Generalized projective synchronization of chaotic satellites problem using linear matrix inequality

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Abstract In this paper, a systematic design procedure for generalized projective synchronization between two identical chaotic satellites systems based on feedback control theory is proposed. This method is developed based on suitable feedback control, combined with the Lyapunov stability theory and linear matrix inequality formulation as a solution of the optimal problem. Two necessary and sufficient conditions for the asymptotic stability of the error dynamic system are obtained. Compared with the predictive-based control method, the proposed method shows comparable and superior performance. The effectiveness of the obtained results is illustrated by numerical simulations.

Keywords Generalized projective synchronization (GPS) · Chaotic satellite systems · Lyapunov stability theory · Linear matrix inequality (LMI)

1 Introduction

Chaos is a very interesting nonlinear phenomenon. The fundamental characteristic of a chaotic system is its sensitive dependence on initial conditions; which means that a small shift in the initial states can lead to extraordinary perturbation in the system states.

During the past two decades, many chaotic systems have been proposed and their complex behaviors have also been

widely studied [1–3]. In recent years, many engineering applications of chaotic systems are discussed such as secure communication, digital communication, power electronic devices, power quality, biological systems, chemical reactions analysis and information processing [4–6]. As a special case, in space technology progresses, the need for improved satellite systems by better understanding of satellite dynamics has continuously kept attention [7]. Recently, non-linear dynamics, especially the chaotic attitude dynamics of a satellite have attracted the attention of many scientists [8–10]. For instance it is possible to achieve synchronization among the systems, so that chaos-based communication can be implemented properly on a satellite formulation flying environment [8]. Also both the chaos and optimal control of the steady rotations of a satellite-gyrostator on a circular orbit [10].

The problem of synchronization of a dynamical system is one of the classic topics in engineering science. Synchronization phenomena in chaotic systems have attracted much attention since the work of Pecora and Carroll [11]. Up to now, various types of synchronization phenomena are being reported for chaotic systems, such as complete synchronization (CS) [12], phase synchronization (PS) [13], anti-phase synchronization (APS) [14], lag synchronization (LS) [15], and generalized synchronization (GS) [16], and projective function synchronization [17].

In 1999, projective synchronization has been first reported by Mainieri and Rehacek [18] in partially linear systems that the master and slave vectors synchronize up to a constant scaling factor (α proportional relation). CS [12] can be regarded as a special case of projective synchronization characterized at $\alpha = 1$, as well as anti-phase synchronization [14] at $\alpha = -1$.

Many methods have been presented for the synchronization of chaotic system such as periodic parametric perturbation method [19], drive-response synchronization method

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Fig. 1 **a** Chaotic attractor of satellite system, **b** trajectories projected on the $\omega_x - \omega_y$ plane, **c** trajectories projected on the $\omega_x - \omega_z$ plane, **d** trajectories projected on the $\omega_y - \omega_z$ plane

[20], active control method [21], adaptive control method [14], sliding mode control method [22], and H^∞ control method [23].

The synchronization problem is interpreted as a stabilization one. The goal is to stabilize, at the origin, the discrepancy between the drive and response systems. Discrepancy is defined as the dynamical differences between the drive and response systems. Recently, Sadaoui et al. [24] presented a predictive control strategy for synchronization of chaotic satellites systems.

Most recently, a new synchronization, called generalized projective synchronization, has been observed in the chaotic systems [25–27]. For example nonlinear controller is applied to investigate the generalized projective synchronization for a controlled chaotic gyroscope with a periodic gyroscope dynamical system [25]. Also the generalized projective synchronization of a class of hyperchaotic systems is studied [27].

Linear matrix inequalities (LMIs) have been playing an increasingly important role in the field of optimization and control theory because of the widespread problems (linear and convex quadratic inequalities, matrix norm inequalities, convex constraints etc.) can be written as LMIs [28,29]. In addition, LMI technique for chaos control is concerned; Chen et al. [30] proposed a LMI-based linear-state-feedback control design to achieve the synchronization of uncertain chaotic systems. The control system is transformed into a LMI system whose feedback gain can be directly solved by a LMI solver. Wang et al. [31] with designing a feedback controller based on sliding mode technique investigated the chaos control problem for a general class of chaotic systems, where controller's parameter is obtained from a LMI. And Singh in [32] is presented new criteria for the global robust stability of interval Hopfield-type delayed neural networks.

