Efficiency analysis of two-period asymmetric gaits

Yuji Harata · Koji Iwano · Fumihiko Asano ·
Takashi Ikeda

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Abstract This paper investigates the properties and optimality of asymmetric passive-dynamic gaits of a rimless wheel and a compass-like biped. Two types of asymmetric gaits are discussed; one is based on two different restored mechanical energies and the other is based on two different strides. The relation between the gait asymmetry and the gait efficiency in terms of walking speed is analyzed through numerical simulations. It is shown that the constraint on the stride is effective for achieving high-speed passive-dynamic walking.

Keywords Asymmetric gait · Limit cycle walk · Rimless wheel · Passive dynamic walk

1 Introduction

Limit cycle walkers inspired by passive dynamic walking (PDW) [1] generate the walking gait as a stationary orbit with impulsive effects or discontinuous state jumps. From a perspective of mechanical energy balance, a stationary walking motion can be divided into two phases; one is the stance phase for energy restoration and the other is the collision phase of heel strike that causes energy dissipation. Therefore

limit cycle walkers that walk on level ground must restore mechanical energy during the stance phases to continue walking. The principle is the simplest but the generated walking pattern often exhibits asymmetry or multiple-periodic motion even if the applied control law is fixed.

It is well known that passive-dynamic walkers often exhibit period-doubling bifurcation and chaotic behavior according to the system parameters. Goswami et al. first showed that a passive compass-gait biped exhibits period-doubling bifurcation with the increase of the slope angle [2]. After that, Osuka et al. experimentally confirmed this phenomenon [3]. Safa showed that the bifurcation occurs when a passive biped walks down a stairs [4]. Moon and Spong numerically showed that the difference between the right and the left legs' physical parameters causes bifurcation [5]. Garcia et al. performed an approximation analysis of the simplest walking model and showed that an unstable period-one gait exists in a period-two gait [6].

Several methods for controlling bifurcated limit-cycle gaits have been proposed [7–10]. Harata et al. [10] investigated the effect of bifurcation on the gait properties and showed that a bifurcation improves the walking speed of PDW. On the other hand, Asano showed that a symmetric gait is the most efficient in terms of walking speed through mathematical analysis of a passive rimless wheel that can be symmetrized [11]. His result implies that a period-doubling bifurcation has the potentiality of decreasing the gait efficiency.

Passive-dynamic and limit-cycle walkers walk with a long step and a short step alternately in a period-two bifurcated gait. Therefore, there are accordingly two different relations between the lost and the restored mechanical energies. The steady energy behavior in this case can be specified by two recurrence formulas of kinetic energy [11], and this technique is useful for understanding the gait efficiency and optimality.

Y. Harata (✉) · T. Ikeda
Hiroshima University, 1-4-1 Kagamiyama,
Higashi-Hiroshima, Hiroshima 739-8527, Japan
e-mail: harata@hiroshima-u.ac.jp

K. Iwano
Mitsubishi Electric Corporation, 5-1-14, Yadaminami,
Higashi-ku Nagoya-shi, Aichi, 461-8670, Japan

F. Asano
Japan Advanced Institute of Science and Technology,
1-1 Asahidai, Nomi, Ishikawa 923-1292, Japan
e-mail: fasano@jaist.ac.jp

The purpose of this paper is to clarify the effect of period-doubling bifurcation on the properties of passive-dynamic and limit-cycle walking gaits. We consider two types of asymmetric bifurcated gaits; one is generated according to two different restored mechanical energies and the other is generated according to two different strides created by using a mechanical stopper. We compare the efficiency of the generated asymmetric gaits with that of a symmetric gait in terms of walking speed.

This paper is organized as follows. Sect. 2 describes the model of a rimless wheel with knee joints. Sect. 3 introduces two methods for generating a period-two gait and discusses the gait efficiency mathematically and numerically. Sect. 4 explains the model of a passive compass-like biped on periodic stairs, and the efficiency of the period-one gait is compared with that of the period-two gait in Sect. 5. Finally, Sect. 6 concludes this paper and describes future research directions.

