.eu; rittee07@gmail.com

Ansgar Traechtler
ansgar.traechtler@hni.upb.de

1 NEC Laboratories Europe GmbH, Kurfuersten Anlage 36, Heidelberg, Germany

2 Control and Mechatronics Engineering Department, University Paderborn, Heinz Nixdorf Institute, Fuestenallee 11, Paderborn, Germany

Introduction

Traffic signal is the general method to manage vehicles crossing an intersection. We can say that this method can totally prevent an accident when all drivers strictly follow the signal and traffic rule. However, the driving habit is different and impossible to control. Apart from traffic safety, traffic congestion is increasingly becoming the important problem, waits for solving in particular in the big cities. It is going severe in many metropolitan cities since the amount of vehicles has been increased whilst road capacity is remaining the same. Over two decades, many solutions focused in optimizing a traffic signal control. In [1] is introduced the real-time traffic signaling using fuzzy logic. The aim was to adaptively change a signal period to match the local traffic demand. This method installed sensors at the entrances of intersection to estimate a queue length on each road as inputs of fuzzy system. The effective timing of green signal is determined using the supervised rule. The commercial intelligent traffic signal controllers SCAT in [2] and SCOO in [3] proposed a different optimization method to minimize the queue length and maximize the throughput.

Recently, the advance of wireless communication technology makes a huge contribution to road transportation. It breaks the limitation in transmitting data, flexibility, range, speed, and no hard wiring is required. Therefore, intelligent transportation systems uses the advantageous of wireless communication to improve the traffic safety and efficiency. The fully autonomous system of road transportation can be basically made in practice by integrating wireless communication with a current autonomous vehicle.

With the fast development of autonomous driving, a work in [4] showed the first vehicle that drives itself throughout the dessert in 2005. A few years later in 2008, an autonomous vehicle in [5] showed the ability to drive in the urban environment with multiple modes e.g., parking and crossing an intersection. Many works contributed to the autonomous vehicle such as real-time motion planning in structured and unstructured environment [6,7], navigation and control algorithm [8–10] sensor fusion technique and localization system. In addition, an autonomous vehicle, Bertha in [11] has proved itself by running over 200 km throughout many cities in Baden Wuerttemberg, Germany without human driver. This shows a big step toward the future of intelligent transportation. Litman [12] predicts the effect of autonomous vehicle technology to the transport planning and [13] surveys of its impact to the vehicle demand and usage.

Based on deployment of two core technologies above, the fully autonomous system on the road transport expects to achieve a secure traffic safety and a higher efficiency. To target to zero accident and enhance the traffic throughput, autonomous vehicle needs sufficient information for making a better decision. On-board sensors can typically provide the local information only where sensors' capacity can reach but a vehicle to be a part of managing a macroscopic traffic requires extra information of a traffic situation around it and its neighborhood. To obtain those interconnection data, vehicle to vehicle (V2V) and vehicle to infrastructure (V2I) communication plays an important role in exchanging information between each other.

The communication standard of IEEE 802.11p, dedicated short-range communication (DSRC) was presented in [14]. It allocates the specific communication frequency of spectrum 5.9 GHz band for using only with vehicle communication. Several research works used V2V for improving traffic safety, e.g., [15] demonstrated of exchanging the local information like position, orientation and speed among the adjacent vehicles to improve the vehicles' collision avoidance system, where a vehicle in the blind spot fails to detect using the line of sight sensor but it can be detected only using wireless communication. Another work [16] used V2V to coordinate vehicles driving through the roundabout-type intersection. Besides the on-board vehicle application, it also implemented for optimizing traffic signal to reduce traffic congestion. It is proposed in [17] the V2V to adapt the effective green light to achieve the short queue by optimizing many different parameters, e.g., delay time, queue length, and number of stops, based on Websters equation and [18,19] used V2I to perform like a centralized controller to adjust the traffic signal. The idea is to equip a traffic light infrastructure with a wireless communication device, which makes it able to receive the data from the approaching vehicle. Hence, it can estimate the amount of vehicles or queue length by counting incoming messages and then determine the cycle length of the signal. But the limitation of this work is that this V2I communication is only in one direction. There is no return message from infrastructure back to vehicle. Moreover, [20] extended the network communication, infrastructure to traffic control center, to distributed control the traffic signal not only at a single intersection but also covered multiple intersections in the city area.

To maximize traffic throughput, intersection management is the most important part because it is the place which causes the most congestion due to waiting in the traffic signal and potential accidents. Many works attempts to mitigate traffic congestion by improving the effectiveness of traffic signaling. However, there are still independent factors that we cannot totally control and they have an influence on the overall traffic situation such as human driver and routing problem. Adaptive traffic signaling basically tries to optimize the period of red and green light, based on the real-time traffic data but the practical problem is that there is unbalanced demand in a network when most vehicles need to go in the same direction. Adaptive traffic signaling alone cannot handle such situation.

Autonomous intersection management (AIM) will allow rerouting and network balancing to stabilize the demand at each intersection. There are some intensive works focusing on autonomous intersection management. The first idea proposed [21,22] two decades ago before autonomous driving and wireless communication technology are broadly known. They introduced the idea of decentralized intersection collision avoidance using the concept of token ring and communication management of autonomous vehicle-included security. The work has been summarized in [23].

Another interesting work [24] proposed an idea of autonomous intersection using multi-agent system. A series of works [25–27] intensively studied the motion planning for AIM and crossing an intersection policy. They introduced the method call ahead for reserving an intersection space before it can enter. The traffic light is replaced by the infrastructure called intersection manager. It holds the rule to approve or reject the request of an intersection reservation from a vehicle, which is dependent on the initial condition e.g., speed, size of vehicle and desired destination. Vehicles agents communicate to an intersection manager to reserve the area. The successful reservation will have no confliction with the others. Otherwise, the reservation will be rejected. They in addition extended their work from managing a single intersection to multiple intersections in [28]. The Braesss paradox was observed and the global throughput was investigated. There are still many creative methods for tackling traffic congestion problem but what they represented one thing in common is that the cooperation and coordination among either vehicles or infrastructures is incontrovertible [29–31].

We organized this paper into seven sections. The first section starts with the introduction and the literature review of the state of the art of autonomous system in transportation and traffic management methods. The second section presents our contribution and our proposed solution method. The third section describes the problem statement. The forth section introduces our Autonomous Intersection Management platform which includes three subsections, network modeling, traffic flow model, and consensus algorithm, following with the simulation and evaluation results in the fifth section. The sixth section will lead to the conclusion of this work. The last section briefs the discussion and future work.

Contribution

From the literature researches, the trend of using autonomous system is rising in the transportation range from a microscopic level for instance, improving the intelligent level to an autonomous vehicle, to a macroscopic level for instance, adaptive traffic signal and autonomous intersection management. Hence, we saw the space for improving the efficiency of transportation in particular in the urban area like at intersection. We introduce the hybrid concept of using multiagent with the traffic flow model. Moreover, we contribute in balancing and maximizing traffic flow rate throughout a network.

In this paper, the authors focus on the coordination algorithm to mitigate traffic congestion by balancing traffic density in the network. In our previous work [32,33], we implemented the AIM for a single intersection. We used V2I with dynamic programming to find an optimal trajectory to safely cross an intersection. The extension work in [34,35] studied the coordination method for multiple intersection scenarios.

The control strategy is divided into 2 layers. Refer to [32,33], the lower layer is a single autonomous intersection. We use the centralized control principle with V2I communication to manage a vehicle crossing an intersection. On the other hand, the upper layer is the infrastructure network. We implement the decentralized control principle with the network communication between intersections, infrastructure to infrastructure (I2I) communication to manage traffic flow in the network. The traffic information, e.g., traffic density, traffic flow rate and average traffic velocity, is shared among their connected neighborhoods. We use the multi-agents approach to represent the multiple intersections where a single autonomous intersection is considered as an intelligent agent. Each agent can exchange information in both direction and we, therefore, use the undirected graph for modeling our communication network topology. In addition, the infrastructure-to-infrastructure (I2I) communication protocol is designed.

With a graph model, an intersection is considered as a node and the interconnection between each node is an edge. Thus, the traffic is physically the flow on the edge. To maintain the continuity flow, we introduce the traffic flow model, which is composed of three parameters. The free flow velocity, traffic density and traffic flow is investigated using the Greenshields model. The discrete version of the consensus algorithm is used to coordinate the traffic information on the edge in the intersection network. The simulation of a multiple autonomous intersection management is presented. The results are plotted and evaluated with the Greenshields model.

The aim of this paper is to use the local information to control the traffic flow of each intersection in the network. We present the communication network and the consensus coordination for balancing traffic in the network with the stability analysis of the proposed method.

Problem statement

To achieve the zero accident, all car manufactures focus on improving the advanced driving assistance system (ADAS). However, the statistic reported that the road accidents are caused mostly by human driver [51]. Thus, the best effective way is to replace a driver behind the wheel with an autonomous system. Autonomous vehicle i.e., [4,5,8,11] shows an impressive ability of safety driving and robustness for a long hour driving when human can have stress and fatigue.

To improve the traffic throughput, the current technology focuses on optimizing traffic signal as reviewed in many literature research, i.e., [1–3]. The fully autonomous system like AIM [21–28], however, will increase the traffic throughput not only in a single intersection but also in the network, since the traffic in the real-world environment is a network composed of many streets and intersections. Vehicles will drive themselves and communicate with the roadside infrastructure through wireless communication for crossing intersections. Information from V2I communication is used to plan the safe trajectory of incoming vehicles and information from I2I communication will be used to coordinate a traffic state in the network to maximize a throughput. Therefore, the effective coordination algorithm will play a crucial role in AIM for managing a macroscopic traffic.

