Radial-basis-functions neural network sliding mode control for underactuated mechanical systems

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Abstract This paper presents an indirect adaptive neural network sliding mode Control (IANSMC) technique and a neural network sliding mode control (NNSMC) for underactuated robot manipulators. The adaptive neural network (NN) based on radial basis functions (RBF) is used to estimate the equivalent control and to compensate model uncertainties. In IANSMC, the adaptive learning algorithms are derived using Lyapunov stability analysis. Sliding mode control and indirect adaptive technique are combined to deal with modeling parameter uncertainties and bounded disturbances. The stability of the mixed controller is then proved. NN parameters are tuned on-line, without an off-line learning phase. For the NNSMC, the NN control is used to learn the equivalent control due to the unknown nonlinear system dynamics and the robust sliding mode control (SMC) is designed for a trajectory tracking control. Simulation results show that the NNSMC and IANSMC are better than the classical SMC to control underactuated manipulators. Although the proposed controllers can eliminate the chattering phenomena and estimate matching uncertainties. The IANSMC can also reject mismatched perturbations. Discussions and comparisons between proposed controllers are presented.

Keywords Neural networks · Sliding mode control · Lyapunov stability · Indirect adaptive · Underactuated manipulator

1 Introduction

Underactuated mechanical systems (UMS) are increasingly present in the field of robotics. They have less actuators than degrees of freedom. In these systems we find: manipulators, vehicles and humanoids with some passive joints. The underactuation arises by deliberate design for the purpose of reducing the weight of the manipulator or might be caused by actuator failure [1].

The difficulty in controlling underactuated mechanisms is due to the fact that techniques developed for fully actuated systems cannot be directly used. These systems are not feedback linearizable, yet they exhibit nonholonomic constraints and nonminimum phase characteristics. Because of this, the class of underactuated mechanical systems present challenging control problems. It is moreover shown that this class of systems can not be stabilized by means of continuous control algorithms. Sliding mode control (SMC) approaches are one of the most common techniques used to control UMS. The effectiveness of the SMC for nonlinear systems in general has been clearly proved by the control engineering community in virtue of its robustness against external perturbations and model parameters variations [2, 3]. However, in practice such an approach results in a undesirable chattering phenomena, which may excite high-frequency dynamics. Moreover, specifications of the system modeling approximations, as well as the boundaries of model uncertainties and unknown disturbances are usually difficult tasks in most situations. To avoid chattering effects, the boundary layer of the switching surface can be reduced. However this approach does not ensure

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the convergence of the state trajectories of the system on the sliding surface. Moreover, the dynamic analysis within the boundary layer is complicated [4]. On the other hand, the design of sliding-mode controllers needs the knowledge of the mathematical model of the plant, which decreases the performance in some applications where the mathematical modeling of the system is not well defined or when the knowledge of the parameters is incomplete. To overcome this problem, several researchers have used soft computing methodologies like fuzzy logic [5] or artificial neural networks (ANN) [6–9]. The combination between the intelligent control algorithms and the sliding mode approach attracted the attention of many researchers [8, 10, 11]. The idea of combining robust control and NN methods is used as a way to improve the performance and robustness to model imprecisions of the control system.

The purpose of the present study is to design an indirect adaptive neural network sliding mode Control (IANSMC) and a neural network sliding mode control (NNSMC) for underactuated systems. In the first approach, an RBF neural network (NN) control is proposed to compensate the effect of uncertainties and parameter variations with an on-line adaptation of the proposed controller. The adaptive law is employed to on-line adjust the weights of neural, algorithm is derived using a suitable Lyapunov equation. Then the stability of the control system can be guaranteed. In the second approach the NN control is used to learn the equivalent control due to the unknown nonlinear system dynamics and the robust control SMC is designed for trajectory tracking. These proposed controllers are used to control the UMS cart-pendulum prototype.

This paper is organized as follows: we firstly introduce in Sect. 2 the system dynamical model. In Sect. 3, we describe the SMC. Then, an IANSMC is proposed later in Sect. 4. In Sect. 5 the NNSMC is presented. Finally in Sect. 6, simulation results are presented and discussed.

