RESEARCH ARTICLE

Air turbine optimization for a bottom-standing oscillating-water-column wave energy converter

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Abstract The oscillating-water-column wave energy device equipped with an air turbine is widely regarded as the simplest and most reliable, and the one that was object of the most extensive development effort. The aerodynamic performance of the air turbine plays a major role in the success of the technology. A case study was selected to investigate these issues: the existing bottom-standing plant on the shoreline of the island of Pico, in Azores Archipelago. The overall performance of the OWC plant was modelled as an integrated hydrodynamic and aerodynamic process. The hydrodynamic modelling was based on linear water wave theory. Published results from model testing of Wells and biradial turbines, together with well-known tools from dimensional analysis, were employed to determine the aerodynamic performance, and to optimize the turbine size and rotational speed. Single- and two-stage Wells turbines were considered. Unlike the biradial turbine, the performance of the Wells turbine, especially the single-stage version, was found to be severely affected by the constraints on rotor-blade tip speed (to avoid excessive centrifugal stresses and shock waves in the more energetic sea states). A power law, relating instantaneous values of the electromagnetic torque and the rotational speed, was found to apply to both types of turbines as a rotational speed control algorithm. The large runaway speed of the Wells turbine was found to be a risk to turbine integrity by excessive centrifugal stresses that requires safety measures. The much lower runaway speed of the biradial turbine is not expected to be a major problem.

Keywords Wave energy · Oscillating water column · Air turbine · Optimization · Control

List of symbols

Roman letters

A_W	Regular wave amplitude
B	Radiation susceptance
D	Turbine rotor diameter
$f_p(p)$	Probability density function of p
g	Acceleration of gravity
G	Radiation conductance
H_s	Significant wave height
k	Polytropic exponent
K	Turbine damping coefficient
L	Capture width
L_t	Aerodynamic torque of turbine
Ma	Mach number
p	Pressure oscillation in air chamber
p_{at}	Atmospheric pressure
P	Complex amplitude of p
$\bar{P}_{abs}$	Time-averaged power absorbed from the waves
P_t	Aerodynamic power of turbine
$\bar{P}_{wave}$	Time-averaged wave power per unit crest length
q	Volume flow rate
Q	Complex amplitude of q
$Re(\cdot)$	Real part of
$S_p(\omega)$	Spectral distribution of p
$S_\omega(\omega)$	Variance density spectrum of waves
t	Time
T_e	Energy period of waves

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V	Volume of air in chamber
V_0	Undisturbed value of V
w	Mass flow rate of air

Greek letters

α	Utilization factor
$\Gamma = Q_e /A_w$	Excitation flow rate coefficient
η	Turbine efficiency
Λ	See Eq. (17)
Ξ	Dimensionless damping coefficient of turbine
Π	Dimensionless power of turbine
ρ	Water density
ρ_{at}	Density of air in atmosphere
ρ_0	Reference density of air
σ_p	Standard deviation of p
σ_Ψ	Standard deviation of Ψ
ϕ	Frequency of occurrence of sea state
Φ	Dimensionless flow rate of turbine
Ψ	Dimensionless pressure head of turbine
ω	Radian frequency of waves
Ω	Rotational speed (radians per unit time)

Subscripts

annual	Annual average
at	Atmospheric conditions
e	Excitation
max	Maximum
r	Radiation
run	Runaway

Superscript

overbar	Time average
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1 Introduction

A wide variety of concepts has been proposed and studied for wave energy conversion. In most cases, the devices are of oscillating-body or of oscillating-water-column (OWC) types (Falnes 2007; Falcão 2010; López et al. 2013). The OWC device is widely regarded as the simplest and most reliable, and the one that was object of the most extensive development effort, with the largest number of prototypes deployed into the sea (Heath 2012; Falcão and Henriques 2016). The OWC comprises a partly submerged (fixed or

floating) structure, open below the water surface, inside which air is trapped above the water free surface. The oscillating motion of the internal free surface produced by the incident waves makes the air to flow through a turbine that drives an electrical generator. Self-rectifying air turbines have the advantage of not requiring rectifying valves. The Wells turbine is the best known self-rectifying air turbine for wave energy applications, and equipped most OWC prototypes, but there are others, mostly of impulse type (Setoguchi and Takao 2006; Curran and Folley 2008; Falcão and Gato 2012; Falcão and Henriques 2016).

Apart from small navigation buoys, most OWC prototypes deployed into the sea were of fixed structure, standing on the sea bottom at the shoreline or near the shore, or incorporated into a breakwater; some floating OWC converters, appropriate for offshore deployment, also reached the stage of prototype (Falcão and Henriques 2016).

The success of an OWC converter depends largely on its wave-to-wire conversion efficiency. Here, the aerodynamic performance of the air turbine plays a major role. It should not be forgotten that there is a coupling between the hydrodynamic process of wave energy absorption and the aerodynamic process that takes place in the turbine. More precisely, this coupling is affected by the relationship between pressure in the air chamber and air flow rate through the turbine (the so-called damping effect). This relationship in turn depends on turbine geometry, size and rotational speed. For these reasons, the overall performance of the OWC plant should be modelled as an integrated hydrodynamic and aerodynamic process. The performance of the electrical power equipment and the constraints that such equipment may introduce into the overall system are important, but are not analysed here.

A case study was selected to investigate these issues: the existing bottom-standing OWC plant constructed on the shoreline of the island of Pico, in Azores Archipelago, northern Atlantic Ocean. This seems to have been the first full-sized wave energy plant designed and constructed to permanently supply an electrical grid. The plant was completed in 1999 and is still operational. In the last few years it has been an experimental infrastructure owned and operated by the Wave Energy Centre, Lisbon.

Not much reliable and accurate information from field measurements is available on the hydrodynamics of the plant or on the aerodynamic performance of the Wells turbine that has equipped the plant since the beginning. In the study presented here, theoretical modelling is used based on linear water wave theory for the hydrodynamics of wave energy absorption. The hydrodynamic coefficients were computed with the aid of a boundary-element-method (BEM) code that accurately models the inner free surface (spatially uniform air pressure rather than the rigid piston assumption).

Two different types of self-rectifying air turbines are considered here: the Wells turbine and the biradial turbine (Falcão and Henriques 2016). Results from model testing of both turbine types, together with well-known tools from dimensional analysis, are employed to determine the aerodynamic performance, and to optimize the turbine size and rotational speed. This allows a comparison to be made between the two types of turbine in this kind of application. The Wells turbine is known to be approximately a linear turbine. In the case of the biradial turbine, a linearization was introduced to allow the frequency-domain analysis to be employed in regular waves, and a stochastic analysis to be used in irregular waves.

