Correction for “Collateral premia and risk sharing under limited commitment” [Econ. Theory 46, 475–501 (2011)]

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Abstract This note provides a correction for Lemma 4 in Kilenthong (Econ Theory 46:475–501, 2011). As pointed out in Zierhut (Econ Theory Bull, 2013), Assumption 2 in Kilenthong (Econ Theory 46:475–501, 2011) is not sufficient to guarantee that optimal allocations will be full risk sharing as claimed in Lemma 4 in Kilenthong (Econ Theory 46:475–501, 2011). This note shows that the main result still holds for an economy in which risk sharing rule is linear by providing a corrected sufficient condition for full risk sharing.

Keywords Collateral premium · Collateralized contracts · Limited commitment · Full risk sharing

JEL Classification D52 · D53

Consider a collateral economy as in Kilenthong (2011). There are two periods. There are S states in the second period. There is no loss of generality to focus only on an economy with consumption and collateralizable capital goods only. Thus, the commodity space is given by $\mathbb{R}^{S+2}$. Let $R_s > 0$ be the payoff of the capital good in state $s = 1, \dots, S$. Let $\mathbf{e}^h = (e_0^h, e_1^h, \dots, e_S^h, e_{\text{coll}}^h)$ be the endowments of an agent of type h . The aggregate collateral is denoted by $\bar{e}_{\text{coll}} = \sum_h e_{\text{coll}}^h$.

There is no loss of generality to focus on collateralized Arrow or contingent securities only because the objective is only to find a sufficient condition. Let D_{sj} be the payoff of the j th-contingent security in state s . As in Kilenthong (2011), its payoff in state s is $D_{ss} = R_s$ and zero in other states.

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Consider the collateral constraint:

$$k_{\text{coll}}^h + \sum_s \min(\theta_s^h, 0) \geq 0, \quad (1)$$

where $k_{\text{coll}}^h \geq 0$ is the amount of collateral held by an agent type h , and $\theta_s^h \in \mathbb{R}$ is the amount of the s th-contingent security held by an agent of type h . Since we focus only on the collateralized contingent securities, the collateral constraint can be rewritten as S state-contingent collateral constraints as follows:

$$k_s^h + \min(\theta_s^h, 0) \geq 0, \quad \forall s = 1, \dots, S, \quad (2)$$

where $k_{\text{coll}}^h = \sum_s k_s^h$. These collateral constraints hold if

$$k_s^h + \theta_s^h \geq 0, \quad \forall s = 1, \dots, S. \quad (3)$$

The budget constraint for an agent of type h in state s is given by

$$c_s^h = e_s^h + R_s k_{\text{coll}}^h + R_s \theta_s^h, \quad \forall h, s. \quad (4)$$

Note that there are only collateralized contingent securities, and the payoffs of each security are as follows: $D_{ss} = R_s$ and $D_{sj} = 0$ when $s \neq j$. Combining (3) and (4) gives

$$c_s^h \geq e_s^h, \quad \forall h, s. \quad (5)$$

In words, the collateral constraint for an agent of type h holds if condition (5) holds. Therefore, we only now need to find the aggregate amount of collateral under which the first-best consumption allocations for all types satisfy condition (5). This aggregate collateral is not the necessary level to achieve full risk sharing but it is a sufficient level nonetheless.

Let $\lambda = (\lambda^1, \dots, \lambda^H)$ be the vector of the Pareto weights such that $\lambda^h > 0$ and $\sum_h \lambda^h = 1$.

Lemma 1 Suppose that the utility functions across households yield a linear risk sharing rule such that the first-best consumption allocation for an agent of type h in state s is as follows:

$$c_s^h = a^h(\lambda) + b^h(\lambda) [\bar{e}_s + R_s \bar{e}_{\text{coll}}], \quad \forall s, h, \quad (6)$$

where $b^h(\lambda) > 0$ for all h . If aggregate collateral

$$\bar{e}_{\text{coll}} \geq \max_h \max_s \left\{ \frac{\frac{e_s^h - a^h(\lambda)}{b^h(\lambda)} - \bar{e}_s}{R_s} \right\}, \quad (7)$$

then an optimal allocation under the collateral constraints exhibits full risk sharing.

