SMAA-AD Model in Multicriteria Decision-Making Problems with Stochastic Values and Uncertain Weights

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Abstract The current paper considers the stochastic multicriteria decision-making (MCDM) problems with multiple alternatives, stochastic criterion values and uncertain criterion weights. We propose SMAA-AD model and illustrate how SMAA-AD model is used in such stochastic MCDM problems. In SMAA-AD model, absolute dominant method is used to turn stochastic criterion values into deterministic absolute dominant values, and stochastic multicriteria acceptability analysis (SMAA) is used to rank the alternatives without foreknowing the decision maker's preference on criterion weights. SMAA-AD model provides three indices, i.e., rank acceptability index, holistic acceptability index and central weight vector, to support the decision in the stochastic MCDM problems. SMAA-AD model overcomes some shortcomings of traditional MCDM methods. For example, it needs not to predefine any parameters and functions. We use a case of technology competition for cleaning polluted soil in Helsinki to illustrate our method.

Keywords Multicriteria decision-making (MCDM) · Stochastic multicriteria acceptability analysis (SMAA) · Absolute dominant method (ADM) · Stochastic value · Uncertain weight

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1 Introduction

The current paper focuses on the stochastic multicriteria decision-making (MCDM) problems with more than one alternatives where the decision maker (DMer) is requested to rank the alternatives according to multiple criteria. Different from the traditional MCDM problems, in the stochastic MCDM problems, the *criterion values* (i.e., the values of different criteria for any given alternative) is stochastic, and the *criterion weights* (i.e., the weights or importance degrees of different criteria) is uncertain. The forthcoming two paragraphs show that stochastic criterion values and uncertain criterion weights are often observed in many real-world MCDM problems.

When dealing with many MCDM problems, we often feel it difficult to provide deterministic measurements for the criterion values. For example, in some cases with limitation of time and capability, the DMer cannot provide deterministic criterion values. Hence the use of accurate criterion values in such a decision making procedure means an oversimplification of the problem. If multiple DMers are involved in such a procedure, it turns more difficult to reach consensus about exact measurement of each criterion. As a result, the criterion measurements are usually obtained in the form of stochastic values or multiple deterministic values with the corresponding probabilities. For the case of stochastic values, the DMer just needs to specify the upper and lower bounds of each criterion value. Considering the total lack of knowledge about the probability distributions of the criteria measurements, we assume that the criteria measurements distribute uniformly within the specific bound.

Another aspect of the actual MCDM problems is related to the difficulty of obtaining the preference information of different DMers about the criterion weights. Sometimes the DMers are not willing to express their preferences explicitly. Sometimes the DMers are unable to estimate the importance degrees of different criteria especially when some criteria are monetary while the others are nonmonetary. In addition, the relative importance of different criteria will not keep steady with changing contexts. As a result, it is difficult for the DMer to gain the explicit importance weight about the criteria. Thus only inaccuracy or incomplete weights are available. Lahdelma and Salminen [9] proposed five types of restrictions on the weight space: (1) partial or complete ranking of criteria, (2) intervals for weights, (3) intervals for weight ratios, (4) linear inequality constraints for weights, and (5) nonlinear inequality constraints for weights.

Several methods have been proposed for dealing with MCDM problems with uncertain criterion weights. Some methods are based on multiattribute utility theory [7] where the uncertainty of the criterion weights is represented by probability distributions and the alternatives are evaluated according to their expected utility. In ELECTRE methods [15], the uncertainty is handled by the threshold model. In PROMETHEE methods [13], flow calculations with intervals are performed as extensions of the current calculations. The PRIME method [16] represents the uncertainty by intervals for weight ratios. In addition, [2] introduced Fuzzy MCDM method to represent the uncertainty by fuzzy numbers.