2 Asymmetric rimless wheel model

2.1 Equation of motion

Figure 1 illustrates the model of a rimless wheel (RW) with knees. The RW consists of lumped mass m at the center (hip) and eight massless legs in which the lower legs with length l_1 and the upper legs with length l_2 are connected on knee joints with angle γ_i . Hip joint angles between leg i and leg $i + 1$ are set as α_i . θ_1 and θ_2 are the inclination angles of the lower and the upper legs of support leg measured from vertical line, respectively. The equation of motion of the RW becomes

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \mathbf{S}u, \quad (1)$$

where $\boldsymbol{\theta} = [\theta_1, \theta_2]^T$ is the generalized coordinate vector, $\mathbf{M}(\boldsymbol{\theta}) \in \mathbb{R}^{2 \times 2}$ is the inertia matrix, $\mathbf{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \in \mathbb{R}^2$ includes

Fig. 1 Rimless wheel with knee joints

the Coriolis force, the centrifugal force and the gravity force, $\mathbf{S} = [1 \ -1]^T$, and u is the control input.

The inertia matrix is given by

$$\mathbf{M}(\boldsymbol{\theta}) = \begin{bmatrix} ml_1^2 & ml_1l_2 \cos(\theta_1 - \theta_2) \\ ml_1l_2 \cos(\theta_1 - \theta_2) & ml_2^2 \end{bmatrix}, \quad (2)$$

and the matrix is singular when a joint of the support leg is stretched to a straight posture. To avoid the singularity, the small angle, ϵ , is introduced, and the joint angle is restricted in $\theta_1 - \theta_2 \geq \epsilon$.

2.2 Impact equation at the heel strike

Figure 2 illustrates the configuration at the heel strike. It is assumed that the heel strike is a completely inelastic collision.

At the heel strike, the joint of the support leg is mechanically locked and the distance, L_i , between a support point, S, and the hip, H, is given by

$$L_i = \sqrt{l_1^2 + l_2^2 + 2l_1l_2 \cos \gamma_i}. \quad (3)$$

The absolute angle of a straight line connecting the points S and H is defined as θ_s , and the angle of a line between an impact point I and H is defined as θ_1 . Since the joint of the support leg is locked, $\dot{\theta}_s^- = \dot{\theta}_1^- = \dot{\theta}_2^-$ holds. In this paper, the superscripts “-” and “+” represent immediately before and immediately after the heel strike, respectively.

The support leg is exchanged after the heel strike. The angular positions of the new support leg are given by

$$\begin{aligned} \theta_3^+ &= \theta_2^- - \alpha_{i+1}, \\ \theta_4^+ &= \theta_2^- - \alpha_{i+1} + \gamma_{i+1}, \end{aligned} \quad (4)$$

At the heel strike, the joint of the swing leg is locked as $\theta_3 - \theta_4 = \gamma_{i+1}$, and hence, the angular velocities are given by $\dot{\theta}_1 = \dot{\theta}_3^+ = \dot{\theta}_4^+$.

Fig. 2 Posture at the heel strike

The angular velocity immediately after the heel strike is derived from the principle of conservation of angular momentum. The angular velocity immediately after the heel strike is given by

$$\dot{\theta}_1^+ = \frac{L_i \cos \bar{\alpha}_i}{L_{i+1}} \dot{\theta}_S^- := F_i(\bar{\alpha}_i) \dot{\theta}_S^-, \quad (5)$$

where $\bar{\alpha}_i$ is the angle between the lines HS and HI.

3 Asymmetric gait for rimless wheel

In a passive period-two gait, a biped robot walks with long and short steps, alternately. Energy restoration of the long step is larger than that of the short step, that is, more potential energy is transformed to the kinetic energy. On the other hand, short step dissipates smaller kinetic energy than long step. Therefore, in the period-two gait there are the differences of energy dissipation and energy restoration between long and short steps.

In this paper, the effects of the differences of the energy dissipation and the energy restoration are separated into the effect of the energy restoration and the effect of the energy dissipation. To do that, a gait on level ground, in which there is no transformation of the potential energy to the kinetic energy, is considered and two types of asymmetric gaits are investigated. For simplicity, the ballistic walking for the RW is introduced.

3.1 Ballistic walking for rimless wheel

In this paper, the energy restoration is achieved by the support leg kicking the ground at foot-off period. It is assumed that the support leg kicks the ground immediately after the heel strike, and the mechanical energy is restored instantaneously immediately after the support leg is exchanged.