The aim of this paper is to present a LMI-based feedback controller for the generalized projective synchroniza-

Fig. 2 State trajectories of the satellites drive and response systems with the scaling factor $\alpha = 0.5$

tion between two identical chaotic satellite systems, where the stability of the proposed method is guaranteed by using Lyapunov stability theory. It will be shown that the performance of the proposed scheme is improved compared to a recently published paper [24].

2 System description and problem formulation

2.1 Satellite system with chaotic dynamics

Recently, the dynamic equations of several spacecraft/satellite systems with the aim of attitude/ congestion control have

been studied [33–35]. The dynamical equations of a typical satellite are as [24]:

$$\begin{cases} I_x \dot{\omega}_x = \omega_y \omega_z (I_y - I_z) + h_x + u_x \\ I_y \dot{\omega}_y = \omega_x \omega_z (I_z - I_x) + h_y + u_y \\ I_z \dot{\omega}_z = \omega_x \omega_y (I_x - I_y) + h_z + u_z \end{cases} \quad (1)$$

where I_x , I_y and I_z are the principal moments of inertia; ω_x , ω_y and ω_z are the angular velocities of the satellite; h_x , h_y and h_z are perturbing torques; and u_x , u_y and u_z are the three control torques. Earlier papers [24,36] have taken $I_x = 3,000 \text{ kg m}^2$, $I_y = 2,000 \text{ kg m}^2$ and $I_z = 1,000 \text{ kg m}^2$

Fig. 3 GPS state errors with the scaling factor $\alpha = 0.5$

with the perturbing torques defined by:

$$\begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = \begin{bmatrix} -1200 & 0 & 1000\sqrt{6}/2 \\ 0 & 350 & 0 \\ -1000\sqrt{6} & 0 & -400 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (2)$$

These torques are chosen such that the satellite is forced into chaotic motion. The chaotic attractor and chaotic trajectories of the system (1) are shown in Fig. 1a and b–d, respectively.

2.2 Synchronization problem formulation

Consider the following two identical satellites systems, where the drive system and response system are denoted with x and y , respectively.

2.2.1 Drive system

$$\begin{cases} \dot{x}_1 = \frac{1}{3}x_2x_3 - 0.4x_1 + \frac{\sqrt{6}}{6}x_3 \\ \dot{x}_2 = -x_1x_3 + 0.175x_2 \\ \dot{x}_3 = x_1x_2 - \sqrt{6}x_1 - 0.4x_3 \end{cases} \quad (3)$$

where x_i ($i = 1, 2, 3$) is drive system state.

2.2.2 Response system

$$\begin{cases} \dot{y}_1 = \frac{1}{3}y_2y_3 - 0.4y_1 + \frac{\sqrt{6}}{6}y_3 + u_1 \\ \dot{y}_2 = -y_1y_3 + 0.175y_2 + u_2 \\ \dot{y}_3 = y_1y_2 - \sqrt{6}y_1 - 0.4y_3 + u_3 \end{cases} \quad (4)$$

where y_i ($i = 1, 2, 3$) is response system state.

Definition 1 The projective synchronization error between drive system and response system is defined as $e_i = y_i - \alpha x_i$; $i = 1, 2, 3$, where $\alpha \neq 0$ is called a scaling factor.

The objective of this study is to propose an approach to designing a suitable controller $u_i(t)$; $i = 1, 2, 3$ such that

$$\lim_{t \rightarrow \infty} \|e_i(t)\| = \lim_{t \rightarrow \infty} \|y_i(t) - \alpha x_i(t)\| = 0; \quad i = 1, 2, 3 \quad (5)$$

where $\|\cdot\|$ denotes the Euclidean norm of a vector. If Eq. (5) is satisfied, then it can be said that systems (3) and (4) have obtained projective synchronization.

