Morison coefficients for a circular cylinder oscillating with dual frequency in still water: an analysis using independent-flow form of Morison's equation

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Abstract In this study, a set of experimental results for wave forces acting on a cylinder oscillating with dual frequency in still water are reported. The experiment was designed to mimic a cylinder slowly oscillating in regular waves, with the high-frequency oscillation representing the wave motion and the low-frequency oscillation, the slow drift motion. The inline forces acting on the cylinder were analyzed using the independent-flow form of Morison's equation. Our experimental results showed that it was not appropriate to simply use in the independent-flow form of Morison's equation the added-mass and drag coefficients obtained for the cylinder oscillating with a single frequency in still water. A new dimensionless parameter was introduced to describe each of the wave force coefficients used in the independent-flow form of Morison's equation, and empirical expressions for the wave force coefficients were proposed using the new dimensionless parameters.

Keywords Wave force · Morison's equation · Dual-frequency oscillation · Keulegan–Carpenter number · Reduced velocity · Drag · Inertia

1 Introduction

When studying the responses of offshore structures to ocean waves, the wave forces acting on slender members are frequently modeled by Morison's equation, which has been widely used to model wave loadings on cylinders fixed in waves or cylinders oscillating in still water or waves (e.g., Sarpkaya and Storm 1985; Najafian et al. 1995; Liu and Bergdahl 1996; DNV 2010). Even though the original Morison's equation was proposed for regular waves, it has been widely used for random waves in practice (Najafian et al. 1995; Burrows et al. 1997). When performing frequency-domain analysis using Morison's equation, the quadratic drag force needs to be linearized (e.g., Gudmestad and Connor 1983). Recent evaluations of the effectiveness of Morison's equation using the data from two small-scale field tests can be found in Boccotti et al. (2012, 2013).

For problems involving two frequencies, two forms of Morison's equation have been proposed: (1) the relative-velocity form of Morison's equation (e.g., Chakrabarti 1987) and (2) the independent-flow form of Morison's equation (e.g., Laya et al. 1984). In the relative-velocity form of Morison's equation, which is the most commonly used one, the drag force is quadratically related to the velocity difference between the ambient flow and the cylinder. In the independent-flow form of Morison's equation, which is used less often, the total force is the sum of the wave force due to the ambient flow field acting on a fixed cylinder and the force on the cylinder vibrating in an otherwise still water.

For a vertical cylinder of diameter D moving in ocean waves, the relative-velocity form of Morison's equation expresses the in-line drag force acting on the cylinder of unit length, f_x , as a quadratic function of the relative velocity $u_w - \dot{x}$, where u_w is the horizontal component of the velocity of the undisturbed ambient flow, and $\dot{x}$ the velocity of the

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moving cylinder. In this formulation, the total force acting on the cylinder of unit length can be written as

$$f_x = \frac{1}{2} \rho D C_D (u_w - \dot{x}) |u_w - \dot{x}| + \rho \frac{\pi D^2}{4} C_M^w \frac{du_w}{dt} - \rho \frac{\pi D^2}{4} C_A^s \frac{d\dot{x}}{dt} \quad (1)$$

where ρ is the density of water, and the superscripts w and s refer to the wave motion and the cylinder motion, respectively. The inertia force contains two terms: one term proportional to the water particle acceleration associated with the wave motion through an inertia coefficient C_M^w and the other term proportional to the acceleration of the cylinder through an added-mass coefficient C_A^s .