The air chamber thermodynamics and the turbine aerodynamics are analysed in Sects. 2 and 3 respectively. The OWC hydrodynamics is addressed in Sect. 4, in the frequency domain for regular waves (Sect. 4.1) and stochastically for irregular waves (Sect. 4.2). The Pico plant, the local wave climate and the two types of air turbines are described in Sect. 5. Section 6 presents the numerical results for the power performance (Sect. 6.1) and for the rotational speed control and runaway speed (Sect. 6.2). Conclusions appear in Sect. 7.

2 Air chamber thermodynamics

We consider an OWC converter with a bottom-fixed structure. The spring-like effect of air compressibility was first modelled as an isentropic process (Sarmiento and Falcão 1985; Jefferys and Whittaker 1986), and more realistically in Falcão and Justino (1999).

Let $p_{at} + p(t)$ (p_{at} = atmospheric pressure) be the pressure (assumed uniform) of the air inside the chamber, $q(t)$ the volume flow rate displaced by the motion of the inside free surface of water (positive for upward motion), and $m(t)$ the mass of air inside the chamber. We may write $m = \rho_{ch} V$, where $\rho_{ch}(t)$ and $V(t)$ are respectively the density and volume of air inside the chamber. Then, taking into account that $dV/dt = -q$, we obtain, for the mass flow rate through the turbine $w = -dm/dt$ (positive for outward flow),

$$w = -V \frac{d\rho_{ch}}{dt} + \rho_{ch} q. \quad (1)$$

The discharge process ($p > 0$, $w > 0$), when air is flowing out of the chamber to the atmosphere, may be regarded as approximately isentropic, i.e. the specific entropy of the air remaining in the chamber is unchanged (although in general different from the atmospheric air entropy). The filling process ($p < 0$, $w < 0$) is more difficult to model, since the air specific entropy increases (from its atmospheric value) due to viscous losses in the turbine, valves and connecting ducts; the air specific enthalpy also changes due to work per-

formed in the turbine. A polytropic relationship between the pressure $p + p_{at}$ and the density ρ_{ch} in the air chamber was proposed by Falcão and Henriques (2014a)

$$\frac{p_{at} + p}{\rho_{ch}^k} = \frac{p_{at}}{\rho_{at}^k}, \quad (2)$$

where ρ_{at} is the density of atmospheric air and k is the polytropic exponent that is related to the average efficiency $\bar{\eta}$ of the turbine. A good approximation to the relationship proposed by Falcão and Henriques (2014a) is (Falcão and Henriques 2016)

$$k = 0.13\bar{\eta}^2 + 0.27\bar{\eta} + 1. \quad (3)$$

From Eq. (2), we obtain

$$\frac{d\rho_{ch}}{dt} = \frac{\rho_{at}}{k p_{at}^{1/k} (p_{at} + p)^{(k-1)/k}} \frac{dp}{dt}. \quad (4)$$

If the absolute value $|p|$ of the pressure oscillation is much smaller than the atmospheric pressure p_{at} , Eq. (4) may be linearized as

$$\frac{d\rho_{ch}}{dt} = \frac{\rho_{at}}{k p_{at}} \frac{dp}{dt} \quad (5)$$

and Eq. (1) becomes

$$w = -V_0 \frac{\rho_{at}}{k p_{at}} \frac{dp}{dt} + \rho_{at} q, \quad (6)$$

where V_0 is the volume of air in the chamber in the absence of waves.

3 Air turbine aerodynamics

Unlike the turbines in most other applications, the air turbine in an OWC is subject to a highly unsteady flow rate (in fact bidirectional and largely random). Hysteretic effects are known to occur (they are particularly significant in the Wells turbine, see Setoguchi et al. 2003); such effects are ignored here.

If the effects of variations in Reynolds number are ignored (which is in general a reasonable assumption), the turbine performance can be represented in dimensionless form as (Dixon and Hall 2014; Falcão and Henriques 2016)

$$\Phi = f_{\Phi}(\Psi, Ma), \quad (7)$$

$$\Pi = f_{\Pi}(\Psi, Ma), \quad (8)$$

where Ma is a Mach number and Φ , Ψ , Π are dimensionless coefficients of flow rate, pressure head and power, respectively, defined as

$$\Phi = \frac{w}{\rho_0 \Omega D^3}, \quad (9)$$

$$\Psi = \frac{p}{\rho_0 \Omega^2 D^2}, \quad (10)$$

$$\Pi = \frac{P_t}{\rho_0 \Omega^3 D^5} = \frac{L_t}{\rho_0 \Omega^2 D^5}, \quad (11)$$

$$Ma = \frac{\Omega D}{c_0}. \quad (12)$$

Here, D is rotor diameter, Ω is rotational speed (in radians per unit time), $P_t = \Omega L_t$ is turbine power output, L_t is turbine torque, p is pressure head (assumed to be equal to the pressure oscillation in the OWC chamber), and ρ_0 and c_0 are air density and speed of sound at reference conditions (normally turbine entrance stagnation conditions). In most cases, especially if these dimensionless relationships are obtained from model testing at relatively low air speeds, the Mach number effect is ignored. This assumption is adopted here. Note that functions f_Φ and f_Π characterize a given turbine geometry, independently of turbine size, rotational speed or gas density.

4 OWC hydrodynamics

The hydrodynamic modelling of an OWC converter in which the boundary condition at the inner free surface is correctly accounted for (uniform air pressure rather than rigid piston model) was done, in the frequency domain, by [Falcão and Sarmento \(1980\)](#) and generalized by [Evans \(1982\)](#).

Assuming linear water wave theory to be applicable, we may write, for the volume flow rate displaced by the motion of the inner free surface,

$$q(t) = q_r(t) + q_e(t). \quad (13)$$

Here, $q_r(t)$ is the radiation flow rate due to the air pressure oscillation $p(t)$ inside the chamber in the absence of incident waves, and $q_e(t)$ is the excitation flow rate due to the incident waves if it were $p = 0$.

In the simple case of incident regular waves of frequency ω and amplitude A_w , we may write, after the transients related to the initial conditions have died out, $q_e(t) = \text{Re} \left(Q_e e^{i\omega t} \right)$, where $\text{Re}(\cdot)$ stands for real part of, and $Q_e(\omega)$ is an (in general complex) excitation flow rate coefficient that is assumed known for the structure and surrounding wall geometry and for incident wave amplitude and direction.