Stochastic multicriteria acceptability analysis (SMAA) is a family of methods that tackle various classifications of MCDM problems with uncertain, imprecise, or missing information about criterion weights. A lot of extended SMAA methods have been pro-

posed in choosing, ranking, or sorting candidates in the MCDM problems [5]. SMAA is based on the overall compromise criterion method and comparative hypervolume criterion. Bana e Costa [1] introduced the overall compromise criterion method to calculate the amount of difference between the preferences of various DMers by defining a joint probability density function for the weight space. Comparative hypervolume criterion by Charnetski [3] is based on computing the volume of the multidimensional weight space that makes each alternative the most preferred one. In the original SMAA method, inverse weight space analysis is introduced to identify for each alternative the weights that make it most preferred based on an additive utility or value function. SMAA-2 extends the analysis to a general utility or value function. SMAA-2 method also provides some useful tools for ranking the alternatives, and those tools include rank acceptability indices, central weight vectors, confidence factors and others. Lahdelma and Salminen [10] developed cross confidence factors which were based on computing confidence factors for alternatives by using each other's central weight vectors when the problem involved a large set of efficient alternatives. Lahdelma et al. [8] introduced a method to handle dependent uncertainties in stochastic multicriteria group decision-making problems based on SMAA-2. SMAA-O method [14] extends SMAA-2 for treating mixed ordinal and cardinal criteria. SMAA-3 [17] uses the out-ranking pseudo-criteria, not a utility function, in ELECTRE III procedure. SMAA-D [11] is a variant of SMAA-2 that employs the efficiency scores of data envelopment analysis instead of a value/utility function. SMAA-P [12] assumes piecewise linear difference functions based on prospect theory instead of a utility or value function model. Ref-SMAA (SMAA-A) method [4] allows multiple DMers to use reference points and Wierzbicki's achievement scalarizing function to provide descriptive information for each alternative.

In the original SMAA, a utility or value function models the DMer's preference structure. However, the utility or value function is not linear because of diminishing marginal utility when dealing with the real-life problems. It is difficult to define the exact function. In SMAA-3, ELECTRE III-type pseudo-criteria are used instead. The DMer needs to provide deterministic thresholds and cutting levels. The results will vary greatly if the DMer chooses different parameters. SMAA-P assumes linear prospect theory type difference functions and avoids the drawback of linear utility or value function model, while the DMer needs to choose reference points and coefficients of loss aversion in advance. It is difficult to predefine the parameters, and the results depend on the assumed parameters. In summary, all of the above methods need to foreknow the DMer's preference on utility function, value function, threshold, parameters and so on.

In this paper, we introduce a new method to transform stochastic criteria measurements to deterministic ones without needing the foreknowledge of the DMer's preference. The basis of the approach is to calculate how much one alternative is better than all other alternatives absolutely by accumulated pairwise integrating without presetting any parameters. When we obtain the deterministic absolute dominant values, the inverse weight space analysis is used to describe each alternative what kind of preferences makes it the most preferred one, or places it on any particular rank. We will use three indices to support the decision. The first and second are the acceptability indices which are to measure the variety of different preferences that support an alter-

Table 1 A typical multicriteria decision matrix with bounds of stochastic criterion values

	Criterion 1	Criterion 2	...	Criterion m
Alternative 1	$[x_{11}^l, x_{11}^u]$	$[x_{12}^l, x_{12}^u]$	...	$[x_{1m}^l, x_{1m}^u]$
Alternative 2	$[x_{21}^l, x_{21}^u]$	$[x_{22}^l, x_{22}^u]$	...	$[x_{2m}^l, x_{2m}^u]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Alternative n	$[x_{n1}^l, x_{n1}^u]$	$[x_{n2}^l, x_{n2}^u]$	...	$[x_{nm}^l, x_{nm}^u]$

native, and the third is the central weight vectors representing the typical preferences which make an alternative preferred.

This paper is organized as follows. In Sect. 2, we introduce absolute dominant method (ADM) which transforms stochastic criteria measurements to deterministic ones. In Sect. 3, we propose SMAA-AD method that combines the fundamental principle of SMAA-2 with ADM to obtain rank acceptability index and central weight vector of each alternative. In Sect. 4, we demonstrate the use of the SMAA-AD method in a technology competition for cleaning polluted soil in Helsinki. We end this paper with conclusions in the last section.

