Optimisation algorithm for sampled-data control systems with delay by $\mathcal{L}_2$ -norm estimations

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Abstract The paper considers the problem of numerical optimisation regarding the quality index of the transient processes in SISO sampled-data systems. Here the mathematical model of the system allows for computational delay time, which is assumed to be nearly constant. As cost function for the system optimisation, an expression for the $\mathcal{L}_2$ norm of the output signal is applied by implementing the delay as a constant quantity. The cost function is constructed by a special approach, which does not need the system description in state space. This approach is based on the application of the parametric transfer function concept and the Laplace transformation in continuous time. Finally an example arising from a maritime control task illustrates the method.

Keywords Sampled-data systems · Time-delays · Transfer functions · Optimal control systems · Genetic algorithms

1 Introduction

Nearly all modern control systems for continuous processes contain pure delay due to computational time or propagation time. As well for theoretical considerations as for computational realisations, the delay is challenging [1–3].

One of the fundamental problems in the theory of sampled-data systems consists in the optimisation of the transient processes in the system by minimising an appropriate $\mathcal{L}_2$ norm. Most of the work in this direction, e.g. [4–6] and the

references therein, are devoted to the optimal $\mathcal{L}_2$ design. This problem includes the search for a stabilising discrete controller, which minimises the value of the $\mathcal{L}_2$ norm of the output signal. However, as investigations of the last years show, the practical application of the derived methods for $\mathcal{L}_2$ design of sampled-data systems is difficult in many cases, due to the following reasons:

- The solutions of the optimal $\mathcal{L}_2$ design are not always sufficiently robust against uncertainties in the system parameters or external disturbances.
- In comparison with the minimal order of the stabilising controllers, the order of the $\mathcal{L}_2$ optimal controller is relatively high, so that the effort for its technical realisation increases.
- As it was shown in [7], among the poles of the $\mathcal{L}_2$ optimal system, we always find so called fixed poles, which are only defined by the properties of the continuous parts of the system. This fact limits the achievable quality of the transient processes.

In many cases a numerical optimisation of the $\mathcal{L}_2$ norm for sampled-data systems helps to overcome the above mentioned difficulties, while assuming limitations on the system structure, especially regarding the order of the controller. For applying the corresponding numerical procedures, the construction of a closed expression for the $\mathcal{L}_2$ norm of the system is needed. In [4–6] various methods for the construction of such expressions are described on the basis of mathematical system models in state space.

For sampled-data systems with delay, in many cases the transformation of the given block diagram to a minimal realisation in state space is connected with honest technical difficulties, which also lead to difficulties in realising the methods for $\mathcal{L}_2$ optimal design, described in [4–6].

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This paper presents an approach for the construction of the $\mathcal{L}_2$ norm for SISO sampled-data systems with delay, which is not connected with the transfer into state space, and it allows to generate closed expressions for the $\mathcal{L}_2$ norm of sampled-data systems on basis of their block diagrams. This approach bases on the relation between the parametric transfer function (PTF) of the system and the Laplace transform in continuous time, ascertaining in [5]. Applying this approach, we formulate the cost function for the transient processes of the closed sampled-data system with delay, and we consider an algorithm for its numerical optimisation according to the formulated criterion under limitations on the order of the controller.

As an example for the application of the derived methods, we consider the optimisation of a system for the course control of an underwater vehicle.

The computational work for the present paper has been performed using MATLAB vers. 2010.

2 System description and problem

The present paper investigates a sampled-data system with a structure shown in Fig. 1. This kind of structure corresponds approximately to a wide area of practical problems.

In Fig. 1 the following notations are used. The continuous object to control is given by its transfer function (TF) $F(s)$. The system output $\alpha(t)$ is measured by the sensor $L(s)$. Then the measuring signal is processed by a low-pass prefilter $K(s)$ [8]. The prefiltered signal comes to an ideal sampler (i.e. quantisation errors of the magnitude and the conversion time are neglected), which works with a non-pathological sampling period T [9]. Then, the control sequence ψ_k is generated by the digital controller given by its backward model

$$C(\zeta) = \frac{\sum_{r=0}^R a_r \zeta^r}{\sum_{r=0}^R b_r \zeta^r} = \frac{A(\zeta)}{B(\zeta)}, \quad (1)$$

where $A(\zeta)$, $B(\zeta)$ are polynomials, and hereby $b_0 \neq 0$. Moreover $R > 0$ is an integer, called the order of the controller. The symbol $\zeta = e^{-sT}$ stands for the backward shift operator. The digital to analog converter $G(s)$ transforms