In this work, we introduce the completed autonomous platform which coordinates information between autonomous vehicles and intelligent traffic infrastructures to encompass the aforementioned problems.

Autonomous intersection management

Autonomous intersection management (AIM) is the traffic signal-less platform which uses the communication between a vehicle and an intersection manager instead of driver perception to cross an intersection.

Traffic in a single intersection is usually modeled as a microscopic level. It considers the dynamic of vehicles rather than an intersection, which means the traffic flow of a single intersection, will have no influence to the others. On the other hand, a normal traffic in the real-world environment is generally represented by the network of multiple intersections, which traffic characteristic can be expressed by the macroscopic traffic. To manage traffics, the flow of each connected intersection will be taken into account to maximize the networks throughput.

At the ground level of modeling AIM, we create the intersection manager. It responds to control autonomous vehicles crossing an intersection and is capable of exchanging traffic information in the network. Hence, the intersection manager must have two levels of communication.

1. Intersection level: intersection manager uses vehicle to infrastructure (V2I) communication to exchange information with autonomous vehicles for planning safe trajectories.
2. Network level: intersection manager uses infrastructure to infrastructure (I2I) communication to exchange traffic information with the neighborhoods for balancing and maximizing traffic throughput of the network.

Under this section, we describe our AIM platform that composes of multiple intersections and every intersection is assumed identical. In this work, we modeled the intersection network, connecting nine intersections, where each intersection has four ways. We applied the distributed control structure with the idea of multi-agents system in such a way that a single intersection is considered as an autonomous agent that has ability to control itself, whilst the control command is dependent on the feedback information of its neighborhoods.

Graph theory [50] is used to visualize and interpret the interaction of a network. The intersection manager is technically assigned by a node and the connection between each node is represented by an edge. The network model of AIM is explained in the following section.

Network modeling

As we explained earlier, we conducted two levels in our AIM platform, the intersection level and the network level. In [33] is presented the model of a single autonomous intersection. At the network level, we used a graph model representing AIM and we defined two network problems, street network and communication network, based on their functionalities.

Street network

Street network is a physical model representing in the real-world traffic environment. We can physically classify a street network into three levels. The smallest level is a street level. It has basically two functions, incoming and outgoing street. The middle level is an intersection level, where multiple streets are joined and vehicles can change a direction of travel. The last level is the network level, which multiple intersections are connected. Therefore, based on using the real physical connection of intersections, the street network can be modeled as illustrated in Fig. 1

We simplify a street network by connecting nine intersections in a square grid. Every intersection is a standard four-way intersection, which has two lanes of incoming and outgoing on each leg. The state of an intersection is defined with the vector of three traffic parameters. For the incoming traffic to an intersection i from the neighborhoods intersection j or the external input outside the network l, the state of a traffic toward an intersection inside the network can be expressed as:

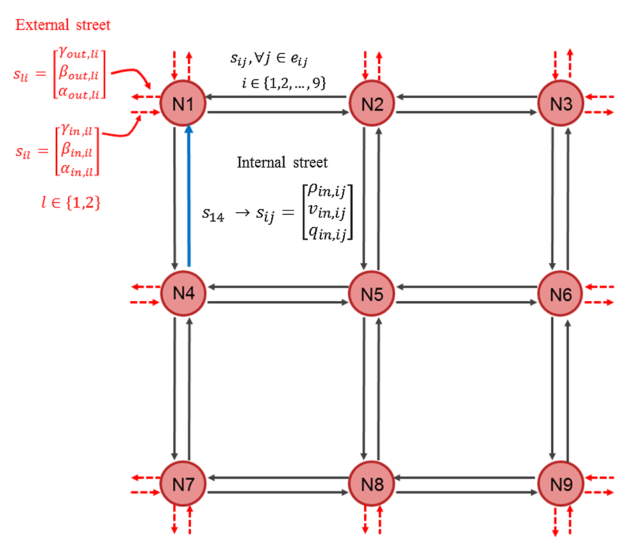

Fig. 1 Street network model

$$s_{ij} = \begin{bmatrix} \rho_{in,ij} \\ v_{in,ij} \\ q_{in,ij} \end{bmatrix}; \quad \forall j \in e_{ij} \quad \text{and} \quad i \in 1, \ldots N_i \tag{1}$$

where s_{ij} is the state of traffic toward an intersection i from the neighborhoods intersection j, $\rho_{in,ij}$ is the incoming traffic density (traffic on the edge), $v_{in,ij}$ is the traffic velocity (velocity on the edge) and $q_{in,ij}$ is the traffic flow rate (flow on the edge) on the same direction to an intersection i from an intersection j.

The second component is the traffic generated outside the network. We express the state of a traffic connected to an intersection from the outside of network as:

$$\begin{aligned} s_{il} &= \begin{bmatrix} \gamma_{\text{in,il}} \\ \beta_{\text{in,il}} \\ \alpha_{\text{in,il}} \end{bmatrix}; \quad \forall l \in (1, 2) \\ s_{li} &= \begin{bmatrix} \gamma_{\text{out,li}} \\ \beta_{\text{out,li}} \\ \alpha_{\text{out,li}} \end{bmatrix}; \quad \forall l \in (1, 2) \end{aligned} \tag{2}$$

where s_{il} is the state of street toward an intersection i from the external source l, where l is a set of input direction (1: horizontal, 2: vertical), $\gamma_{\text{in,il}}$, $\beta_{\text{in,il}}$ and $\alpha_{\text{in,il}}$ are the traffic density, traffic velocity and traffic flow rate with the direction to an intersection i from the external source l $(in : incoming)$, respectively, and in the opposite direction s_{li} is the state of street outward an intersection i to an external source l, $\gamma_{\text{out,il}}$, $\beta_{\text{out,il}}$ and $\alpha_{\text{out,il}}$ are the traffic density, traffic velocity and traffic flow rate from an intersection i to an external source l $(out : outgoing)$, respectively.

In addition, the outgoing traffic's state can be defined in the same way. However, we assume that inside the street network, an outgoing traffic is practically an incoming traffic to a neighborhood intersection. Hence, s_{ij} represents the state of an incoming traffic to an intersection i from an intersection j can refer to an outgoing traffic of an intersection j to an intersection i.

Associating to the graph, an intersection is defined by a node and street connected between each intersection is defined by an edge. The amount of vehicles driving on each street can be represented as the flow on the edge. Thus, each single intersection in AIM can know the local traffic density by counting the requested messages that are transmitted from the incoming vehicles over the V2I communication. The collected traffic density information of each intersection can be determined by summing the traffic density of all incoming streets to intersection. We define the intersection's state with the gross incoming traffic to an intersection as the density on the node.

$$\rho_{\text{in},i} = \sum_{j \in e_{ij}} \rho_{\text{in},ij} + \sum_{l \in (1,2)} \gamma_{\text{in,il}} \tag{3}$$

where $\rho_{\text{in},i}$ is the gross incoming traffic density of the intersection i (density on the node). $\rho_{\text{in},ij}$ is the traffic density from an internal network, neighborhood intersection j (density on the internal edge), and $\gamma_{\text{in,il}}$ is the traffic density from an external source l (density on the external edge).

We used the incoming traffic density to indicate the traffic condition of an intersection, traffic on the node. In addition, we normalize traffic of each street with the gross incoming traffic to an intersection. Thus, we can define the distribution of traffic on each connected street.

$$P(\rho_{\text{in},ij}|\rho_{\text{in},i}) = \frac{P(\rho_{\text{in},i}|\rho_{\text{in},ij}) \cdot P(\rho_{\text{in},ij})}{\rho_{\text{in},i}} \tag{4}$$

$$\sum_{j \in e_{ij}} P_j(\rho_{\text{in},ij}) = 1 \tag{5}$$

where $P(\rho_{\text{in},ij}|\rho_{\text{in},i})$ is the distribution of traffic on each incoming street toward intersection i given by the gross incoming traffic of an intersection i, $P(\rho_{\text{in},i}|\rho_{\text{in},ij}) \cdot P(\rho_{\text{in},ij})$ is the direct measured value of traffic on each street toward intersection i, the normalized term is the gross incoming traffic of an intersection i obtained from Eq. 3 and the summation of total probability is 1. The observed parameters v and q also represent with the corresponded probability distribution.

Apart from traffic density that we can have a direct measurement, we estimate the average of traffic velocity and traffic flow rate using traffic model. We will introduce the traffic flow model in the next section.

Communication network

Different from the street network function which has a physical interaction, the communication network is responsible for signal interaction, exchanging traffic data. An intersection is an autonomous agent represented as a node and the communication flow between each node is represented as an edge. We model the communication only within the network, in which an intersection is not allowed to communicate with external sources because we assume that the external source is independent and not allowed to control. The communication topology of the intersection network is illustrated in Fig. 2.

The data flow on the edge uses the bi-directional communication. Each node, which represents an intersection manager, can either receive or transmit the data package to their destination node.

$$c_{ij} = c_{ji}; \quad \forall j \in e_{ij} \tag{6}$$

where c_{ij} and c_{ji} is the valid bi-directional communication link between a pair of node.

We use the properties of a graph theory to model the interaction of the communication in the intersection network.