2 Absolute dominant method (ADM)

We consider stochastic MCDM problems, where the DMer wants to identify one or a few ‘best’ alternatives from a set of n alternatives $\{x_1, x_2, \dots, x_n\}$. The alternatives are evaluated in terms of m criteria. The criterion values are represented by stochastic variables x_{ik} with estimated probability distributions and density functions $f(x_{ik})$ in the criterion space $X \subseteq R^{n \times m}$. The upper bound of x_{ik} is x_{ik}^u and the lower bound is x_{ik}^l . The stochastic MCDM problem is described as Table 1.

We define absolute dominant function $V(\cdot)$. It represents the expectation that how much alternative i is better than alternative j . For the criterion to be maximized, the absolute dominant value that alternative i is better than alternative j on criterion k is

$$V_{ijk} = \int \int_D (x_{ik} - x_{jk}) f(x_{ik}, x_{jk}) dx_{ik} dx_{jk} \quad (1)$$

Here $D = \{(x_{ik}, x_{jk}) | x_{ik}^l \leq x_{ik} \leq x_{ik}^u, x_{jk}^l \leq x_{jk} \leq x_{jk}^u, x_{ik} > x_{jk}\}$.

While for the criterion to be minimized

$$V_{ijk} = \int \int_{D'} (x_{jk} - x_{ik}) f(x_{ik}, x_{jk}) dx_{ik} dx_{jk} \quad (2)$$

where $D' = \{(x_{ik}, x_{jk}) | x_{ik}^l \leq x_{ik} \leq x_{ik}^u, x_{jk}^l \leq x_{jk} \leq x_{jk}^u, x_{jk} > x_{ik}\}$.

Here $f(x_{ik}, x_{jk})$ is the joint density function of x_{ik} and x_{jk} . Usually x_{ik} and x_{jk} are independent stochastic variables, thus $f(x_{ik}, x_{jk}) = f(x_{ik})f(x_{jk})$. It is assumed without loss of generality that every criterion is to be maximized and the alternatives

Fig. 1 The integral area if the values of x_{ik} and x_{jk} have a cross area

Fig. 2 The integral area if $x_{ik}^l > x_{jk}^u$

are independent. The following four cases describe the calculation of the absolute dominant value.

Case 1. Both x_{ik} and x_{jk} are stochastic

If the stochastic values of x_{ik} and x_{jk} have a cross area, the integral area can be illustrated graphically as in Fig. 1. If $x_{ik}^l > x_{jk}^u$, the integral area can be illustrated as in Fig. 2. Otherwise if $x_{ik}^u < x_{jk}^l$, the integral area is null-space, and the absolute dominant value is zero.

Case 2. x_{ik} is stochastic and x_{jk} is deterministic

If x_{ik} is stochastic and x_{jk} is deterministic, the joint destiny function is

$$f(x_{ik}, x_{jk}) = f(x_{ik})\sigma(x_{jk} - x_{jk}^*) \quad (3)$$

Here deterministic criterion value x_{jk}^* is a special case of independent stochastic value x_{jk} with density function

$$f(x_{jk}) = \sigma(x_{jk} - x_{jk}^*) \quad (4)$$

Table 2 The overall absolute dominant values on each criterion

	Criterion 1	Criterion 2	...	Criterion m
Alternative 1	V_{11}	V_{12}	...	V_{1m}
Alternative 2	V_{21}	V_{22}	...	V_{2m}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Alternative n	V_{n1}	V_{n2}	...	V_{nm}

where σ is Dirac's delta function and $f(x_{jk})$ is the impulse function.

