Gain-scheduled controllers design for interceptor parameter-varying system with multi-saturated constraint

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Abstract Based on homogeneous polynomial parameter dependent Lyapunov function (HPPDLF) theory and the algorithm that the nonlinear matrix differential equations converted into convex polyhedron state equations, the robust optimal control system is designed, for the interceptor with time-varying parameters and actuator nonlinear multi-saturated constraint. First, the model of the actuator saturation operator is established, which provides the control system topology. The nonlinear differential equations with time-varying parameters converted into the linear polytopic vertex system by introduced the normalization factors of the amplitude and rate actuator saturation, and then it converted into HPPD matrix differential equations, based on HPPDLF theory and the extension of Pólya's theorem to the case of matrix-valued polynomials. The dynamic performance of the closed-loop control system is obtained by setting the minimum allowable value of the scaling factor, when solving the minimum of the maximum homothetic set in the iterative processing. Finally the simulations of the interceptor have demonstrated the effectiveness of the approach proposed.

Keywords Actuator saturation · Pólya's theorem · Near-space interceptor · Convex polyhedron of matrix · Homogeneous polynomial parameter dependent Lyapunov function

1 Introduction

Requiring direct collision way to destroy the high dynamic aircraft [1], in order to make the minimize damage to the ground, when intercept high dynamic aircraft in near-space, which are put forward higher requirements for the interceptor time-varying parameters system with actuator saturated constraints. Compared with conventional aircraft flying in the atmosphere, the control system of the near space interceptor has greater impact with the actuator saturation constraints, that is because the natural frequency of the interceptor is low in near space, need high-gain controller to enhance the system responses speed when design the control system. The control efficiency of interceptor significantly reduced with the increase of altitude, in the process of the control system dynamic adjustment, when considering the actuator saturation nonlinearity. The gain-scheduled controllers can ensures the control system stability and improve the system response speed, under certain preconditions are met (time-varying parameters can be obtained in real time).

Actuator saturation problem is an old and new problem, the negligence of both amplitude and rate saturation bounds can be source of limit cycles, parasitic equilibrium points and even instability of the close loop system; especially rate saturation is responsible by a phase lag that has a high destabilizing effect [2]. As a result of saturation, the actual plant input is different from the controller output and the controller states are wrongly updated, this discrepancy between the controller output and the plant input due to actuator saturation is called controller wind-up [3].

Robust stability and performance analysis of polytopic systems via parameter-dependent Lyapunov functions and LMI-based methods have been extensively studied in the last few years leading to three classes of results [4]: I. "Slack variables" method [5,6]: This methodology is based on a convex

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of polynomial parameter-dependent LMIs by introduction of additional variables using the elimination lemma backward. This structural relaxation LMI approach, by its essence is to expand the searching optimization space, thereby obtaining less conservative robust stability condition; II. “Sum-of-squares” method [7–10]: This approach considers a general form of PPDLFs and shows that for polytopic systems these polynomial parameter dependent functions may always be chose homogeneous. The main result consists in writing conditions for positive definite polynomial parameter-depend matrices to be sum-of-squares leading to sufficient parameter independent LMI conditions for the original parameter dependent problem. Results have no proof of convergence towards exact robustness results as the degree of the PPDLF grows. III. “Positive polynomials with positive coefficients” method [11–13]: This method handles the positivity of polynomial parameter-dependent LMI over the set of positive uncertain parameters by testing that all matrix coefficients are positive. In the case of affine PDLFs, a sequence of such conditions involving exponentially many LMI constraints is proved to have asymptotic convergence properties. The present paper adopts this same framework to solve robust analysis problems by means of PPDLF and the extension of Pólya’s theorem, the conservatism of the derived LMI conditions reduce with the increase the degree of the PPDLF and the relaxation of Pólya’s.

Gain-scheduled control theory is a hot field of the control community in recent years [14–23], the literature [21, 23] research parameter time-varying system controller design with the amplitude actuator saturated constraint. Gain-scheduled control system is designed with multi-saturated constraints in this paper. Firstly, set up interceptor mathematical model with actuator saturation constraints and gives the longitudinal plane normal overload stable tracking four loop autopilot topology, based on the classic three-loop autopilot topology [24] and the mathematical model of the amplitude and rate saturation constraint operator [25]. The nonlinear time-varying parameters differential equations converted into the linear polytopic vertex differential equations, by introduced literature [2] the normalization factors of saturated constraint. Then design the parameters time-varying system controller based on parameter dependent Lyapunov stability theory. The linear polytopic vertex differential equations converted into HPPD matrix differential equations, based on homogeneous polynomial parameter dependent Lyapunov function (HPPDLF) theory and the extension of Pólya’s theorem to the case of matrix-valued polynomials; The dynamic performance of the closed-loop control system is obtained by setting the minimum allowable value of the scaling factor, when solving the minimum of the maximum homothetic set in the iterative processing; Finally, the simulation of the fixed typical flight point and the guidance and control systems for longitudinal plane show that the control sys-

tem time domain performance is improved with the increase the degree of the PPDLF and the relaxation of Pólya’s.

2 Problem formulation

2.1 Model and topology of the system

Based on the classic three-loop autopilot topology [24], the establishment of interceptor missile longitudinal plane short cycle kinetics linear mathematical model:

$$\dot{x} = Ax + Bu \quad (1)$$

where

$$x = \begin{bmatrix} n_y \\ \omega \\ \dot{\omega} \end{bmatrix}, \quad A = \begin{bmatrix} -a_{34} & a_{34}v/g & 0 \\ 0 & 0 & 1 \\ a_{24}g/v & -a_{24} & -a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} a_{35}v/g \\ 0 \\ -a_{25} \end{bmatrix},$$

$n_y, \omega, v, g, \dot{\omega}$ are normal overload, pitch rate angular, the speed of interceptor, the gravitational acceleration, rate angular of actuator, the definition of the aerodynamic coefficients $a_{22}, a_{24}, a_{25}, a_{34}, a_{35}$ see the literature [17].