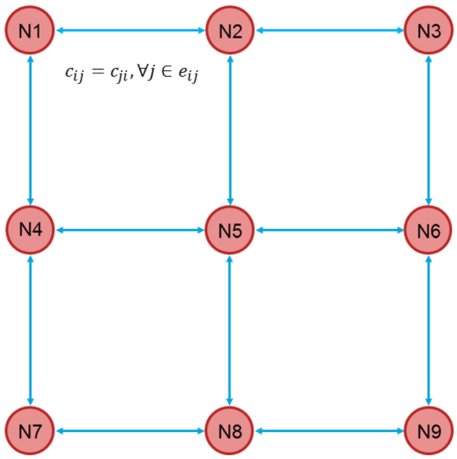

Fig. 2 Communication network model

A graph (G) with N elements is able to define into a set $G = (V, E)$, where (V) is denoted by a finite set of vertices. (E) is the finite set of edges and represents the connection between couple of nodes. Hence, the adjacency element, will have value 1 when there is an edge between each node, otherwise the value is equal to 0. The adjacency matrix can be expressed as the following equation.

$$a_{ij} = \begin{cases} 1, & (n_i, n_j) \in E \\ 0, & \text{Otherwise} \end{cases}$$
$$\underline{A} = [a_{ij}]; \quad i,j \in N \tag{7}$$

The second component is the degree of a graph. It describes the number of connections at each intersection. The degree matrix is a diagonal matrix, where the degree element d_{ij} is equal to the row summing of adjacency elements and it can be expressed as:

$$d_{ij} = \begin{cases} \sum_{i=1}^{n}, & (n_i, n_j) \in E \\ 0, & \text{Otherwise} \end{cases}$$
$$\underline{D} = [d_{ij}]; \quad i,j \in N \tag{8}$$

From the graph theory, an interaction of a graph can explain through a Laplacian graph. The Laplacian matrix describes the complete relationship of the intersection network. The simple way to determine the Laplacian matrix is subtracting the degree matrix with the adjacency matrix.

$$\underline{L} = \underline{D} - \underline{A}$$
$$= \begin{bmatrix} d_{ij} & -a_{ij+1} & \dots & -a_{iN} \\ -a_{i+1j} & d_{i+1j+1} & \dots & -a_{i+1N} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{Nj} & -a_{Nj+1} & \dots & d_{NN} \end{bmatrix} \tag{9}$$

where i is the row element of the matrix, j is the column element of the matrix, N is the number of node, $\underline{A}$ is the adjacency matrix, a_{ij} is the adjacency element, $\underline{D}$ is the degree matrix, d_{ij} is the degree element, and $\underline{L}$ is the Laplacian matrix.

We defined two different networks for modeling AIM and will explain the conditions that both networks are stable. Since the street network represents the actual traffic, the stability of network is dependent on the traffic flow characteristic. The maximum capacity refers to the maximum number of vehicles that can drive on a street. It is used to indicate the stability of a network; when the number of vehicles are below the maximum capacity the network will be stable and vice versa.

In the second network, we used a graph to model the communication topology. Thanks to the properties of the algebraic graph, the stability of a communication network can be guaranteed through the Laplacian matrix. More details of the stability and boundary conditions will be explained in the next section.

Traffic flow model

We introduce the traffic flow model to interpret whether the traffic situation in the street network is congestion or not. Refer to the traffic flow model [47,48]; it is a general topic in the field of transportation engineer. It is always used to plan and monitor a macroscopic traffic. In the real-world traffic environment, there are many vehicles driving with various levels of velocity in many streets. The traffic flow is considered in the macroscopic level rather than the motion of each single vehicle. Thus, the parameters that are used to indicate traffic flow are composed of three parameters.

- Traffic density (ρ): is the number of vehicles per one kilometer per lane.
- Traffic flow rate (q): is the number of vehicles passed a particular point per hour.
- Traffic velocity (v): is the average velocity of vehicles on the observed street.

Traffic flow model defines that traffic flow rate is the product of traffic velocity and traffic density. It is derived by the empirical data, which are observed and collected in the real traffic circumstances and its relationship is basically used to represent the macroscopic traffic. The continuity equation of a general traffic flow can be written as the following equation.

$$q = v \cdot \rho \tag{10}$$

where q is traffic flow rate, v is traffic velocity, and ρ is traffic density.

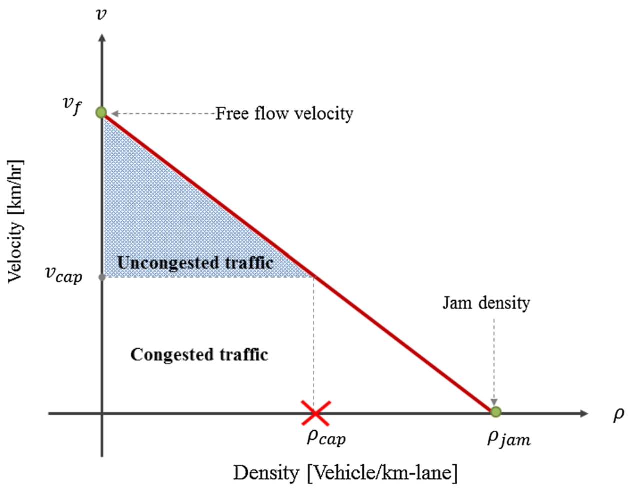

Fig. 3 Relationship between velocity and density

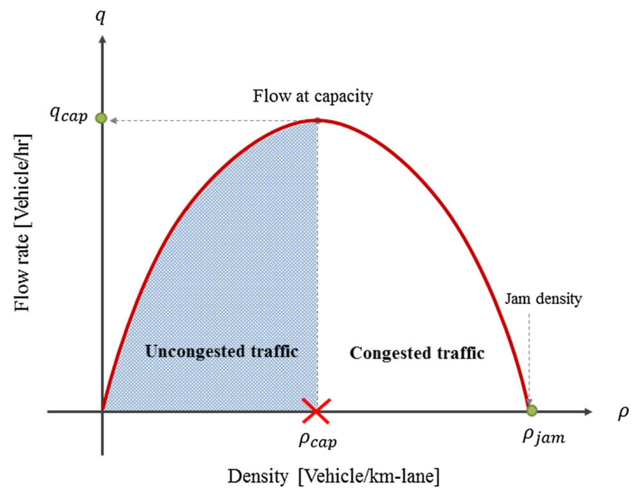

Fig. 4 Relationship between flow rate and density

In this work, we introduced the classic traffic flow model, Greenshields model, [49] as the referent traffic model. This model is the approximation function of the empirical data of the aforementioned three parameters. Greenshields model defines three corresponding relationship between those three parameters.

Traffic velocity and traffic density

It is modeled with the linear function. The traffic velocity is the inverse proportional of traffic density. Greenshield classified the traffic situation with two simple groups which are congested and uncongested traffic. The relationship and the boundary conditions are illustrated in Fig. 3.

According to the parameters of the Greenshields model [49], the average of free flow velocity (v_f) is given at 91 km/h and the jamming density (ρ_jam) is given at 78 vehicles/km/lane. The average velocity at capacity (v_cap) is given at 46 km/h and is the lower boundary of the average velocity that vehicles can still drive under the uncongested traffic condition. Technically, the traffic will begin to congest if the vehicles cannot keep the driving velocity, at least at this level. Consequently, the traffic density at capacity (ρ_cap) is the maximum number of vehicles on the street that still keeps the average velocity within this boundary condition. The relationship between traffic velocity and traffic density can be written as:

$$\rho_\mathrm{cap} = \rho_\mathrm{jam} \cdot \left(1 - \frac{v_\mathrm{cap}}{v_\mathrm{f}}\right) \tag{11}$$

The traffic density at capacity (ρ_cap) is round up to 38 vehicles/km/lane. The boundary condition of the uncongested traffic can be summarized into these following equations.

$$v_\mathrm{cap} \le v \le v_\mathrm{f} \tag{12}$$

$$0 \le \rho \le \rho_\mathrm{cap} \tag{13}$$

The uncongested traffic condition is satisfied when the average of traffic velocity is higher than the average velocity at capacity and less than the average of free flow velocity, as well as the traffic density being greater than zero and less than the traffic density at capacity. On the other hand, the congested traffic conditions are vice versa.

Traffic flow rate and traffic density

The relationship between traffic flow rate and traffic density is a nonlinear function. Greenshield's model defined using a parabolic function and of course classified the traffic situation in the similar interpretation into congested and uncongested traffic.

With the inverse U-shape parabola, it has an equilibrium point on the top, where traffic flow rate will reach the maximum (q_cap) when traffic density reaches its capacity (ρ_cap) and traffic flow rate will fall to zero after this point. The plot is shown in Fig. 4.

With the provided parameters above, the traffic flow at capacity (q_cap) can be determined as:

$$q_\mathrm{cap} = v\mathrm{f} \cdot \left(\rho_\mathrm{cap} - \frac{\rho_\mathrm{cap}^2}{\rho_\mathrm{jam}}\right) \tag{14}$$

The traffic flow rate at the capacity (q_cap) will be approximately 1800 vehicles/h/lane and it can be said that this point is the boundary of the uncongested traffic.

Traffic velocity and traffic flow rate

Greenshields model defined the relationship between traffic velocity and traffic flow rate with parabolic function as

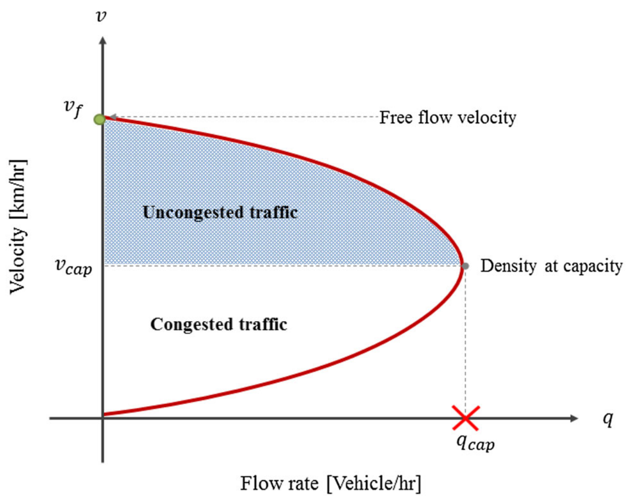

Fig. 5 Relationship between velocity and flow rate

illustrated in Fig. 5. In addition, the uncongested traffic is represented in the upper part of the graph and the lower part is the congested traffic condition. A model gives the average of traffic velocity at the equilibrium point, where the traffic flow rate can reach the maximum. The velocity below this point will represent the congested condition.