Then the absolute dominant function can be simplified as

$$V_{ijk} = \int_{x_{ik} > x_{jk}^*} (x_{ik} - x_{jk}^*) f(x_{ik}) dx_{ik} \quad (5)$$

Case 3. x_{ik} is deterministic and x_{jk} is stochastic

If x_{ik} is deterministic and x_{jk} is stochastic, we have

$$V_{ijk} = \int_{x_{ik}^* > x_{jk}} (x_{ik}^* - x_{jk}) f(x_{jk}) dx_{jk} \quad (6)$$

Case 4. Both x_{ik} and x_{jk} are deterministic

If both x_{ik} and x_{jk} are deterministic, the joint destiny function is defined as

$$f(x_{ik}, x_{jk}) = \sigma(x_{ik} - x_{ik}^*) \sigma(x_{jk} - x_{jk}^*) \quad (7)$$

Then absolute dominant function is simplified as

$$V_{ijk} = x_{ik}^* - x_{jk}^* \quad (8)$$

According to the above equations, we can calculate the absolute dominant value that that alternative i is better than another alternative j . Next we consider the absolute dominant value of one alternative relative to all of other alternatives. The overall absolute dominant value that alternative i is better than all of the other alternatives on criterion k is

$$V_{ik} = \sum_{j \neq i} \int_D (x_{ik} - x_{jk}) f(x_{ik}, x_{jk}) dx_{ik} dx_{jk} \quad (9)$$

The larger V_{ik} , the better alternative i than all of other alternatives on criterion k . The overall absolute dominant values of each alternative are expressed as Table 2.

The absolute dominant value on each criterion can be normalized as

$$v_{ik} = \frac{V_{ik}}{\max_i V_{ik}} \quad (10)$$

Table 3 presents the normalized overall absolute dominant values on each criterion.

Table 3 The normalized overall absolute dominant values

	Criterion 1	Criterion 2	...	Criterion m
Alternative 1	v_{11}	v_{12}	...	v_{1m}
Alternative 2	v_{21}	v_{22}	...	v_{2m}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Alternative n	v_{n1}	v_{n2}	...	v_{nm}

After we get the normalized overall absolute dominant values on each criterion, the stochastic MCDM problem is converted to a deterministic MCDM problem. Then the overall absolute dominant value of alternative i on all the criteria is expressed as

$$v_i = \sum_{k=1}^m w_k v_{ik} \quad (11)$$

where w_k is the weight of criterion k . The larger v_i , the better alternative i will be better than all the other alternatives. Thus the DMer will choose the alternative with the largest v_i . However, in the real world decision problems, the DMer can not provide exact weight of each criterion w_k due to some reasons. Consequently, we propose a method that combined ADM and inverse weight space method to deal with such problems in the next section.

3 SMAA-AD method

The current section continues the stochastic MCDM problem in Sect. 2. According (1)–(10), we have obtained the overall absolute dominant values on each criterion for any alternative. However, the DMer's preference structure on the criteria is still unknown, which makes the calculation of (11) difficult. The current section extends SMAA-2 model to SMAA-AD model by introducing ADM as discussed in the previous section. SMAA-2 uses inverse weight space analysis to describe for each alternative what kind of preferences makes it the most preferred one, or places it on any particular rank.

We assume that the DMer's preference is unknown or partially known, represented by a weight distribution with density function $f(w)$ in the set of feasible weights W . Here the weight space W is defined as

$$W = \left\{ w \in R^m : w \geq 0 \text{ and } \sum_{k=1}^m w_k = 1 \right\} \quad (12)$$

The weight space is an $(n - 1)$ -dimensional, and can be added some constraints to represent different kinds of preference information according to different circumstances. For example, the DMer can specify precise weights (w_k), a priority order for the criteria ($w_j > w_k$, for some j, k), weight intervals ($w_k^{\min} \leq w_k \leq w_k^{\max}$), intervals for weight ratios ($w_j^{\min} \leq w_j/w_k \leq w_j^{\max}$), and arbitrary linear or non-linear constraints

for weights ($Aw \leq c$ or $g(w) \leq 0$). Total lack of preference information is represented by a uniform weight distribution in W . The density function of uniform distribution is expressed as