Proof An aggregate amount of the consumption good in state s is

$$\sum_h e_s^h + R_s \sum_h e_{\text{coll}}^h = \bar{e}_s + R_s \bar{e}_{\text{coll}}. \quad (8)$$

The linear risk sharing rule (see e.g., [Townsend 1993](#) for utility functions yielding linear risk sharing rules) can be expressed as follows:

$$c_s^h = a^h(\lambda) + b^h(\lambda) [\bar{e}_s + R_s \bar{e}_{\text{coll}}], \quad \forall s, h, \quad (9)$$

where $\lambda = (\lambda^1, \dots, \lambda^H)$ is the vector of the Pareto weights. Condition (5) can now be rewritten as follows:

$$a^h(\lambda) + b^h(\lambda) [\bar{e}_s + R_s \bar{e}_{\text{coll}}] \geq e_s^h, \quad \forall s, h, \quad (10)$$

which can be rearranged as

$$R_s \bar{e}_{\text{coll}} \geq \frac{e_s^h - a^h(\lambda)}{b^h(\lambda)} - \bar{e}_s, \quad \forall s, h. \quad (11)$$

This condition holds if

$$\bar{e}_{\text{coll}} \geq \frac{\frac{e_s^h - a^h(\lambda)}{b^h(\lambda)} - \bar{e}_s}{R_s}, \quad \forall s, h. \quad (12)$$

As a result, to ensure that this condition holds for all s and all h , the aggregate collateral must satisfy the following condition:

$$\bar{e}_{\text{coll}} \geq \max_h \max_s \left\{ \frac{\frac{e_s^h - a^h(\lambda)}{b^h(\lambda)} - \bar{e}_s}{R_s} \right\}. \quad (13)$$

We now consider the counter example in [Zierhut \(2013\)](#), where $e_1^h = 1$, $e_2^h = 0$ for all $h = 1, 2$, and $R = (1, 2)$. It is not difficult to show that the risk sharing rule in that example is as follows:

$$c_s^h = \lambda^h [\bar{e}_s + R_s \bar{e}_{\text{coll}}]. \quad (14)$$

That is, $a^h(\lambda) = 0$ for all $h = 1, 2$, and $b^1(\lambda) = \lambda^1 = \frac{7}{8}$ and $b^2(\lambda) = \lambda^2 = \frac{1}{8}$. As a result, the critical level of aggregate collateral is

$$\max \left\{ \frac{\frac{1-0}{\frac{7}{8}} - 2}{1}, \frac{\frac{0-0}{\frac{1}{8}} - 0}{2}, \frac{\frac{1-0}{\frac{1}{8}} - 2}{1}, \frac{\frac{0-0}{\frac{1}{8}} - 0}{2} \right\} = 6.$$

This lemma then suggests that if the aggregate collateral is $\bar{e}_{\text{coll}} \geq 6$, then the optimal allocation is full risk sharing. In fact, with $\bar{e}_{\text{coll}} = 6$, the optimal allocation is $k^1 = 6, k^2 = 0, \theta_1^1 = \theta_1^2 = 0, \theta_2^1 = -\theta_2^2 = -\frac{3}{2}, c_1^1 = 7, c_2^1 = \frac{21}{2}, c_1^2 = 1, c_2^2 = \frac{3}{2}$. It is clear that the collateral constraints hold for all s and all h . That is, for given Pareto weights, it is possible to find a finite level of aggregate collateral where nobody faces binding collateral constraints and risk sharing is complete.

In conclusion, this lemma is a corrected version of Lemma 4 in Kilenthong (2011). Accordingly, it can be used to prove Proposition 4 in Kilenthong (2011). Lemma 4 and Proposition 4 hold under a sufficient condition (7) for an economy in which risk sharing rule is linear.

References

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