Dynamic performance of the interceptor body change with the environment (height, density) and flight speed change, establish interceptor system mode with dynamic pressure as time-varying parameter

$$\dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u(t) \quad (2)$$

Here $x \in R^n$ is the state vector, $u \in R^m$ is the control vector, time-varying parameter vector $\theta(t) \in R^N$ ($N = 2$ variable parameters for the dynamic pressure head) in the unit simplex [23]:

$$\Lambda = \left\{ \lambda \in R^N : \sum_{j=1}^N \lambda_j = 1, \quad \lambda_j \geq 0, \quad j = 1, \dots, N \right\} \quad (3)$$

Matrices $A(\theta(t)) \in R^{n \times n}$, $B(\theta(t)) \in R^{m \times m}$ in the polyhedron:

$$P = \left\{ (A, B)(\cdot) : (A, B) = \sum_{j=1}^N \lambda_j(t)(A, B)_j, \quad \lambda(t) \in \Lambda \right\}.$$

The modeling issue for the amplitude-plus-rate constraint turns out to be nontrivial because of the dynamics in the constraints [25]; by introduce the standard amplitude plus rate saturation operator $\sigma_{\Psi_P, \Omega_R}(\cdot)$. For any measurable scalar signal $v(t)$, $\sigma_{\Psi_P, \Omega_R}(\cdot)$ is defined by:

$$\sigma_{\Psi_P, \Omega_R}(v(t)) \triangleq \lim_{\Gamma \rightarrow +\infty} x_{\Gamma}(t; \Psi_P, \Omega_R) \quad (4)$$

Here $x_{\Gamma}(t; \Psi_P, \Omega_R)$ is the unique solution of the state space model.

$$\begin{aligned}\dot{x}_\Gamma(t) &= \sigma_{\Omega_R}(\Gamma(\sigma_{\Psi_P}(v(t)) - x_\Gamma)), \quad x_\Gamma(0) \\ &= \sigma_{\Psi_P}(v(0))\end{aligned}\quad (5)$$

So that $x_\Gamma = u$, $\sigma_{\Omega_R} = \text{sat}_R$, $\sigma_{\Psi_R} = \text{sat}_P$, the actuator saturation constraint order inertia model:

$$\dot{u}_{(i)} = \text{sat}_{R(i)}(\tau_{(i)}(-u_{(i)}(t) + \text{sat}_{P(i)}(v_{(i)}(t) + r(t)))) \quad (6)$$

Here $(v_{(i)} + r)$, $u_{(i)}$ respectively the elevator angle command and the elevator angle, r represents the input command, $\tau_{(i)} > 0$ and

$$\text{sat}_{R(i)}(\cdot) \triangleq \text{sgn}(\cdot) \min(\Omega_{(i)}, |\cdot|)$$

$$\text{sat}_{P(i)}(\cdot) \triangleq \text{sgn}(\cdot) \min(\Psi_{(i)}, |\cdot|)$$

where $\Omega_{(i)}$ and $\Psi_{(i)}$ denote respectively the rate and amplitude bounds, the interceptor nonlinear control system topology shown in Fig. 1; With multi-saturated constraint pitch channel stable track overload command autopilot topology shown in Fig. 2, by introducing the integration of the state feedback controller, combined with the classic three-loop autopilot topology [24].

By introduce the following state feedback:

$$v(t) = K_x(\theta(t))x(t) + K_u(\theta(t))u(t) \quad (7)$$

The longitudinal plane closed-loop control system:

$$\begin{aligned}\dot{x}(t) &= A(\theta(t))x(t) + B(\theta(t))u(t) \\ \dot{u}(t) &= \text{sat}_R\left(-Tu(t) + T\text{sat}_P(v(t)) + \int r(t)dt\right)\end{aligned}\quad (8)$$

2.2 Problem statement

Define the regions R_{LP} and R_{LR} respectively as the region where is no occurrence of amplitude saturation and the region

where there is no occurrence of rate saturation. For simple, write $*(\theta(t))$ to $*(\cdot)$.

$$R_{LP} \triangleq \left\{x \in R^n, u \in R^m \mid \left\| \begin{bmatrix} K_x(\cdot) & K_u(\cdot) \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \right\| \leq \Psi \right\}$$

$$R_{LR} \triangleq \{x \in R^n, u \in R^m \mid -Tu + T\text{sat}_P(v(t)) \leq \Omega\}$$

It follows that $R_L \triangleq R_{LP} \cap R_{LR}$ is the region of linear behavior of system (8), i.e. where no saturation occurs.

Define $z(t) \triangleq [x^T(t) u^T(t)]^T \in R^{n+m}$. According to the second equation of (8), the input command can convert into the allowable initial state $z_0 = [r \ 0 \ 0 \ 0]^T$. This set can be viewed as the zone of operation of the system (8).

From the definitions and considerations above the problem we intend to solve is stated as follow:

Problem 1 Find feedback matrices $K_x(\cdot)$ and $K_u(\cdot)$ such that:

- I. System (8) is locally asymptotically stable in z_0 , that is $\forall r$, the corresponding trajectories converge asymptotically to the origin;
- II. When the system operates inside the linearity region R_L , a certain time-domain performance specification is satisfied.