2 System dynamical model

The cart-pendulum system is a typical benchmark of nonlinear and UMS [12]. For this system, the control input is the force u that moves the cart horizontally, the outputs are the angular position of the pendulum θ and the linear position of the cart. However, despite its simple mechanical structure, this prototype is not easy to control and requires sufficiently sophisticated control designs. Indeed it is proven that the system is not feedback linearizable and has no constant relative degree [13]. Moreover, Zhao and Spong [14] have shown that many geometric properties of the system are lost when the pendulum moves through horizontal positions. The principal control task considered for the cart pendulum is to swing up the pendulum from the stable equilibrium point to the unstable equilibrium point, and to stabilize the

Fig. 1 Cart-pendulum system

cart in a desired position. Numerous control techniques have been developed to stabilize the inverted Pendulum such as; proportional-integral-derivative (PID) controller where the parameters control gains are adjustable and updated online with a stable adaptation mechanism [15]. Fantoni et al. [12] proposed an energy approach. Using this control the system can be stabilized around its homoclinic orbit. Olfati [1] transformed the system in cascade form, then a fixed point backstepping procedures has been developed for global and semi-global stabilization. A hybrid controller has been presented by Spong [14], which globally asymptotically stabilizes the system for all initial conditions. Backstepping procedure [16], SMC [17, 18], NN [19, 20], fuzzy control [21] are also used to control this system. The dynamical behavior of the inverted pendulum can be described by [12]:

$$\begin{aligned} (m + M)\ddot{y} + mL(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) &= u \\ \ddot{y} \cos \theta + L\ddot{\theta} - g \sin \theta &= 0 \end{aligned} \quad (1)$$

where L is the length of the pendulum, m is the pendulum mass, M is the cart mass, u is the horizontal force action, θ is the angular deviation, y is the horizontal displacement of the cart (Fig. 1).

Letting $x_1 = y$, $x_2 = \dot{y}$, $x_3 = \theta$, $x_4 = \dot{\theta}$, we can transform these equation into the following state space canonical representation:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1 + b_1 u \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2 + b_2 u \end{aligned} \quad (2)$$

where $X = [x_1, x_2, x_3, x_4]^T$ is the state variable vector and u is the control input. f_1 , f_2 , b_1 and b_2 are nonlinear functions defined by:

$$\begin{aligned} f_1 &= \frac{mLx_4 \sin x_3 - mg \sin x_3 \cos x_3}{M + m \sin^2 x_3}; \\ f_2 &= \frac{(m + M)g \sin x_3 - mLx_4^2 \cos x_3}{L(M + m \sin^2 x_3)} \\ b_1 &= \frac{1}{M + m \sin^2 x_3}; b_2 = \frac{-\cos x_3}{L(M + m \sin^2 x_3)} \end{aligned} \quad (3)$$

3 Sliding mode control

We can transform the system into two subsystems with state variable groups (x_1, x_2) and (x_3, x_4) for which we construct the following linear functions as sliding surfaces. Those functions are defined to be the first level sliding mode and are expressed by

$$\begin{aligned} s_1 &= c_1 e_1 + e_2 \\ s_2 &= c_2 e_3 + e_4 \\ e &= \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} x_1 - x_{1d} \\ x_2 - x_{2d} \\ x_3 - x_{3d} \\ x_4 - x_{4d} \end{bmatrix} \end{aligned} \quad (4)$$

c_1 and c_2 are some real positive design parameters and $x_{1d}, x_{2d}, x_{3d}, x_{4d}$ are the desired values. Differentiating (4) with respect to time and setting $\dot{s}_1 = 0$ and $\dot{s}_2 = 0$ gives the equivalent control laws as:

$$u_{eq1} = -\frac{c_1 \dot{e}_1 + f_1}{b_1}; \quad u_{eq2} = -\frac{c_2 \dot{e}_3 + f_2}{b_2} \quad (5)$$

The choice of the sliding surface influences to the behavior of the control performance. It can minimize chattering, improving the tracking speed and the error convergence. In this paper a second-level sliding surface S is defined as a linear combination of first level sliding surfaces s_1 and s_2 . In addition, all system states are attracted to this surface and the switching control is determined based on stability of S by deriving this surface with respect to time.

For the whole system (2) we define the second-level sliding surface as:

$$S = \alpha_1 s_1 + \alpha_2 s_2 \quad (6)$$

where α_1 and α_2 are the sliding control parameters.

The control of the system can be written as:

$$u = u_{eq} + u_{sw} \quad (7)$$

where the equivalent control u_{eq} is designed in such a way that we have $\dot{S} \leq 0$, hence:

$$u_{eq} = \frac{\alpha_1 b_1 u_{eq1} + \alpha_2 b_2 u_{eq2}}{\alpha_1 b_1 + \alpha_2 b_2} \quad (8)$$

In the following we use the Lyapunov stability theory to find the switching control law u_{sw} .

