The birth of a School

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Abstract At the end of the 1880s, Corrado Segre directed Guido Castelnuovo to the study of geometry on algebraic curves: this heralded the birth of the Italian School of algebraic geometry, and Castelnuovo was to become the first of Segre's many disciples, direct or indirect. Just a short time later, Federico Enriques came into contact with Segre, attracted by the young master's reputation and by the liveliness of the Torino milieu. He was instead destined for Rome, where his study of surfaces was supervised by Castelnuovo. Segre followed his results at a distance, while in the meantime attempting to derive a rigorous proof of the resolution theorem of singularities. The acquaintances of Segre, Castelnuovo and Enriques transformed into a scientific fellowship and they even planned to collaborate on the writing of a general treatise, but this was never realised.

Keywords Castelnuovo G · Enriques F · Higher geometry · Singularities · Algebraic surfaces

Dear Guido, I have received your affectionate letter and I thank you for it. Since Monday you have been gone and I feel it keenly. You mention that bit of good that my company did you in these 4 years. If this is true, it is also true that I have been repaid in full by you, and that your acute mind, like your goodness of heart, rendered continuously useful and pleasurable the many hours that we spent together. I

was accustomed to thinking of you as a little brother: little, I mean, in terms of years of experience (which justified my tendency to counsel you on various occasions). Or perhaps, exaggerating in my mind the differences in our age and characters, there were sometimes in me towards you the feelings of a father towards a son: and I felt proud of the praise you received, proud to see you esteemed as you deserved to be ... You did me good, I repeat, not only intellectually but morally as well. And now that I miss you I feel a genuine emptiness, which no one can fill. Let us at least preserve, even at a distance, our friendship unaltered: the friendship of two youths who, above the egotism of the Philistines, place their ideals in goodness, honesty and worship of science (Segre to Castelnuovo, 12 November 1891).¹

We are in the autumn of 1891: Guido Castelnuovo (1865–1952) has transferred to Rome, having won a competition for a chair, and Segre sends him words of esteem, affection and nostalgia that reveal feelings of profound friendship. Castelnuovo had arrived to Torino in 1887 thanks to the good offices of Segre, who had helped him obtain a position as an assistant after the year of post-graduate studies spent in Rome. Just 2 years older, Segre (then an assistant to the chair of projective and descriptive geometry) took responsibility for the young man whose progress he had begun to follow when he was still a student at the University of Padua. Castelnuovo was certain that he found in Segre an attentive and reliable guide. He had proof of this when he had sent him his first articles on the

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¹ Digital reproductions of the letters cited herein are available on the website of the Accademia dei Lincei, http://archivi-matematici.lincei.it/Castelnuovo/Lezioni_E_Quaderni/menu.htm (accessed 9 December 2013).

projective geometry of hyperspace, of which Segre did such a detailed revision that he was rather awestruck: ‘You are right: that of praising works without having read them is a system as terrible as it is widespread. Instead, I have the habit of reading all the memoirs that I receive and of sending my criticisms to the authors: I do unto others as I would have them do unto me’ (Segre to Castelnuovo, 29 July 1886). This was the Segre’s response to him, sending his extracts of some of his own works and advising him to study several recent publications. The tone of this young *maestro*, at once serious and severe, is a reflection of the tragic events that had afflicted his family, and which had caused him to mature quickly and forged some of his traits, even while preserving his youthful romantic idealism.

In a brief article of 1887, Segre placed the properties of a linear systems of plane curves in relation to those of points cut out by the curves of the system on the generic curve of the system, thus shifting attention to the ‘linear series of groups of points’ and showing interest in considering what would come to be called the ‘characteristic series’ of a the system. Upon his arrival in Torino, Castelnuovo was directed by Segre to the study of linear series, that is, towards the study of geometry on a curve. Together they gave a hyperspatial interpretation of the algebraic theory of linear series, set forth by Alexander Brill (1842–1935) and Max Nöther (1882–1935), illustrating the originality and potential of the methods that characterised what would come to be identified as the Italian School of algebraic geometry. Geometry on a curve, following the new line of study, was the object of the Course in Higher Geometry given by Segre—who in the meantime had been appointed to the chair—during the year 1890–1891. The lessons were attended by Gino Fano (1871–1952), who had enrolled in mathematics in 1888, and to whom Segre had entrusted the task of translating an old article by Felix Klein (1849–1925), which would become known in the history of mathematics by the name of the Erlangen Program, and remembered for the interpretation of mathematics that it presented. The young Segre’s adhesion to the modern concept of geometry as expressed in the Program is testified to by the first works of his scientific production, and by his letters to Klein. In November 1889, Segre wrote to Klein about his intention to publish a translation with commentary of the inaugural address that the German mathematician had given on the occasion of his installation at the University of Erlangen in 1872, because in his opinion it had not received the recognition it deserved. In entrusting its translation to Fano, he oriented his first early research works.