the sequence ψ_k into the continuous control signal $\mu(t)$. In this paper we assume a zero-order hold for it. In the system a delay acts on the control signal with the delay time $\tau \neq T$, which further on is assumed to be nearly constant. After that, the delayed signal $\mu_\tau(t)$ reaches the actuator with transfer function $W(s)$, which affects the object with transfer function $F(s)$. Moreover, in Fig. 1 $M(s)$ is a given stable filter, where its TF is a strictly proper rational fraction. Here, we also assume $F(s)$ to be strictly proper, and the product $W(s)F(s)L(s)$ at least proper rational functions. As outputs of the system, we consider the signals $e(t)$ and $u(t)$. We also assume that the system in Fig. 1 is asymptotically stable. The reference signal $\beta(t)$ is generated as response of the strictly proper forming filter $Q(s)$ to a Dirac impulse $\delta(t)$.

The dynamic behaviour of the above system in continuous time with input $\delta(t)$ and outputs $u(t)$ and $e(t)$, is described by the parametric transfer functions $W_u(s, t)$ and $W_e(s, t)$, respectively.

Let the vector Z contain all design parameters of the system, which uniquely define the TF of the discrete controller (1). Then, the signals $e(t)$ and $u(t)$ continuously depend on the time t , and at the same time they depend on the parameter vector Z , i.e. $u(t) = u(t, Z)$, $e(t) = e(t, Z)$. Moreover, the PTF also depend on Z : $W_u(s, t) = W_u(s, t, Z)$, $W_e(s, t) = W_e(s, t, Z)$.

The traditional problem of $\mathcal{L}_2$ optimal design for a system with a structure given in Fig. 1 consists in finding a vector $Z = Z_{opt}$ in such a way, that the corresponding controller becomes causal and together with the asymptotic stability of the system it ensures a minimal value of the cost function

$$\begin{aligned} L(Z) &= \int_0^\infty \left(e^2(t, Z) + \rho u^2(t, Z) \right) dt \\ &= L_e(Z) + \rho L_u(Z) \rightarrow \min_Z, \end{aligned} \quad (2)$$

where $\rho > 0$ is a real coefficient, and $L_u(Z)$, $L_e(Z)$ are the $\mathcal{L}_2$ norms of the signals $u(t, Z)$ and $e(t, Z)$, respectively. Notice, that due to the suppositions on the TF of the continuous parts of the system in Fig. 1, the integral in (2) converges absolutely.

Fig. 1 Block diagram for sampled-data system optimisation according to minimal $\mathcal{L}_2$ norm

Under the taken assumptions and for given vector Z , the quantities $L_u(Z)$ and $L_e(Z)$ can be calculated by the Parseval formula

$$\begin{aligned} L_e(Z) &= \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} E(s, Z) E(-s, Z) ds, \\ L_u(Z) &= \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} U(s, Z) U(-s, Z) ds, \end{aligned} \quad (3)$$

where $E(s, Z)$, $U(s, Z)$ are the Laplace transforms of the outputs $e(t, Z)$ and $u(t, Z)$

$$\begin{aligned} E(s, Z) &= \int_0^\infty e(t, Z) e^{-st} dt, \\ U(s, Z) &= \int_0^\infty u(t, Z) e^{-st} dt. \end{aligned} \quad (4)$$

Moreover, in (3) and further on $j = \sqrt{-1}$. The functions $E(s, Z)$, $U(s, Z)$ can be calculated with the help of the PTF $W_e(s, t, Z)$ and $W_u(s, t, Z)$ by formulae presented in [7]

$$\begin{aligned} E(s, Z) &= \int_0^T \frac{1}{T} \sum_{k=-\infty}^\infty W_e(s + kj\omega, t, Z) e^{kj\omega t} dt \\ U(s, Z) &= \int_0^T \frac{1}{T} \sum_{k=-\infty}^\infty W_u(s + kj\omega, t, Z) e^{kj\omega t} dt. \end{aligned} \quad (5)$$

In (5) $\omega = 2\pi/T$ is the sampling frequency. An algorithm for choosing the vector Z_{opt} is provided in [5].