$$f(w) = 1/\text{vol}(W) \quad (13)$$

According to ADM, we define the rank of each alternative as an integer from the best rank (=1) to the worst rank (= n) by means of a ranking function

$$\text{rank}(i, w) = 1 + \sum_{j \neq i} \rho(v_j > v_i) \quad (14)$$

where $\rho(\text{true}) = 1$ and $\rho(\text{false}) = 0$. The favorable weights sets for each alternative to get rank r is defined by

$$W_i^r = \{w \in W : \text{rank}(i, w) = r, \quad r = 1, 2, \dots, n\} \quad (15)$$

Any weight $w \in W_i^r$ makes alternative i obtain rank r .

Next we use three descriptive indices to appraise all of the alternatives in the stochastic MCDM problem.

(1) Rank Acceptability Index b_i^r

The first descriptive measure of SMAA-AD model is *rank acceptability index* b_i^r , which measures the variety of different preferences that grant alternative x_i to get rank r . b_i^r is computed numerically as a multidimensional integral over the favorable rank weights by

$$b_i^r = \int_{W_i^r} f(w)dw \quad (16)$$

In fact, the rank acceptability index can be regarded as the probability for which alternative x_i gets rank r if the weights distribute uniformly in the feasible weight space. Evidently, the rank acceptability indices are in the range $[0, 1]$ where 0 indicates that the alternative will never obtain a given rank and 1 indicates that it will obtain the given rank always with any choice of weights.

Among all of b_i^r , we focus most attention on the first rank acceptability index b_i^1 . The first rank acceptability index represents the share of all the weights space for which the alternative i is the best choice. If b_i^1 is near to 1, we conclude that the alternative will be selected as the best one with very high probability. If b_i^1 is zero or near-zero, we conclude that the alternative is hardly to be regarded the best one. In summary, the most acceptable alternatives are those with high values of b_i^1 .

(2) Holistic Acceptability Index a_i^h

The holistic acceptability index a_i^h aims at measuring the overall acceptability of each alternative, and it is computed as

$$a_i^h = \sum_{r=1}^n \alpha^r b_i^r \quad (17)$$

Here α_r is the so-called meta-weight, and $1 \geq \alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_n \geq 0$. How to determine the meta-weights has been discussed in [9]. In general, the metaweights are normalized, nonnegative and nonincreasing when the rank increases. Usually we set the forms of α_r as linear weights $\alpha^r = (n - r)/(n - 1)$, inverse weights $\alpha^r = 1/i$, or centroid weights $\alpha^r = \sum_{i=r}^n 1/i / \sum_{i=r}^n 1/i$.

(3) Central Weight Vector w_i^c

Central weight vector w_i^c is regarded as the expected centroid of the favorable weight space. It is the most likely weight vector that represents a typical DM prefers an alternative. It is computed as an integral of the weight vector over the weight distributions by

$$\begin{aligned} w_i^c &= \int_{w_i^1} w f(w) dw / \int_{w_i^1} f(w) dw \\ &= \int_{w_i^1} w f(w) dw / b_i^1 \end{aligned} \quad (18)$$

4 Illustration

In the current section we demonstrate the use of SMAA-AD method in the context of multicriteria decision support in a technology competition for cleaning polluted soil in Helsinki [6]. The city of Helsinki in Finland was planning to clean ex-industrial contaminated areas. There were nine candidates to compete for the business. The DMers were eight experts from different fields, and they were responsible for evaluating the candidates. In the first step, the DMers would choose three best candidates to test-clean a small part of the contaminated region. Then the winner of the test-cleaning phase will get the contract for cleaning the whole area. As a result, the problem was converted to how the DMers would choose the three best candidates. To deal with this problem, the DMers chose five criteria to evaluate the nine alternatives: (1) costs, (2) environment, (3) innovation, (4) credibility and (5) project management (Table 4).