3 System representation

3.1 Linearization

To deal with the nonlinear matrix differential equations in (8), consider the following definitions:

Fig. 1 Interceptor nonlinear control system topology

Fig. 2 Pitch channel four loop autopilot topology

I. Amplitude saturated normalized vectors factor:

$$\eta_{(i)}(t) \triangleq \min \left(1, \frac{\Psi_i}{|K_{(i)}(\cdot) z(t)|} \right)$$

II. Rate saturation normalized vectors factor:

$$\lambda_{(i)}(t) \triangleq \min \left(1, \frac{\Omega_i}{|\dot{u}_{(i)}(t)|} \right)$$

III. The matrices:

$$A(\lambda(t), \cdot) = \begin{bmatrix} A(\cdot) & B(\cdot) \\ \mathbf{0} & -D(\lambda(t))T \end{bmatrix};$$

$$B(\lambda(t)) = \begin{bmatrix} \mathbf{0} \\ D(\lambda(t))T \end{bmatrix};$$

$$K(\cdot) = [K_x(\cdot) \quad K_u(\cdot)]$$

where $D(\lambda(t))$ are the real diagonal matrices that the diagonal elements are $\lambda_{(i)}(t)$, $i = 1, \dots, m$.

From the definitions above, it follows that the closed-loop system (8) is equivalent to the following one:

$$\dot{z}(t) = (A(\lambda(t), \cdot) + B(\lambda(t))D(\eta(t))K(\cdot))z(t) \quad (9)$$

Define $\underline{\eta} \triangleq [\underline{\eta}_{(1)} \cdots \underline{\eta}_{(m)}]^T$ and $\underline{\lambda} \triangleq [\underline{\lambda}_{(1)} \cdots \underline{\lambda}_{(m)}]^T$ be respectively a vector of lower bounds for $\eta(t)$ and for $\lambda(t)$. Consider now all the possible m -order vectors such that the i^{th} entry takes the value 1 or $\underline{\eta}_{(i)}$ and respectively 1 or $\underline{\lambda}_{(i)}$. Hence, there exist 2^m different vectors. By denoting each one of these vectors by $r_j(\underline{\eta})$ and, respectively, $r_j(\underline{\lambda})$, $j = 1, \dots, 2^m$, define the following matrices:

$$D_j(\underline{\eta}) = \text{diag}(\gamma_j(\underline{\eta})), D_j(\underline{\lambda}) = \text{diag}(\gamma_j(\underline{\lambda})) \quad (10)$$

Associated to $\underline{\eta}$ and $\underline{\lambda}$ we can now define the following regions in the space R^{n+m} :

$$R_{LP}(\underline{\eta}) = \{z \in R^{n+m} \mid |K(\cdot)z| \leq \Psi(\underline{\eta})\} \quad (11)$$

$$R_{LR}(\underline{\lambda}) = \bigcap_{j=1}^{2^m} R_{LR}(\underline{\lambda})_j \quad (12)$$

where the definition $R_{LR}(\underline{\lambda})_j$ of see the literature [2],

$$\Psi(\underline{\eta})_{(i)} \triangleq \Psi_{(i)}/\underline{\eta}_{(i)}, \quad \Omega(\underline{\lambda})_{(i)} \triangleq \Omega_{(i)}/\underline{\lambda}_{(i)}.$$

Notice that for all $z(t) \in R_{LP}(\underline{\eta}) \cap R_{LR}(\underline{\lambda})$ one has

$$\begin{aligned} 0 < \underline{\eta}_{(i)} \leq \eta_{(i)}(t) \leq 1 \\ 0 < \underline{\lambda}_{(i)} \leq \lambda_{(i)}(t) \leq 1 \end{aligned} \quad \forall i = 1, \dots, m \quad (13)$$

Consider now the variable change

$$v_{(i)}(t) = \eta_{(i)}(t) - 1 + 0.5 \left(1 - \underline{\eta}_{(i)} \right) \quad (14)$$

Then, if $\eta_{(i)}(t)$ verifies (13) it follows that

$$|v_{(i)}(t)| \leq 0.5 \left(1 - \underline{\eta}_{(i)} \right) \triangleq \underline{v}_{(i)}$$

Defining the vector $v(t) \triangleq [v_{(1)}(t) \cdots v_{(m)}(t)]^T$ and $\underline{v} \triangleq [\underline{v}_{(1)} \cdots \underline{v}_{(m)}]^T$, $\forall z(t) \in R_{LP}(\underline{\eta})$ from (10) to (14), $\dot{z}(t)$ can be computed by:

$$\begin{aligned} \dot{z}(t) = & \left(A(\lambda(t), \cdot) + B(\lambda(t))D_1(\underline{\eta})K(\cdot) \right) z(t) \\ & + B(\lambda(t))D(v(t))K(\cdot)z(t) \end{aligned} \quad (15)$$

where $|v_{(i)}(t)| \leq \underline{v}_{(i)}$ and $D_1(\underline{\eta}) \triangleq D(0.5(1_m + \underline{\eta}))$.

Define now matrices:

$$A_j(\underline{\lambda}, \cdot) = \begin{bmatrix} A(\cdot) & B(\cdot) \\ \mathbf{0} & -D_j(\underline{\lambda})T \end{bmatrix}; \quad B_j(\underline{\lambda}) = \begin{bmatrix} \mathbf{0} \\ D(\underline{\lambda})T \end{bmatrix}.$$

Note that the matrices $A_j(\underline{\lambda}, \cdot)$ and $B_j(\underline{\lambda})$, are the vertices of convex poly-topes of matrices, i.e.

$$A(\lambda(t), \cdot) \in \text{Co} \{A_1(\underline{\lambda}), \dots, A_{2^m}(\underline{\lambda})\},$$

$$B(\lambda(t)) \in \text{Co} \{B_1(\underline{\lambda}), \dots, B_{2^m}(\underline{\lambda})\}.$$