The relative-velocity form of Morison's equation is rather heuristic even though it may seem to be an obvious extension of the original Morison's equation. Laya et al. (1984) examined the applicability of the relative-velocity assumption using two dimensionless parameters: the so-called reduced velocity V_R' and the Keulegan–Carpenter number KC' , defined by

$$V_R' = \frac{U_w T_s}{D}, \quad KC' = \frac{U_w T_w}{D} \quad (2)$$

where T_s is the period of the cylinder motion, and T_w the period of wave motion, and U_w the amplitude of the horizontal velocity of wave motion. Laya et al. (1984) pointed out that the relative-velocity assumption was valid only when $V_R' > 10 \sim 15$ and $KC' > 10 \sim 15$. Based on the experimental data of Moe and Verley (1980) for a steady current past an oscillating cylinder, Laya et al. (1984) have argued that for the case of a rapidly vibrating cylinder in a slowly oscillating external flow or a cylinder oscillating with a low-frequency in a high-frequency flow, the water particles near the cylinder's field-of-influence follow a highly disorganized and unsteady flow pattern, and thus the suitability of the relative-velocity form of Morison's equation is highly suspected. Therefore, Laya et al. (1984) expressed the in-line force on a cylinder oscillating in waves as

$$f_x = \frac{1}{2} \rho D C_D^w u_w |u_w| + \rho \frac{\pi D^2}{4} C_M^w \frac{du_w}{dt} - \frac{1}{2} \rho D C_D^s \dot{x} |\dot{x}| - \rho \frac{\pi D^2}{4} C_A^s \frac{d\dot{x}}{dt} \quad (3)$$

which is the so-called “independent-flow form of Morison's equation”. For the four wave force coefficients, (C_D^w , C_M^w , C_D^s , C_A^s), Laya et al. (1984) proposed the following functional relationships,

$$C_D^w, C_M^w = f \left(\frac{U_w D}{v}, \frac{U_w T_w}{D} \right), \quad (4)$$

$$C_D^s, C_A^s = f \left(\frac{U_s D}{v}, \frac{U_s T_s}{D} \right) \quad (5)$$

where U_s is the velocity amplitude of the cylinder motion. Eqs. (4) and (5) basically assume that the wave force coefficients obtained for a cylinder fixed in waves and a cylinder oscillating in still water can be used in Eq. (3).

Koterayama (1984) studied the wave force coefficients for a circular cylinder moving with a constant speed in regular waves; the Keulegan–Carpenter numbers for the wave motion ranged from 1.3 to 100 and the reduced velocity defined by the constant speed and the wave period ranged from 0 to 60. For a circular cylinder oscillating sinusoidally with a very low frequency in regular waves and a cylinder oscillating with dual frequency in still water (the ratio of the two frequencies was an integer), Koterayama and Nakamura (1988) measured and analyzed the wave forces using the method similar to that of Koterayama (1984); their Keulegan–Carpenter numbers for the wave motion ranged from 1.1 to 15.7 and the reduced velocity ranged from 0.5 to 5.2. In the data analysis of both Koterayama (1984) and Koterayama and Nakamura (1988), the in-line force was first written in terms of the relative velocity form, and then a harmonic analysis was applied to the drag force to obtain a drag coefficient for the steady motion and a pair of drag and inertia coefficients for each frequency component. All the results were presented as functions of the reduced velocity, but the data points were widely scattered in some results. Even though the ranges of the Keulegan–Carpenter number and reduced velocity covered a wide range in Koterayama (1984) and Koterayama and Nakamura (1988), but the definitions of the added-mass and drag coefficients used in these two studies are different from those in the independent-flow form of Morison's Equation: their drag coefficients include the contributions from nonlinear interactions among different frequencies.

In this study, a cylinder oscillating with dual frequency in still water is used to simulate a cylinder rapidly oscillating in low-frequency waves or a cylinder slowly oscillating in regular waves. The loading on the cylinder (after removing the inertia force due to the weight of the test ring) of unit length is analyzed using the following independent-flow form of Morison's equation

$$f_x = -\frac{1}{2} \rho D C_D^H \dot{x}_H |\dot{x}_H| - \rho \frac{\pi D^2}{4} C_A^H \frac{d\dot{x}_H}{dt} - \frac{1}{2} \rho D C_D^L \dot{x}_L |\dot{x}_L| - \rho \frac{\pi D^2}{4} C_A^L \frac{d\dot{x}_L}{dt} \quad (6)$$

where the subscript H refers to the high-frequency oscillation and L the low-frequency oscillation. The total drag force is the sum of the drag forces associated with the two

oscillation frequencies through two drag coefficients: the drag coefficient C_D^L for the low-frequency component of the dual-frequency oscillation and C_D^H for the high-frequency component of the dual-frequency oscillation. The objectives of this study are to understand how the low-frequency motion affects the wave force coefficients for the high-frequency motion and vice versa.