Real (unidirectional) irregular waves in a given sea state may be represented with good approximation as a superposition of regular waves, by defining a one-dimensional variance density spectrum $S_\zeta(\omega)$ (in the more realistic case of directional spread, not considered here, a two-dimensional spectrum would be required, see [Holthuijsen 2007](#)).

4.1 Frequency-domain analysis

We consider first the simple case of regular incident waves of frequency ω and unit amplitude A_w . The air turbine is assumed linear at constant rotational speed, i.e. we may write $\Phi = \Xi \Psi$, where Ξ is a dimensionless constant, and assume the reference density ρ_0 to be the atmospheric air density ρ_{at} . It follows, from Eqs. (9) and (10), that $w = K \rho_{at} p$, where $K = \Xi D \rho_{at}^{-1} \Omega^{-1}$. Note that, unlike Ξ , K is not independent of turbine size or rotational speed. Equation (6) becomes

$$q = \frac{V_0}{k p_{at}} \frac{dp}{dt} + K p. \quad (14)$$

This relationship between the volume flow rate q displaced by the OWC free surface and the air pressure oscillation p represents a linear power take-off system, provided that the rotational speed Ω , and hence K , are independent of time. This requires a sufficiently large rotational inertia of the rotating elements.

In this case we may employ the frequency domain analysis and write

$$\{q(t), q_e(t), q_r(t), p(t)\} = \text{Re} \{Q, Q_e, Q_r, P\} e^{i\omega t}. \quad (15)$$

Here $\text{Re}(\cdot)$ stands for real part of (a notation that will be omitted in what follows) and Q , Q_e , Q_r and P are complex amplitudes. We further write $Q_r = -(G + iB)P$, where G and B are real coefficients that depend on OWC geometry and on wave frequency ω (but not on wave amplitude), and G is non-negative. We call G the radiation conductance and B the radiation susceptance ([Falnes 2002](#)). These hydrodynamic coefficients are assumed known as functions of the frequency ω for the structure and surrounding wall geometry, from theoretical or numerical modelling or from experiments. The linearized frequency-domain version of Eq. (14) becomes

$$P = \Lambda Q_e, \quad (16)$$

where

$$\Lambda = \left[K + G + i \left(\frac{\omega V_0}{k p_{at}} + B \right) \right]^{-1}. \quad (17)$$

Here V_0 is the volume V of the air chamber in calm water. The power absorbed from the waves is $P_{abs}(t) = p(t)q(t)$. Its

time-averaged value is $\bar{P}_{\text{abs}} = \frac{1}{2} \text{Re}(QP^*)$ (where the asterisk denotes complex conjugate). Taking into account that $Q = Q_e - (G + iB)P$, this can be written (if $G \neq 0$) as

$$\bar{P}_{\text{abs}} = \frac{1}{8G} |Q_e|^2 - \frac{G}{2} \left| P - \frac{Q_e}{2G} \right|^2. \quad (18)$$

For a given OWC plant and given incident regular waves, the coefficients G (real non-negative) and Q_e (in general complex) are fixed. Then the absorbed power $\bar{P}_{\text{abs}}$ depends on the complex amplitude P of the pressure oscillation, i.e. on the linear turbine constant K and the air chamber volume V_0 . Equation (18) shows that its maximum value, equal to $|Q_e|^2 / (8G)^{-1}$, occurs for

$$\frac{Q_e}{P} = 2G, \quad (19)$$

which, together with Eqs. (16) and (17), gives two optimal conditions involving real quantities:

$$K = G(\omega) \quad (20)$$

and

$$\omega = -kp_{\text{at}} B(\omega) V_0^{-1} \quad (21)$$

(note that B may be negative). Equation (19) shows that, under optimal conditions, the pressure oscillation p is in phase with the excitation flow rate q_e (not with the total flow rate q).

4.2 Stochastic model for irregular waves

We assume now that the local wave climate may be represented by a set of sea states, each being a stationary stochastic ergodic process. In a sea state, the surface elevation ζ at a given observation point is supposed Gaussian. We assume a one-dimensional variance density spectrum $S_\zeta(\omega)$ (Holthuijsen 2007). Within the framework of linear water wave theory and if, in addition, a linear turbine is assumed, the wave-to-pneumatic conversion process may also be regarded as linear. Consequently, the pressure oscillation $p(t)$ in the chamber is also a Gaussian process, and a pressure spectrum $S_p(\omega)$ may be defined. It is related to the wave spectrum $S_\zeta(\omega)$ by (Falcão and Rodrigues 2002)

$$S_p(\omega) = \Gamma^2(\omega) |\Lambda(\omega)|^2 S_\zeta(\omega), \quad (22)$$

where $\Lambda(\omega)$ is given by Eq. (17), and the excitation flow rate coefficient $\Gamma(\omega)$ is equal to the value of $|Q_e(\omega)|$ for waves of unit amplitude. We denote by σ_p and σ_p^2 the standard

deviation (or root mean square) and the variance of the air pressure oscillations, respectively. It is

$$\sigma_p^2 = \int_0^\infty S_p(\omega) d\omega. \quad (23)$$

The probability density function $f_p(p)$ of the Gaussian pressure oscillation is given by

$$f_p(p) = \frac{1}{\sqrt{2\pi}\sigma_p} \exp\left(-\frac{p^2}{2\sigma_p^2}\right). \quad (24)$$

The time-averaged power output of the turbine $\bar{P}_t$ may be obtained by integration

$$\begin{aligned} \bar{P}_t &= \int_{-\infty}^{\infty} f_p(p) P_t(p) dp = \frac{\rho_{\text{at}} \Omega^3 D^5}{\sqrt{2\pi}\sigma_p} \\ &\times \int_{-\infty}^{\infty} \exp\left(-\frac{p^2}{2\sigma_p^2}\right) f_\Pi\left(\frac{p}{\rho_{\text{at}} \Omega^2 D^2}\right) dp. \end{aligned} \quad (25)$$

