Erratum to: A Donsker Delta Functional Approach to Optimal Insider Control and Applications to Finance

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Unfortunately, in the original publication of the article some equations appeared incorrectly. The corrected equations are given below:

- In (1.1) it should be added that the filtration $\mathbb{H}$ is assumed to be right-continuous, i.e., that

$$\mathcal{H}_t = \mathcal{H}_{t+} = \bigcap_{s>t} \mathcal{H}_s.$$

- The third line of (1.4) should read

$$\gamma(t, x, u, y, \zeta) = \gamma(t, x, u, y, \zeta, \omega) : [0, T_0] \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \Omega \mapsto \mathbb{R}$$

- In (2.5), which reads

$$Y = Y(T_0); \text{ where } Y(t)$$

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$$= \int_0^t \beta(s) dB(s) + \int_0^t \int_{\mathbb{R}} \psi(s, \zeta) \tilde{N}(ds, d\zeta), \text{ for } t \in [0, T_0],$$

we should add the assumption that $\beta \neq 0$.

- In the formulas (2.7)–(2.14) and in (6.107), (6.109), we should replace β by $i\beta$, where $i = \sqrt{-1}$.

When this is done, the correct version of (2.7) becomes

$$\begin{aligned} \delta_Y(y) = & \frac{1}{2\pi} \int_{\mathbb{R}} \exp^{\diamond} \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s, \zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s) dB(s) \right. \\ & \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s, \zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds - ixy \right] dx, \end{aligned}$$

and similarly, the correct versions of (2.8)–(2.14) become as follows:

$$\begin{aligned} \mathbb{E}[\delta_Y(y) | \mathcal{F}_t] &= \frac{1}{2\pi} \int_{\mathbb{R}} \mathbb{E} \left[\exp^{\diamond} \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s, \zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s) dB(s) \right. \right. \\ &\quad \left. \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s, \zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds - ixy \right] \middle| \mathcal{F}_t \right] dx \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} \exp^{\diamond} \left[\mathbb{E} \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s, \zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s) dB(s) \right. \right. \\ &\quad \left. \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s, \zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds - ixy \middle| \mathcal{F}_t \right] \right] dx \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} \exp^{\diamond} \left[\int_0^t \int_{\mathbb{R}} (e^{ix\psi(s, \zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^t ix\beta(s) dB(s) \right. \\ &\quad \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s, \zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds - ixy \right] dx \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} \left\{ \exp^{\diamond} \left[\int_0^t \int_{\mathbb{R}} (e^{ix\psi(s, \zeta)} - 1) \tilde{N}(ds, d\zeta) \right] \right\} \diamond \left\{ \exp^{\diamond} \left[\int_0^t ix\beta(s) dB(s) \right] \right\} \\ &\quad \diamond \left\{ \exp^{\diamond} \left[\int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s, \zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds - ixy \right] \right\} dx \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} \exp \left[\int_0^t \int_{\mathbb{R}} ix\psi(s, \zeta) \tilde{N}(ds, d\zeta) + \int_0^t ix\beta(s) dB(s) \right. \\ &\quad \left. + \int_t^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s, \zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) ds - \int_t^{T_0} \frac{1}{2} x^2 \beta^2(s) ds - ixy \right] dx \end{aligned} \quad (2.8)$$

Here we have used that (see e.g. [11, Lemma 3.1])

$$\exp^\diamond \left[\int_0^{T_0} ix\beta(s)dB(s) \right] = \exp \left[\int_0^{T_0} ix\beta(s)dB(s) + \frac{1}{2} \int_0^{T_0} x^2\beta^2(s)ds \right] \quad (2.9)$$

and

$$\begin{aligned} & \exp^\diamond \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) \right] \\ &= \exp \left[\int_0^{T_0} \int_{\mathbb{R}} ix\psi(s, \zeta) \tilde{N}(ds, d\zeta) - \int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s, \zeta))v(d\zeta) \right] \end{aligned} \quad (2.10)$$