According to the Definition 1, the error dynamic system can be obtained from (3) and (4) as follows:

$$\begin{cases} \dot{e}_1 = \frac{1}{3}(y_2y_3 - \alpha x_2x_3) - 0.4(y_1 - \alpha x_1) + \frac{\sqrt{6}}{6} \times (y_3 - \alpha x_3) + u_1 \\ \dot{e}_2 = -(y_1y_3 - \alpha x_1x_3) + 0.175(y_2 - \alpha x_2) + u_2 \\ \dot{e}_3 = (y_1y_2 - \alpha x_1x_2) - \sqrt{6}(y_1 - \alpha x_1) - 0.4(y_3 - \alpha x_3) + u_3 \end{cases} \quad (6)$$

Base on Definition 1, it is directly obtained that

$$\begin{cases} y_2y_3 - \alpha x_2x_3 = \alpha x_3e_2 + \alpha x_2e_3 + e_2e_3 + \alpha(\alpha - 1)x_2x_3 \\ y_1y_3 - \alpha x_1x_3 = \alpha x_3e_1 + \alpha x_1e_3 + e_1e_3 + \alpha(\alpha - 1)x_1x_3 \\ y_1y_2 - \alpha x_1x_2 = \alpha x_2e_1 + \alpha x_1e_2 + e_1e_2 + \alpha(\alpha - 1)x_1x_2 \end{cases} \quad (7)$$

With substituting Eq. (7) into Eq. (6), it yields:

$$\begin{cases} \dot{e}_1 = -0.4e_1 + \frac{1}{3}\alpha x_3e_2 + \left(\frac{\sqrt{6}}{6} + \frac{1}{3}\alpha x_2\right)e_3 + \frac{1}{3}e_2e_3 \\ \quad + \frac{1}{3}\alpha(\alpha - 1)x_2x_3 + u_1 \\ \dot{e}_2 = -\alpha x_3e_1 + 0.175e_2 - \alpha x_1e_3 - e_1e_3 - \alpha(\alpha - 1)x_1x_3 + u_2 \\ \dot{e}_3 = (\alpha x_2 - \sqrt{6})e_1 + \alpha x_1e_2 - 0.4e_3 + e_1e_2 + \alpha(\alpha - 1)x_1x_2 + u_3 \end{cases} \quad (8)$$

where u_i ($i = 1, 2, 3$) are control inputs that will be designed by LMI-based feedback control method.

3 LMI-based feedback controller design and it's stability analysis

The LMI-based feedback controller is a combination of two design procedure: the feedback controller design and the LMI relation formulation, which are given, respectively.

Fig. 4 State trajectories of the satellites drive and response systems with the scaling factor $\alpha = -1$

For the error dynamic system, Eq. (8), to design the controller, set the control law as:

$$\begin{cases} u_1 = -\frac{1}{3} \frac{(\alpha-1)}{\alpha} (y_2 - e_2)(y_3 - e_3) - \gamma_1 e_1 \\ u_2 = \frac{(\alpha-1)}{\alpha} (y_1 - e_1)(y_3 - e_3) - \gamma_2 e_2 \\ u_3 = -\frac{(\alpha-1)}{\alpha} (y_1 - e_1)(y_2 - e_2) - \gamma_3 e_3 \end{cases} \quad (9)$$

where γ_i ($i = 1, 2, 3$) are positive feedback gains that will be obtained by LMI.

For the error dynamic system in Eq. (8), the existence of a quadratic Lyapunov function $V(t) = e^T(t)Pe(t)$, $P > 0$, $P \in \mathbb{R}^{3 \times 3}$ is a necessary and sufficient condition for asymptotic stability [37]. Define a Lyapunov function candidate of the Eq. (8) as follows:

$$V(t) = \rho_1 e_1^2(t) + \rho_2 e_2^2(t) + \rho_3 e_3^2(t) \quad (10)$$

Fig. 5 GPS state errors with the scaling factor $\alpha = -1$

where $\rho_i (i = 1, 2, 3)$ are positive constants and therefore this Lyapunov function is positive definite. Then, taking the derivative of Eq. (10) with respect to time, and with applying the controller (9) in to error dynamic Eq. (8), the result will be as follows:

$$\begin{aligned} \dot{V} &= 2\rho_1 e_1 \dot{e}_1 + 2\rho_2 e_2 \dot{e}_2 + 2\rho_3 e_3 \dot{e}_3 \\ &= -2(0.4 + \gamma_1)\rho_1 e_1^2 - 2(-0.175 + \gamma_2)\rho_2 e_2^2 \\ &\quad - 2(0.4 + \gamma_3)\rho_3 e_3^2 + 2\left(\frac{1}{3}\rho_1 - \rho_2\right)\alpha x_3 e_1 e_2 \\ &\quad + 2\left(\left(\frac{\sqrt{6}}{6} + \frac{1}{3}\alpha x_2\right)\rho_1 + (\alpha x_2 - \sqrt{6})\rho_3\right)e_1 e_3 \\ &\quad + 2[-\rho_2 + \rho_3]\alpha x_1 e_2 e_3 + 2\left[\frac{1}{3}\rho_1 - \rho_2 + \rho_3\right]e_1 e_2 e_3 \end{aligned} \quad (11)$$