Typically, in consistence with [5], the algorithm for the $\mathcal{L}_2$ optimal system consists in finding a vector $Z = Z_{min}$, which minimises criterion (2) by numeric optimisation for controller order $R \leq R_{max}$, where R_{max} is a given integer. The problem in this case can be formulated as follows:

Let be given all transfer functions of the continuous blocks in Fig. 1, the non-pathological sampling period T , and also the delay $\tau \neq T$. Moreover, let be given the number R_{max} - the highest allowable order of the controller. Then find TF (1) of a stabilising causal controller of order $R \leq R_{max}$ ensuring the least value of criterion (2).

3 Parametric transfer functions

Since the PTF concept is fundamental for the above mentioned methods, the algorithm for building the PTF $W_e(t, s, Z)$

and $W_u(t, s, Z)$ for the system in Fig. 1 should be considered in more detail.

Hereby, although the PTF depend on the parameter vector Z , for compactness reasons, it will be omitted. This dependence is not essential from the mathematical point of view. Moreover, we will introduce the notations $W_\tau(s) = W(s)e^{-s\tau}$ and $n(s, \tau) = K(s)W_\tau(s)F(s)L(s)$.

Let us start with the construction of the PTF $W_e(s, t, Z)$.

Obviously

$$W_e(s, t) = Q(s) [W_\alpha(s, t) - M(s)] \quad (6)$$

where $W_\alpha(s, t)$ is the PTF of the system in Fig. 1 from the input $\beta(t)$ to the output $\alpha(t)$. At first we consider the auxiliary PTF $W_z(s, t)$ from the input $\beta(t)$ to the output $z(t)$. For this purpose we assume $\beta(t) = e^{st}$. Then, by definition [6], we obtain

$$z(t) = W_z(s, t) e^{st}, \quad (7)$$

and thus

$$W_z(s, t) = z(t) e^{-st}. \quad (8)$$

Hereby, due to the periodicity of the system in Fig. 1

$$W_z(s, t) = W_z(s, t + T). \quad (9)$$

On the other side, owing to the stroboscopic property of the sampler, the signal $z(t)$ can be determined by

$$z(t) = [K(s)e^{st} - W_z(s, 0)e^{st}] \tilde{C}(s) \varphi_{Gn}(T, s, t, \tau), \quad (10)$$

where $\tilde{C}(s) = C(\zeta)|_{\zeta=e^{-sT}}$. Moreover, in (10) $\varphi_{Gn}(T, s, t, \tau)$ is the displaced pulse frequency response (DPFR). For functions of the form $X_\tau(s) = X(s)e^{-s\tau}$, where $X(s)$ is an arbitrary strictly proper rational function, the DPFR $\varphi_{X_\tau}(T, s, t, \tau)$ is defined as the sum of the convergent series

$$\varphi_{X_\tau}(T, s, t, \tau) = \frac{1}{T} \sum_{k=-\infty}^\infty X(s + kj\omega) e^{-(s + kj\omega)\tau} e^{kj\omega t}. \quad (11)$$

Closed expressions for the DPFR of form (11) are provided in [5].

Mapping (7) and (10), we can write

$$W_z(s, t) = [K(s) - W_z(s, 0)] \tilde{C}(s) \varphi_{Gn}(T, s, t, \tau). \quad (12)$$

Hence, for $t = 0$

$$W_z(s, 0) = [K(s) - W_z(s, 0)] \tilde{C}(s) \varphi_{Gn}(T, s, 0, \tau), \quad (13)$$

and finally

$$W_z(s, t) = \frac{K(s) \tilde{C}(s) \varphi_{Gn}(T, s, t, \tau)}{1 + \tilde{C}(s) \varphi_{Gn}(T, s, 0, \tau)}. \quad (14)$$

Now, we are in the position to directly achieve the PTF $W_\alpha(s, t)$. Assume $\beta(t) = e^{-st}$, then in analogy to (7)–(14), we find

$$W_\alpha(s, t) = [K(s) - W_z(s, 0)] \tilde{C}(s) \varphi_{GW_\tau F}(T, s, t, \tau), \quad (15)$$

that, using (13) yields

$$W_\alpha(s, t) = \frac{K(s) \tilde{C}(s) \varphi_{GW_\tau F}(T, s, t, \tau)}{1 + \tilde{C}(s) \varphi_{Gn}(T, s, 0, \tau)}. \quad (16)$$

Hence for the PTF $W_e(s, t)$ we eventually obtain

$$W_e(s, t) = \frac{Q(s) K(s) \tilde{C}(s) \varphi_{GW_\tau F}(T, s, t, \tau)}{\lambda(s)} - Q(s) M(s), \quad (17)$$

where

$$\lambda(s) = 1 - \tilde{C}(s) \varphi_{Gn}(T, s, 0, \tau) \quad (18)$$

is the characteristic function, providing information about the stability behaviour of the system.