We proceed to find

$$\mathbb{E}[D_{t,z}\delta_Y(y)|\mathcal{F}_t],$$

where $D_{t,\zeta}$ denotes the Hida–Malliavin derivative at $(t, \zeta) \in [0, T] \times \mathbb{R}$ with respect to the Poisson random measure N :

First, note that

$$\begin{aligned} D_{t,z}\delta_Y(y) &= \frac{1}{2\pi} \int_{\mathbb{R}} D_{t,z} \exp^\diamond \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s)dB(s) \right. \\ &\quad \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s, \zeta))v(d\zeta) - \frac{1}{2}x^2\beta^2(s) \right\} ds - ixy \right] dx \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} \exp^\diamond \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s)dB(s) \right. \\ &\quad \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s, \zeta))v(d\zeta) - \frac{1}{2}x^2\beta^2(s) \right\} ds - ixy \right] \\ &\quad \diamond D_{t,z} \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s)dB(s) \right. \\ &\quad \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s, \zeta))v(d\zeta) - \frac{1}{2}x^2\beta^2(s) \right\} ds - ixy \right] dx \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} \exp^\diamond \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s)dB(s) \right. \\ &\quad \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s, \zeta))v(d\zeta) - \frac{1}{2}x^2\beta^2(s) \right\} ds - ixy \right] \\ &\quad \times (e^{ix\psi(t,z)} - 1) dx. \end{aligned} \quad (2.11)$$

Here we have used that

$$D_{t,z} \int_0^T \beta(s) dB(s) = 0,$$

which follows from our assumption that B and $\tilde{N}$ are independent, so for $D_{t,z}$ the random variable $B(s)$ is like a constant.

Using Eq. (2.11) and the Wick chain rule we get

$$\begin{aligned} & \mathbb{E}[D_{t,z} \delta_Y(y) | \mathcal{F}_t] \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} \mathbb{E} \left[\exp^{\diamond} \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s) dB(s) \right. \right. \\ & \quad \left. \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s,\zeta)) v(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds - ixy \right] \right. \\ & \quad \left. \times (e^{ix\psi(t,z)} - 1) dx \middle| \mathcal{F}_t \right] dx \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} \mathbb{E} \left[\exp^{\diamond} \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s) dB(s) \right. \right. \\ & \quad \left. \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s,\zeta)) v(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds - ixy \right] \middle| \mathcal{F}_t \right] \\ & \quad \times (e^{ix\psi(t,z)} - 1) dx \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} \exp \left[\int_0^t \int_{\mathbb{R}} ix\psi(s,\zeta) \tilde{N}(ds, d\zeta) + \int_0^t ix\beta(s) dB(s) \right. \\ & \quad \left. + \int_t^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s,\zeta)) v(d\zeta) ds - \int_t^{T_0} \frac{1}{2} x^2 \beta^2(s) ds - ixy \right] \\ & \quad \times (e^{ix\psi(t,z)} - 1) dx. \end{aligned} \quad (2.12)$$

Next we want to find

$$\mathbb{E}[D_t \delta_Y(y) | \mathcal{F}_t],$$

where D_t denotes the Hida–Malliavin derivative at t with respect to Brownian motion B :

Note that:

$$\begin{aligned} & D_t \delta_Y(y) \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} D_t \exp^{\diamond} \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s) dB(s) \right. \\ & \quad \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s,\zeta)) v(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds - ixy \right] dx \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2\pi} \int_{\mathbb{R}} \exp^{\diamond} \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s) dB(s) \right. \\
&\quad \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds - ixy \right] \\
&\quad \diamond D_t \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s) dB(s) \right. \\
&\quad \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds - ixy \right] dx \\
&= \frac{1}{2\pi} \int_{\mathbb{R}} \exp^{\diamond} \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s) dB(s) \right. \\
&\quad \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds - ixy \right] ix\beta(t) dx.
\end{aligned} \tag{2.13}$$

Here we have used that

$$D_t \int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) = 0,$$

which follows from the assumption that B and $\tilde{N}$ are independent, so for D_t the random variable $\tilde{N}(s, \zeta)$ is like a constant.