Only the costs criterion could be quantified directly, thus the DMers had to define scales for the other criteria and assess criterion values for each alternative to compare among the alternatives. The following scale of criteria was agreed on after discussion. Only the costs criterion was to be minimized, the other four criteria were all to be maximized. The cost criterion measurements were provided by the candidates in the contract offers and were measured in million FIM. Hokkanen [6] described the criteria scales as follows:

Table 5 presents the criterion values for each alternative that the DMers have reached consensus. All the criterion values are the average evaluations of the DMers. Each criteria measurement is given as an interval $x_{ik} \pm \Delta x_{ik}$ to represent the imprecision or uncertainty of each criterion. The bottom row of Table 5 shows the intervals that the DMers defined for each criterion. The uncertainty of the cost criterion is assumed to be $\pm 10\%$ for all alternatives except X_1 and X_6 for which much larger intervals are used.

Table 4 The scales of four criteria

Environment	
1	Environmental effects were controlled by isolation, i.e. the chemicals were trapped or encapsulated between tight layers of soil and/or concrete materials.
2	Environmental effects were controlled by strobilization, i.e. binding the harmful chemicals to an insoluble form.
3	Environmental effects were controlled by degradation, i.e. by composting or burning the material.
Innovation	
1	The method had been already commercially used in Finland.
2	The method was new in Finland but in commercial use somewhere.
3	The method was based on a new idea in this context and was new in Finland.
4	The method was universally new.
Credibility	
1	It was doubtful that the suggested method worked in this task.
2	It was possible that the suggested method fitted this problem.
3	It was no doubt that the suggested method worked in practice.
Project management	
1	These factors* were not described in the proposal.
2	These factors* were partly taken into account in the proposal.
3	These factors* were all taken into account in the proposal.

* Quality control, control of permission procedure, and cooperation with the authorities

Table 5 The criterion values and associated uncertainty intervals

Alternative	Criterion 1 cost	Criterion 2 environment	Criterion 3 innovation	Criterion 4 credibility	Criterion 5 proj. mngr
X ₁	19 ± 5.9*	2	3	1	1
X ₂	2.3	1	1.8	2	2
X ₃	6	2	1.9	2	1.1
X ₄	3.6	2.3	2	1.5	1.2
X ₅	8.4	2.2	2.7	2	2
X ₆	13.6 ± 2.85*	2.8	2.8	2	2
X ₇	5.2	1	1	2.8	3
X ₈	5.6	2	2	2.7	2.9
X ₉	6	2	1	2.5	3
Δx	±10%	±0.5	±0.3	±0.3	±0.3

* Specified intervals apply for costs of X₁ and X₆ instead of ±10 %

The weights of different criteria are presented in Table 6. Since the DMers do not reach an agreement on the importance of each criterion, the weight intervals are given in the form of DMers' minimum and maximum weights. In our analysis, the criteria and weights all follow uniform distributions in the corresponding intervals.

Table 7 presents the results based on 500,000 simulations of the normalized overall absolute dominant values and weight vectors. The aim is to choose three best

Table 6 DMers' minimum and maximum weights

Weights	Criterion 1 costs	Criterion 2 environment	Criterion 3 innovation	Criterion 4 credibility	Criterion 5 project management
min	0.08	0.19	0.20	0.06	0.06
max	0.38	0.33	0.42	0.22	0.15

Table 7 The holistic acceptability index a^h and rank acceptability index b^r (%)

Alternative	a^h	b^1	b^2	b^3	b^4	b^5	b^6	b^7	b^8	b^9
X ₁	32.03	0	0	5.95	23.39	9.52	9.34	11.03	15.42	25.35
X ₂	25.36	0.06	0.92	1.23	6.58	11.18	12.21	24.59	25.60	17.68
X ₃	6.85	0	0	0	0	0	0.18	4.31	45.68	49.83
X ₄	53.93	0.50	5.00	16.27	22.20	26.55	18.62	10.85	0	0
X ₅	69.15	0	21.68	44.56	12.46	12.98	3.24	5.09	0	0
X ₆	97.34	83.71	13.39	1.20	1.39	0.18	0.12	0	0	0
X ₇	34.63	0	1.03	6.70	8.73	13.12	19.13	31.98	12.17	7.14
X ₈	85.10	15.78	57.98	17.93	7.87	0.45	0	0	0	0
X ₉	45.61	0	0	6.16	17.38	26.03	37.16	12.15	1.13	0