The derived boundary conditions are used to indicate the traffic situation. With these three relationships, it shows the corresponding relationship which gives a common point to classify the congested and uncongested traffic. The maximum traffic flow rate is defined at the equilibrium point of two relationships with traffic velocity and traffic density. This point in principle is the flow at capacity (q_{cap}), which still be in the uncongested traffic, guaranteed by the velocity at capacity (v_{cap}) and density at capacity (ρ_{cap}). The relationship function can be expressed as:

$$q_{\text{cap}} = \rho_{\text{jam}} \cdot \left(1 - \frac{v_{\text{cap}}}{v_{\text{f}}}\right) \tag{15}$$

Therefore, the explanation is that the uncongested condition is the situation where the traffic density on the observed street is lower than the traffic density at capacity (ρ_{cap}) that makes vehicles drive with the average velocity, at least the velocity at capacity (v_{cap}), and the traffic flow rate less or equal to the flow at capacity (q_{cap}).

We introduced the street network model in the previous section. Using Greenshields traffic model, we can classify the traffic condition on each street and also the intersection. The total density of incoming streets toward the intersection can be computed through Eq. 3. To maximize the traffic throughput in the street network, the traffic relationship shows that traffic density and traffic velocity in each street must be under the uncongested condition, which is bounded by the proposed boundary conditions. As well, the uncongested traffic can refer to the stability of the street network since the congested traffic or traffic jamming represents the instability.

In this section, we investigated the traffic model to explain the traffic condition of a street network. We will use these traffic relationships for managing traffic in the AIM intersection network. Next section, we will explain the coordination technique for AIM that is used to balance the traffic throughput of the intersection network. The mentioned traffic parameters will be distributed, coordinated and we will apply their relationships in feedback control structure.

Consensus coordination technique for AIM

We implement the discrete consensus algorithm for coordinating the traffic information in AIM to balance the overall traffic flow in the network. Consensus has a distributed structure. It uses only local information to coordinate with their neighborhoods in the network and make them reach the common agreement. Robot applications such as cooperative robots in [37,38] and robots formation [39], distributed motion control of robotics network [40,41] and flocking [42,43] have been recently studied in multi-agents system [36]. In addition, it is used in distributed sensors network [44].

Naturally, consensus algorithm is the distributed control that gives the convergence property, which fits for the large-scale system. We illustrate the system architecture of the AIM for multiple intersections in Fig. 6.

An intersection acts as the centralized controller. Basically, it determines safe trajectories and manages vehicles crossing an intersection [32,33]. Using traffic flow model, an intersection manager collects the traffic density on the street network because it is countable and uses to indicate the traffic condition. In practice, an intersection manager knows the traffic density on a street by counting a number of requested messages received from V2I communication. We do not con-

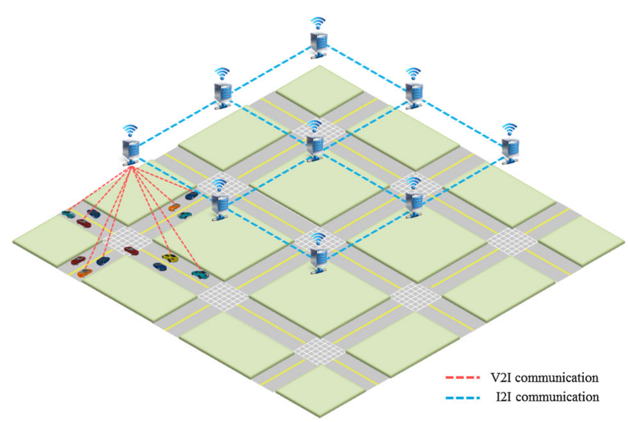

Fig. 6 The system architecture of AIM for multiple intersections

cern the traffic density of outgoing vehicles because they are normally the incoming vehicles to other intersections.

The traffic density information is distributed to its neighborhoods in the intersection network through I2I communication. AIM mechanism in an intersection manager will coordinate traffic density of itself and its neighborhoods to compute the control output and update the state, using consensus algorithm.

We used the traffic density as the coordinated information, as well as representing the state of an intersection. Consensus algorithm expresses the dynamics of a local intersection as:

$$\dot{\rho}_i = \sum_{j \in Ni} a_{ij}(\rho_j - \rho_i) \tag{16}$$

With the communication network topology, the dynamics of the intersection network is expressed as:

$$\dot{\rho} = -L\rho \tag{17}$$

The street network provides the gross traffic density of each intersection, and then consensus coordinates this information, based on the communication topology. Using Eq. 17, the global dynamics of AIM network can be derived as:

$$\begin{bmatrix} \dot{\rho}_1 \\ \vdots \\ \dot{\rho}_{N_i} \end{bmatrix} = -L \cdot \begin{bmatrix} \rho_1 \\ \vdots \\ \rho_{N_i} \end{bmatrix} = -L \cdot \begin{bmatrix} \sum_i \xi_{ji} + \sum_i \gamma_{ji} \\ \vdots \\ \sum_{N_i} \xi_{jN_i} + \sum_i \gamma_{jN_i} \end{bmatrix} \tag{18}$$

Concerning the communication frequency, the data update rate is a discrete time. Hence, we implement the discrete time consensus using the difference equation. Then, the discrete time consensus for a local intersection of Eq. 16 can be derived as the following equation.

$$\rho_i(k+1) = \rho_i(k) + \varepsilon \sum_{j \in Ni} a_{ij}(\rho_j(k) - \rho_i(k)) \tag{19}$$

As well, the discrete version of the dynamics of the intersection network in Eq. 20 is expressed as:

$$\rho(k+1) = P \cdot \rho(k) \tag{20}$$

where P is a Perron matrix $P = I - \varepsilon L$ and ε is the step size $\varepsilon > 0$. P must be satisfied a non-negative matrix. For $P = I - \varepsilon L$, it can be written as $P = I - \varepsilon D + \varepsilon A$. The Perron matrix will be a non-negative matrix, if $I - \varepsilon D$ is non-negative. Therefore, the sufficient condition of the step size is defined by $0 < \varepsilon < 1/\Delta$, where Δ is the maximum degree of a Perron matrix. In addition, the rows' summation of the Laplacian matrix is equal to zero $\sum_j l_{ij} = 0$ so that L has a zero eigenvalue, $\lambda_1 = 0$, and its corresponding eigenvector $\underline{1}$, $L\underline{1} = 0$. Hence, a Perron matrix can be written as $P\underline{1} = I - \varepsilon L\underline{1}$, which means the summation of all rows is 1. In addition, 1 is a trivial eigenvalue of a Perron matrix since, the simple root of a Perron matrix is determined by $\mu_j = 1 - \varepsilon\lambda_j$, $\lambda_j = 0$. The sufficient conditions for the stability of a consensus in the network are provided in [45].

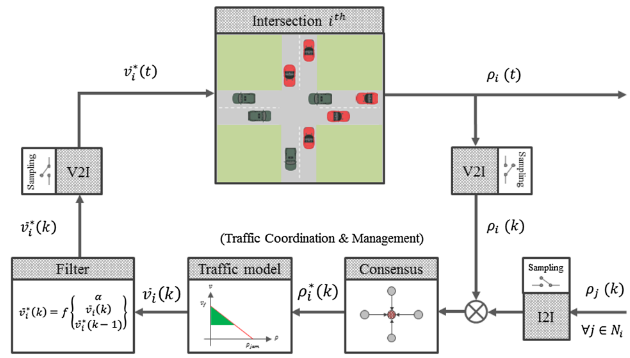

Fig. 7 Closed loop control block diagram of an intersection manager

The control system of a multiple autonomous intersections is composed of nine intersection managers. We design the distributed control structure as a hierarchical level, starting from an intersection level to a city level, which contains multiple intersections. Moreover, it can expand into larger scale because the distributed consensus has the advantage of scalability. It requires only a coordination of a local information. The intersection control strategy is identical for every intersection manager. Figure 7 shows the closed loop control block diagram of AIM of a single intersection.

AIM practically prioritizes the timing of crossing an intersection for autonomous vehicles. The control variable is the incoming time which can be transformed to the average velocity when the distance between a vehicle and intersection is known. In general, every vehicle has to send the requesting message to AIM in a particular range of communication before crossing an intersection. With this process, the traffic density of streets toward an intersection is counted through the V2I communication. However, the information is updated in a discrete time after sampling.

To control the traffic flow rate of an intersection, we input the feedback traffic density of the neighborhoods from I2I communication. Consensus algorithm coordinates the traffic density from the V2I measurement with its neighborhood from I2I feedback. With the discrete consensus in Eqs. 19 and 20, the desired value of traffic density is computed $\rho_i^*(k) = \rho_i(k+1)$. Therefore, the residual density of an intersection is the difference between the desired traffic density and the current traffic density. The error can be expressed as the following equation.

$$e_i(k) = \rho_i^*(k) - \rho_i(k) \tag{21}$$

Consensus algorithm determines the common level of traffic density in the local intersection network to balance traffic density of the neighborhoods close to each others. Theoretically, the error term must be minimized and approach zero in the finite time to make the current traffic density equal to the desired traffic density.