Towards the end of the 1880s and the early years of the 1890s, Torino became a magnet for brilliant young scholars. Segre was only 28 years old when, in 1891, he wrote the article entitled “Su alcuni indirizzi delle

investigazioni geometriche”, with the aim of guiding young people who wished to undertake research. He wrote and worked with the awareness of a master whose objectives transcended his personal ambitions and aimed at creating new stimuli. Communication about science was helped by his young age, which allowed him to establish relationships that were less formal. The conversations continued even outside the strict confines of academia, in Torino’s coffeehouses. The climate was made even more lively by the polemics regarding the problem of foundations and rigour in mathematics in which Giuseppe Peano (1858–1932) was a leading figure, and which even Segre was involved.

At the end of his post-degree studies, Federigo Enriques (1871–1946), who had graduated from Pisa’s prestigious Scuola Normale Superiore in 1891, made contact with Segre with the aim of moving to Torino to study under his guidance. The young Enriques was attracted to the Torinese milieu, the stimulating air that was breathed there, saturated with discussions that flowed into the philosophy of science, for which Enriques had nourished a passion since high school: ‘Enriques is a nice young man, who comes each evening to wait for me nearby (with whom I then find myself with d’Ovidio, Korsenberg, Foà, etc.) to discuss various things’. To Enriques’s disappointment he was assigned to Rome, where his steps would be guided by Castelnuovo, to whom Segre spoke openly of the young man’s defects: ‘He has great need of more studies and above all of someone who would insist in making him acquire greater circumspection. He makes errors that are truly gross as an effect of thinking too lightly. Negligent in research, as in exposition, you will have a lot to do with this young man’ (Segre to Castelnuovo, 16 November 1892). This is certainly not a promising letter of introduction: had it been written by anyone else, it would have been a complete and perhaps irremediable criticism. However, Castelnuovo was able to govern Enriques’s impulsiveness, charging him with the task of extending what had been done for curves and founding geometry on a surface. Enriques’s almost visionary intuition, nourished by much study, flanked by the collaboration of Castelnuovo and reassured by his encouraging and well-meaning attitude, produced extraordinary results in just a few months. Segre was appreciative of the work done in the semester spent in Rome, which, in the form of a memoir entitled “Ricerche di geometria sulle superficie algebriche”, he presented to the Academy of Science in Torino in June 1893 with glowing terms of praise, in spite of the many questions that were still open. Part of what made Segre so well-disposed was undoubtedly the contribution of his friend Castelnuovo: ‘The sympathy that Enriques inspires in me and what you have already told me about his work are such that I quite willingly accept to present it, if you

wish me to' (Segre to Castelnuovo, 16 May 1893). Nevertheless, Enriques in this years remained somewhat anxious about how Segre judged him.

Given the rapid developments of the theory of algebraic surfaces, Segre harboured the idea of dedicating the Course of Higher Geometry to it in 1894–1895, but in September, setting himself to work on drafting the notes, he became aware that Castelnuovo and Enriques would have rendered 'outdated in a few months what could be done now' (Segre to Castelnuovo, 13 September 1894). He thus decided to choose 'some chapter that is not as imperfect as others'. With this in mind, though it would prove illusory, he decided to dedicate his course to the singularities of surfaces, in part to see 'how far it can be kicked away'. With regard to plane algebraic curves, techniques had already been formalised to analyse singularities, and the relative resolution theorems had already been proved. With regard to algebraic surfaces, there was a feeling that, roughly speaking, those techniques would work, or were easily extendible and adaptable to singularities of surfaces. Enriques was convinced of this, but he was quite happy to leave a more detailed investigation to Segre. The question to be dealt with, in a definitive way, was the possibility of birationally transforming any surface into a surface without singularities into a projective space P_n , of a suitable dimension n , from which by projection is obtained a surface of P_3 with only *ordinary singularities*. That is what Castelnuovo and Enriques were pressing for, because this is the assumption that the theory of the linear systems of curves on surfaces is founded on. Segre first wanted to verify if it was possible 'with Cremonian transformations to reduce the surface to only ordinary singularities'. The question had already been dealt with by Pasquale Del Pezzo (1859–1936), 'with remarkable superficiality of thought', in Segre's opinion, and by G. Kobb, who seemed to him, however, to 'not have exhausted the question' (Segre to Castelnuovo, 23 September 1894). In the space of just a few days he wrote again to his friend: 'I must immediately tell you that I have not concluded anything!': in effect, Kobb, in applying successive quadratic transformations in way that was similar to what had been done for curves, did not notice 'that these can produce higher singularities' and the 'same oversight' made by Del Pezzo 'is in itself sufficient (without mentioning others) to invalidate all of his so-called proof of the reducibility of higher singularities by means of birational transformations of space'. He thus reiterates his promise to 'look again at the other, older work of Del Pezzo on the reduction, by means of only birational correspondences between surfaces, to a surface with double line and triple points [i.e., ordinary singularities]' only in the hope of being able to derive something, adding 'but I greatly fear I cannot' (Segre to Castelnuovo, 2 October 1894). Just over a week later he wrote, 'From