alternatives, thus the metaweights of top three ranks should not be different greatly. Therefore, we choose the linear weights $\alpha^r = (n - r)/(n - 1)$ to calculate the holistic acceptability index.

Based on the first rank acceptability indices, the top three alternatives are X₆, X₈ and X₄. X₆ has the highest acceptability index (83.71 %) for the best rank, and almost zero acceptability for ranks from five to nine. Alternatives X₈ receive reasonably high acceptability for the top three ranks, while X₄ is not. Even though the first rank acceptability index of X₅ is zero, the second and third acceptability indices are strikingly high. Therefore, we also need to consider the holistic acceptability index of each alternative.

The first three alternatives (X₆, X₈ and X₅) obtain markedly higher holistic acceptability indices (97.34, 85.10 and 69.15 %) and also higher rank acceptability indices for the first three ranks than the remaining alternatives. As a result, alternative X₆, X₈ and X₅ are definitely the three finalists.

The central weights for each alternative are presented in Table 8. For X₁, X₃, X₅, X₇ and X₉, the first rank acceptability indices are all zero, which means they can not be ranked first. Therefore their central weights do not exist. The first rank acceptability index of X₂ is approximately zero; however, X₂ still has the opportunity to rank first when the number of simulations is large enough. The central weight is a reference vector that the DMers prefer to if they choose the alternative as the best one.

5 Conclusions

The main contribution of this paper is to propose ADM that compared the criteria measurements of each alternative and obtain the absolute dominant values. Then we

Table 8 The central weights for the alternatives

Alternative	b^1	w_1	w_2	w_3	w_4	w_5
X ₁	0	–	–	–	–	–
X ₂	0.06	0.38	0.19	0.26	0.06	0.11
X ₃	0	–	–	–	–	–
X ₄	0.50	0.38	0.24	0.22	0.08	0.08
X ₅	0	–	–	–	–	–
X ₆	83.71	0.19	0.26	0.31	0.13	0.11
X ₇	0	–	–	–	–	–
X ₈	15.78	0.27	0.22	0.23	0.16	0.12
X ₉	0	–	–	–	–	–

combine ADM with SMAA-2 and use the deterministic values together with inverse weight space analysis to calculate the probability that each alternative could be acceptable for a particular rank.

We present a revisit of a case study showing the application of the SMAA-AD method. In the case study, uniform distributions are used to represent absence of information both in criteria measurements and preferences. The results of our re-analysis demonstrate the effectiveness of SMAA-AD model.

ADM has many advantages. Especially when the criterion weights are given, the stochastic MCDM problems are transformed into deterministic single-criterion decision problems, and the DMer can choose the best alternative directly. When dealing with the stochastic MCDM problems with multiple DMers, the criteria are defined in intervals subjected to specific distributions. If the weight of each criterion is also not deterministic, it can also be determined as intervals which contain the preferences of all DMers, or be given as partial (or complete) ranking of criteria. Compared with other SMAA-type methods, the advantage of the SMAA-AD method is that we need not to predefine utility function (i.e. liner utility function) and other parameters (i.e. thresholds, cutting levels reference points and coefficients of loss aversion as other SMAA-type methods). This will save a lot of time and cost in decision.

Our method is the first one to calculate the absolute dominant values by pairwise integration, and can obtain more persuasive result than traditional methods. The proposed method could be applied into more fields to achieve robust conclusions when dealing with stochastic MCDM problems. One of the unsolved problems in SMAA-AD is how to solve problems which criteria measurements for some or all criteria are ordinal, which will be considered in future.

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