To manage the current traffic density to meet the desired traffic density, the Greenshields relationship of traffic velocity and traffic density is implemented. Since the model gives the direct relationship between them, it is obvious that changing the traffic velocity is the way to minimize the traffic density error of an intersection. The average of traffic velocity in the discrete time can be derived as:

$$\bar{v}_i(k) = \bar{v}_i(k-1) - \frac{v_{\mathrm{f}}}{\rho_{\mathrm{jam}}} e_i(k) \tag{22}$$

In the control block diagram, we implement the filter to remove the short-term fluctuation for smoothing the output response. We assumed that the communication is ideal. There is no package loss and delay. The technique of moving average is applied by weighting the value between the current, computed value and the previous, desired value. The weighting coefficient is called the degree of filtering and the summation of them is unity. It is called the exponential moving average filter, responded speed of which is dependent on the weighting coefficient. In practice, the function of this filter is identical to the first-order, low-pass filter in the electronic circuit, suppressing the amplitude of a signal so that the frequency is higher than the cutoff frequency. The desired traffic velocity suggested to vehicles on each street in Eq. 22 will be updated as:

$$\bar{v}_i^*(k) = \alpha \bar{v}_i(k) + (1-\alpha)\bar{v}_i^*(k-1) \tag{23}$$

where $v_i^*(k)$ is the desired average of traffic velocity for an intersection i at time step k, $v_i(k)$ is the computed average traffic velocity from the Greenshields model at time step k, $\bar{v}_i^*(k-1)$ is the previous time step *k-1* of a desired average traffic velocity and α is the weight coefficient, $\alpha \in [0, 1]$.

The traffic velocity in Eq. 23 is sent to vehicle to use for planning trajectory by V2I communication. The frequency of the feedback loop must be set corresponding to the requirement of stability condition of a communication graph.

The pseudo algorithm of consensus-based local traffic information coordination for the networked control of the Autonomous Intersection Management is shown in Algorithm 1. Separate algorithms for street state function, intersection state function, traffic flow model and filter are provided in Algorithms 2, 3, 4 and 5, respectively.

Algorithm 1 AIM: Consensus coordination of the local traffic information

define:
i: Observed intersection
j: The neighborhood intersection
k: Discrete time update
l: The external street
m: Numbers of vehicle on a street in the range of V2I communication
n: Numbers of street connected to an intersection
t: Continuous time update

procedure MAIN($(a_{ij}, d_{ij} \in la_{ij}), c_{ij}$)
  $S_i \leftarrow$ *Initialization*
  while not S_i is empty **do**
    Check for connection to external streets s_{il}
    $a_{il} \leftarrow 0$
    $n \leftarrow d_{ij}$
    $m \leftarrow$ counting V2I messages
    Street State Function(n, m)
    if $(\exists \rho_i \geq \rho_{jam})$ **then**
      Congested: Exit
      (Reroute**)
    else
      Intersection State Function $(\forall_{i=1}^{N} S_i, \epsilon, a_{ij})$
      Greenshield's model $(\forall_{i=1}^{N} \rho_{[i]}^*(k), \forall_{i=1}^{N} \rho_{[i]}(k))$
      Filter $(\forall_{i=1}^{N} \bar{v}_{[i]}(k), \alpha)$
    De-normalizing
    $v_{[i][j]}^*(k) \leftarrow \frac{v_i^*(k)}{x[i][j]}$
    $\rho_{[i][j]}^*(k) \leftarrow \frac{\rho_i^*(k)}{y[i][j]}$
    $q_{[i][j]}^*(k) \leftarrow \frac{q_i^*(k)}{z[i][j]}$

    //Distribute the desired velocity of each street to vehicles using V2I communication with a specific update rate.
    $v_{[i][j]}^*(t) \leftarrow v_{[i][j]}^*(k)$

Controlling only traffic velocity might not be sufficient, when the number of vehicles on the street is exceeded. Even though consensus attempts to release vehicles out of an intersection as fast and as much as possible, it is still limited with the capacity of the street (ρ_{jam}). Hence, we propose the rerouting process for the further improvement to assist the consensus coordination when the traffic condition is become congested. It supports an intersection manager to distribute excessive vehicles to the neighborhoods that have lower density. The proposed concept will be described in the discussion and future work.

Simulation results

We present the simulation results of multiple autonomous intersection managements, which were implemented, based on the discrete consensus algorithm with the Greenshields traffic model. The input traffic flow rate of all 12 sources is independent and assigned randomly also for the travel-

Algorithm 2 An intersection manager: Street State Function

//Through using V2I communication for each intersection manager i and set the radius of communication at 200 m.
define:
o: A vehicle on a particular street communicated to an intersection manager

procedure STREET STATE FUNCTION(n,m)
  for $j = 1 : n$ **do**
    for $o = 1 : m$ **do**
      $v_{[i][j]} \leftarrow \frac{\sum_{o=1}^{m} v_{[i][j][o]}}{m}$
      $\rho_{[i][j]} \leftarrow \frac{\sum_{o=1}^{m} \rho_{[i][j][o]}}{m}$
      $q_{[i][j]} \leftarrow v_{[i][j]} \cdot \rho_{[i][j]}$
    $s_{[i][j]} \leftarrow [v_{[i][j]}; \rho_{[i][j]}; q_{[i][j]}]$
  Normalizing
  $norm(v_{[i][j]}) \leftarrow \frac{v_{[i][j]}}{\max(v_{[i][j]})}$
  $norm(\rho_{[i][j]}) \leftarrow \frac{\rho_{[i][j]}}{\max(\rho_{[i][j]})}$
  $norm(q_{[i][j]}) \leftarrow \frac{q_{[i][j]}}{\max(q_{[i][j]})}$
  $v_{[i]} \leftarrow x_{[i][j]} \cdot v_{[i][j]}$ and
  $\sum_{j=1}^{n} x_{[i][j]} = 1$
  $\rho_{[i]} \leftarrow y_{[i][j]} \cdot \rho_{[i][j]}$ and
  $\sum_{j=1}^{n} y_{[i][j]} = 1$
  $q_{[i]} \leftarrow z_{[i][j]} \cdot q_{[i][j]}$ and
  $\sum_{j=1}^{n} z_{[i][j]} = 1$
  where $1 \leq j \leq n$, and (x, y, z) is the normalized coefficient.
  $S_{[i]} \leftarrow [v_{[i]}; \rho_{[i]}; q_{[i]}]$

Algorithm 3 The intersection network: Intersection State Function

//Through using I2I communication with the communication graph.
define:
k: Discrete Time step
ϵ: Update rate
a_{ij}: The existing connection to an intersection
N: Total numbers of neighborhood intersection

procedure INTERSECTION STATE FUNCTION($\forall S_i$,ϵ,a_{ij})
  $a_{ij} \leftarrow c_{ij}$ //defined the communication graph
  $\epsilon \leftarrow \Delta k$ //set the sufficient discretized step
  $\rho_{[i]}(k) \leftarrow \rho_{[i]}(t)$
  $\rho_{[j]}(k) \leftarrow \rho_{[j]}(t)$
  while $(i > 0, j > 0)$ **do**
    if $(\rho_i(k) \leq 0)$ **then**
      $\rho_{[i]}(k) \leftarrow \hat{\rho_{[i]}}(k-1)$
    if $(\rho_{[j]}(k) \leq 0)$ **then**
      $\rho_{[j]}(k) \leftarrow \hat{\rho_{[j]}}(k-1)$
    $\rho_{[i]}(k+1) \leftarrow \rho_{[i]}(k) + \epsilon \cdot \sum_{j \in Ni} a_{ij}(\rho_{[j]}(k) - \rho_{[i]}(k))$
    Vectorized:
    $P \leftarrow$ Perron matrix
    $\underline{\rho}(k) \leftarrow \forall_{i=1}^{j} \rho_{[i]}$
    $\underline{\rho}(k+1) = P \cdot \underline{\rho}(k)$
    Update:
    $\rho_{[i]}^{*}(k) \leftarrow \rho_{[i]}(k+1)$
  Memory:
  $\hat{\rho_{[i]}}(k-1) \leftarrow \rho_{[i]}^{*}(k)$

ing route. According to the traffic model, maximum traffic flow rate (q_{cap}) for a single lane is 1800 vehicles/h. Therefore, we set the range of the traffic flow rate between

Algorithm 4 Traffic model

//Greenshield's model
procedure GREENSHIELD'S MODEL($\rho_{[i]}^{*}(k)$, $\rho_{[i]}(k)$)
  while $(i > 0, j > 0)$ **do**
    if $(\rho_{[i]}^{*}(k) \leq 0)$ **then**
      $\rho_{[i]}^{*}(k) \leftarrow \rho_{[i],measured}(k)$
    if $(\rho_{[i]}(k) \leq 0)$ **then**
      $\rho_{[i]}(k) \leftarrow \rho_{[i],measured}(k)$
    $e_{[i]}(k) \leftarrow \rho_{[i]}^{*}(k) - \rho_{[i]}(k)$
    $\bar{v}_{[i]}(k) \leftarrow \bar{v}_{[i]}(k-1) - \frac{v_f}{\rho_{jam}} e_{[i]}(k)$
    $\bar{q}_{[i]}(k) \leftarrow \bar{v}_{[i]}(k) \cdot \rho_{[i]}^{*}(k)$

Algorithm 5 Filter

//Moving weight average
define:
α: Filter coefficient

procedure FILTER($\bar{v}_{[i]}(k)$, α)
  while $(i > 0, j > 0)$ **do**
    if $(\bar{v}_{[i]}(k) \leq 0)$ **then**
      $\bar{v}_{[i]}(k) \leftarrow \bar{v}_{[i],measured}(k)$
    if $(\bar{v}_{[i]}^{*}(k-1) \leq 0)$ **then**
      $\bar{v}_{[i]}^{*}(k-1) \leftarrow \bar{v}_{[i],measured}(k)$
    $\alpha \leftarrow f(\epsilon)$ //set the size of alpha corresponding to ϵ
    $\bar{v}_{[i]}^{*}(k) \leftarrow \alpha \bar{v}_{[i]}(k) + (1-\alpha)\bar{v}_{[i]}^{*}(k-1)$

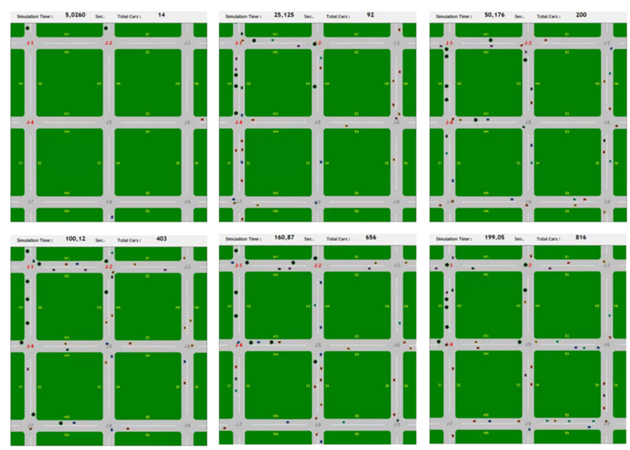

Fig. 8 Screenshot of the AIM simulation of multiple intersection management

1000 and 2000 vehicles/h/lane to test the system with the maximum capacity. The snapshot of the AIM simulator for multiple intersection management is shown in Fig. 8.