It follows that at instant t , if $z(t) \in R_{LP}(\underline{\eta}) \cap R_{LR}(\underline{\lambda})$ there exist nonnegative scalars, $|v_{(i)}(t)| \leq \underline{v}_{(i)}$, such that $\dot{z}(t)$ can be computed as follows:

$$\dot{z}(t) = \sum_{j=1}^{2^m} \lambda_j \left[B_j(\lambda)D_1(\eta)K(\cdot) + A_j(\underline{\lambda}, \cdot) + B_j(\underline{\lambda})D(v(t))K(\cdot) \right] z(t) \quad (16)$$

3.2 Controller design

Theorem 1 *If there exists a positive definite symmetric matrix $W(\cdot) > 0$, $W(\cdot) \in R^{(n+m) \times (n+m)}$, a diagonal matrix $S(\cdot) > 0$, $S(\cdot) \in R^{m \times m}$, matrices $Y(\cdot) \in R^{m \times (n+m)}$ and vectors $\underline{\eta} \in R^m$, $\underline{\lambda} \in R^m$ ($0 < \underline{\eta}_{(i)}, \underline{\lambda}_{(i)} \leq 1$), $\mu \in R$, satisfying the following matrix inequalities (The matrix inequalities and the proof see "Appendix 1").*

Then $K(\cdot) \triangleq Y(\cdot)W^{-1}(\cdot)$ are solves of the Problem 1.

The Proof of Theorem 1 see "Appendix 2".

4 Mathematical preliminaries

Lemma 1 [7] *The set A is Hurwitz if and only if there exists a HPD-QLF $v_m(x; \theta)$ such that:*

$$\begin{cases} P_m(\theta) > 0 \\ A^T(\theta)P_m(\theta) + P_m(\theta)A(\theta) < 0 \end{cases} \quad \forall f(\theta) \in \Lambda \quad (17)$$

The Proof of Lemma 1 see the literature [7], from the proof of Lemma 1, an upper bound on the degree m of the homogeneous matricial for $P_m(\theta)$ defining the HPD-QLF can be derived.

According to Polya's theorem [26,27] and Theorem 1, the following lemmas give necessary and sufficient conditions for the Hurwitz stability of A in terms of a generic parameter-dependent Lyapunov function $P(\theta) = P^T(\theta) > 0, \forall f(\theta) \in \Lambda$, but the conditions must be fulfilled for all $\theta \in \Lambda$ thus resulting in problems of infinite dimension.

Lemma 2 [26] *The set A is Hurwitz stable if and only if there exists a symmetric positive definite parameter dependent matrix $P(\theta) \in R^{n \times n}$ such that one of the following equivalent conditions holds $\forall \theta \in \Lambda$:*

- I. $\Gamma(\theta) \triangleq A(\theta)^T P(\theta) + P(\theta) A(\theta) < 0$
- II. $\Gamma_d(\theta, \lambda) \triangleq (\lambda_1 + \lambda_2 + \dots + \lambda_N)^d \Gamma(\theta) < 0; \quad \forall d \in Z_+$

Note that $P(\theta)$ in Lemma 2 does not have a special structure and the verification of stability is based on the existence of a positive definite Lyapunov matrix for any choice of $\theta \in \Lambda$, which is a well known result.

The aim here is to investigate necessary and sufficient conditions for the existence of $P(\theta)$, a class of Lyapunov function that can provide less conservative results for the stability analysis of A than quadratic stability. The algebraic properties of condition II of Lemma 2, which defines a family of polynomials whose number of monomials is parametrized on $d \in Z_+$, will be used to provide a complete characterization of the existence of $P(\theta)$ assuring the Hurwitz stability of A in terms of linear matrix inequalities formulated only at the vertices of Λ .

Theorem 2 [21] *Consider a parameter-dependent LMI generally written as*

$$\ell(\xi, \theta) = \ell_0(\theta) + \xi_1 \ell_1(\theta) + \dots + \xi_M \ell_M(\theta) > 0 \quad (18)$$

Here $\theta \in \Lambda$. Assume that $\ell_i(\theta), i = 0, \dots, M$ are continuous functions of θ . if $\forall \theta(t) \in \Lambda$ there exists a parameter-dependent solution $\xi(\theta) \in R^M$ such that $\ell(\xi(\theta), \theta) > 0$, then there exists a homogeneous polynomial solution $\xi^*(\theta) : \Lambda \rightarrow R^M$ such that, $\forall \theta \in \Lambda, \ell(\xi^*(\theta), \theta) > 0$.

5 Main results

Define [26] $\kappa(d)$ as the set of N -tuples obtained as all possible combinations of $k_1 k_2 \dots k_N, k_i \in Z_+, i = 1, \dots, N$ such that $k_1 + k_2 + \dots + k_N = d$. $\kappa_\ell(d)$ is the ℓ th N -tuple of $\kappa(d)$ which is lexically ordered, $\ell = 1, \dots, J(d)$. For a fixed N , the number of elements in $\kappa(d)$ is given by $J(d) = (N + d - 1)! / d! (N - 1)!$ and the associated standard multinomial coefficients are $\varepsilon^\ell(d) = \frac{d!}{k_1! k_2! \dots k_N!}, k_1 k_2 \dots k_N = \kappa_\ell(d), \ell = 1, \dots, J(d)$. As an example consider $N = 3$ and $d = 2$, which yields $J(2) = 6, \kappa(2) = \{002, 011, 020, 101, 110, 200\}$ and the coefficients $\varepsilon^\ell(d) = \{1, 2, 1, 2, 2, 1\}$.