Choosing the Lyapunov candidate function as:

$$V = \frac{1}{2} S^T S \quad (9)$$

the convergence of the sliding mode requires that

$$\dot{V} = S \dot{S} \leq -\eta |S| \quad (10)$$

where η is a positive constant. Computing $\dot{V}$

$$\begin{aligned} \dot{V} &= S \dot{S} = S(\alpha_1 \dot{s}_1 + \alpha_2 \dot{s}_2) \\ &= S(\alpha_1 (c_1 \dot{e}_1 + \dot{e}_2) + \alpha_2 (c_2 \dot{e}_3 + \dot{e}_4)) \\ &= S(\alpha_1 (c_1 \dot{e}_1 + f_1 + b_1 u) + \alpha_2 (c_2 \dot{e}_3 + f_2 + b_2 u)) \end{aligned} \quad (11)$$

Substituting (7), (8) in (11) gives:

$$\begin{aligned} \dot{V} &= S[\alpha_1 (c_1 \dot{e}_1 + f_1 + b_1 (u_{eq} + u_{sw})) \\ &\quad + \alpha_2 (c_2 \dot{e}_3 + f_2 + b_2 (u_{eq} + u_{sw}))] \\ &= S[u_{sw}(\alpha_1 b_1 + \alpha_2 b_2)] \leq -\eta |S| \end{aligned} \quad (12)$$

then:

$$u_{sw}(\alpha_1 b_1 + \alpha_2 b_2) \leq -\eta \text{sign}(S)$$

so u_{sw} can be defined by the following term

$$u_{sw} = -\frac{\eta \text{sign}(S)}{\alpha_1 b_1 + \alpha_2 b_2}$$

Consequently the control law (7) becomes

$$u = \frac{\alpha_1 b_1 u_{eq1} + \alpha_2 b_2 u_{eq2}}{\alpha_1 b_1 + \alpha_2 b_2} - \frac{\eta \text{sign}(S)}{\alpha_1 b_1 + \alpha_2 b_2} \quad (13)$$

4 Indirect adaptive technique neural network sliding mode control (IANSMC)

Consider now the perturbed model of system (2) defined as :

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1 + b_1 u + d_1 + \Delta f_{u1} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2 + b_2 u + d_2 + \Delta f_{u2} \end{aligned} \quad (14)$$

where d_1, d_2 are the matched uncertainties, $\Delta f_{u1}, \Delta f_{u2}$ the unmatched uncertainties [11], f_1, f_2, b_1 and b_2 are expressed by:

$$\begin{aligned} f_1 &= f_{10} + \Delta f_1; \quad f_2 = f_{20} + \Delta f_2; \\ b_1 &= b_{10} + \Delta b_1; \quad b_2 = b_{20} + \Delta b_2 \end{aligned}$$

where $\Delta f_1, \Delta f_2, \Delta b_1, \Delta b_2$ denote the uncertain parts, $f_{10}, f_{20}, b_{10}, b_{20}$ are nominal values of f_1, f_2, b_1 and b_2 expressed in Eq. (3).

4.1 Model description of NN

Neural networks have the capacity to approximate nonlinear functions and to learn through examples of the system in control engineering [6]. They can be used to approximate the optimal controller which is designed based on the assumption that all dynamics of the system are known. Then a novel adaptive neural controller is developed to compensate the plant uncertainties, smooth the control signal and increase the system robustness. Radial basis function neural networks (RBFNN) have been used in this control structure to compensate the uncertainties in the design of the SMC. An RBFNN has an input layer, a hidden layer and an output layer. Neurons in the hidden layer have gaussian transfer functions for which outputs are inversely proportional to the distance from the center of the neuron. Compared to the standard feedforward backpropagation networks, RBFNNs have the advantage of faster learning speed and less chance of falling into local minimal convergence.