2 Experimental setup and test conditions

2.1 Integrated cylindrical force model

All experiments in this study were conducted in a wave flume located in the Hydraulics Modelling Laboratory in Nanyang Technological University, Singapore. The wave flume was 0.55-m wide, 0.6-m deep and 36-m long. The model was placed in the middle section of the flume to minimize any possible influence of the two ends.

An integrated cylindrical force model was designed and fabricated to measure the hydrodynamic force acting on a circular section. The model was constructed using a 3-mm thick aluminum pipe, and segmented into three parts: a test section (or test ring), an upper dummy section and a lower dummy section. The test section was 50 mm in length and 50 mm in diameter and had a smooth outer surface. All these sections were connected by specially designed inner interlocking fixtures (see Fig. 1). The test section, which is separated from the two dummy sections, is connected a three-dimensional piezoelectric force transducer (Kistler). The hydrodynamic forces acting on this test section were measured in both in-line and transverse directions. To confirm that the force acting on test section can be measured accurately, both static and dynamic calibrations were performed by applying known static and dynamic forces on the test section. These tests have confirmed that the integrated force model used in this study is able to measure dynamic loads accurately. The details of these calibrations can be found in [Yuan and Huang \(2010\)](#).

The test section was chemically treated (coating) to prevent the metal from reacting with water so that its surface was always kept smooth during the experiment. We stress that the surface roughness may affect the wave force coefficients ([Wolfram and Naghipour 1999](#)). The upper dummy section was 300-mm long and the length of lower dummy section was adjustable. All these three sections were carefully assembled so that their center lines were aligned. For the details of this instrumented cylinder model, the reader is referred to [Yuan and Huang \(2010\)](#) and [Yuan and Huang \(2011\)](#), who used the same cylindrical force model in their study of hydrodynamic forces on a cylinder fixed in regular waves or oscillating transversely in regular waves.

Referring to Fig. 1, the integrated cylindrical force model was firmly attached to a stiff frame, which itself was firmly

Fig. 1 Sketch of the experimental setup. Not drawn to scale

attached to a belt-driven type linear actuator (RG-MB40, HISAKA). The actuator was attached to the two walls of the wave flume used in the experiment. The actuator can be programmed and provide any specified oscillations. In this study, the actuator was programmed to move according to a superposition of a slow-frequency motion $x_L(t)$ and a high-frequency motion $x_H(t)$, i.e., the displacement of the cylinder is specified by $x = x_L(t) + x_H(t)$, with $x_L(t)$ and $x_H(t)$ being given by

$$x_L(t) = A_L \cos\left(\frac{2\pi}{T_L}t\right), x_H(t) = A_H \cos\left(\frac{2\pi}{T_H}t\right), \quad (7)$$

where A_L and A_H are the amplitudes for the two periods T_L and T_H . The motion of the cylindrical force model was also monitored by a manufacture-calibrated Ultralab sensor.

Each individual component in the experimental system was estimated to have a nominal error bound less than 3%. Since the overall system error bound cannot be known a priori, the reliability of the test results can only be assured by comparing our force measurements with some theoretical values and experimental data published in the literature.