Consider also the modified multinomial coefficients used in this paper

$$\varepsilon_i^\ell(d, a) = \begin{cases} \frac{d!}{k_1! \dots (k_i - a)! \dots k_N!} & \text{if } k_i - a \in Z_+ \\ 0 & \text{otherwise} \end{cases}$$

5.1 Predetermined gain controller design

Corollary 1 [23] *If there exist parameter dependent matrices $W(\cdot), Y(\cdot)$ and $S(\cdot)$ solving Theorem 1 then, without loss of generality, there exist HPPD matrices $W_g(\cdot), Y_g(\cdot)$ and $S_g(\cdot)$, of arbitrary degree g , solving Theorem 1.*

Theorem 3 *There exist HPPD symmetric positive definite matrix $W_g(\cdot) \in R^{(n+m) \times (n+m)}$, HPPD matrices $Y_g(\cdot) \in R^{m \times (n+m)}$ and a diagonal HPPD matrix $S_g(\cdot) \in R^{m \times m}$ of arbitrary degree g solving Theorem 1, if and only if, there exist a symmetric positive definite matrix $W_k \in R^{(n+m) \times (n+m)}$, matrices $Y_k \in R^{m \times (n+m)}$, diagonal matrices $S_k \in R^{m \times m}$ and a sufficiently large $d \in Z_+$ solving following matrix inequalities (the matrix inequalities see "Appendix 3")*

$$\text{Here } D_2\left(\frac{\eta}{1}\right) = D\left(0.5\left(1_m - \frac{\eta}{1}\right)\right).$$

In this case,

$$K_g(\cdot) = \sum_{k \in K(g)} \lambda_1^{k_1} \lambda_2^{k_2} \dots \lambda_N^{k_N} Y_g \left(\sum_{k \in K(g)} \lambda_1^{k_1} \lambda_2^{k_2} \dots \lambda_N^{k_N} W_g \right)^{-1} \\ k = k_1 k_2 \dots k_N.$$

Proof Notice that $W(\cdot) = W_g, Y(\cdot) = Y_g, S(\cdot) = S_g$, then R_1 with can be rewritten as

$$\left(\sum_{i=1}^N \lambda_i \right)^d R_1(\cdot) = \sum_{\substack{k \in K(g+d+1) \\ k = k_1 k_2 \dots k_N}} \lambda_1^{k_1} \lambda_2^{k_2} \dots \lambda_N^{k_N} R_{1k}(\cdot),$$

For $d \in Z_+$ since $(\lambda_1 + \lambda_2 + \dots + \lambda_N)^d = 1$, it is clear that if $R_{1k}(\cdot) > 0$, then $R_1(\cdot) > 0, \forall f(\cdot) = \lambda \in \Lambda$. Conversely, based on the extension of Polya's theorem to the case of matrix-valued polynomial, one has that if $R_1(\cdot) > 0, \forall f(\cdot) = \lambda \in \Lambda$, then exists a sufficiently large $d \in Z_+$ such that all the matrix-valued coefficients $R_{1k}(\cdot)$ are positive definite. Similar steps can be used to show sufficient and necessary to ensure that Theorem 1 is feasible. $\square$

5.2 The controller calculates

We discuss now how to compute the matrix gains $K_g(\cdot)$ in order to solve Problem 1 form the conditions stated in Theorem 3.

One way that can given gain-scheduled controller matrix method is LMIs iterative algorithm, First fixed $\underline{\eta}, \underline{\lambda}$ and the value μ , so that all inequalities $R_{1k}(\cdot) \sim R_{6k}(\cdot)$ convent

Fig. 3 Overload loop response curve when $g = 2, d = 0$

Fig. 4 Actuator loop response curve when $g = 2, d = 0$

into linear matrix inequalities. Given initial status and performance index, the controller solution can be obtained by solving Theorem 3 $R_{1k}(\cdot) \sim R_{6k}(\cdot)$. In fact, considering a small scaling factor $\delta > 0$, the maximum homothetic set to Z_0 , denoted $\delta * Z_0$, can be obtained by solving the following convex optimization problem with LMI constraints:

$$\begin{aligned} & \max_{\delta, W, Y} \delta \\ & \text{s.t.: } \begin{cases} \begin{bmatrix} 1 & \delta v_i^T \\ \delta v_i & W \end{bmatrix} > 0, \quad \forall i = 1, \dots, n_v \\ \text{LMIs: } R_{1k} \sim R_{2k}, R_{4k} \sim R_{6k} \end{cases} \end{aligned} \quad (19)$$

If the optimal value of δ greater than or equal to 1, it indicates that for a given (z_0, D_p) , the controller can be obtained by fixing the values of vectors $\underline{\eta}$ and $\underline{\lambda}$. The smaller of the vector elements $\underline{\eta}$ and $\underline{\lambda}$ the larger of the optimal value, that to say can be allowed to a large area for the initial state. To improve dynamic characteristics of close-loop system, it can be assumed initial state of the closed-loop control system is not in range ε , that is, allowing the emergence of saturation phenomenon in iterative process, this by setting the minimum allowable value to improve the system fast with less conservative; Fixed two or three variables at each step of the algorithm, so the problem can view as

Fig. 5 Overload response curve with Pólya's relaxation d

Table 1 Miss-distance relationship with g and d (unit: m)

DELTA	$d = 0$	$d = 1$	$d = 2$
$g = 1$	4.8108	3.3544	1.7703
$g = 2$	2.0180	1.9753	1.7920
$g = 3$	1.8416	1.7402	1.4412
$g = 4$	1.6189	1.4379	1.3736
$g = 5$	1.4159	1.4005	1.3366

a convex linear matrix inequality constrained optimization problem:

Iterative algorithm:

- Step 0: Initialization vectors $\underline{\eta}$, $\underline{\lambda}$ and μ ;
 Step 1: fixed $\underline{\eta}$, $\underline{\lambda}$ and given poles of the closed loop system $\mu_0 < 0$, the solution (19) obtained W , Y , δ , when δ greater than or equal to the set value into the Step 2, otherwise reduce the closed-loop system pole boundary values $\mu_0 = \mu_0 - \Delta\mu$, ($\Delta\mu > 0$) and continue to solve (19) until there are no less than the set value;

Fig. 6 The missile and target relationship when $g = 1$, $d = 0$

Step 2: fixed $\underline{\lambda}$ and Y , solving the following optimization problem W , $\underline{\eta}$, δ :

$$\begin{aligned} & \max_{\underline{\eta}, W, Y} \eta \\ & \text{s.t.} \begin{cases} \begin{bmatrix} 1 & \delta v_i^T \\ \delta v_i & W \end{bmatrix} > 0, \quad \forall i = 1, \dots, n_v \\ \text{LMIs: } R_{1k} \sim R_{2k}, R_{4k} \sim R_{6k} \end{cases} \end{aligned} \quad (20)$$

Step 3: Fixed W , Y , S and $\underline{\eta}$ solving following optimization problem:

$$\begin{aligned} & \min_{\underline{\lambda}} \sum_{i=1}^m \lambda_{(i)} \\ & \text{s.t. LMIs } R_{1k} \sim R_{2k}, R_{4k}, R_{5k} \end{aligned} \quad (21)$$

Step 4: Go to Step 1 until no significant change in value is very small $\underline{\eta}$ or $\underline{\lambda}$ stop iterating.

6 Simulation of the control system

Controllers design: According to the changes of the dynamic pressure, choose the first typical fight point of the interceptor: flight Mach 5.0Ma, 30km altitude, flying angle of attack is 20° ; the second typical fight point: Mach 4.0Ma, 35 km altitude, flying angle of attack is 20° as the polyhedral matrix vertices ($N = 2$), get a group controller gains with the degree of the homogeneous polynomial Lyapunov matrix and level of Pólya's relaxations based on the iterative algorithm that given in the section 5.2.

Check typical fight point: 4.0Mach, at 33 km altitude, flying angle of attack is 20° , the structure of the overload commands are square wave, when the time is < 2 s, the overload command is $4g$, when the time is equal to 2s, the over-

Fig. 7 The missile and target relationship when $g = 1$, $d = 2$

Fig. 8 Segment curve of actuator angle and overload

load command has suddenly turned into $-3g$, until 5 s the overload command turned into $4g$ again. Through the simulation the overload response curve and actuator response curve are shown in Figs. 3 and 4, respectively. When $g = 1$, Fig. 5 shows the overload response curve with the Pólya's laxity d .

The simulation of the control system: Select guidance law coefficient $K = 1.4$, based on HPPD theory of the Theorem 3 and the iteration algorithm, get a group controller gain with degree of the homogeneous polynomial Lyapunov matrix and level of Pólya's relaxations. Assumption when the relative distance < 30 km, the target to escape with $0.4g$ constant acceleration, Table 1 given the Miss-distance relationship with g and d . Figures. 6 and 7 given the missile and target trajectory curve respectively. Figure 8 given the actuator angle and overload segment curve.

The Figs. 3 and 4 show that compare with the common way, the HPPD way can improve the control system time domain performance, From Figs. 5, 6, 7, 8 and Table 1 have

demonstrated the control system performance is improved (Miss-distance reduced) with g and d are increased.

7 Conclusion

The robust optimal control system is designed for interceptor time-varying parameter system with multi-saturated constraint based on HPPDLFs and the algorithm of the nonlinear matrix differential equations converted into convex polyhedron. Built a mathematical model of interceptor with actuator saturation constraints and gives normal overload autopilot topology; establish linear polyhedron differential equations and HPPD mathematical model based on saturated normalization factor, HPPDL theory and expand Pólya's theorem; The dynamic performance of the closed-loop control system is obtained by setting the minimum allowable value of the scaling factor, when solving the minimum of the maximum homothetic set, in the iterative processing; the numerical simulations demonstrated the effectiveness of the approach proposed.

8 Appendix 1

The matrix inequalities of Theorem 1 as follow:

$$\begin{aligned}
 R_1(\cdot) &= \begin{bmatrix} X_j(\cdot) - B_j(\underline{\lambda}) S(\cdot) B_j^T(\underline{\lambda}) & * \\ D_2(\underline{\eta}) Y(\cdot) & S(\cdot) \end{bmatrix} > 0, \\
 &\quad j = 1, \dots, 2^m \\
 R_2(\cdot) &= \begin{bmatrix} W(\cdot) & * \\ \underline{\eta}_{(i)} Y_{(i)}(\cdot) & \Psi_{(i)}^2 \end{bmatrix} \geq 0, \quad i = 1, \dots, m \\
 R_3(\cdot) &= \begin{bmatrix} 1 & v_s^T \\ v_s & W(\cdot) \end{bmatrix} \geq 0, \quad s = 1, \dots, n_v \\
 R_4(\cdot) &= \begin{bmatrix} W(\cdot) & * \\ (-N_{(i)}(\cdot) + \underline{\eta}_{(i)} M_{(i)}(\cdot)) \left(\frac{\Omega_{(i)}}{\underline{\lambda}_{(i)}} \right)^2 \end{bmatrix} \geq 0, \\
 &\quad i = 1, \dots, m \\
 R_5(\cdot) &= \begin{bmatrix} W(\cdot) & * \\ (-N_{(i)}(\cdot) + M_{(i)}(\cdot)) \left(\frac{\Omega_{(i)}}{\underline{\lambda}_{(i)}} \right)^2 \end{bmatrix} \geq 0, \quad i = 1, \dots, m \\
 R_6(\cdot) &= Q(\underline{\lambda}, \cdot) + 2\mu W(\cdot) < 0, \quad j = 1, \dots, 2^m \quad (22)
 \end{aligned}$$