The effect of the kick is calculated as follows. Integrating Eq. (1) during the interval between immediately before and immediately after kicking,

$$\int_{t^+}^{t^{++}} (\mathbf{M}(\theta) \ddot{\theta} + \mathbf{h}(\theta, \dot{\theta})) dt = \int_{t^+}^{t^{++}} \mathbf{S} u dt, \quad (6)$$

where the superscript “++” represents immediately after kicking the ground. It is assumed that the wheel can kick instantaneously, angles of the new support leg do not change when kicking, the Coriolis and the centrifugal force are finite during the kicking, and u is impulsive. Therefore, Eq. (6) is rewritten as

$$\mathbf{M}(\theta) \dot{\theta}^{++} - \mathbf{M}(\theta) \dot{\theta}^+ = \mathbf{S} \lambda_u, \lambda_u = \int_{t^+}^{t^{++}} u dt \quad (7)$$

The energy restoration by $\mathbf{S} \lambda_u$ is defined as ΔE_i . In our model, a joint of the support leg at kicking is locked, and then,

$$\frac{1}{2} m L_i^2 (\dot{\theta}_1^+)^2 + \Delta E_i = \frac{1}{2} m L_{i+1}^2 (\dot{\theta}_1^{++})^2, \quad (8)$$

and the angular velocity immediately after kicking can be solved as

$$\dot{\theta}_1^{++} = \sqrt{\frac{2}{m L_{i+1}^2} \left(\frac{1}{2} m L_{i+1}^2 (\dot{\theta}_1^+)^2 + \Delta E_i \right)}. \quad (9)$$

3.2 Asymmetry of energy restoration

3.2.1 Asymmetric gait

First, the case where the different energy restoration, ΔE_1 and ΔE_2 , are previously given is considered. The values of the parameters of the RW are set as the lower leg length $l_1 = 0.6$ m, the upper leg length $l_2 = 0.4$ m, and the hip mass $m = 1.0$ kg, the angles $\alpha_i = \pi/6$, $\gamma_i = 0.05$ rad, and the restored energies $\Delta E_1 = 0.2$ J and $\Delta E_2 = 0.3$ J. The distance, $L_i = L$ (const.), because the knee angle is constant.

Figure 3 illustrates the simulation results. Here, (a) is the angular position, (b) the mechanical energy, and (c) the phase

Fig. 3 Asymmetric gait with two different energy restoration

portrait in the $(\theta_1, \dot{\theta}_1)$ plane. It can be seen from Fig. 3 that a gait of the RW converges to period-two. In this case, the hip angle and the knee angle are constant, and hence, leg angles immediately before the heel strike are constant in Fig. 3(a). Fig. 3(b) demonstrates that the RW walks at the different energy levels because of the difference of energy restoration between ΔE_1 and ΔE_2 .

3.2.2 Comparison of the symmetric gait with the asymmetric gaits

In this subsection, the asymmetric gaits are compared with the symmetric gait. Since potential energy immediately before the heel strike equals to that immediately after energy restoration,

$$\frac{1}{2}mL^2(\dot{\theta}_S^-)^2 - \frac{1}{2}mL^2(\dot{\theta}_I^+)^2 = \Delta E_i. \quad (10)$$

Substituting Eq. (5) into Eq. (10),

$$\frac{1}{2}mL^2(1 - F^2)(\dot{\theta}_S^-)^2 = \Delta E_i. \quad (11)$$

Note that F is constant because hip angle α_i is constant.

When the two different values of energy restoration, ΔE_1 and ΔE_2 , are given, Eq. (11) is rewritten as

$$\begin{aligned} \frac{1}{2}mL^2((\dot{\theta}_{S1}^-)^2 - F^2(\dot{\theta}_{S2}^-)^2) &= \Delta E_1, \\ \frac{1}{2}mL^2((\dot{\theta}_{S2}^-)^2 - F^2(\dot{\theta}_{S1}^-)^2) &= \Delta E_2, \end{aligned} \quad (12)$$

where θ_{S1} and θ_{S2} are the angles of the lines between S and H for ΔE_1 and ΔE_2 , respectively. In the case of the symmetric gait, Eq. (11) is rewritten as

$$\frac{1}{2}mL^2(1 - F^2)(\dot{\theta}_a^-)^2 = \frac{\Delta E_1 + \Delta E_2}{2} = \Delta E_a, \quad (13)$$

where θ_a is θ_S in the case of the average energy restoration, ΔE_a . From Eq. (12), kinetic energies of the asymmetric gait immediately before the heel strike are given by

$$\begin{aligned} K_1^- &= \frac{1}{2}mL^2(\dot{\theta}_{S1}^-)^2 = \frac{\Delta E_1 + F^2\Delta E_2}{1 - F^4}, \\ K_2^- &= \frac{1}{2}mL^2(\dot{\theta}_{S2}^-)^2 = \frac{\Delta E_2 + F^2\Delta E_1}{1 - F^4}, \end{aligned} \quad (14)$$

and the mean value of K_1^- and K_2^- becomes

$$\begin{aligned} K_m^- &= \frac{K_1^- + K_2^-}{2} = \frac{(1 + F^2)(\Delta E_1 + \Delta E_2)}{2(1 - F^4)} \\ &= \frac{\Delta E_1 + \Delta E_2}{2(1 - F^2)}. \end{aligned} \quad (15)$$