2.2 Data analysis

In the experiment, the force sensor and the Ultralab sensor were synchronized. Before calculating the force coefficients, a low pass filter was used to remove high frequency noises from the measured force signals. All unwanted high-frequency signals were filtered out by using a zero phase shift fast Fourier transform (FFT) low-pass filter. The cut-off frequency was at least 4 times the oscillation frequency. The measured in-line forces were analyzed using the independent-flow form of Morison's equation, Eq. (6). The added-mass (C_A^H, C_A^L) and drag coefficients (C_D^H, C_D^L) were obtained by performing a nonlinear least-square fitting of Eq. (6) to the time series of the measured in-line wave force, with the displacement of the cylinder provided by the Ultralab sensor. For reference, the independent-flow Morison equation, Eq. (6), can be written as

$$f(t) = C_D^L F_D^L(t) + C_A^L F_A^L(t) + C_D^H F_D^H(t) + C_A^H F_A^H(t) \quad (8)$$

Fig. 2 Comparison of the added-mass and drag coefficients measured for $\beta = 500$ with those reported in Obasaju et al. (1988) for $\beta = 485$

where

$$F_D^L(t) = -\frac{1}{2}\rho DC_D^L \dot{x}_L |\dot{x}_L| \quad (9)$$

$$F_A^L(t) = -\rho \frac{\pi D^2}{4} C_A^L \frac{d\dot{x}_L}{dt} \quad (10)$$

$$F_D^H(t) = -\frac{1}{2}\rho DC_D^H \dot{x}_H |\dot{x}_H| \quad (11)$$

$$F_A^H(t) = -\rho \frac{\pi D^2}{4} C_A^H \frac{d\dot{x}_H}{dt} \quad (12)$$

The four force coefficients ($C_D^L, C_A^L, C_D^H, C_A^H$) are calculated using the standard nonlinear least-square method. In this method, the error function is defined by

$$\begin{aligned} E^2 &= \sum_{i=1}^N \{f_m(t_i) - f(t_i)\}^2 \\ &= \sum_{i=1}^N \left\{ f_m(t_i) - \left[C_D^L F_D^L(t_i) + C_A^L F_A^L(t_i) \right. \right. \\ &\quad \left. \left. + C_D^H F_D^H(t_i) + C_A^H F_A^H(t_i) \right] \right\}^2 \end{aligned} \quad (13)$$

where $f_m(t_i)$ is the measured force acting on a section of the cylinder of unit length at an instant $t = t_i$, and N is the total number of the data points used in the analysis. The four coefficients are obtained by solving the following set of linear algebraic equations:

$$\frac{\partial E^2}{\partial C_D^L} = 0, \frac{\partial E^2}{\partial C_A^L} = 0, \frac{\partial E^2}{\partial C_D^H} = 0, \frac{\partial E^2}{\partial C_A^H} = 0. \quad (14)$$

In this study, the nonlinear least-square fitting was performed on a time series of length 5–10 cycles.

2.3 Verification of the integrated cylindrical force model

In the literature, there are a lot of results about a cylinder oscillating sinusoidally in still water. In particular, Obasaju et al. (1988) presented the wave force coefficients as functions of a Keulegan–Carpenter number KC and a frequency number β , defined by.

$$KC = \frac{UT}{D}, \quad \beta = \frac{D^2}{\nu T} \quad (15)$$

where T is the wave period, U the horizontal component of the wave orbital velocity, and ν the kinematic viscosity of water. The accuracy of our instrumented force model was verified by comparing our results with those reported in Obasaju et al. (1988) and Sarpkaya (1986) with similar values of β . As an example, Fig. 2 shows a comparison of our results for $\beta = 500$ with those of Obasaju et al. (1988) for $\beta = 485$. The small difference between our results and those of Obasaju et al. (1988) for larger values of KC is due to the small difference in the values of β and the different force models used in the two tests.

2.4 Test conditions

When the cylinder oscillates with dual frequency in still water, each frequency component can define its own values of KC and β . For the low-frequency component, we have

$$KC_L = \frac{U_L T_L}{D}, \quad \beta_L = \frac{D^2}{\nu T_L} \quad (16)$$

where the subscript L refers to “low-frequency component”. Similarly, for the high-frequency component, we have

$$KC_H = \frac{U_H T_H}{D}, \quad \beta_H = \frac{D^2}{\nu T_H} \quad (17)$$

where subscript H refers to “high-frequency component”.