The Gaussian-type function can be expressed as:

$$\varphi_j(s) = \exp \left[-\frac{(s - m_j)^2}{\sigma_j^2} \right], \quad j = 1, 2, \dots, k \quad (15)$$

s , is used as the input signal for a RBF NN, k is the number of nodes in the hidden layer, m_j and σ_j are the center and the variance of the j th basis function. In this section, the IANSMC algorithm for nonlinear underactuated systems is described, the adapting rule is derived, and the stability analysis is discussed. In the indirect case, two approximations are used to estimate the dynamics of the controlled system, which will be used in the control law [8]. An arbitrary choice of the initial value of adjustable parameters cannot always ensure the convergence of the adaptive algorithm due to the singularity problem. Differentiating (9) with respect to time, yields:

$$\begin{aligned} \dot{V} &= S[\alpha_1(c_1 e_2 + f_{10} + \Delta f_1 + (b_{10} + \Delta b_1)u + d_1 + \Delta f_{u1}) \\ &\quad + \alpha_2(c_2 e_4 + f_{20} + \Delta f_2 + (b_2 + \Delta b_{20})u + d_2 + \Delta f_{u2})] \\ &= S[\alpha_1(c_1 e_2 + f_{10} + b_{10}u) \\ &\quad + \alpha_2(c_2 e_4 + f_{20} + b_{20}u) + \Psi] \end{aligned} \quad (16)$$

where

$$\Psi = \alpha_1(\Delta f_1 + \Delta b_1 u + d_1 + \Delta f_{u1}) + \alpha_2(\Delta f_2 + \Delta b_2 u + d_2 + \Delta f_{u2}) \quad (17)$$

An RBFNN will be presented to estimate and compensate the effect of uncertainties and $\bar{\Psi}$ is the output of NN estimating Ψ [20].

The proposed control u is taken as:

$$u = \frac{\alpha_1 b_{10} u_{eq1} + \alpha_2 b_{20} u_{eq2}}{\alpha_1 b_1 + \alpha_2 b_2} - \frac{\bar{\Psi} + \eta \text{sign}(S)}{\alpha_1 b_1 + \alpha_2 b_2} \quad (18)$$

where η is a positive constant.

Moreover, an adaptive adjusting law for parameters will be designed. As a second approach, an estimation of system parameters can be done.

The output of the linear layer RBF NN is:

$$\bar{\Psi} = \bar{\Psi}(s, m, \sigma) = \sum_{j=1}^k W_j \varphi_j(s) = W^T \varphi(s) \quad (19)$$

where:

$$W = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_k \end{bmatrix}; \quad \varphi(s) = \begin{bmatrix} \varphi_1(s) \\ \varphi_2(s) \\ \vdots \\ \varphi_k(s) \end{bmatrix}; \quad m = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_k \end{bmatrix}; \quad \sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_k \end{bmatrix}$$

The sliding variable s is an input signal to the RBF neural network, W is the output weight vector of the RBFNN.

An optimal NN controller will be designed to estimate the uncertainties (17) such that:

$$\Psi = \hat{\Psi} + \varepsilon_1 = \hat{W}^T \hat{\varphi}(s) + \varepsilon_1 \quad (20)$$

where ε_1 is a reconstructed error vector. $\hat{\Psi}$ is an optimal output of NN, function $\hat{W}$ and $\hat{\varphi}$ corresponds to optimal parameters of the NN.

The expression of the control law becomes:

$$u = u_{eq} + u_{sw} + u_{NN} \quad (21)$$

where

$$\begin{aligned} u_{NN} &= \frac{-\bar{\Psi}}{\alpha_1 b_1 + \alpha_2 b_2}; \quad u_{sw} = \frac{-\eta \text{sign}(S)}{\alpha_1 b_1 + \alpha_2 b_2}; \\ u_{eq} &= \frac{\alpha_1 b_1 u_{eq1} + \alpha_2 b_2 u_{eq2}}{\alpha_1 b_1 + \alpha_2 b_2} \end{aligned} \quad (22)$$

with $\bar{\Psi} \in \mathbb{R}$. Substituting (21) in (16) gives:

$$\begin{aligned} \dot{V} &= S[\Psi - \bar{\Psi} - \eta \text{sign}(S)] \\ &= -\eta |S| + o(\Delta W, \Delta m, \Delta \sigma) \end{aligned} \quad (23)$$

where $\Delta W = \tilde{W} = \hat{W} - W$, $\Delta m = \tilde{m} = \hat{m} - m$, $\Delta \sigma = \tilde{\sigma} = \hat{\sigma} - \sigma$