where

$$\begin{aligned}
 D_2(\underline{\eta}) &= D(0.5(1_m - \underline{\eta})), \quad N_{(i)}(\cdot) = [\mathbf{0} \quad T_{(i)}] W(\cdot), \\
 M_{(i)}(\cdot) &= T_{(i)} Y(\cdot) \\
 X_j &= -W(\cdot) A_j^T(\underline{\lambda}, \cdot) \\
 &\quad - B_j(\underline{\lambda}) D_1(\underline{\eta}) Y(\cdot) - A_j(\underline{\lambda}, \cdot) W(\cdot) - Y^T(\cdot) D_1(\underline{\eta}) B_j^T(\underline{\lambda}) \\
 Q(\underline{\lambda}, \cdot) &= A_j(\underline{\lambda}, \cdot) W(\cdot) + W(\cdot) A_j^T(\underline{\lambda}, \cdot) + B_j(\underline{\lambda}) Y(\cdot) \\
 &\quad + Y^T(\cdot) B_j^T(\underline{\lambda})
 \end{aligned}$$

9 Appendix 2

The Proof of Theorem 1 as follow:

Proof For notational simplicity consider: $A_j(\underline{\lambda}, \cdot) = A_j$, $B_j(\underline{\lambda}) = B_j$ and $\lambda_j(z(t)) = \lambda_j$.

Consider the set $\varepsilon \triangleq \{z \in \mathbb{R}^{n+m} \mid z^T W^{-1} z \leq 1\}$, $(P \triangleq W^{-1})$ and the regions $R_{Lp}(\underline{\eta})$ and $R_{Lr}(\underline{\lambda})$ defined respectively in (11) and (12), and $K(\cdot) \triangleq Y(\cdot) W^{-1}(\cdot)$. Satisfaction of conditions R_1 to R_6 lead to the following facts:

- I. Pre and post-multiplying R_1 by $\text{diag}(P(\cdot), I)$, considering $K(\cdot) = Y(\cdot) W^{-1}(\cdot)$ then:

$$\begin{bmatrix} P(\cdot) X_j(\cdot) P(\cdot) - P(\cdot) B_j(\underline{\lambda}) S(\cdot) B_j^T(\underline{\lambda}) P(\cdot) & * \\ D_2(\underline{\eta}) K(\cdot) & S(\cdot) \end{bmatrix} > 0$$

Applying Schur's complement it follows that R_1 is equivalent to

$$\begin{bmatrix} L_j - K^T(\cdot) D_2(\underline{\eta}) S^{-1}(\cdot) D_2(\underline{\eta}) K(\cdot) & P(\cdot) B_j(\underline{\lambda}) \\ B_j^T(\underline{\lambda}) P(\cdot) & S^{-1}(\cdot) \end{bmatrix} > 0 \quad (23)$$

Here $L_j = -(A_j(\underline{\lambda}, \cdot) + B_j(\underline{\lambda}) D_1(\underline{\eta}) K(\cdot))^T P(\cdot) - P(\cdot) (A_j(\underline{\lambda}, \cdot) + B_j(\underline{\lambda}) D_1(\underline{\eta}) K(\cdot))$.

Since R_1 holds for $j = 1, \dots, 2^m$ by convexity one obtains:

$$\sum_{j=1}^{2^m} \lambda_j \left(\begin{aligned} & z^T(t) L_j(\cdot) z(t) + z^T(t) P(\cdot) B_j(\underline{\lambda}) q(t) \\ & + q^T(t) B_j^T(\underline{\lambda}) P(\cdot) z(t) - \\ & \sum_{i=1}^m \sigma_{(i)}(\cdot) [z^T(t) \underline{\theta}_{(i)} K_{(i)}^T(\cdot) K_{(i)}(\cdot) \\ & \quad \times \underline{v}_{(i)} z(t) - q_{(i)}^2(t)] \end{aligned} \right) > 0 \quad (24)$$

$\forall z(t), q(t)$ such that $\sum_{j=1}^{2^m} \lambda_j = 1$, with $\sigma_{(i)}(\cdot)$ denoting the i^{th} diagonal component of $S^{-1}(\cdot)$.

- II. Applying the complement of Schur R_2 and R_4 ensure that $\varepsilon \subset R_{Lp}(\underline{\eta}) \cap R_{Lr}(\underline{\lambda})$:

- III. R_3 ensures that $z_0 \subset \varepsilon$.

- IV. R_6 ensures that the poles of the close-loop system are placed in the region D_p defined in (26) [26].

Using the variable change (14), if (13) is verified, that is $\forall z(t) \in (R_{Lr}(\underline{\lambda}) \cap R_{Lp}(\underline{\eta}))$, $\dot{z}(t)$ can be computed by:

$$\begin{aligned} \dot{z}(t) &= \sum_{j=1}^{2^m} \lambda_j (A_j(\cdot) + B_j D_1(\underline{\eta}) K(\cdot)) z(t) \\ &\quad + \sum_{j=1}^{2^m} \lambda_j B_j D(v(t)) K(\cdot) z(t) \mid v_{(i)}(t) \mid \leq v_{(i)} \quad (25) \end{aligned}$$

Suppose now that $z(t) \in z_0$, since $R_2 \sim R_5$ are verified we have that $z(t) \in \varepsilon \subset (R_{Lr}(\underline{\lambda}) \cap R_{Lp}(\underline{\eta}))$ such that $|v_{(i)}(t)| \leq v_{(i)}, i = 1, \dots, m$ and therefore $\dot{z}(t)$ can be computed by (25). Consider now that $q(t) = -D(v(t)) K(\cdot) z(t)$. Since, $|v_{(i)}(t)| \leq v_{(i)}$ it follows that $z^T(t) \underline{v}_{(i)} K_{(i)}^T(\cdot) K_{(i)}(\cdot) \underline{v}_{(i)} z(t) \geq q_{(i)}^2(t)$ (26)