If the relation between Lyapunov function's parameters are chosen as

$$\rho_1 = 3(\rho_2 - \rho_3) \quad (12)$$

the following result can be obtained

$$\begin{aligned} \dot{V}(t) &= -6(0.4 + \gamma_1)(\rho_2 - \rho_3)e_1^2 \\ &\quad - 2(-0.175 + \gamma_2)\rho_2 e_2^2 \\ &\quad - 2(0.4 + \gamma_3)\rho_3 e_3^2 - 2\rho_3 \alpha x_3 e_1 e_2 \\ &\quad + 2\left(\left(\frac{\sqrt{6}}{2} + \alpha x_2\right)\rho_2 - \frac{3\sqrt{6}}{2}\rho_3\right)e_1 e_3 \\ &\quad + 2(\rho_3 - \rho_2)\alpha x_1 e_2 e_3 \leq -6(4 + \gamma_1)(\rho_2 - \rho_3)e_1^2 \\ &\quad - 2(-0.175 + \gamma_2)\rho_2 e_2^2 - 2(0.4 + \gamma_3)\rho_3 e_3^2 \\ &\quad + 2\rho_3 |\alpha| |x_3| |e_1| |e_2| + 2\left(\left(\frac{\sqrt{6}}{2} + |\alpha| |x_2|\right)\rho_2 \right. \\ &\quad \left. + \frac{3\sqrt{6}}{2}\rho_3\right)|e_1| |e_3| + 2(\rho_3 + \rho_2)|\alpha| |x_1| |e_2| |e_3| \end{aligned} \quad (13)$$

In LMI relation formulation, it can be said that because the trajectories of a drive chaotic system are all bounded, there exist three positive constants M_1 , M_2 and M_3 such that $|x_1| \leq M_1$, $|x_2| \leq M_2$ and $|x_3| \leq M_3$ and so $\dot{V} \leq (|e_1|, |e_2|, |e_3|)\Omega(|e_1|, |e_2|, |e_3|)^T$, where

$$\Omega = \begin{bmatrix} -6(0.4 + \gamma_1)(\rho_2 - \rho_3) & \rho_3 |\alpha| M_3 & \left(\left(\frac{\sqrt{6}}{2} + |\alpha| M_2\right)\rho_2 + \frac{3\sqrt{6}}{2}\rho_3\right) \\ * & -2(-0.175 + \gamma_2)\rho_2 & (\rho_2 + \rho_3) |\alpha| M_1 \\ * & * & -2(0.4 + \gamma_3)\rho_3 \end{bmatrix} \quad (14)$$

Fig. 6 State trajectories of the satellites drive and response systems with the scaling factor $\alpha = 2$

Note that Ω is a symmetrical matrix where $*$ denotes the symmetric terms. To guarantee that the synchronous errors converge to zero, the matrix Ω must be negative definite.

Thus, the condition $\Omega < 0$ is a sufficient condition for the projective synchronization of the satellite chaotic systems. The condition $\Omega < 0$ and $\rho_1, \rho_2, \rho_3 > 0$ can be expressed as

Fig. 7 GPS state errors with the scaling factor $\alpha = 2$

Fig. 8 GPS state errors for stabilization of the systems to the unstable fixed point $(0, 0, 0)$

$$\begin{bmatrix} -6(0.4 + \gamma_1)(\rho_2 - \rho_3) & \rho_3 |\alpha| M_3 & \left(\left(\frac{\sqrt{6}}{2} + |\alpha| M_2 \right) \rho_2 + \frac{3\sqrt{6}}{2} \rho_3 \right) \\ * & -2(-0.175 + \gamma_2) \rho_2 & (\rho_2 + \rho_3) |\alpha| M_1 \\ * & * & -2(0.4 + \gamma_3) \rho_3 \end{bmatrix} < 0 \quad (15a)$$

$$\begin{bmatrix} 3(\rho_3 - \rho_2) & 0 & 0 \\ 0 & -\rho_2 & 0 \\ 0 & 0 & -\rho_3 \end{bmatrix} < 0 \quad (15b)$$

Which are LMIs in γ_1 , γ_2 , γ_3 , ρ_2 and ρ_3 , and it can be solved directly by various LMI solvers such as MATLAB's LMI Control Toolbox. Based on Lyapunov's second method for stability, the system is asymptotically stable.