Kinetic energy of a symmetric gait K_a immediately before the heel strike is given by

$$K_a^- = \frac{1}{2}mL^2(\dot{\theta}_a^-)^2 = \frac{\Delta E_1 + \Delta E_2}{2(1 - F^2)} = K_m^-. \quad (16)$$

As a result, the mean value of kinetic energies of the asymmetric gait equals to that of the symmetric gait.

Next, the mean value of the angular velocities of the asymmetric gait is compared with that of the symmetric gait. The angular velocities are given by

$$\dot{\theta}_a^- = \sqrt{\frac{1}{mL^2(1 - F^2)}(\Delta E_1 + \Delta E_2)}, \quad (17)$$

$$\begin{aligned} \dot{\theta}_m^- &= \frac{1}{2}(\dot{\theta}_{S1}^- + \dot{\theta}_{S2}^-) \\ &= \sqrt{\frac{1}{2mL^2(1 - F^4)}} \\ &\quad \times \left(\sqrt{F^2\Delta E_1 + \Delta E_2} + \sqrt{\Delta E_1 + F^2\Delta E_2} \right). \end{aligned} \quad (18)$$

By subtracting the squared values of Eqs. (17) and (18) after multiplying $A = \sqrt{2mL^2(1 - F^4)}$, we get

$$\begin{aligned} A^2(\dot{\theta}_a^-)^2 - A^2(\dot{\theta}_m^-)^2 &= (1 + F^2)(\Delta E_1 + \Delta E_2) \\ &\quad - 2\sqrt{(\Delta E_1 + F^2\Delta E_2)(\Delta E_2 + F^2\Delta E_1)}, \end{aligned} \quad (19)$$

and by subtracting the squared values of the first term of the right hand side of Eq. (19) from that of the second term,

$$\begin{aligned} (1 + F^2)^2(\Delta E_1 + \Delta E_2)^2 \\ - 4(\Delta E_1 + F^2\Delta E_2)(\Delta E_2 + F^2\Delta E_1) \\ = (1 - F^2)^2(\Delta E_1 - \Delta E_2)^2 \geq 0. \end{aligned} \quad (20)$$

Therefore, $\dot{\theta}_a^- \geq \dot{\theta}_m^-$.

The walking speed of the symmetric gait is also compared with those of the asymmetric gaits by the numerical simulation. The walking speed is defined as the stride length divided by the walking period. In the simulation, the data after the RW walks 50 steps are used. This is because a steady gait is difficult to obtain analytically, and hence, it is assumed that a gait converges to a stable gait when the RW walks 50 steps.

Here, the walking speed is calculated when the restored energy difference $\Delta E_1 - \Delta E_2$ increases. The values of the parameters are chosen as $\alpha_i = \pi/6$, and $\gamma_i = 0.05$ rad. Figure 4 shows the simulation result. In Fig. 4, the horizontal axis represents $\Delta E_1 - \Delta E_2$ and the vertical axis represents the walking speed. The simulation results for $\Delta E_a = (\Delta E_1 + \Delta E_2)/2 = 0.3, 0.4$ and 0.5 J are plotted by circles, squares and crosses, respectively. The walking speed increases as the restored energy becomes large, while the difference of the walking speed of the asymmetric gait decreases. It can be seen from Fig. 4 that the walking speed varies linearly, and the mean value of the walking speed varies

Fig. 4 Walking speed versus energy difference

little. Therefore, the existence of the asymmetry does not affect the walking speed significantly. Note that the symmetric gait corresponds to the result when $\Delta E_1 - \Delta E_2 = 0$.

3.3 Asymmetry of the stride

3.3.1 Influence of the hip angle

In this subsection, the case of a RW which has the two different hip angles, $\alpha_{2n-1} = \alpha_1$ and $\alpha_{2n} = \alpha_2$ is considered. The parameters are set as $\alpha_1 = 5\pi/36$, $\alpha_2 = 7\pi/36$, $\gamma_i = 0.05$ rad, and $\Delta E_i = 0.25$ J.