Substituting $q(t)$ in (24) and take into account (23) and the fact that $\sigma_{(i)} \geq 0$ one obtains

$$z^T(t) \left[\begin{aligned} & \sum_{j=1}^{2^m} \lambda_j (A_j(\cdot) + B_j D_1(\underline{\eta}) K(\cdot) + \\ & \quad B_j D(v(t)) K(\cdot)) P(\cdot) \\ & + P(\cdot) \sum_{j=1}^{2^m} \lambda_j (A_j(\cdot) + B_j D_1(\underline{\eta}) K(\cdot) \\ & \quad + B_j D(v(t)) K(\cdot)) P(\cdot) \end{aligned} \right] z(t) < 0.$$

And therefore, from (25) one has

$$\dot{z}^T(t) P(\cdot) z(t) + z^T(t) P(\cdot) \dot{z}(t) < 0.$$

If conditions $R_1 \sim R_6$ are satisfied with $K(\cdot) = Y(\cdot) W^{-1}(\cdot)$, it follows that $\forall z(0) \in z_0$ the corresponding trajectory dose not leave ε and converges asymptotically to the origin. Furthermore the poles of $(A(\cdot) + BK(\cdot))$ are placed in D_p guaranteeing the performance specification in the linearity region R_L . $\square$

10 Appendix 3

The matrix inequalities of Theorem 3 as follow:

$$R_{1k}(\cdot) := \sum_{\substack{k' \in \mathbf{K}(d) \\ k \geq k'}} \sum_{\substack{i \in \{1, \dots, N\} \\ k_i \geq k'_i}} \frac{d!}{\pi(k')} \\ \times \begin{bmatrix} -A_{ji}(\underline{\lambda}) W_{k-k'-e_i} - W_{k-k'-e_i} A_{ji}^T(\underline{\lambda}) \\ -B_j(\underline{\lambda}) D_1(\underline{\eta}) Y_{k-k'-e_i} - Y_{k-k'-e_i}^T D_1(\underline{\eta}) B_j^T(\underline{\lambda}) \\ -B_j(\underline{\lambda}) S_{k-k'-e_i} B_j^T(\underline{\lambda}) \\ D_2(\underline{\eta}) Y_{k-k'-e_i} \end{bmatrix} \begin{matrix} * \\ * \\ * \\ S_{k-k'-e_i} \end{matrix} > 0, \\ \forall k \in \mathbf{K}(g+d+1), j=1, \dots, 2^m$$

$$R_{2k}(\cdot) := \sum_{\substack{k' \in \mathbf{K}(d) \\ k \geq k'}} \frac{d!}{\pi(k')} \begin{bmatrix} W_{k-k'} & * \\ \underline{\eta}_{(l)} [Y_{k-k'}]_{(l)} & \varepsilon_i^\ell(g, k') \Psi_{(l)}^2 \end{bmatrix} \geq 0,$$

$$\forall k \in \mathbf{K}(g+d), l=1, \dots, m$$

$$R_{3k}(\cdot) := \sum_{\substack{k' \in \mathbf{K}(d) \\ k \geq k'}} \frac{d!}{\pi(k')} \begin{bmatrix} \varepsilon_i^\ell(g, k') & * \\ \varepsilon_i^\ell(g, k') v_s & W_{k-k'} \end{bmatrix} \geq 0,$$

$$\forall k \in \mathbf{K}(g+d)$$

$$R_{4k}(\cdot) := \sum_{\substack{k' \in \mathbf{K}(d) \\ k \geq k'}} \frac{d!}{\pi(k')} \\ \times \begin{bmatrix} W_{k-k'} \\ -[\mathbf{0} \ T_{(l)}] W_{k-k'} + \underline{\eta}_{(l)} T_{(l)} Y_{k-k'} \end{bmatrix} \begin{matrix} * \\ \varepsilon_i^\ell(g, k') (\Omega/\underline{\lambda})_l^2 \end{matrix} \geq 0,$$

$$\forall k \in \mathbf{K}(g+d), l=1, \dots, m$$

$$R_{5k}(\cdot) := \sum_{\substack{k' \in \mathbf{K}(d) \\ k \geq k'}} \frac{d!}{\pi(k')} \\ \times \begin{bmatrix} W_{k-k'} \\ -[\mathbf{0} \ T_{(l)}] W_{k-k'} + T_{(l)} Y_{k-k'} \end{bmatrix} \begin{matrix} * \\ \varepsilon_i^\ell(g, k') (\Omega/\underline{\lambda})_l^2 \end{matrix} \geq 0,$$

$$\forall k \in \mathbf{K}(g+d), l=1, \dots, m$$

$$R_{6k}(\cdot) := \sum_{\substack{k' \in \mathbf{K}(d) \\ k \geq k'}} \sum_{\substack{i \in \{1, \dots, N\} \\ k_i \geq k'_i}} \frac{d!}{\pi(k')}$$

$$\times \begin{bmatrix} A_{ji}(\underline{\lambda}) W_{k-k'-e_i} + W_{k-k'-e_i} A_{ji}^T(\underline{\lambda}) + B_j(\underline{\lambda}) Y_{k-k'-e_i} \\ + Y_{k-k'-e_i}^T B_j^T(\underline{\lambda}) + 2\mu W_{k-k'-e_i} \end{bmatrix} < 0$$

$$\forall k \in \mathbf{K}(g+d+1), j=1, \dots, 2^m$$

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