Its dimensionless value is defined as

$$\bar{\Pi} = \frac{\bar{P}_t}{\rho_{\text{at}} \Omega^3 D^5}. \quad (26)$$

The function $f_\Pi()$ in Eq. (25) is defined by Eq. (8), with the dependence on Mach number Ma ignored. The time-averaged power available to the turbine is $\bar{P}_{\text{avai}} = K \sigma_p^2$. The average efficiency of the turbine is defined as $\bar{\eta} = \bar{P}_t / \bar{P}_{\text{avai}}$. It is convenient to consider $\bar{\Pi}$ and $\bar{\eta}$ as functions of the dimensionless value of the standard deviation of the pressure oscillation (see Falcão and Rodrigues 2002)

$$\sigma_\Psi = \frac{\sigma_p}{\rho_{\text{at}} \Omega^2 D^2}. \quad (27)$$

5 Test case: the Pico plant

5.1 Plant description

The OWC plant on the island of Pico (Falcão 2000) was chosen as a test case for the numerical simulations based on the theory presented above. Pico, with a population of about 15 thousand inhabitants, is one of the nine islands of the archipelago of the Azores, in the Northern Atlantic (Fig. 1). The plant is located on the northern shoreline of the island, at a site named Porto Cachorro (38°32'N, 28°32'W). The coastline at Porto Cachorro (Fig. 2) is very indented over a distance of a few hundred metres, and was formed by volcanic activity in the 18th century. It comprehends several small harbours and recesses with almost vertical basaltic walls and water depths of about 6–9 m. There is no sand.

Fig. 1 Island of Pico, showing the plant location at Porto Cachorro and the neighbouring islands of São Jorge and Faial

Fig. 2 Porto Cachorro site, showing the location of the plant

The exact site where the plant was actually built was chosen because of natural wave energy concentration (hot spot) and also because of relatively easy access by land. The absence of a continental shelf makes it possible for the waves to reach the vicinity of the shoreline with relatively little energy dissipation. Detailed information on wave climate was available for the chosen site from measurements offshore (Waverider buoys in 100 m-deep water) as well as close to shore (ultra-sonic probes suspended by cables from the deeply indented shoreline). The site is partly shielded by the presence of the two neighbouring islands of Faial and São Jorge (Fig. 1), which however does not substantially prevent the propagation of the waves from the predominant N-NW direction. The average tidal amplitude is about 0.5 m.

Fig. 3 Longitudinal cross-section of the plant

Wave tank model tests, at scales 1:35 and 1:25, were performed about 1993–1994 to find a suitable configuration for the structure, as well as to provide information on the optimum design specifications for the power take-off equipment. The optimum value for the turbine damping coefficient was found to be about $8 \times 10^{-3} \text{ m}^3 \text{ s}^{-1} \text{ Pa}^{-1}$.

The concrete structure of the chamber (square planform with inside dimensions of 12 m $\times$ 12 m at mean water level) was built in-situ on rocky bottom (about 8 m water depth), spanning a small natural harbour (gully) (Fig. 3). The power plant is equipped with a horizontal-axis Wells turbine-generator set rated 400 kW. Room was left, for testing purposes, for a second turbine-generator set, which however was never installed. The plant is connected to the island's electrical grid. It operated for the first time in October 1999. Although affected by technical problems, sometimes for long periods, the plant has been operational for most of the time over the last 10 years.

The radiation conductance G , the radiation susceptance B and the excitation flow coefficient $\Gamma = |Q_e|/A_w$ were computed, as functions of frequency ω , by a modified version of the code AQUADYN, based on the boundary element method (BEM) (Brito-Melo et al. 2001). In the definition of Γ , A_w is the wave amplitude in deep water; the dominant wave direction offshore was assumed. The discretization of the plant structure and of the surrounding rocky bottom and side walls are shown in Fig. 4. The hydrodynamic coefficients G , B and Γ are plotted versus wave frequency ω in Fig. 5. The irregularity of the curves reflects the very irregular configuration of the rocky bottom and side walls in the vicinity of the plant. Note that radiation susceptance $B(\omega)$ changes sign at $\omega \cong 0.61 \text{ rad/s}$, which has implications with respect to the resonance condition (21). The volume of air in the chamber, in the absence of waves and under mid-tidal conditions, is $V_0 = 1050 \text{ m}^3$.

Fig. 4 Discretization of the plant and surrounding rocky bottom and coastline

Fig. 5 Hydrodynamic coefficients G (radiation conductance), B (radiation susceptance) and $\Gamma = |Q_e|/A_w$ (excitation flow rate coefficient) for the plant

5.2 Wave climate

Wave measurements were performed off the plant site with a Waverider buoy. A simplified description of the local wave climate was adopted here for the calculations. It consists of a set of 9 sea states, each defined by the significant wave height H_s , the energy period T_e and the frequency of occurrence ϕ (Table 1), together with a Pierson-Moskowitz variance density spectrum (Goda 2000)

$$S_\omega(\omega) = 262.6 H_s^2 T_e^{-4} \omega^{-5} \exp \left[-1052 (T_e \omega)^{-4} \right], \quad (28)$$

where H_s is the significant wave height and T_e is the energy period, defined in the usual way. The energy flux of the waves per unit crest length in deep water is

$$\bar{P}_{\text{wave}} = \frac{\rho g^2}{64\pi} H_s^2 T_e. \quad (29)$$

Table 1 Energy period $T_{e,i}$, significant wave height $H_{s,i}$ and frequency of occurrence ϕ_i of the nine sea states $i = 1$ to 9

i	$T_{e,i}$ (s)	$H_{s,i}$ (s)	ϕ_i	$\bar{P}_{\text{wave},i}$ (kW/m)
1	9.0	0.8	0.25	2.82
2	9.5	1.2	0.2	6.70
3	10.0	1.6	0.177	12.5
4	10.5	2.0	0.145	20.6
5	11.0	2.4	0.10	31.0
6	11.5	2.9	0.07	47.4
7	12.0	3.4	0.045	68.0
8	12.5	4.0	0.007	98.0
9	13.0	4.5	0.006	129.0

Taking $\rho = 1025 \text{ kg/m}^3$ and $g = 9.8 \text{ m/s}^2$, the annual-averaged wave power is

$$\bar{P}_{\text{wave}} = \sum_{i=1}^9 \bar{P}_{\text{wave},i} \phi_i = 18.2 \text{ kW/m}. \quad (30)$$

5.3 Air turbine

Two types of self-rectifying air turbines were considered in this simulation. The first one is the well-known Wells turbine, equipped with guide vanes. Both the more usual version with a single rotor plane and a row of guide vanes of each side of the rotor, and the two-stage version were modelled. The other type is the recent biradial impulse turbine, described in (Falcão et al. 2013a, b), Fig. 6. In both cases, results from model testing are used here for the turbine performance curves.