4.2 Adaptation law

In this part, a learning algorithm for tuning parameters W , m and σ is given. The linearization technique is used to transform the nonlinear basis function into a partially linear form of the active function [9]. Differentiating (19) yields:

$$\begin{aligned} \Delta \bar{\Psi}_j(s) &= \Delta W_j \varphi_j(s) + W_j \frac{\partial \varphi_j}{\partial m_j} \Delta m_j \\ &\quad + W_j \frac{\partial \varphi_j}{\partial \sigma_j} \Delta \sigma_j + \varepsilon_{2j} \end{aligned} \quad (24)$$

where $\bar{\Psi}_j(s) = W_j \Psi_j(s)$ and $\bar{\Psi}(s) = \sum_{j=1}^k \bar{\Psi}_j(s)$. Then $\Delta \bar{\Psi}(s) = \sum_{j=1}^k \Delta \bar{\Psi}_j(s)$ and $\varepsilon_2 = \sum_{j=1}^k \varepsilon_{2j} \in \mathbb{R}$ denotes

the error vector due to the neglect of high-order terms of the Taylor expansion of $\bar{\Psi}(s)$ with a maximum bound given by $|\varepsilon_2| \leq \varepsilon_{2max}$.

where $\frac{\partial \varphi_j}{\partial m_j}$ and $\frac{\partial \varphi_j}{\partial \sigma_j}$ are given by:

$$\begin{aligned} \varphi'_m = \varphi'^T_m &= \begin{bmatrix} \frac{\partial \varphi_1}{\partial m_1} & 0 & 0 & \dots & 0 \\ 0 & \frac{\partial \varphi_2}{\partial m_2} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \frac{\partial \varphi_k}{\partial m_k} \end{bmatrix} \in \mathbb{R}^{k \times k} \\ \varphi'_\sigma = \varphi'^T_\sigma &= \begin{bmatrix} \frac{\partial \varphi_1}{\partial \sigma_1} & 0 & 0 & \dots & 0 \\ 0 & \frac{\partial \varphi_2}{\partial \sigma_2} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \frac{\partial \varphi_k}{\partial \sigma_k} \end{bmatrix} \in \mathbb{R}^{k \times k} \end{aligned} \quad (25)$$

The difference between the system uncertainties and the NN output is expressed as:

$$\tilde{\Psi} = \Psi - \bar{\Psi} = \hat{\Psi} + \varepsilon_1 - \bar{\Psi} \quad (26)$$

Then (26) can be expressed as:

$$\tilde{\Psi} = \Delta \bar{\Psi} + \varepsilon_1 \quad (27)$$

with $\Delta \bar{\Psi} = \hat{\Psi} - \bar{\Psi}$.

Equations (27) and (24) give:

$$\tilde{\Psi}(s) = \tilde{W}^T \varphi(s) + W^T \varphi'_m(\tilde{m}) + W^T \varphi'_\sigma(\tilde{\sigma}) + \varepsilon_1 + \varepsilon_2 \quad (28)$$

where the uncertain term $\varepsilon_1 + \varepsilon_2$ is assumed to be bounded by $2|\varepsilon_1 + \varepsilon_2| < \eta$. The adaption laws of the NN controller are designed using the following equations:

$$\begin{cases} \dot{\tilde{W}} = -\dot{W} = \eta_1 S \varphi(s) \\ \dot{\tilde{m}} = -\dot{m} = \eta_2 S \varphi'_m W \\ \dot{\tilde{\sigma}} = -\dot{\sigma} = \eta_3 S \varphi'_\sigma W \end{cases} \Rightarrow \begin{cases} \dot{\tilde{W}}_j = -\dot{W}_j = \eta_1 S \varphi_j(s) \\ \dot{\tilde{m}}_j = -\dot{m}_j = \eta_2 S \frac{\partial \varphi_j}{\partial m_j} W_j \\ \dot{\tilde{\sigma}}_j = -\dot{\sigma}_j = \eta_3 S \frac{\partial \varphi_j}{\partial \sigma_j} W_j \end{cases} \quad (29)$$

where η_i are some positive constants, $i = \{1, 2, 3\}$ and for $j = \{1, \dots, k\}$.