In analogy, we find expressions for the PTF $W_u(s, t)$

$$W_u(s, t) = Q(s) \left(\frac{K(s) \tilde{C}(s) \varphi_{GW_\tau F}(T, s, t, \tau)}{\lambda(s)} \right). \quad (19)$$

Functions (17), (19) are essentially used for the calculation of cost function (2).

4 Calculation and numerical optimisation

The optimisation criterion of investigated system (2) is calculated with the help of the Laplace transformation in continuous time. In order to calculate this Laplace transform without a conversion to the minimal realisation in state space, it is convenient to apply the results of [7], where the relation between PTF and Laplace transform is stated. Using these results, we obtain

$$E(s, Z) = M(s) Q(s) - \frac{F(s) W(s) G(s) \tilde{C}(s)}{\lambda(s)} D_{KQ}(T, s, 0), \quad (20)$$

$$U(s, Z) = \frac{W(s) G(s) \tilde{C}(s)}{\lambda(s)} D_{KQ}(T, s, 0), \quad (21)$$

where

$$D_{KQ}(T, s, t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} K(s + kj\omega) Q(s + kj\omega) e^{(s+kj\omega)t} \quad (22)$$

is the discrete Laplace transform (DLT) of the function $K(s) Q(s)$. For the strictly proper rational product $Q(s) K(s)$ series (22) converges, and [6] supplies closed expressions for the DLT.

Substituting in (20), (21) $s = j\nu$, where ν is the frequency, then we find the real frequency responses

$$\begin{aligned} \bar{E}(\nu, Z) &= E(s, Z) E(-s, Z)|_{s=j\nu}, \\ \bar{U}(\nu, Z) &= U(s, Z) U(-s, Z)|_{s=j\nu}, \end{aligned} \quad (23)$$

and we achieve a real form of integral (3) (for the lower integration limit we used that $\bar{E}(\nu, Z)$, $\bar{U}(\nu, Z)$ are even functions)

$$\begin{aligned} L_e(Z) &= \frac{1}{\pi} \int_0^\infty \bar{E}(\nu, Z) d\nu, \\ L_u(Z) &= \frac{1}{\pi} \int_0^\infty \bar{U}(\nu, Z) d\nu. \end{aligned} \quad (24)$$

Since integrals (24), due to the assumptions on the TF of the continuous parts of the system, converge absolutely, their values for a given vector Z can be determined by computation, when the infinite upper limit is replaced with a finite number. Thus, we obtain the value of criterion (2).

The selection of the vector Z minimizing criterion (2) over the set of stabilizing controllers, can be performed numerically, in analogy to the procedure presented in [10]. Hereby, the set of stabilizing controllers is constructed with the help of the characteristic function $\lambda(\zeta)$, which in the given case becomes

$$\tilde{\lambda}(\zeta) = \lambda(s)|_{e^{-sT}=\zeta} = 1 + \frac{A(\zeta) a_\tau(\zeta)}{B(\zeta) b_\tau(\zeta)}. \quad (25)$$

where, due to the non-pathological sampling period T , $a_\tau(\zeta)$ and $b_\tau(\zeta)$ are coprime polynomials, determined by the TF of the continuous system parts and the value of the delay τ , in the form

$$\frac{a_\tau(\zeta)}{b_\tau(\zeta)} = \varphi_{Gn}(T, s, 0, \tau)|_{e^{-sT}=\zeta}. \quad (26)$$

Since the fraction on the right side of (25) is irreducible, the set of roots of the function $\tilde{\lambda}(\zeta)$ coincides with the set of roots of the characteristic polynomial

$$A(\zeta) a_\tau(\zeta) + B(\zeta) b_\tau(\zeta) = 0, \quad (27)$$

and the set of stabilizing controllers is determined by the set of solutions $\{A^*(\zeta), B^*(\zeta)\}$ of the Diophantine equation

$$A(\zeta) a_\tau(\zeta) + B(\zeta) b_\tau(\zeta) = \Delta_+(\zeta), \quad (28)$$

where $\Delta_+(\zeta)$ is any stable polynomial, i.e. it does not possess roots inside or on the periphery of the unit circle. This set can be parametrised in the form

$$\begin{aligned} A^*(\zeta) &= A_0(\zeta) + b_\tau(\zeta) \xi(\zeta) \\ B^*(\zeta) &= B_0(\zeta) - a_\tau(\zeta) \xi(\zeta). \end{aligned} \quad (29)$$