Therefore, if the conditions (15a) and (15b) are satisfied, the error functions e_i ($i = 1, 2, 3$) converges to zero as the time t tends to infinity, and projective synchronization between systems (3) and (4) is achieved under the controller (9).

4 Simulation

In this section, computer simulations are used to verify and demonstrate the effectiveness of the above presented LMI-based control method. The simulation results are carried out using MATLAB software. The fourth order Runge–Kutta integration algorithm was performed to solve the differential equations.

Table 1 Comparison between two different methods of stabilization.

Criteria	SSSE	SEST (Sec.)
Predictive-based control method [24]	SSSE (e_1) = 248.94, SSSE (e_2) = 260.54, SSSE (e_3) = 90.75	≈ 15.20
Proposed method	SSSE (e_1) = 36.22, SSSE (e_2) = 65.41, SSSE (e_3) = 24.46	≈ 0.8764

For simulation with the scaling factor $\alpha = 0.5$, the design parameters are taken as $\gamma_1 = 2.2669$, $\gamma_2 = 3.7156$, and $\gamma_3 = 14.5751$. For simulation with the scaling factor $\alpha = -1$ (APS), the design parameters are taken as $\gamma_1 = 4.5378$, $\gamma_2 = 8.2017$, and $\gamma_3 = 21.6570$. For simulation with the scaling factor $\alpha = 2$, the design parameters are taken as $\gamma_1 = 10.9989$, $\gamma_2 = 16.5075$, and $\gamma_3 = 34.5318$. As it is said before all these parameters can be obtained directly by solving Eq. (15) in MATLAB's LMI Control Toolbox. The initial values of satellite systems are taken as $x_1 = 5$, $x_2 = 3$, $x_3 = -1$ and $y_1 = 3$, $y_2 = 4.1$, $y_3 = 2$.

The simulation results with the scaling factor $\alpha = 0.5$ are shown in Figs. 2 and 3. The simulation results with the scaling factor $\alpha = -1$ (APS) are shown in Figs. 4 and 5. The simulation results with the scaling factor $\alpha = 2$ are shown in Figs. 6 and 7. From Figs. 3, 5, and 7, it is clear that for an initial error between drive and response systems, after a finite period of time, the initial error will converge to zero asymptotically and GPS between drive and response systems will be achieved.

To present a quantitative comparison between the proposed method and Sadaoui method [24] in stabilization of unstable fixed point $[0 \ 0 \ 0]^T$, two following criteria are used:

- (a) Synchronization error settling time (SEST). It is the time at which $\|e\| < 0.005$.
- (b) Sum of squared synchronization error (SSSE) up to SEST.

The simulation result for synchronization of unstable fixed point $[0 \ 0 \ 0]^T$ is shown in Fig. 8. A comparative result between proposed method and Sadaoui method [24] is given in Table 1. As it is known, the less the SEST, the convergence is sooner. With the less the SSSE the better stabilization is achieved. Referring to the Table 1, it can be concluded that the speed of synchronization with the proposed method is better than that with predictive-based control method.

Because the anti-synchronization ($\alpha = -1$), and projective synchronization ($\alpha = 0.5, 2$) are included in generalized projective synchronization, this paper results are applicable for different α scales (full range GPS). How to realize the generalized projective synchronization of two uncertain and noisy chaotic satellite systems will be presented in our future work.

5 Conclusion

In this paper, Lyapunov stability theory and LMI technique have been employed to obtain optimal solution for generalized projective synchronization of the chaotic satellite systems. It extends the control capability to achieve a full range GPS of all state variables in a feedback way. This method had two advantages in comparison with other result in the same field. It was illustrated that the speed of synchronization of the states was very fast and it was better than that with predictive-based control methodology. The errors had been greatly reduced and had converged very rapidly to zero. Numerical simulations were illustrated to demonstrate the effectiveness of the proposed approach.

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