Figure 5 illustrates the simulation results. Here, (a) is the angular position, (b) the mechanical energy, and (c) the phase portrait in the $(\theta_1, \dot{\theta}_1)$ plane. It can be seen from Fig. 5(a) that the RW walks with long and short steps, alternately. Compared with Fig. 3, the difference between two gaits of Fig. 5 is larger than that of Fig. 3, while mechanical energy of Fig. 3 is higher than that of Fig. 5.

Next, the walking speed of the symmetric gait is compared with that of the asymmetric gait. Here, the walking speed is calculated when the hip angle difference $\alpha_2 - \alpha_1$ increases. The parameters are chosen as $\gamma_i = 0.05$ rad and $\Delta E_i = 0.35$ J. Figure 6 illustrates the simulation result. The results for

$\alpha_a = (\alpha_1 + \alpha_2)/2 = 7\pi/45$, $\pi/6$ and $8\pi/45$ are plotted by circles, crosses and squares, respectively. It can be seen from Fig. 6 that the average walking speed decreases with the increase of the difference of the hip angles.

3.3.2 Comparison between the symmetric gait and the asymmetric gaits

In this case, the distance, $L_i = L(\text{const.})$, and the energy restoration $\Delta E_i = \Delta E(\text{const.})$.

The relations between the kinetic energy and the potential energy are given by

$$\begin{aligned} P_1 + K_1^+ + \Delta E &= P_2 + K_2^-, \\ P_2 + K_2^+ + \Delta E &= P_1 + K_1^-, \end{aligned} \quad (21)$$

where P_i and K_i ($i = 1, 2$) represent the potential energy and kinetic energy when the hip angle is α_i , respectively. Because of the difference of the hip angles, there is the difference on F_i in Eq. (5), F_1 and F_2 . Equation (21) is rewritten as

$$\begin{aligned} P_1 + F_1^2 K_1^- + \Delta E &= P_2 + K_2^-, \\ P_2 + F_2^2 K_2^- + \Delta E &= P_1 + K_1^-, \end{aligned} \quad (22)$$

Fig. 6 Walking speed versus hip angle difference

Fig. 5 Asymmetric gait with two different hip angles

and kinetic energies immediately before the heel strike are given by

$$\begin{aligned} K_1^- &= \frac{(P_2 - P_1)(1 - F_2^2) + (1 + F_2^2)\Delta E}{1 - F_1^2 F_2^2}, \\ K_2^- &= \frac{(P_1 - P_2)(1 - F_1^2) + (1 + F_1^2)\Delta E}{1 - F_1^2 F_2^2}. \end{aligned} \quad (23)$$

On the other hand, in the case of the symmetric gait with the average hip angles, kinetic energy becomes

$$K_a^- = \frac{\Delta E}{1 - F_a^2}, \quad (24)$$

where $F_a = \cos \bar{\alpha}_a$ and $\bar{\alpha}_a = (\bar{\alpha}_1 + \bar{\alpha}_2)/2$.

The mean value of kinetic energy immediately before the heel strike K_m^- is given by

$$\begin{aligned} K_m^- &= \frac{K_1^- + K_2^-}{2} \\ &= \frac{(P_2 - P_1)(F_1^2 - F_2^2) + (2 + F_1^2 + F_2^2)\Delta E}{2(1 - F_1^2 F_2^2)}. \end{aligned} \quad (25)$$

It is assumed that $\bar{\alpha}_1 > (\bar{\alpha}_1 + \bar{\alpha}_2)/2 > \bar{\alpha}_2$, and then the relations $F_1^2 < F_a^2 < F_2^2$ and $P_1 < P_2$ hold. Therefore,

$$\frac{(P_2 - P_1)(F_1^2 - F_2^2)}{2(1 - F_1^2 F_2^2)} < 0. \quad (26)$$

From convexity of cosine and the relation between arithmetic average and geometric mean, the following inequality

$$F_a^2 \geq \frac{F_1^2 + F_2^2}{2} \geq \sqrt{F_1^2 F_2^2} \quad (27)$$

holds in the range of $\bar{\alpha}_1, \bar{\alpha}_2 < \pi/4$. Then, we get

$$K_m^- \leq \frac{2(1 + F_a^2)\Delta E}{2(1 - (F_1^2 + F_2^2)/4)} \leq \frac{2(1 + F_a^2)\Delta E}{2(1 - F_a^4)} = K_a^-. \quad (28)$$

In this case, kinetic energy immediately before the heel strike of the symmetric gait is larger than that of the asymmetric gait.