The chosen Wells turbine geometry was that of the model, code-named GV6, tested at Universität Siegen, Germany (Starzmann 2012), Fig. 7. This may be regarded as a highly efficient state-of-the-art Wells turbine. The tested turbine had a rotor of diameter $D = 0.4 \text{ m}$, hub-to-tip ratio 0.43, 5 blades

Fig. 6 Schematic representation of the biradial turbine with sliding guide vanes

Fig. 7 Rotor and guide vanes of the GV6 Wells turbine

Fig. 8 Dimensionless plot of the flow rate Φ versus the pressure head Ψ for the single-stage Wells turbine

of increasing chord from hub to tip, and solidity at hub equal to 0.67. The turbine had a row of guide vanes on each side of the rotor. Dimensionless performance curves of flow rate Φ , power output Π and aerodynamic efficiency η versus pressure head Ψ are given in Figs. 8, 9, 10. The average values $\bar{\Pi}$ and $\bar{\eta}$, in random waves, of the dimensionless power output and of the efficiency are plotted in Figs. 9 and 10 versus the standard deviation σ_Ψ of the pressure oscillation.

Figure 8 shows that the adopted Wells turbine is very approximately linear, with a flow rate versus pressure head relationship given, in dimensionless form, by $\Phi = \Xi\Psi$,

Fig. 9 Dimensionless plot of the turbine power output Π versus the pressure head Ψ for the single-stage Wells turbine (solid line). The broken line represents the average power $\bar{\Pi}$ versus σ_Ψ (rms of Ψ)

Fig. 10 Dimensionless plot of the turbine efficiency η versus the pressure head Ψ for the single-stage Wells turbine (solid line). The broken line represents the average efficiency $\bar{\eta}$ versus σ_Ψ (rms of Ψ)

with $\Xi \cong 1.0$. Figure 9 shows that the power output drops sharply when the pressure head exceeds a critical value ($\Psi_{\text{crit}} \cong 0.072$). This is due to strong aerodynamic stalling at the rotor blades and is typical of most Wells turbines.

A two-stage Wells turbine was also considered, with guide-vanes between rotors, possibly as described in Falcão and Gato (2012) and Arlitt et al. (2013). Since no data from model testing are available, it was assumed that the dimensionless performance curves for the two-stage turbine can be obtained from the curves for the corresponding single-stage turbine, $\Pi = f_\Pi(\Psi)$, $\Phi = f_\Phi(\Psi)$ and $\eta = f_\eta(\Psi)$, by setting $\Pi_2 = f_{\Pi,2}(\Psi) = 2f_\Pi(\Psi/2)$, $\Phi_2 = f_{\Phi,2}(\Psi) = f_\Phi(\Psi/2)$ and $\eta = f_{\eta,2}(\Psi) = f_\eta(\Psi/2)$. Identical transformations apply to the time-averaged quantities $\bar{\Pi}$ and $\bar{\eta}$ as functions of σ_Ψ .

The corresponding curves for the biradial turbine were obtained from testing a model, as described in detail in Falcão et al. (2013b), and are plotted in Figs. 11, 12, 13. The curves (dotted lines) in Figs. 12 and 13 for time-averaged

Fig. 11 Dimensionless plot of the flow rate Φ versus the pressure head Ψ for the biradial turbine (solid line). The broken straight line represents $\Phi = 0.282\Psi$

Fig. 12 Dimensionless plot of the turbine power output Π versus the pressure head Ψ for the biradial turbine (solid line). The broken line represents the average power $\bar{\Pi}$ versus σ_Ψ (rms of Ψ)

Fig. 13 Dimensionless plot of the turbine efficiency η versus the pressure head Ψ for the biradial turbine (solid line). The broken line represents the average efficiency $\bar{\eta}$ versus σ_Ψ (rms of Ψ)

values of power and efficiency were computed assuming a Gaussian distribution for the pressure oscillation. Figure 11 shows that the flow-versus-pressure-head curve of the bira-

dial turbine is not rectilinear. Since the frequency-domain analysis and the stochastic approach require the turbine to be linear, in the computations the experimental curve was replaced by a straight line through the origin $\Phi = \Xi\Psi$, with $\Xi = 0.282$. This straight line crosses the experimental line at abscissa $\Psi = 0.35$, for which the efficiency η is maximum. This is also approximately equal to the value of σ_Ψ (i.e. the root-mean square of Ψ) for which the average efficiency $\bar{\eta}$ is maximum. The time-averaged power output from the linearized turbine was computed from $\bar{\Pi} = \bar{\eta}(\sigma_\Psi)\Xi\sigma_\Psi^2$.

It may be of interest to assess the runaway speed of the turbine if the electrical generator torque vanishes, due possibly to malfunction of the electrical equipment or to failure in the connection to the electrical grid. In such case, the rotational speed will increase until the time-averaged power output of the turbine vanishes, i.e. until $\bar{\Pi}(\sigma_\Psi) = 0$. We denote by $\sigma_{\Psi,0}$ the root of this equation. In the case of the single stage Wells turbine, it can be found that $\sigma_{\Psi,0} = 0.009$. For the two-stage Wells turbine we have $\sigma_{\Psi,0} = 2 \times 0.009 = 0.018$. In the case of the biradial turbine it is $\sigma_{\Psi,0} = 0.155$.

It is important to know how the turbine accelerates when the load suddenly vanishes. From basic dynamics, we have

$$P_t(t) = \frac{I}{2} \frac{d\Omega^2}{dt}, \quad (31)$$

where I is the rotational inertia of the rotating elements. The integration of this equation would require a time-domain approach of the whole process, which is outside the scope of the present analysis. An approximate method will be adopted instead. By integration with respect to time in the interval $(t - \Delta t, t + \Delta t)$, we obtain

$$\bar{P}_{t,\Delta t}(t) = \frac{I}{2} \frac{d}{dt} \bar{\Omega}_{\Delta t}^2(t), \quad (32)$$

where

$$\bar{P}_{t,\Delta t}(t) = \frac{1}{2\Delta t} \int_{t-\Delta t}^{t+\Delta t} P_t(\tau) d\tau \quad (33)$$

is the averaged value of the turbine aerodynamic power in the time interval $(t - \Delta t, t + \Delta t)$ and

$$\Omega_{\Delta t}(t) = \left[\frac{1}{2\Delta t} \int_{t-\Delta t}^{t+\Delta t} \Omega^2(\tau) d\tau \right]^{1/2} \quad (34)$$

is the root-mean-square of the rotational speed in the same interval. As an approximation, we extend the concept of duration of the sea state under consideration, from a few hours (as for oceanographers), to a few wave periods, and consider that $\bar{P}_{t,\Delta t}(t)$ is approximately equal to $\bar{P}_t$ as given by Eq. (25) with the rotational speed Ω replaced by $\Omega_{\Delta t}$. Note that