Two types of tests were conducted: single-frequency tests and dual-frequency tests. Two high frequencies were selected: $T_H = 1.2$ and 1.6 s; for each frequency, several oscillation amplitudes were tested in both single frequency tests and dual-frequency tests. Three low frequencies were selected: $T_L = 4.0, 6.0$ and 8.0 s; for each frequency, a range of oscillation amplitudes were tested in both single frequency tests and dual-frequency tests. The detailed conditions for the dual-frequency tests are listed in Table 1.

Table 1 Test conditions

Test code	T_L (s)	T_H (s)	A_H (cm)	A_L (cm)	β_L	β_H	KC_H	KC_L
(a)	4.0	1.2	3.25	3.90–10.40	625	2083	4.1	4.9–13.1
(b)	4.0	1.2	3.90	3.90–9.15	625	2083	4.9	4.9–11.5
(c)	4.0	1.6	3.90	3.90–10.40	625	1563	4.9	4.9–13.1
(d)	4.0	1.6	4.55	3.90–10.40	625	1563	5.7	4.9–13.1
(e)	6.0	1.2	3.25	5.17–14.30	417	2083	4.1	6.3–18.0
(f)	6.0	1.2	3.90	3.90–14.30	417	2083	4.9	4.9–18.0
(g)	6.0	1.6	3.90	3.90–14.30	417	1563	4.9	4.9–18.0
(h)	6.0	1.6	4.55	3.90–12.97	417	1563	5.7	4.9–16.3
(i)	8.0	1.2	3.25	3.90–18.20	313	2083	4.1	4.9–22.9
(j)	8.0	1.2	3.90	3.90–15.59	313	2083	4.9	4.9–19.6
(k)	8.0	1.6	3.90	3.90–18.20	313	1563	4.9	4.9–22.9
(l)	8.0	1.6	4.55	3.90–18.20	313	1563	5.7	4.9–22.9

3 Results

In this section, we will first present wave force coefficients as functions of $(KC_L, KC_H, \beta_L, \beta_H)$. Empirical formulas for the wave force coefficients will be proposed in Sect. 4 when we discuss our results.

3.1 Effects of high-frequency motion on wave force coefficients for low-frequency motion

Figures 3, 4, 5, 6, 7 and 8 show the wave force coefficients for the low-frequency component in the presence and absence of high-frequency motion for three values of β_L (625, 471 and 313) and two values of β_H (1563 and 2083). In these figures, the results for $KC_H = 0.0$ are obtained for the cylinder oscillating with a single frequency in still water. Note that the Keulegan–Carpenter number for the high-frequency component ranges from 4.1 to 5.7. According to Laya et al. (1984), the assumption adopted by the relative-velocity form of Morison's equation is expected to be invalid for this range of Keulegan–Carpenter number. The wave force coefficients for the low-frequency component are presented as functions of the Keulegan–Carpenter number for low-frequency component, KC_L , defined in Eq. (16). It can be seen that the presence of the high-frequency motion can significantly increase both the added-mass and drag coefficients for the low-frequency component, and with increasing KC_L both C_A^L and C_D^L tend to approach the wave force coefficients for the cylinder oscillating with a single frequency. However, due to the limitation of facility, the range of KC_L for the dual frequency motion is not as wide as that for the single-frequency oscillation. When $KC_L < KC_H$, both the added-mass and drag coefficients can be 2 times larger than the corresponding values obtained for the single frequency motion. For the same value of KC_L , increasing KC_H increases both C_A^L and C_D^L .