Next, the mean value of the angular velocities of the asymmetric gait is compared with that of the symmetric gait. The angular velocities are given by

$$\dot{\theta}_a^- = \sqrt{\frac{2}{mL^2} K_a^-}, \quad (29)$$

$$\begin{aligned} \dot{\theta}_m^- &= \frac{1}{2}(\dot{\theta}_{S1}^- + \dot{\theta}_{S2}^-) \\ &= \sqrt{\frac{2}{mL^2} \frac{\sqrt{K_1^-} + \sqrt{K_2^-}}{2}}. \end{aligned} \quad (30)$$

By subtracting the squared values of Eqs. (29) and (30) after multiplying $B = \sqrt{mL^2/2}$, we get

$$B^2(\dot{\theta}_a^-)^2 - B^2(\dot{\theta}_m^-)^2 = K_a^- - \frac{K_1^- + K_2^- + 2\sqrt{K_1^- K_2^-}}{2}. \quad (31)$$

From Eq. (28), the following inequality is derived.

$$\begin{aligned} K_a^- - \frac{K_1^- + K_2^- + 2\sqrt{K_1^- K_2^-}}{4} \\ \geq \frac{K_1^- + K_2^-}{2} - \frac{K_1^- + K_2^- + 2\sqrt{K_1^- K_2^-}}{4} \\ = \frac{K_1^- + K_2^- - 2\sqrt{K_1^- K_2^-}}{4} \geq 0 \end{aligned} \quad (32)$$

Therefore, $\dot{\theta}_a^- \geq \dot{\theta}_m^-$.

From above analyses, the asymmetric gait in the case with the two different energy restoration does not affect the walking speed significantly, while the stride difference decreases the average walking speed. This result implies that constraint on the impact posture achieves high-speed walking.

4 Passive dynamic walker model

4.1 Equation of motion

Figure 7 shows the model of a planar biped robot on the periodic stair whose height is defined as h_i . The robot consists of support and swing legs. Each leg with lumped mass m , length l and semicircular feet with radius R is connected with each other on a hip joint. θ_1 and θ_2 are the inclination angles of the support and the swing legs measured from vertical lines, respectively. The torso is represented as a lumped mass m_H on the hip joint. The robot has a stopper on the hip joint

Fig. 7 Planar passive bipedal robot on stairs

Table 1 Physical parameters of biped robot

a	0.5 m	l	1.0 m	m	5.0 kg
b	0.5 m	R	0.5 m	m_H	10.0 kg

to constrain the heel strike posture, which means that the walker falls down as a 1-DoF rigid body in the same posture immediately before the heel strike. The values of the robot parameters shown in Table 1. The robot walks down the stairs. Period- n stairs is illustrated in Fig. 7, where the relation, $h_{i+n} = h_i$ is hold.

The gait pattern of the robot consists of the following two phases.

- Phase I : The support leg rotates around the contact point between the semicircular foot and the ground. In this phase, the hip joint is free ($\lambda = 0$). When the hip angle becomes a certain value, the impact at hip joint occurs and pattern goes to Phase II.
- Phase II : The support leg rotates around the contact point between the semicircular foot and the ground. The hip joint angle is constrained on the certain angle ($\lambda \neq 0$), and then the system becomes 1-DoF rigid body. After the heel strike occurs, the pattern goes to Phase I.

We assume that the heel strike occurs instantaneously, and then the support and swing legs change roles. It is also assumed that the wheel does not jump.

The equation of motion during the single-support phase is given by

$$\mathbf{M}(\theta)\ddot{\theta} + \mathbf{h}(\theta, \dot{\theta}) = \mathbf{J}^T \lambda, \quad (33)$$

where $\theta = [\theta_1 \ \theta_2]^T$, $\mathbf{M}(\theta) \in \mathbb{R}^{2 \times 2}$ is the inertia matrix, and $\mathbf{h}(\theta, \dot{\theta}) \in \mathbb{R}^2$ is the Coriolis force, the centrifugal force and the gravity vectors. $\mathbf{J}^T \lambda$ in Eq. (33) is the constraint force vector given by

$$\mathbf{J}^T \lambda = [1 \ -1]^T \lambda, \quad (34)$$

where λ is the constraint force on the hip joint due to the stopper.

Note that foot scuffing is ignored in this paper.

4.2 Impact equation

In this model, there are two kinds of the impacts: the impact on the hip joint with the stopper and the heel strike. The impact equation is modeled on the assumption that the impacts are completely inelastic.