$\Omega_{\Delta t}(t)$ may be regarded as an oscillation-free value of the rotational speed. Upon integration, Eq. (32) yields

$$t - t_0 = I \int_{\Omega_{0,\Delta t}}^{\Omega_{\Delta t}} \frac{\Omega_{\Delta t}}{\bar{P}_t(\Omega_{\Delta t})} d\Omega_{\Delta t}. \quad (35)$$

Here t_0 is the instant when the electromagnetic torque of the generator vanishes, and $\Omega_{0,\Delta t}$ is the corresponding value of $\Omega_{\Delta t}$. Obviously it is $t \rightarrow \infty$ as $\Omega_{\Delta t} \rightarrow \Omega_{\text{run}}$, the runaway speed Ω_{run} being such that $\bar{P}_t(\Omega_{\text{run}}) = 0$.

6 Numerical results

The following values were adopted: $\rho = 1025 \text{ kg/m}^3$ for the sea water density, $\rho_{\text{at}} = 1.25 \text{ kg/m}^3$ for the atmospheric air density, $g = 9.8 \text{ m/s}^2$ for the acceleration of gravity and $k = 1.25$ for the polytropic exponent.

6.1 Power performance

Figure 14 shows a plot of the capture width L in regular waves versus wave frequency ω , for three different values of the turbine damping $K = 0.007, 0.01$ and $0.015 \text{ m}^4/\text{s/kg}$. The curve $K = G(\omega)$ for optimal damping at each frequency (see Eq. 20) is also shown. Here, the capture width in regular waves is defined as $L = \bar{P}_{\text{abs}}/\bar{P}_{\text{wave}}$, where $\bar{P}_{\text{abs}}$ is the time-averaged power absorbed from the waves given by Eq. (18), and

$$\bar{P}_{\text{wave}} = \frac{\rho g^2 A_{\text{wave}}^2}{4\omega} \quad (36)$$

is the wave power in deep water per unit wave crest length. It can be seen that the curves for the three different values of turbine damping K are not far below the optimal theoretical

Fig. 14 Capture width L versus wave frequency ω , for three different values of the turbine damping K . The curve for optimal damping for each frequency is also shown

Fig. 15 Annual-averaged power output $\bar{P}_{t,\text{annual}}$ versus turbine rotor diameter D for the single-stage Wells turbine (Wells 1), the two-stage Wells turbine (Wells 2) and the biradial turbine

curve $K = G(\omega)$ in the range $0.6 < \omega < 1.2 \text{ rad/s}$. This is particularly true close to the peak at $\omega \cong 0.63 \text{ rad/s}$ ($T \cong 10 \text{ s}$). We recall that the turbine damping K depends on the rotational speed, and so is expected to change with the sea state. This point is examined in more detail later in this section. The figure also shows that, in the range $0.6 < \omega < 1.2 \text{ rad/s}$, all curves give values for the capture width substantially larger than the width of the OWC itself (12 m). This is probably due to the gully concentration effect, in addition to resonance close to the peak $\omega \cong 0.63 \text{ rad/s}$.

Figure 15 shows results for the annual-averaged power output $\bar{P}_{t,\text{annual}}$ of the Pico plant, when equipped with a biradial turbine (rotor diameter $D = 1.5, 1.75, 2.0, 2.25, 2.5, 2.75$ and 3.0 m) and Wells turbine (1 and 2 stages, rotor diameter $D = 2.0, 2.25, 2.5, 2.75, 3.0, 3.25$ and 3.5 m). A constraint of maximum allowed rotor blade tip speed $\Omega D/2 = 180 \text{ m/s}$ was imposed, in order to avoid excessive centrifugal stresses in the turbine rotor, and to avoid shock waves in the air flow about the rotor blades. (This constraint applies particularly to Wells turbines.) In every case, the rotational speed was optimized for the turbine and sea state under consideration with the aid of the subroutine FindMaximum of Mathematica.

It can be seen in Fig. 15 that $\bar{P}_{t,\text{annual}}$ reaches a maximum of 252 kW for a biradial turbine with rotor diameter $D = 2.6 \text{ m}$, 185 kW for a single-stage Wells turbine with $D = 3.35 \text{ m}$, and 230 kW for a two-stage Wells turbine with $D = 3.5 \text{ m}$. In no case (rotor diameter and sea state) did the rotor blade tip speed of the biradial turbine reach 180 m/s which was set as a limit for the Wells turbines. This constraint results in a reduction in power output of the single- and two-stage Wells turbines of all diameters (2.0–3.5 m) for several of the nine sea states of Table 1. This is more severe for the single-stage Wells turbine than for the two-stage Wells turbine, and for the turbines with smaller rotor diameter D . Obviously, in the case of the two-stage Wells turbine, the pressure head

p is divided equally between the two stages, which are less loaded and less affected by the blade tip speed constraint than is the single-stage turbine.

Maximum energy production is not in general the most appropriate criterion for the investor, who will be looking for maximum profit (rather than maximum produced energy). Profit is related, not only to the amount of energy produced annually (which in turn depends on turbine size and generator rated power capacity), but also to the investment required to purchase, install and maintain the equipment (which increases with turbine and generator size), and to the price at which the unit electrical energy is paid to the producer. These issues were addressed in detail in Falcão (2004) where several scenarios were analysed. Here we will simply assume that the best choices are turbine sizes somewhat smaller than the sizes that maximize the annual energy production. For the more detailed analysis of power performance and turbine control that follows, we selected $D = 2.0$ m for the biradial turbine rotor diameter and $D = 2.5$ m for the single- and two-stage Wells turbine rotor diameter.

The effect of the rotational speed constraints on the efficiency (and hence on power output) of the Wells turbines is clearly shown in Fig. 16. This constraint negatively affects the performance of the single-stage turbine in all but the two least energetic sea states, whereas the two-stage turbine is affected only in sea states $i = 5$ to 9.