We plotted three observed traffic parameters with their relationships compared with the Greenshields traffic model. Figure 9 shows the plot of a relationship between traffic velocity and traffic density. Figures 10 and 11 show the plots of a relationship between traffic velocity and traffic flow rate, and traffic flow rate and traffic density, respectively. The results showed that with the randomized input traffic

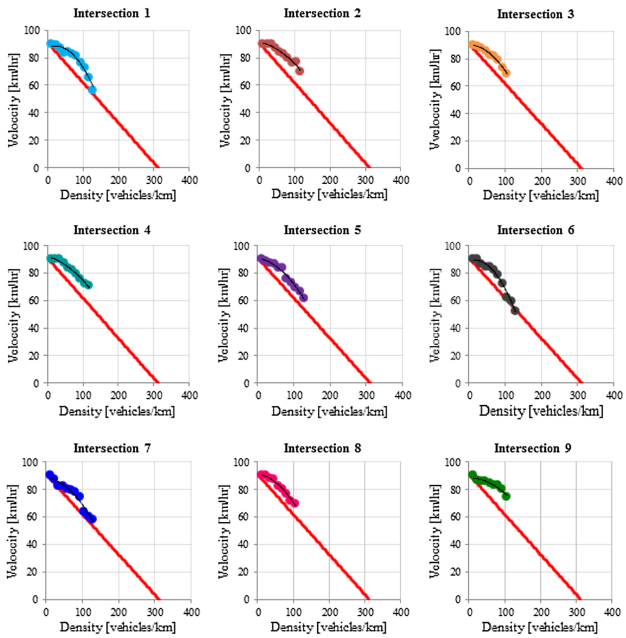

Fig. 9 The comparison of traffic velocity and the traffic density relationship of each intersection in the network

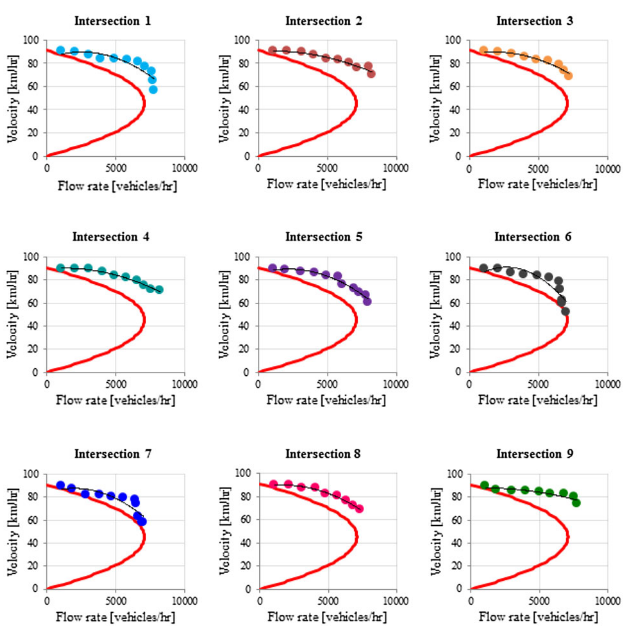

Fig. 10 The comparison of traffic velocity and the traffic flow rate relationship of each intersection in the network

from the outside of network, consensus showed the ability to manage all traffics in the network work under the uncongested condition. It represented the corresponded trend to the Greenshields model.

Figure 12 shows the collecting plot of all intersections in the network. The results showed all intersections can maintain the level of traffic density, traffic velocity and the traffic flow rate, within the uncongested condition. In addition, AIM

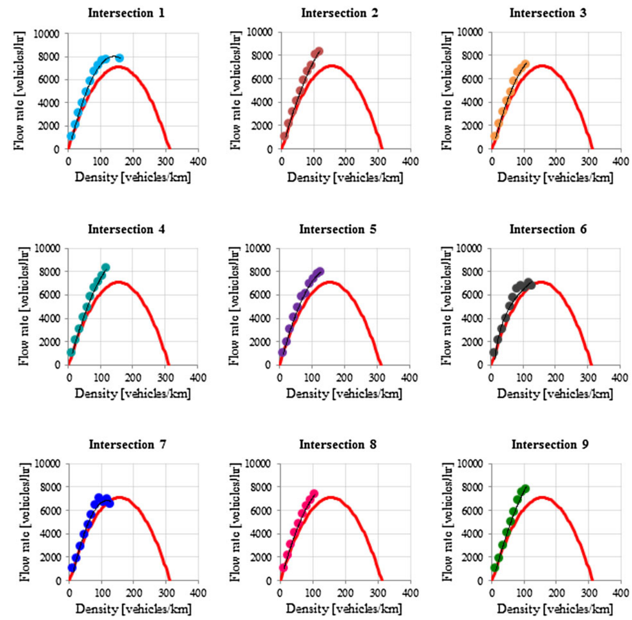

Fig. 11 The comparison of traffic flow rate and the traffic density relationship of each intersection in the network

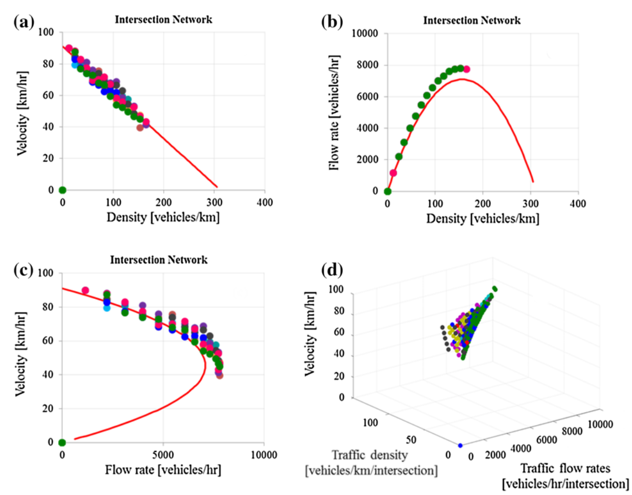

Fig. 12 Collecting plot of all traffic relationships of the intersection network, compared with the Greenshields model (**a**). Traffic velocity and traffic density (**b**). Traffic flow rate and Traffic density (**c**). Traffic velocity and traffic flow rate and (**d**). Triple plot of all traffic parameters

provides better efficiency in traffic flow rates, compared to the theoretical value given by Greenshields model.

In addition, we analyzed the variation of traffic velocity, traffic density and traffic flow rate of each intersection in the network as shown in Figs. 13, 14 and 15, respectively. The plots show that with the consensus coordination, all traffic parameters in the network maintain a similar level with small variation. The exceeded congestion has been distributed to the nearest neighbored intersection to balance the overall throughput of the entire network.

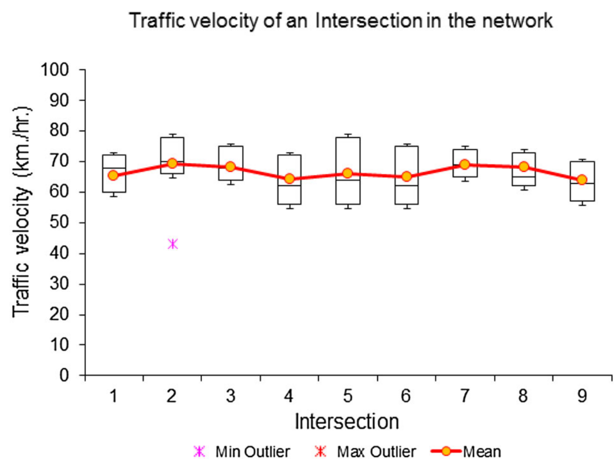

Fig. 13 Variance plot of traffic velocity in the network

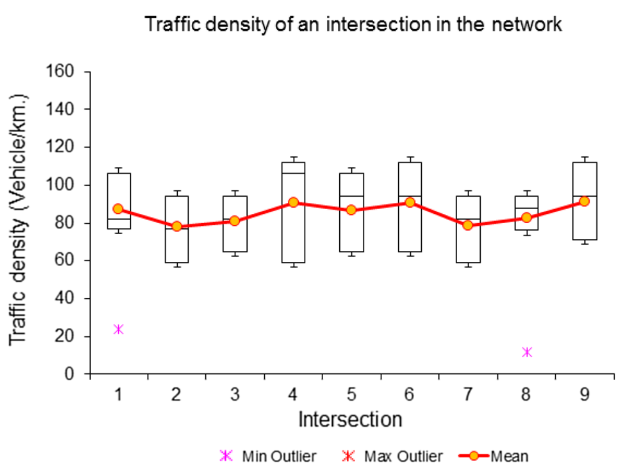

Fig. 14 Variance plot of traffic density in the network

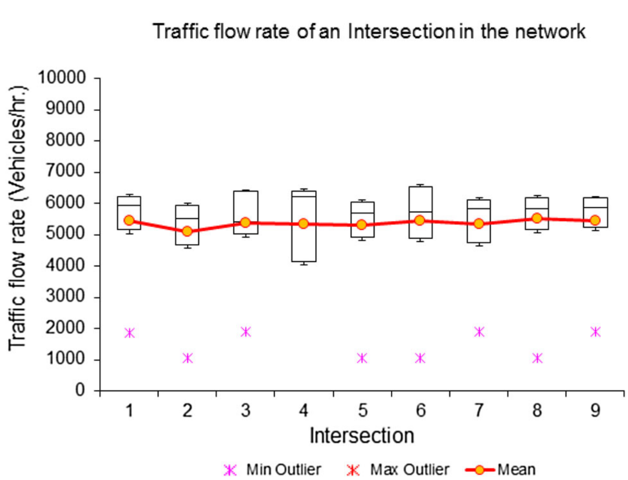

Fig. 15 Variance plot of traffic flow rate in the network

We also compared the performance of the consensus-based AIM with the traffic light signal system with the same configurations. Benchmarks of three traffic parameters are