Del Pezzo's work on univocal transformation of a surface in a surface in a suitable hyperspace without multiple points ... there is nothing to be derived' (Segre to Castelnuovo, 10 October 1894) and assures his friend that he will continue to think about it. The month of October passed without results, and a certain impatience was felt on the part of Castelnuovo. Segre was distracted by exams and the preparations for his course, but also by fatherhood, whose pleasures he wasn't willing to give up: 'When I arrive home I find an adorable little girl who holds out her arms and cries if I don't pick her up and sit to play with her'. The drafting of the 'work that you desire', he wrote, 'is perhaps for the end of the year' (Segre to Castelnuovo, 29 October 1894). In effect, by the end of December Segre was convinced that he had gotten to the bottom of the thorny problem. He wrote to Castelnuovo that in his paper 'there will also be made explicit the theorem that a surface can be referred to a surface in a hyperspace without multiple points', telling him to alert Enriques and to ask the journal in which their work was to be published to 'save 12 to 20 pages of space: because I would be very pleased to have it come out in your company. That way all of our school would be represented' (Segre to Castelnuovo, 9 February 1895). But the proofs did not stand up to the final revisions and, embittered, he could only say, 'Don't wait for me any longer you two!' (Segre to Castelnuovo, 18 March 1895). Singularities of surfaces continued to engage his attention for a long time, until at the end of 1896, when the memoir 'Sulla scomposizione dei punti singolari delle superficie algebriche' finally went to print: 'My dearest friends. I am writing to Enriques and Castelnuovo at the same time. ... I think you also for the opinion that you sent me regarding my drafts, but in this regard I have to ask you to then read the whole of my work, when you receive the extracts (I hope in just a few days)' (Segre to Castelnuovo, 30 December 1896). Segre was able to extend to surfaces the concept of 'infinitely near multiple points', a fundamental instrument for the analysis of singular points. With regard to the resolution theorem, as his letter shows, he came very close but decided to leave it the merit to his new student, Beppo Levi (1875–1961), who had helped him in revising the galleys and to whom he had assigned the task of 'exhausting a certain reasoning', and who, he maintained, had succeeded. Levi's proof was published the following year: it would be held satisfactory until the criticisms of Oscar Zariski (1899–1986) appeared in 1935.

This dramatic unfolding of events impeded Segre from entering into the heart of research in algebraic surfaces and contributing to the theory that in the meantime his friends had constructed. The international acclaim for the results on algebraic curves and surfaces, the awareness of having created a School, led him to think of compiling 'a great treatise on higher geometry' (Segre to Castelnuovo, 30

December 1896), planned since Castelnuovo's years in Torino. 'Now that a new, energetic element is introduced, the thing becomes much easier', he wrote in response to Enriques's queries, adding:

I completely approve the idea, and believe that between the three of us could be made a good treatise, harmonic and original. Whatever form of collaboration is adopted, I already have a sure way to take part in the work: and that is with the summaries (compiled by me) of various of my courses: that is, on some general courses on algebraic curves, surfaces, etc., on special courses on higher singularities, on the geometry on a curve, on rational surfaces and linear systems of plane curves, etc. If the undertaking is delayed by a few years, some new course of mine might also serve. ... I then fully approve that the treatise be by the three of us and not others: because we three are fully uniform of ideas; and others might perhaps perturb the harmony of the work.

The treatise never saw the light, perhaps because Segre's concerns about Enriques's career advised 'delaying the undertaking still a few years more'; perhaps because 'it is better that he dedicate his scientific efforts only to original research'; perhaps because he was distracted by his work for

the German encyclopaedia, *Encyklopädie der Mathematischen Wissenschaften*, work on which had begun in this period.

Translated from the Italian by Kim Williams.

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