The transition equations at the impact on the hip joint are given by

$$\theta^+ = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \theta^-, \quad \dot{\theta}^+ = \mathbf{H}(\alpha) \dot{\theta}^-, \quad (35)$$

where $\mathbf{H}(\alpha)$ is the function of the constraint angle of the hip joint, α rad.

The transition equations at the heel strike are given by

$$\theta^+ = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \theta^-, \quad \dot{\theta}^+ = \mathbf{\Xi} \dot{\theta}^-, \quad (36)$$

where the superscripts “+” and “−” represent immediately before and immediately after the heel strike. The constant matrix $\mathbf{\Xi}(\alpha)$ is derived from the geometric constraints at the heel strike.

4.3 Hip joint constraint

The hip joint constraint force is derived from the equation of motion and Jacobian $\mathbf{J}$. On the hip constraint, $\theta_1 - \theta_2 = 0 = \mathbf{J}\dot{\theta}$. The sufficient condition of the equation is given by $\mathbf{J}\ddot{\theta} = 0$. From Eq. (33) and the sufficient condition, the constraint force is given by

$$\lambda = \left(\mathbf{J} \mathbf{M}(\theta)^{-1} \mathbf{J}^T \right)^{-1} \mathbf{J} \mathbf{M}(\theta)^{-1} \mathbf{h}(\theta, \dot{\theta}). \quad (37)$$

5 Asymmetric gait for passive dynamic walker

In subsection 5.1, different stride gaits on the period-one stairs are investigated, while in subsection 5.2, constant stride gaits on the period-two stairs are investigated.

5.1 Different stride gaits on the period-one stairs

To realize different stride, two kinds of the constraint hip joint angles α_1 and α_2 rad are given. By choosing the different angles, period-two gait emerges on the period-one stairs shown in Fig. 8, where the height of stairs is $h_1 = h_2 =$

Fig. 8 Gait pattern of period-two gait by using two kind of stride

Fig. 9 Bifurcated gait by difference hip angle

$\bar{h} = \text{const.}$ Therefore, the value of transformation of potential energy to kinetic energy is almost constant due to the period-one stairs. The angles has the following relation.

$$\alpha_0 = \frac{\alpha_1 + \alpha_2}{2}, \quad (38)$$

where α_0 is the initial hip joint angle.

Figure 9 shows the simulation result where $\alpha_1 = 0.85$ rad, $\alpha_2 = 0.95$ rad ($\alpha_0 = 0.9$ rad) and $\bar{h} = 0.05$ m. In this figure, (a) is the time histories of the support, the swing and the hip joint angles, (b) the time histories of the support, the swing and the hip joint angular velocities, and (c) the time histories of the mechanical energy. It can be seen from Fig. 9 that the period-two gait emerges. The time-interval of the Phase II differs according to the hip angle mechanically locked. In the case that the hip angle is locked to α_2 , the time-interval is significantly shorter and the next heel-strike soon occurs. The mechanical energy level during the subsequent Phase I becomes higher than that during the previous Phase I. This is because the kinetic energy loss with the hip angle of α_1 is smaller than that of α_2 .

Figure 10 shows the walking speed when the difference between the constraint hip angles increases. In Fig. 10, hori-

Fig. 10 Walking speed versus hip angle difference ($h_1 = h_2 = \bar{h} = \text{const.}$)

zontal axis is the difference between the hip angles, $\alpha_2 - \alpha_1$, and vertical axis is the walking speed. The values of the stairs height and the initial hip angles are set as $\bar{h} = 0.05$ m and $\alpha_0 = 0.9$ rad in Case1, and $\bar{h} = 0.03$ m and $\alpha_0 = 0.8$ rad in Case 2. $\square$ and $\circ$ represent the walking speed during each step, and $\diamond$ and $\times$ represent the average walking speed.

It can be seen from Fig. 10 that the average walking speed becomes small when the difference between the constraint hip angles increases. The walking speed of Case 1 is larger than that of Case 2, and the biped can walk when the difference is larger in Case 1.

5.2 Constant stride gaits on the period-two stairs

To realize different energy restoration, period-two stairs is introduced, where the height of the stairs h_1 and h_2 m are differently given. A period-two gait then emerges as Fig. 11, where the constraint hip angle is $\alpha_1 = \alpha_2 = \bar{\alpha} = \text{const.}$ The height has the following relation:

$$h_0 = \frac{h_1 + h_2}{2}, \quad (39)$$

where h_0 is the initial height of the stairs.