3.2 Effects of low-frequency motion on wave coefficients for high-frequency motion

Figures 9, 10, 11, 12, 13 and 14 show the wave force coefficients for the high-frequency component in the presence and absence of low-frequency motion for three values of β_L (625, 471 and 313) and two values of β_H (1563 and 2083). In these figures, the lines are the results for the cylinder oscillating with a single high frequency in still water. It can be seen that the presence of the low-frequency motion reduced/increased the added-mass/drag coefficient for the high-frequency component in all tested cases. As expected, the influence of the low-frequency motion tends to be smaller when KC_L is small. When $KC_H \approx KC_L$, the drag coefficients for the high-frequency component are close to those obtained for a cylinder oscillating with a single-frequency in still water; when this happens, the added-mass coefficients are still significantly smaller than those obtained for a cylinder oscillating with a single-frequency in still water.

4 Discussion

When using the independent-flow form of Morison's equation to model the hydrodynamic force acting on a cylinder moving in regular waves, our results showed that it was not appropriate to simply use the added-mass and drag coefficients obtained for a cylinder oscillating with a single-frequency in still water. For dual-frequency oscillations, a simple dimensionless analysis shows that the wave force coefficients depend on four dimensionless parameters. It is desirable to find a single dimensionless parameter for each wave force coefficient so that a simple empirical equation can be proposed for each wave force coefficient.

For each wave force coefficient (C_A^L , C_D^L , C_A^H or C_D^H), if we put all the data in one plot as a function of KC_L ,

Fig. 3 The added-mass and drag coefficients for the low-frequency component. $\beta_L = 625$ and $\beta_H = 2083$

Fig. 4 The added-mass and drag coefficients for the low-frequency component. $\beta_L = 625$ and $\beta_H = 1563$

Fig. 5 The added-mass and drag coefficients for the low-frequency component. $\beta_L = 417$ and $\beta_H = 2083$

Fig. 6 The added-mass and drag coefficients for the low-frequency component. $\beta_L = 417$ and $\beta_H = 1563$

Fig. 7 The added-mass and drag coefficients for the low-frequency component. $\beta_L = 313$ and $\beta_H = 2083$

Fig. 8 The added-mass and drag coefficients for the low-frequency component. $\beta_L = 313$ and $\beta_H = 1563$

Fig. 9 The added-mass and drag coefficients for the high-frequency component (symbols). Lines are for the corresponding values obtained from single-frequency oscillation tests. $\beta_L = 625$ and $\beta_H = 2083$

Fig. 10 The added-mass and drag coefficients for the high-frequency component (symbols). Lines are for the corresponding values obtained from single-frequency oscillation tests. $\beta_L = 625$ and $\beta_H = 1563$

Fig. 11 The added-mass and drag coefficients for the high-frequency component (symbols). Lines are for the corresponding values obtained from single-frequency oscillation tests. $\beta_L = 417$ and $\beta_H = 2083$

Fig. 12 The added-mass and drag coefficients for the high-frequency component (symbols). Lines are for the corresponding values obtained from single-frequency oscillation tests. $\beta_L = 417$ and $\beta_H = 1563$

Fig. 13 The added-mass and drag coefficients for the high-frequency component (symbols). Lines are for the corresponding values obtained from single-frequency oscillation tests. $\beta_L = 313$ and $\beta_H = 2083$

Fig. 14 The added-mass and drag coefficients for the high-frequency component (symbols). Lines are for the corresponding values obtained from single-frequency oscillation tests. $\beta_L = 313$ and $\beta_H = 1563$

Fig. 15 Wave force coefficients C_A^L and C_D^L as functions of KC_L for all test conditions. The values of β_L , β_H and KC_H for each symbol (test code) in the legend are listed in Table 1

we found that KC_L was not an appropriate parameter to describe these wave force coefficients, as shown in Figs. 15 and 16.