It was shown in Sect. 4 that, in regular waves of frequency ω , the turbine damping coefficient K that maximizes the power $\bar{P}_{\text{wave}}(\omega)$ absorbed from the waves is equal to the radiation conductance $G(\omega)$ (see Eq. 20). Figure 17 shows a plot of $G(\omega)$ together with curves of $K(\omega_e)$ (with $\omega_e = 2\pi/T_e$), where $K(\omega_e)$ is the turbine damping coefficient that maximizes the turbine power output $\bar{P}_t$ (subject to

Fig. 16 Average turbine efficiency $\bar{\eta}$ versus wave power $\bar{P}_{\text{wave}}$ for the single-stage Wells turbine $D = 2.5$ m (Wells 1), the two-stage Wells turbine $D = 2.5$ m (Wells 2) and the biradial turbine $D = 2.0$ m. The plotted points on each curve represent the 9 sea states

Fig. 17 Radiation conductance G versus wave frequency ω , and optimized turbine damping K , subject to maximum rotational speed constraint, versus $\omega_e = 2\pi/T_e$, for the single-stage Wells turbine $D = 2.5$ m (Wells 1), the two-stage Wells turbine $D = 2.5$ m (Wells 2) and the biradial turbine $D = 2.0$ m. The plotted points on each curve represent the 9 sea states

maximum rotational speed constraints) in the different sea states. The difference between the curves $G(\omega)$ and $K(\omega_e)$ may be explained (1) by the difference between regular and irregular waves, and (2) by the fact that, in regular or irregular waves, the turbine damping coefficient that maximizes the power absorbed from the waves is in general different from the turbine damping coefficient that maximizes the turbine power output $\bar{P}_t$ (in which the turbine aerodynamic efficiency plays a major role).

Figure 18 shows the utilization factor $\alpha = \bar{P}_{t,\text{annual}}/\bar{P}_{t,\text{max}}$ versus turbine rotor diameter D for the single-stage Wells turbine (Wells 1), the two-stage Wells turbine (Wells 2) and the biradial turbine. Here $\bar{P}_{t,\text{max}}$ is the maximum value of the averaged power output under the nine sea states that appear in Table 1. It is $0.674 \leq \alpha \leq 0.328$ for the single-stage

Fig. 18 Utilization factor $\alpha = \bar{P}_{t,\text{annual}}/\bar{P}_{t,\text{max}}$ versus turbine rotor diameter D for the single-stage Wells turbine (Wells 1), the two-stage Wells turbine (Wells 2) and the biradial turbine

Wells turbine, $0.425 \leq \alpha \leq 0.224$ for the two-stage Wells turbine, and $0.175 \leq \alpha \leq 0.151$ for the biradial turbine. The utilization factor α decreases with increasing rotor diameter D , as should be expected, since the larger turbines can produce more power than the smaller ones especially in the more energetic sea states. The smaller Wells turbines are more severely affected than the larger ones by the maximum blade-tip-speed constraint in the more energetic sea states.

Naturally, a small value of the utilization factor α means that the rated power PTO is much larger than the annual-averaged power. This affects severely the electrical equipment. Apart from the cost increase with rated power, the overall efficiency of the electrical equipment decreases markedly with decreasing loads; small to medium loads occur much more frequently than the largest ones. The electrical equipment driven by a biradial turbine is especially affected by this behaviour, due to the fact that the biradial turbine, unlike the Wells turbine, is capable of converting, with nearly the same efficiency, the wave energy in the less energetic sea states and also in the more energetic ones.

Since the frequency of occurrence of the most energetic sea states is relatively small, it may be wise to limit its power output, or even reducing it to zero, by partially or totally closing a valve in series with the turbine, or by opening a bypass valve (Henriques et al. 2016). The Pico plant is equipped with both types of valves: a vertical sluice gate (closing time about 30 s), a fast multi-element stop valve (closing time about 3 s) in series with the turbine, and a slow horizontally sliding relief valve on the roof of the air chamber. Apart from preventing overloads to the electrical equipment, closing the air flow through the turbine may be regarded as a survival measure in the most energetic sea states.

In the case of a biradial turbine of $D = 2$ m, if the PTO is closed in the two most energetic sea states 8 and 9 of Table 1, the maximum power $\bar{P}_{t,\max}$ decreases from 1419 kW (in sea state 9) to 775 kW (in sea state 7), while the decrease in annual-averaged power $\bar{P}_{t,\text{annual}}$ is only from 236 to 219 kW; the utilization factor α increases markedly from 0.166 to 0.283. In the case of the single-stage Wells turbine of $D = 2.5$ m, the maximum power $\bar{P}_{t,\max} = 283$ kW (in sea state 7) is unaffected, while the annual-averaged power $\bar{P}_{t,\text{annual}}$ and the utilization factor α decrease slightly from 159 to 156 kW and from 0.561 to 0.551, respectively. Finally, in the case of the two-stage Wells turbine of $D = 2.5$ m, the maximum power $\bar{P}_{t,\max}$ decreases from 568 kW (in sea state 9) to 525 kW (in sea state 7), while the decrease in annual-averaged power $\bar{P}_{t,\text{annual}}$ is only from 197 to 190 kW; the utilization factor α increases slightly from 0.348 to 0.362. We may conclude that closing the flow through the biradial turbine in the most energetic sea states reduces strongly (factor 0.55) the maximum turbine power output (and the required rated power of the electrical equipment), at the cost of a modest decrease (factor 0.93) in annual produced energy. The

corresponding effects in the case of the single- and two-stage Wells turbines are much less significant, which is explained by the major effect of the constraints imposed by the maximum allowed rotor blade tip speed.

6.2 Rotational speed control and runaway speed

Figure 19 shows a log-log plot of the optimized average power output $\bar{P}_t$ versus turbine rotational speed Ω for the single- and two-stage Wells turbines and for the biradial turbine. It can be seen that the plotted points are well aligned along straight lines $\bar{P}_t = a \Omega^\beta$ (where a and β are constants), except when the rotational velocity is affected by the maximum allowed rotor blade tip speed constraint (7 and 5 points for the single- and two-stage Wells turbines respectively). If bearing friction losses are neglected, we may write $\bar{P}_t = \bar{P}_{\text{electr}}$, where $P_{\text{electr}} = \Omega L_{\text{electr}}$. Here L_{electr} is the electromagnetic torque on the generator rotor. We may extend the concept of sea state to a few wave periods and write $L_{\text{electr}} = a \Omega^{\beta-1}$ as a rotational speed control law. This power law has been proposed in Falcão (2002) and adopted as a rotational speed control law in other papers (e.g. Falcão and Henriques 2014a; Falcão et al. 2014b; Henriques et al. 2016). The constants of the straight lines in Fig. 19 are $a = 1.31 \times 10^{-6}$, $\beta = 3.863$ for the biradial turbine, and $a = 1.61 \times 10^{-5}$, $\beta = 2.361$ for the two-stage Wells turbine, with L_{electr} in kN and Ω in rad/s.