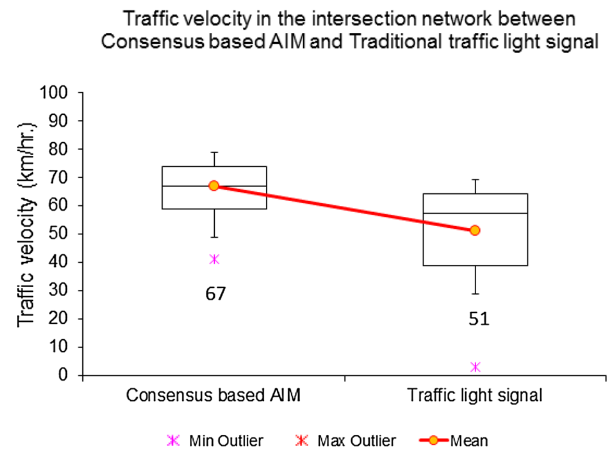

Fig. 16 Comparison plot of traffic velocity between traffic light signal and consensus-based AIM

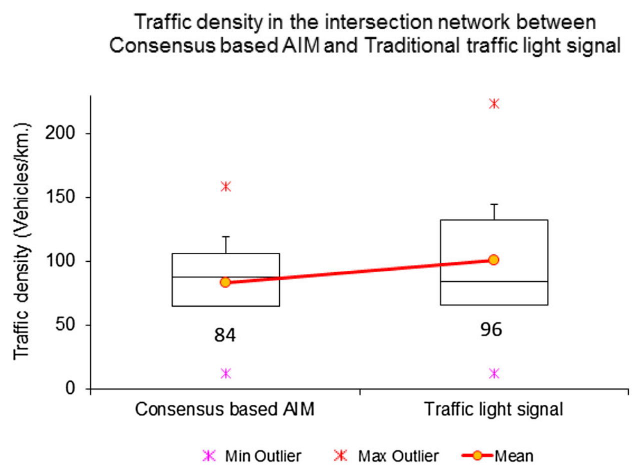

Fig. 17 Comparison plot of traffic density between traffic light signal and consensus-based AIM

plotted in Figs. 16, 17 and 18 and the comparison results are listed in Table 1.

Consensus-based AIM provides a higher average of traffic velocity that allows vehicle driving throughout the network at 67 km/h and traditional traffic light signal gives at 51 km/h. However, the obvious difference is the range of driving speed. The traditional traffic signal showed a larger variation and the worse case is vehicle has completely stopped. In addition, our proposed method showed that the average of traffic density and its variation provided a better performance ca. 32% over the traditional traffic signal system. Last is the average of traffic flow rate. With consensus-based AIM, it gave a higher throughput ca. 16.46% per Hour. The comparison plot of three traffic parameters between consensus-based AIM and the traditional traffic light signal system is shown in Figs. 16, 17 and 18.

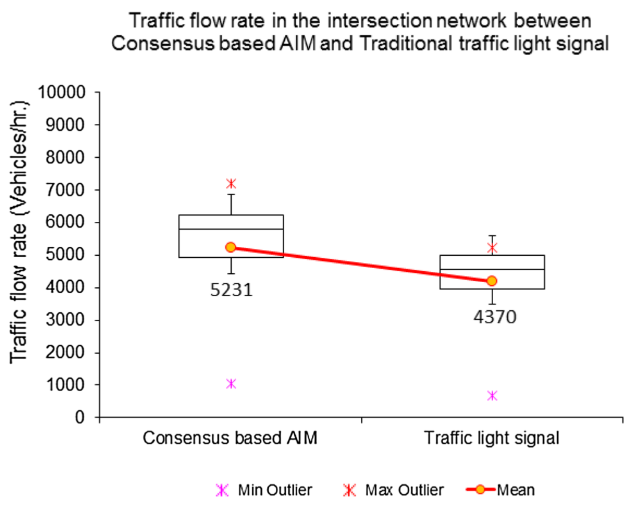

Fig. 18 Comparison plot of traffic flow rate between traffic light signal and consensus-based AIM

Table 1 Comparison table

Parameters	Method			
	Consensus based AIM		Traffic light signal	
	Mean	SD	Mean	SD
Velocity (km/h)	67	11.41	51	19.11
Density (vehicle/km)	84	34	96	50
Flow rate (vehicle/h)	5231	1433	4370	1005

Conclusions

This work introduces the coordination method for multiple, autonomous intersections using discrete consensus algorithm with the Greenshields model. In this paper, the proposed method presents the success performance in autonomous managing the traffic in the network of multiple intersections. The simulation results showed every intersection in the network can operate under the uncongested flow condition and provides a contribution in traffic flow rate capability. In addition, it presents the success driving under the green wave concept that all vehicles can maintain continuous driving and crossing multiple intersections without stop.

Discussion and future work

We would like to test the system at the critical condition where the number of vehicles is over the maximum capacity. In this work, we proved that the consensus can achieve the maximum traffic flow input, but the actual traffic congested is caused by the excessive number of vehicles on street. Thus, instead of generating traffic flow input, we will input the system with the maximum traffic density. We have an implementation plan for the next step. We will implement the route assignment process to help consensus distribute the excessive vehicles to the nearest neighborhoods, where they have a lower traffic density. Referring to the traffic flow model this logic fills the gap of the remaining control parameter, traffic density. Technically, consensus coordinates the traffic density to find the optimal level of traffic velocity and is used to manage the traffic. It has no direct control on the traffic density but with the rerouting process, it provides direct control on the traffic density. The implementation concept is illustrated in Fig. 19.

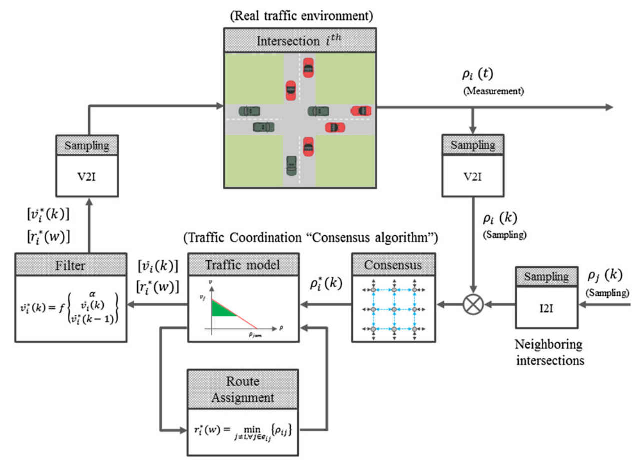

Fig. 19 Closed loop control block diagram of an intersection manager with the route assignment

Acknowledgements This research is funded by Ministry of Innovation, Science, Research, and Technology of the Federal State North Rhine-Westphalia, Germany through the International Graduate School (IGS) of Dynamic Intelligent System. This research work is the continued project of the autonomous intersection management under the supervision of Prof. Ansgar Traechtler, Heinz Nixdorf Institute, Control engineering and Mechatronics department, University of Paderborn.

References

1. Chiu S, Sujeet C (1993) Adaptive traffic signal control using fuzzy logic. In: Fuzzy systems, 1993, second IEEE international conference on, pp 1371–1376. IEEE. doi:10.1109/FUZZY.1993.327593
2. Robertson DI, Bretherton RD (1991) Optimizing networks of traffic signals in real time: the SCOOT method. IEEE transactions on vehicular technology, vol 40, no. 1. doi:10.1109/25.69966
3. Aydos JC, O'Brien A (2014) SCATS ramp metering: strategies, arterial integration and results. In: 17th international IEEE conference on intelligent transportation systems (ITSC), pp 2194–2201. IEEE. doi:10.1109/ITSC.2014.6958028