In searching for a suitable dimensionless parameter to describe their data, Koterayama (1984) and Koterayama and Nakamura (1988) used the so-called “reduced velocity” to correlate their wave force coefficients. Reduced velocity is defined using the velocity of the slow motion and the period of the fast motion, i.e.,

$$V_R = \frac{U_L T_H}{D} \quad (18)$$

Note that in terms of β_L , β_H and KC_L , $V_R = (\beta_L/\beta_H)KC_L$. However, using the reduced velocity did not correlate well with the drag coefficients for the fast motion and the inertia coefficient for the slow motion in Koterayama (1984) and Koterayama and Nakamura (1988). We have also used V_R to analyze our data and found that this is not an appropriate parameter to analyze all four wave force coefficients (C_A^L , C_D^L , C_A^H and C_D^H).

Physically, the amplitude of the high-frequency motion should also play an important role in the determination of wave force coefficients. Therefore, we propose a new dimensionless parameter $V_R + \alpha KC_H$, with α being a fitting parameter, to analyze our data. Our results have shown that

1. A suitable parameter for C_A^H is V_R .
2. A suitable parameter for C_D^H is $V_R + KC_H$.
3. A suitable parameter for C_A^L and C_D^L is $V_R - KC_H/2$.

Based on our data, we propose the following empirical expressions for the four wave force coefficients:

$$C_A^L = 0.393 + 1.199 \exp[-0.351(V_R - 0.5 KC_H)], \quad (19)$$

$$C_D^L = 2.146 + 1.214 \exp[-0.738(V_R - 0.5 KC_H)], \quad (20)$$

$$C_A^H = -0.079 + 0.760 \exp(-0.202 V_R), \quad (21)$$

$$C_D^H = -2.535 + 5.158 \tanh[0.127(V_R + KC_H)], \quad (22)$$

where all the model parameters were obtained by fitting the expressions to our data. We remark that Koterayama (1984)

Fig. 16 Wave force coefficients C_A^H and C_D^H as functions of KC_L for all test conditions. The symbols are the same as in Fig. 15

Fig. 17 Left panel comparison of Eq. (19) with all the measured values of C_A^L . Right panel comparison of Eq. (20) with all the measured values of C_D^L . The lines are the fitting curves and the symbols are the measurements. The symbols are the same as in Fig. 15

Fig. 18 Left panel comparison of Eq. (21) with all the measured values of C_A^H . Right panel comparison of Eq. (22) with all the measured values of C_D^H . The lines are the fitting curves and the symbols are the measurements. The symbols are the same as in Fig. 15

used a parameter similar to $V_R + KC_H$ to discuss the lift force acting on a cylinder moving with a constant speed in regular waves, which is equivalent to $T_L \rightarrow \infty$ in our problem. Figure 17 compares the fitting curves with the data for C_D^L and C_A^L , and Fig. 18 compares the fitting curves with the data for C_D^H and C_A^H . It can be seen from Figs. 17 and 18 that the empirical equations, Eqs. (19) through (22), describe the trends of our data reasonably well.

5 Conclusions

A set of wave flume tests were performed to study the wave force acting on a cylinder oscillating with dual frequency in still water. The experimental results showed that the presence of the high-frequency motion had a significant influence on the wave force coefficients for low-frequency motion, and vice versa. When using the independent-flow form of Morison's equation to model the wave force on a cylinder slowly oscillating in waves or a cylinder rapidly oscillating in low-frequency waves, it is not appropriate to simply use the wave force coefficients obtained for a cylinder oscillating with a

single frequency in still water. Our results have shown that it is not possible to use a single dimensionless parameter to describe all four wave force coefficients. It was suggested that each wave force coefficient should use a different dimensionless parameter to describe its trend, and our results have shown that:

- A suitable parameter for the added-mass coefficient of the high-frequency component is the reduced velocity.
- A suitable parameter for the drag coefficient of the high-frequency component is the sum of the reduced velocity and the Keulegan–Carpenter number for the high-frequency component.
- A suitable parameter for both the added-mass and drag coefficients of the low-frequency component is the reduced velocity minus one half of the Keulegan–Carpenter number for the high-frequency component.

Empirical expressions for the wave force coefficients were proposed using these three dimensionless parameters, and they were found to be in a reasonable agreement with our data.

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