In (29) $\xi(\zeta)$ is zero or any polynomial, and $A_0(\zeta)$, $B_0(\zeta)$ is a particular solution of Eq. (28), for which it is expedient to select the solution of minimal degree. It can be shown that under the condition

$$\deg \Delta_+(\zeta) < \deg a_\tau(\zeta) + \deg b_\tau(\zeta), \quad (30)$$

such a solution always exists, and its degree satisfies

$$\begin{aligned} d_A &= \deg A_0(\zeta) \leq \deg b_\tau(\zeta) - 1, \\ d_B &= \deg B_0(\zeta) \leq \deg a_\tau(\zeta) - 1. \end{aligned} \quad (31)$$

Thus the set of TF of the causal stabilising controllers is parametrised in the form

$$C(\zeta) = \frac{A^*(\zeta)}{B^*(\zeta)} = \frac{A_0(\zeta) + b_\tau(\zeta)\xi(\zeta)}{B_0(\zeta) - a_\tau(\zeta)\xi(\zeta)}. \quad (32)$$

Since for $\tau > 0$, always holds

$$\deg a_\tau(\zeta) > \deg b_\tau(\zeta), \quad (33)$$

the order of the controller should not exceed the number $R_{\max}$, and it is necessary that

$$\deg \xi(\zeta) \leq R_{\max} - \deg a_\tau(\zeta). \quad (34)$$

As a consequence of the above considerations, we build the vector Z as the unification

$$Z = \{Z_\Delta, Z_\xi\}, \quad (35)$$

where the vector Z_Δ has the dimension $1 \times N = (d_A + d_B - 1)$ and it contains the complex numbers ζ_i ($i = 1, \dots, N$), satisfying $|\zeta_i| > 1$, which present the roots of the stable polynomial $\Delta_+(\zeta)$. The vector Z_ξ has the maximal degree (by means of (34)) $1 \times (R_{\max} - \deg a_\tau(\zeta) + 1)$ and it contains real numbers - the coefficients of the arbitrary polynomial $\xi(\zeta)$.

Notice that the minimal order of the controller emerges for $\xi(\zeta) = 0$.

We have to find such a vector $Z = Z_{\min}$, that corresponds to a stable controller, minimizes criterion (2) and does not possess an order higher than $R_{\max}$. Hereby, all controllers generated by parametrisation (32) are causal.

For the search of the vector $Z_{\min}$ numerical optimisation methods can be applied, for instance genetic algorithms, as it was performed in [10].

If the forming filter is not known as concrete TF $Q(s)$, but only its belonging to a certain class, then the given problem can be reformulated as a problem for guaranteed performance of the transient processes in sampled-data systems with delay, analogously to the problem solved in [10]: Minimize the estimation of the $\mathcal{L}_2$ norm of the sampled-data system over the class of permitted excitations. Then the designed system will guarantee a certain quality of the transient processes for a wide range of excitations.

5 Example

As an application example for the method explained in this paper, we consider the optimisation problem of a sampled-data system controlling the motion direction of a high-maneuverable underwater vehicle. We suppose that the ref-

erence for the direction of this vehicle is unknown, but it results from using information about external circumstances. Those conditions for control problems are typical, for example, when the vehicle has to approximate to a second vehicle, which moves independently.

To keep the calculations plain, we restrict to a simple model of the moving device. Using the general methods for mathematical model creation, described in [11, 12], we generate a system with the structure shown in Fig. 1. The following parameters correspond to a small high-speed submersible. The TF of the motion for the vehicle body is described by the TF

$$F(s) = \frac{8.727}{s + 1.60}. \quad (36)$$

The TF of the remaining continuous parts of the system are given by

$$\begin{aligned} W(s) &= \frac{2}{s + 2}, & L(s) &= \frac{1}{0.2s + 1}, \\ K(s) &= \frac{1}{0.1s + 1}, & M(s) &= \frac{1}{0.001s + 1}. \end{aligned} \quad (37)$$

Moreover, we assume for the forming filter the TF $Q(s) = 1$ defining the class of permitted excitations. Let us choose the (non-pathological) sampling period $T = 0.2$ s. Finally we assume for the computation delay $\tau = 0.04$ s.