Figure 12 shows the simulation result where $h_1 = 0.035$ m, $h_2 = 0.065$ m ($h_0 = 0.05$ m) and $\bar{\alpha} = 0.9$ rad. In this figure, (a) is the time histories of the support, the swing and the hip joint angles, (b) the time histories of the support, the swing and the hip joint angular velocities, and (c) the time histories of the mechanical energy, where h_1 and h_2 represent the energy when the robot is on the stairs whose heights are h_1 and h_2 , respectively. It can be seen from Fig. 12 that a period-two gait emerges when keeping the hip angle constant.

Fig. 11 Gait pattern of period-two gait by using two kind of stairs

Fig. 12 Bifurcated gait by difference stairs height

Step period on h_2 is longer than that on h_1 , while the mechanical energy on h_1 is larger than that on h_2 . By comparing Fig. 9, the mechanical energy in Fig. 12 is larger than that in Fig. 9, while the mechanical energy in Fig. 9 varies significantly during the period-two gait.

Figure 13 shows the walking speed with respect to the difference between the heights of the stairs. In Fig. 13, the

Fig. 13 Walking speed versus the stairs height ($\alpha_1 = \alpha_2 = \bar{\alpha} = \text{const.}$)

horizontal axis is the difference between the heights of the stairs, $h_2 - h_1$, and the vertical axis is the walking speed. The values of the initial height of the stairs and the hip angle are set as $h_0 = 0.05$ m and $\bar{\alpha} = 0.9$ rad in Case1, and $h_0 = 0.03$ m and $\bar{\alpha} = 0.8$ rad in Case 2. $\square$ and $\circ$ represent the walking speed during each step, and $\diamond$ and $\times$ represent average of the walking speed.

It can be seen from Fig. 13 that the average of the walking speed increases a little bit as the difference between the heights of the stairs increases. The walking speed of Case 1 is larger than that of Case 2, and the biped can walk when the difference is larger in Case 1.

6 Conclusion and future work

This paper has investigated the properties of asymmetric passive dynamic gaits of a rimless wheel and compass-like biped walker. The results can be summarized as follows:

- The symmetric gait is more efficient than the asymmetric gait.
- The difference between energy restorations has not much effect on the walking speed.
- The difference between strides decreases the walking speed.
- When the robot walks on the uneven ground, the walking speed may become higher by making hip angles at the heel strike constant.

In the future, we will investigate the stability of the bifurcated gaits, and clarify the effect of the bifurcation. We will also investigate the bifurcated gaits of multi-DoF biped

robots with knees, torso and arms, and analyze the gaits theoretically.

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References

1. McGeer T (1990) Passive dynamic walking. *Int J Robotics Res* 9(2):62–82
2. Goswami A, Thuilot B, Espiau B (1996) Compass-like biped robot part I: stability and bifurcation of passive gaits. INRIA Research, Report, no. 2996
3. Osuka K, Kirihaara K (2000) Motion analysis and experiment of passive walking robot Quartet II. *J Robotics Soc Jpn* 18(5):121–126 (in Japanese)
4. Safa T, Saadat MG, Naraghi M (2007) Passive dynamic of the simplest walking model: replacing ramps with stairs. *Mech Mach Theory* 42:1314–1325
5. Moon JS, Spong MW (2011) Classification of periodic and chaotic passive limit cycles for a compass-gait biped with gait asymmetries. *Robotica* 29(7):967–974
6. Garcia M, Chatterjee A, Ruina A, Coleman M (1998) The simplest walking model: stability, complexity, and scaling. *J Biomechanical Eng* 120(2):281–288
7. Suzuki S, Furuta K (2002) Enhancement of stabilization of passive walking by chaos control approach. *Proceedings of the 15th IFAC Triennial World Congress*
8. Yamakita K, Ishimura K, Wada M (2005) Chaos dynamics of biped passive walking and the evaluation by an actual robot. *Trans Jpn Soc Mech Eng C* 71(705):1669–1677
9. Osuka K, Sugimoto Y, Sugie T (2004) Stabilization of semi-passive dynamic walking based on delayed feedback control. *J Robotics Soc Jpn* 22(2):45–51 (In Japanese)
10. Harata Y, Iwano K, Asano F, Ikeda T (2012) Suppression of period-doubling bifurcation in passive dynamic walking with delayed feedback control. *Proceedings of the 2012 ASME International Design Engineering Technical Conferences, DETC2012-70209*, pp 1–10
11. Asano F, Luo Z-W (2009) Asymptotically stable biped gait generation based on stability principle of rimless wheel. *Robotica* 27(6):949–958