As mentioned in Sect. 5.3, if the electromagnetic torque L_{electr} suddenly vanishes due to failure in the grid connection or other failure in the electrical equipment, and no emergency air-valve is operated, then the rotational speed will increase, tending to the runaway speed. This behaviour is well known in hydroelectric plants, in which the ratio between the runaway speed and the normal rotational speed is in the range

Fig. 19 Average power output $\bar{P}_t$ versus turbine rotational speed Ω for the single-stage Wells turbine $D = 2.5$ m (Wells 1), the two-stage Wells turbine $D = 2.5$ m (Wells 2) and the biradial turbine $D = 2.0$ m. The plotted points on each curve represent the 9 sea states

Fig. 20 Turbine rotational speed Ω versus wave power $\bar{P}_{\text{wave}}$ for the single-stage Wells turbine $D = 2.5$ m (Wells 1), the two-stage Wells turbine $D = 2.5$ m (Wells 2) and the biradial turbine $D = 2.0$ m. *Solid lines* rotational speed for maximum turbine power, subject to maximum allowable blade tip speed. *Dotted lines* runaway speed. The plotted points on each curve represent the 9 sea states

1.6–2.9, depending on hydraulic turbine type (Mataix 1975; Raabe 1985).

As above, we assume that the rotational inertia I of the rotating parts is large enough for the amplitude of oscillations in rotational speed to be small compared with the instantaneous rotational speed. The runaway speed of the single- and two-stage Wells turbines ($D = 2.5$ m) and the biradial turbine ($D = 2$ m) in the Pico plant is plotted in Fig. 20 for the nine sea states $i = 1$ to 9. The optimized rotational speed (subject to the rotor blade tip speed constraint) is also plotted in the same figure for comparison. The runaway speed of the biradial turbine in the most energetic sea state 9 is 173 rad/s: the rotor blade tip speed (173 m/s) does not exceed the 180 m/s constraint. The runaway speeds of the single- and two-stage Wells turbines are much higher, and pose serious risks to the turbine (and generator) integrity if a sufficiently fast valve is not actuated to limit or close the flow rate through the turbine. It should be pointed out that, at very high blade tip speeds, transonic flow is expected to occur (Henriques and Gato 2002), resulting in rotor blade drag larger than predicted by the incompressible flow approach adopted here. For this reason, the runaway speeds shown in Fig. 20 for the single- and two-stage Wells turbines could be over-predictions.

Figure 21 gives, for the accelerating unloaded single- and two-stage Wells turbines ($D = 2.5$ m), the time t from initial rotational speed 144 rad/s (blade tip speed 180 m/s) to rotational speed $\Omega_{\Delta t}$ in sea states $i = 4, 7$ and 9. The rotational inertia I of the rotating parts was set equal to 650 kg m² for the single stage Wells turbine and 900 kg m² for the two-stage one. These values can be compared with $I = 595$ kg m² for the rotor of the single-stage Wells turbine ($D = 2.3$ m) actually installed at the Pico plant. If 250 rad/s is set as the

Fig. 21 Time t from initial rotational speed $\Omega_{0,\Delta t} = 144$ rad/s to rotational speed $\Omega_{\Delta t}$ for an unloaded accelerating Wells turbine of $D = 2.5$ m in sea states $i = 4, 7$ and 9. *Thick lines* single-stage turbine; *thin lines* two-stage turbine

maximum rotational speed prior to rotor disintegration, this value is attained, for the single-stage Wells turbine, in 56 s in sea state 4, 24 s in sea state 7 and 21 s in sea state 9. For the two-stage Wells turbine, the corresponding values are 24, 15 and 16 s. These times indicate that a valve should be actuated within a few seconds (or a few tens of seconds) to limit or close the flow rate through the turbine and in this way avoid Wells turbine rotor destruction by excessive centrifugal stresses, especially in the most energetic sea states.

7 Conclusions

Results based on linear water wave theory were obtained for the performance of the OWC plant on Pico island. Non-linear effects due to real fluid (viscosity) effects, non-small wave amplitude and wave breaking were not accounted for. Such effects are known to be important in the more energetic sea states. This is especially the case of wave breaking which is observed to occur due to local water depth of about 8 m in the immediate vicinity of the plant and about 15 m fifty metres away. For these reasons, the numerical results in the more energetic sea states should be regarded as qualitative, rather than quantitative, representations of reality.

Except in the less energetic sea states, the performance of the Wells turbine, especially the single-stage version, was found to be severely affected by the constraints on rotor-blade tip speed. This is not the case of the biradial turbine. Such behaviour confirms what should be expected from Wells turbines, known for their high rotational speed and low torque, compared with self-rectifying impulse turbines, of which the biradial turbine is a radial-flow version.

The fact that, unlike the Wells turbine, the biradial turbine (if its rotational speed is adequately controlled) is capable of performing efficiently even in very energetic sea states could

be taken as a reason to couple it to an electrical generator whose rated power would match the turbine power capability. However, one should not ignore the low frequency of occurrence of the most energetic sea conditions. Besides, unlike the turbine, the efficiency of the electrical equipment is known to be relatively poor at (the frequently occurring) small power loads. For several reasons, including equipment costs, the specified rated power of the electrical equipment should be somewhat lower than the power capacity of the biradial turbine in the most energetic sea states.

The power law of type $L_{\text{electr}} = a \Omega^{\beta-1}$, relating instantaneous values of the electromagnetic torque and of the rotational speed, was found to apply to both types of turbines as a rotational speed control algorithm (outside the range where rotational speed constraints apply).

The large runaway speed of the Wells turbine was found to be a risk to turbine integrity by excessive centrifugal stresses. This requires safety measures, normally a sufficiently fast acting air valve in series with the turbine. The much lower runaway speed of the biradial turbine is not expected to be a serious problem.

In general, the biradial turbine was found to outperform the Wells turbine, especially the single-stage one. Naturally, the decision on which type of turbine to choose should also take into consideration equipment costs, an aspect that is not addressed here.

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