4.3 Stability proof

Consider the Lyapunov function candidate:

$$V = \frac{1}{2} S^2 + \frac{1}{2\eta_1} \tilde{W}^T \tilde{W} + \frac{1}{2\eta_2} \tilde{m}^T \tilde{m} + \frac{1}{2\eta_3} \tilde{\sigma}^T \tilde{\sigma} \quad (30)$$

Fig. 2 Structure of IANSMC

Differentiating (30) with respect to time, leads to:

$$\begin{aligned} \dot{V} &= S\dot{S} + \frac{1}{\eta_1} \tilde{W}^T \dot{\tilde{W}} + \frac{1}{\eta_2} \tilde{m}^T \dot{\tilde{m}} + \frac{1}{\eta_3} \tilde{\sigma}^T \dot{\tilde{\sigma}} \\ &= -\eta|S| + S\tilde{\Psi} + \frac{1}{\eta_1} \tilde{W}^T \dot{\tilde{W}} + \frac{1}{\eta_2} \tilde{m}^T \dot{\tilde{m}} + \frac{1}{\eta_3} \tilde{\sigma}^T \dot{\tilde{\sigma}} \\ &= -\eta|S| + S \left[\tilde{W}^T \varphi(s) + W^T \varphi'_m(\tilde{m}) + W^T \varphi'_\sigma(\tilde{\sigma}) + \varepsilon_1 + \varepsilon_2 \right] \\ &\quad - \frac{1}{\eta_1} \tilde{W}^T \dot{W} - \frac{1}{\eta_2} \tilde{m}^T \dot{m} - \frac{1}{\eta_3} \tilde{\sigma}^T \dot{\sigma} \\ &= \tilde{W}^T \left(\varphi(s)S - \frac{1}{\eta_1} \dot{W} \right) + \tilde{m}^T \left(\varphi'_m(\tilde{m})WS - \frac{1}{\eta_2} \dot{m} \right) \\ &\quad + \tilde{\sigma}^T \left(\varphi'_\sigma(\tilde{\sigma})WS - \frac{1}{\eta_3} \dot{\sigma} \right) - \eta|S| + 2S(\varepsilon_1 + \varepsilon_2) \\ &= -\eta|S| + 2S(\varepsilon_1 + \varepsilon_2) \leq -\eta|S| + 2|\varepsilon_1 + \varepsilon_2||S| \\ &= -(\eta - 2|\varepsilon_1 + \varepsilon_2|)|S| \leq 0 \end{aligned} \quad (31)$$

$\dot{V}$ becomes negative if $2|\varepsilon_1 + \varepsilon_2| < \eta$. Thus the IANSMC control system is stable.

In summary, an indirect RBF SMC law is developed in (21) with parameters W , m and σ adjusted by (29). The objective is to build a RBF NN control in order to compensate the effect of uncertainties and the parameter variations, with an on-line adaptation of the proposed controller. The adaptive NN SMC is stable in the sense of Lyapunov. The representation of the proposed IANSMC is shown in Fig. 2.

5 NN sliding mode controller (NNSMC)

5.1 Control principle

If the knowledge of f and b is very poor, then the computed equivalent control will be too far from the actual equivalent control. The model used is an estimate of the real system. To solve this problem, a NN is proposed to compute the equivalent control to control the underactuated manipulator. A RBFNN is employed to model the relation between the system state vector, x , and the equivalent control, u_{eq} . The model of the con-

Fig. 3 Disposition of neurons

Fig. 4 Evolution of the position error y without perturbation

Fig. 5 Evolution of the angle error θ without perturbation

trol is presented in Fig. 3. The Gaussian function is used as the activation function of each neuron in the hidden layer.

Fig. 6 Evolution of the position error y with perturbations

Fig. 7 Evolution of the angle error θ with perturbations

5.2 NN sliding mode controller

Sliding surfaces are defined as (4) for system (2). The second-level sliding surface is defined by (6) and the equivalent SMC can be chosen as:

$$U_{eq} = k_{11}\dot{e}_1 + k_{12}\dot{e}_2 + k_{21}x_2 + k_{22} \quad (32)$$

where:

$$\begin{aligned} k_{11} &= \frac{-\alpha_1 c_1 l (M + m \sin^2 x_3)}{\alpha_1 l - \alpha_2 \cos x_3}; \\ k_{12} &= \frac{-\alpha_2 c_2 l (M + m \sin^2 x_3)}{\alpha_1 l - \alpha_2 \cos x_3}; \\ k_{21} &= \frac{\alpha_1 m l^2 \sin x_3 + \alpha_2 m g}{\alpha_1 l - \alpha_2 \cos x_3}; \\ k_{22} &= \frac{\alpha_1 m l g \sin x_3 \cos x_3 - \alpha_2 (M + m) g \sin x_3}{\alpha_1 l - \alpha_2 \cos x_3} \end{aligned} \quad (33)$$

In this case the switching control is defined by:

$$u_{sw} = -K \text{sign}(S) \quad (34)$$

Using the neural control, the expression of U_{eq} becomes:

$$U_{eq} = U_{eqNN} + o(\Delta f, \Delta b) \quad (35)$$

where $\Delta f = \Delta f_1 + \Delta f_2$ and $\Delta b = \Delta b_1 + \Delta b_2$.