4. Thrun S, Montemerlo M, Dahlkamp H, Stavens D, Aron A, Diebel J, Fong P et al (2006) Stanley: the robot that won the DARPA grand challenge. J Field Robot 23(9):661–692. doi:10.1002/rob.20147
5. Urmson C, Anhalt J, Bagnell D, Baker C, Robert Bittner MN, Clark JD et al (2008) Autonomous driving in urban environments: boss and the urban challenge. J Field Robot 25(8):425–466. doi:10.1002/rob.20255
6. Ferguson D, Howard TM, Likhachev M (2008) Motion planning in urban environments: Part II. In: 2008 IEEE/RSJ international conference on intelligent robots and systems, pp 1070–1076. IEEE. doi:10.1109/IROS.2008.4651124
7. Dolgov D, Thrun S, Montemerlo M, Diebel J (2010) Path planning for autonomous vehicles in unknown semi-structured environments. Int J Robot Res 29(5):485–501. doi:10.1177/0278364909359210
8. Wuthishuwong C, Silawatchananai C, Parnichkun M (2009) Navigation of an intelligent vehicle by using stand-alone GPS, compass and laser range finder. In: Robotics and Biomimetics, 2008. ROBIO 2008. IEEE international conference on, pp 2121–2126. IEEE. doi:10.1109/ROBIO.2009.4913330
9. Suppachai H , Silawatchananai C, Parnichkun M, Wuthishuwong C (2009) Double loop controller design for the vehicle's heading control. In: Robotics and biomimetics (ROBIO), 2009 IEEE international conference on, pp 989-994. IEEE. doi:10.1109/ROBIO.2009.5420747
10. Hebert MH, Thorpe CE, Stentz A (eds) (2012) Intelligent unmanned ground vehicles: autonomous navigation research at Carnegie Mellon, vol 388. Springer Science and Business Media, Berlin. doi:10.1007/978-1-4615-6325-9
11. Ziegler J, Bender P, Schreiber M, Lategahn H, Strauss T, Stiller C, Dang T et al (2014) Making bertha drivean autonomous journey on a historic route. IEEE Intell Transp Syst Mag 6(2):8–20. doi:10.1109/MITS.2014.2306552
12. Litman T (2015) Autonomous vehicle implementation predictions: implications for transport planning. In: Transportation research board 94th annual meeting, no. 15–3326. doi:10.1177/0739456X15613591
13. Schoettle B, Sivak M (2015) Potential impact of self-driving vehicles on household vehicle demand and usage. The University of Michigan Transportation Research Institute
14. Kenney JB (2011) Dedicated short-range communications (DSRC) standards in the United States. Proc IEEE 99(7):1162–1182. doi:10.1109/JPROC.2011.2132790
15. Hafner MR, Cunningham D, Caminiti L, Del Vecchio D (2011) Automated vehicle-to-vehicle collision avoidance at intersections. In: Proceedings of world congress on intelligent transport systems
16. Azimi R, Bhatia G, Rajkumar R, Mudalige P (2013) V2V-intersection management at roundabouts. SAE international journal of passenger cars-mechanical systems 6, no. 2013-01-0722:681–690. doi:10.4271/2013-01-0722
17. Gradinescu V, Cristian G, Raluca D, Valentin C, Liviu I (2007) Adaptive traffic lights using car-to-car communication. In: 2007 IEEE 65th vehicular technology conference-VTC2007-Spring, pp 21-25. IEEE. doi:10.1109/VETECS.2007.17
18. Cai C, Wang Y, Geers G (2013) Vehicle-to-infrastructure communication-based adaptive traffic signal control. IET Intell Transp Syst 7(3):351–360. doi:10.1049/iet-its.2011.0150
19. Djahel S, Nafaa J, Robert B, John M (2015) Toward V2I communication technology-based solution for reducing road traffic congestion in smart cities. In: Networks, computers and communications (ISNCC), 2015 international symposium on, pp 1–6. IEEE. doi:10.1109/ISNCC.2015.7238584
20. Wongpiromsarn T, Tawit U, Yu W, Emilio F, Danwei W (2012) Distributed traffic signal control for maximum network throughput. In: 2012 15th international IEEE conference on intelligent transportation systems, pp 588–595. IEEE. doi:10.1109/ITSC.2012.6338817
21. Naumann R, Rasche R, Tacken J, Tahedi C (1997) Validation and simulation of a decentralized intersection collision avoidance algorithm. In: Intelligent transportation system, 1997. ITSC'97, IEEE conference on, pp 818–823. IEEE. doi:10.1109/ITSC.1997.660579
22. Naumann R, Rainer R (1997) Intersection collision avoidance by means of decentralized security and communication management of autonomous vehicles. Univ.-GH, SFB 376
23. Naumann R, Rasche R, Tacken J (1998) Managing autonomous vehicles at intersections. IEEE Intell Syst Appl 13(3):82–86. doi:10.1109/5254.683216
24. Dresner K, Peter S (2006) Traffic intersections of the future. In: Proceedings of the national conference on artificial intelligence, vol 21, no 2, p 1593. Menlo Park, CA; Cambridge, MA; London; AAAI Press; MIT Press; 1999, 2006
25. Dresner K, Stone P (2008) A multiagent approach to autonomous intersection management. J Artif Intell Res 31:591–656. doi:10.1613/jair.2502
26. Au T-C, Peter S (2010) Motion planning algorithms for autonomous intersection management. In: Bridging the gap between task and motion planning
27. Au T-C, Shun Z, Peter S (2014) Semi-autonomous intersection management. In: Proceedings of the 2014 international conference on autonomous agents and multi-agent systems, pp 1451–1452. International foundation for autonomous agents and multiagent systems
28. Hausknecht M, Au T-C , Stone P (2011) Autonomous intersection management: multi-intersection optimization. In: 2011 IEEE/RSJ international conference on intelligent robots and systems, pp 4581–4586. IEEE. doi:10.1109/IROS.2011.6094668
29. Qian X, Gregoire J, Moutarde F, De La Fortelle A (2014) Priority-based coordination of autonomous and legacy vehicles at intersection. In: 17th international IEEE conference on intelligent transportation systems (ITSC), pp 1166–1171. IEEE. doi:10.1109/ITSC.2014.6957845
30. Chen L, Englund C (2016) Cooperative intersection management: a survey. IEEE Trans Intell Transp Syst 17(2):570–586. doi:10.1109/TITS.2015.2471812
31. Greenwood D, Burdiliak B, Trencansky I, Armbruster H, Dannegger C (2009) GreenWave distributed traffic intersection control. In: Proceedings of the 8th international conference on autonomous agents and multiagent systems-volume 2, pp 1413–1414. International foundation for autonomous agents and multiagent systems
32. Wuthishuwong C, Traechtler A (2013) Vehicle to infrastructure based safe trajectory planning for Autonomous Intersection Management. In: ITS telecommunications (ITST), 2013 13th international conference on (pp 175–180). IEEE. doi:10.1109/ITST.2013.6685541
33. Wuthishuwong C, Traechtler A, Bruns T (2015) Safe trajectory planning for autonomous intersection management by using vehicle to infrastructure communication. EURASIP J Wirel Commun Netw 2015(1):1–12. doi:10.1186/s13638-015-0243-3
34. Wuthishuwong C, Traechtler A (2013) Coordination of multiple autonomous intersections by using local neighborhood information. In: 2013 international conference on connected vehicles and expo (ICCVE), pp 48–53. IEEE. doi:10.1109/ICCVE.2013.6799768
35. Wuthishuwong C, Traechtler A (2014) Consensus coordination in the network of autonomous Intersection Management. In: Informatics in control, automation and robotics (ICINCO), 2014 11th international conference on, vol 2, pp 794–801. IEEE. doi:10.5220/0005148607940801
36. Ren W, Beard RW, Atkins EM (2005) A survey of consensus problems in multi-agent coordination. In: Proceedings of the 2005, American Control Conference, 2005, pp 1859–1864. IEEE. doi:10.1109/ACC.2005.1470239

37. Ren Wei, Beard RW, Atkins EM (2007) Information consensus in multivehicle cooperative control. IEEE Control Syst Mag 2(27):71–82. doi:10.1109/MCS.2007.338264
38. Ren W, Beard RW (2008) Distributed consensus in multi-vehicle cooperative control. Springer, London. doi:10.1007/978-1-84800-015-5
39. Ren W, Sorensen N (2008) Distributed coordination architecture for multi-robot formation control. Robot Auton Syst 56(4):324–333. doi:10.1016/j.robot.2007.08.005
40. Martinez S, Cortes J, Bullo F (2007) Motion coordination with distributed information. IEEE Control Syst 27(4):75–88. doi:10.1109/MCS.2007.384124
41. Bullo F, Cortes J, Martinez S (2009) Distributed control of robotic networks: a mathematical approach to motion coordination algorithms. Princeton University Press. doi:10.1515/9781400831470
42. Olfati-Saber R, Fax JA, Murray RM (2007) Consensus and cooperation in networked multi-agent systems. In: Proceedings of the IEEE 95, no. 1 (2007), pp 215–233. doi:10.1109/JPROC.2006.887293
43. Olfati-Saber R (2006) Flocking for multi-agent dynamic systems: algorithms and theory. IEEE Trans Autom Control 51(3):401–420. doi:10.1109/TAC.2005.864190
44. Olfati-Saber R, Shamma JS (2005) Consensus filters for sensor networks and distributed sensor fusion. In: Proceedings of the 44th IEEE conference on decision and control, pp 6698-6703. IEEE. doi:10.1109/CDC.2005.1583238
45. Wuthishuwong C, Traechtler A (2014) Stability of the consensus in the network of multiple autonomous intesections management. In: Automation and computing (ICAC), 2014 20th international conference on, pp 295–300. IEEE. doi:10.1109/IConAC.2014.6935503
46. Moreau L (2004) Stability of continuous-time distributed consensus algorithms. In: Decision and control, 2004. CDC. 43rd IEEE conference on, vol 4, pp 3998–4003. IEEE. doi:10.1109/CDC.2004.1429377
47. Hall FL (1996) Traffic stream characteristics. Traffic flow theory. US Federal Highway Administration
48. Maerivoet S, De Moor B (2005) Traffic flow theory. physics/0507126**(preprint)**
49. Rakha H, Crowther B (1802) Comparison of greenshields, pipes, and van aerde car-following and traffic stream models. Transp Res Rec J Transp Res Board 2002:248–262. doi:10.3141/1802-28
50. Murray RM (2009) Introduction to graph theory and consensus. In: Lecture notes, 2009 Caltech control and dynamical systems
51. World Health Organization (2015) Global status report on road safety