Following the above derived method, and applying a genetic algorithm as numerical optimisation method, for $\rho = 10$, we obtain a controller with the TF

$$C(z) = \frac{0.3729z^4 - 0.8496z^3 + 0.6408z^2 - 0.1601z}{z^4 - 2.0996z^3 + 1.2585z^2 - 0.1585z - 0.0003}. \quad (38)$$

Here the TF is written in dependence on the argument $z = \zeta^{-1}$.

Investigate the motion of vehicle model (36) equipped with controller (38) under the condition that the value of its direction of motion currently changes as defined by the location of another vehicle (further on called vehicle $\mathcal{T}$), which moves independently. The manoeuvre ends, when our vehicle bumps $\mathcal{T}$.

Fig. 2 shows the desired (dashed line) and the actual (solid line) of the variable motion direction of vehicle (36) while orienting at vehicle $\mathcal{T}$, which moves rather close to it. The corresponding reference is called type 1 of excitation.

We realise from Fig. 2, that for the assumed excitation the system assures adequate performance for the approximation of the vehicle to the demanded direction.

Now let us consider the variation of the direction of motion after changing the character of the excitation (type 2), which correlates to an approach of the vehicle to the vehicle $\mathcal{T}$, operating in considerable distance. The simulation results are presented in Fig. 3.

Fig. 2 Variation of motion direction of the vehicle under excitation of type 1

Fig. 3 Variation of motion direction of the vehicle under excitation of type 2

It is evident from Fig. 3, that for the changed character of the excitation, the system achieves a rather good performance of the approach to the desired direction. For the considered problem, a similar picture arises, even for a different type of excitation.

Thus, we are hopeful to assume, that the control system of vehicle (36) and controller (38) guarantees principal performance of the transient processes, and it will successfully operate for a wide region and various types of excitations.

6 Conclusions

A method for numerical optimisation of sampled-data control systems with delay has been considered, where an estimate of the $\mathcal{L}_2$ norm of the system has to be minimised over the set of stabilising causal controllers, and the controller order is limited. The optimisation criterion is calculated with the help of the Laplace transformation in continuous time. For the determination of the Laplace transform an approach is used, that bases on a relation between the Laplace transform and the parametric transfer function, and it allows to determine the Laplace transform without transition to state space.

The application of the method has been illustrated on a practical motivated example—the optimisation of a control system for the direction of motion control of an underwater vehicle under the condition of currently changing excitation.

References

1. Gil' MI (2013) Stability of vector differential delay equations. Frontiers in mathematics. Birkhäuser, Basel
2. Insperger T, Stépán G (2011) Semi-discretization for time-delay systems—stability and engineering applications. Applied Mathematical Sciences. Springer, New York
3. Michiels W, Niculescu S-I (2007) Stability and stabilization of time-delay systems—an Eigenvalue-based approach. Advances in design and control. SIAM, Philadelphia, PA
4. Chen T, Francis BA (1995) Optimal sampled-data control systems. Springer-Verlag, Berlin
5. Rosenwasser EN, Lampe BP (2000) Computer controlled systems—analysis and design with process-orientated models. Springer-Verlag, London
6. Rosenwasser EN, Lampe BP (2006) Multivariable computer-controlled systems—a transfer function approach. Springer, London
7. Lampe BP, Rosenwasser EN (2014) $\mathcal{L}_2$ -optimization and fixed poles of multivariable sampled-data systems with delay. Int J Control 1–35
8. Isermann R (1987) Digitale Regelungssysteme. Band I: Grundlagen, deterministische Regelungen. Band II: Stochastische Regelungen, Mehrgrößenregelungen Adaptive Regelungen, Anwendungen, 2nd ed. Springer-Verlag, Berlin
9. Rybinskii VO (2013) Selecting a sampling period for a sampled-data submarine motion control system. Gyroscopy Navig 4(1):57–63
10. Lampe BP, Polyakov KY, Rosenwasser EN, Rybinskii VO, Shilovskii A (2009) Control with guaranteed precision in time-delayed sampled-data systems. In: Proceeding of 3rd MSC: IEEE multi-conference on systems and control, St. Petersburg, Russia, pp 1–6
11. Diomidov MN, Dmitriev AN (1966) Underwater devices (design and construction). Sudostroyeniye, Leningrad (in Russian)
12. Fossen TI (2011) Handbook of marine craft hydrodynamics and motion control. Wiley, Chichester