Fig. 8 Evolution of the position error y with unmatched and matched perturbations (IANSMC)

Fig. 9 Evolution of the angle error θ with unmatched and matched perturbations (IANSMC)

Fig. 10 The simulated torque u without perturbation (SMC) using saturation function

The total control is expressed by:

$$u = U_{eqNN} + u_{sw} \quad (36)$$

Taking into account the Lyapunov function in (7), we have:

$$\begin{aligned} \dot{V} &= S(b_1\alpha_1 + b_2\alpha_2)(-K \text{sign}(S) + o(\Delta f, \Delta b)) \\ &= -K|S|(b_1\alpha_1 + b_2\alpha_2) + o(\Delta f, \Delta b) \end{aligned} \quad (37)$$

Fig. 11 The simulated torque u without perturbation (IANSMC)

Based on Lyapunov stability theorem, if $V > 0$ and $\dot{V} \leq 0$, the stability of the system can be proved. Let $K > 0$ and to guarantee that $\dot{V}$ is negative $-K|S|(b_1\alpha_1 + b_2\alpha_2)$ should be ≤ 0 , so $\alpha_1 > -\frac{b_2\alpha_2}{b_1}$ then $\alpha_1 > \alpha_2 \frac{\cos x_3}{l^2}$. For the simulations, we choose $\alpha_1 = 2/l^2$ and $\alpha_2 = 1$.

6 Simulation results and discussion

The effectiveness of the proposed approaches is evaluated on a 2 degree of freedom underactuated manipulator. For the simulation the following parameters are considered: $M_0 = 20$ kg, $m_0 = 1.8$ kg, $L_0 = 0.3$ m.

The initial conditions are considered to be $(y_0, \dot{y}_0) = (0, 0)$, $(\theta_0, \dot{\theta}_0) = (0.1, 0)$ and the expectations are $y_d = 2$, $\theta_d = 0$ and $\dot{y}_d = \dot{\theta}_d = 0$.

Figures 4 and 5 show the simulation results for each of the three controllers (SMC, IANSMC, NNSMC) when 20 % of mass uncertainty is considered for the pendulum and cart in the dynamics of the system and without perturbations. We can note that the convergence is obtained for all the system states. Figures 6, 7, 12 and 14 show the system response when matched disturbances are considered ($d_1 = 0.2$ and $d_2 = 0.7 \sin \frac{\pi t}{12}$). Unlike the SMC technique, the two controllers IANSMC and NNSMC are robust against these disturbances with slight advantage to the IANSMC.

Figures 8 and 9 present the simulations results for the IANSMC technique when the system is subjected to matched and an unmatched uncertainly defined by: $\Delta f_{u1}(x) = 0.22x_1 \cos x_1$ and $\Delta f_{u2}(x) = 1.2x_3 \cos x_2$. All the considered disturbances are rejected by the designed controller.

As shown in Fig. 10 the phenomena of chattering was solved using a function saturation in SMC. Figures 11, 12 and 13 show the simulation results of the control effort for

Fig. 12 The simulated torque u with perturbation (IANSMC)

Fig. 13 Learning the equivalent control

Fig. 14 The simulated torque u NNSMC with perturbation

both the IANSMC and the NNSMC. It is clear that no chattering phenomenon is present. We remark from Fig. 14 a good learning of equivalent control u_{eq} determined from NN. The curves u_{eq} and u_{eqNN} are almost identical.

7 Conclusion

In this paper, an IANSMC and NNSMC control techniques have been designed for underactuated manipulators. An RBF

NN is used to approximate the unknown uncertainties part. An adaptive learning algorithm is constructed based on Lyapunov stability analysis where the weights of the output layer of RBF and the parameters of the basis function are updated on-line. The proposed controllers guarantee stability and convergence of the system position. Simulations have been carried out to evaluate the performance of the proposed controllers. Simulation results show that the proposed IANSMC is able to effectively compensate the parameter uncertainties of the underactuated manipulator better than NNSMC approach and the SMC.

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