Using Eq. (2.13) and the Wick chain rule we get

$$\begin{aligned}
&\mathbb{E}[D_t \delta_Y(y) | \mathcal{F}_t] \\
&= \frac{1}{2\pi} \int_{\mathbb{R}} \mathbb{E} \left[\exp^{\diamond} \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s) dB(s) \right. \right. \\
&\quad \left. \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds \right. \right. \\
&\quad \left. \left. - ixy \right] ix\beta(t) dx | \mathcal{F}_t \right] dx \\
&= \frac{1}{2\pi} \int_{\mathbb{R}} \mathbb{E} \left[\exp^{\diamond} \left[\int_0^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1) \tilde{N}(ds, d\zeta) + \int_0^{T_0} ix\beta(s) dB(s) \right. \right. \\
&\quad \left. \left. + \int_0^{T_0} \left\{ \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) - \frac{1}{2} x^2 \beta^2(s) \right\} ds \right. \right. \\
&\quad \left. \left. - ixy \right] | \mathcal{F}_t \right] ix\beta(t) dx \\
&= \frac{1}{2\pi} \int_{\mathbb{R}} \exp \left[\int_0^t \int_{\mathbb{R}} ix\psi(s, \zeta) \tilde{N}(ds, d\zeta) + \int_0^t ix\beta(s) dB(s) \right. \\
&\quad \left. + \int_t^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s,\zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) ds - \int_t^{T_0} \frac{1}{2} x^2 \beta^2(s) ds - ixy \right] ix\beta(t) dx.
\end{aligned} \tag{2.14}$$

- (2.16) should be corrected to

$$\|\beta\|_{[t,T]}^2 := \int_t^T \beta(s)^2 ds > 0 \quad \text{for all } t \in [0, T].$$

- The expression after the last equality sign in (2.19) should be corrected to

$$-\left(2\pi\|\beta\|_{[t,T_0]}^2\right)^{-\frac{1}{2}} \exp\left[-\frac{(Y(t)-y)^2}{2\|\beta\|_{[t,T_0]}^2}\right] \frac{Y(t)-y}{\|\beta\|_{[t,T_0]}^2} \beta(t).$$

- Section 2.3 should be deleted, because the formulas (2.2)–(2.22) in Sect. 2.3 are not correct as stated. The reason is that with $\beta = 0$ in (2.7) these integrals do not converge. However, the integrals do converge for all $\beta \neq 0$, and it can be proved that the limit of the quotient $\psi(t, y) := \frac{(2.12)}{(2.8)}$ with $\psi = 1$ exists when $\beta \rightarrow 0$. This limit is the quantity that plays a key role in the corresponding optimal portfolio problem in Sects. 6.3 and 6.4. See (6.62) and (6.82).
- In the last line before (5.3), correct $L^2(\lambda \times P)$ to $L^2(\lambda \times P)$.
- In the statement of Theorem 5.1, and also in the proof, we should write $\frac{d}{da} J((u + a\beta)(\cdot, y))|_{a=0}$ in stead of $\frac{d}{da} J(u + a\beta)|_{a=0}$.
- (6.40) should be corrected to

$$\Pi^*(s) = \frac{b_0(s, Y(T_0))}{\sigma_0^2(s, Y(T_0))} + \frac{(Y(T_0) - Y(s))\beta(s)}{\sigma_0(s, Y(T_0))\|\beta\|_{[s,T_0]}^2}, \quad 0 \leq s \leq T < T_0.$$

- From (6.107) to (6.109) we should write:

Theorem 6.9

Suppose Y is as in (2.5). Then the processes $\phi(t, y)$ and $\psi(t, y, z)$ in the Eq. (6.106) for the optimal portfolio $\pi(t, y)$ have the following expressions:

$$\phi(t, y) = \frac{i\beta(t) \int_{\mathbb{R}} F(t, x, y) x dx}{\int_{\mathbb{R}} F(t, x, y) dx} \quad (6.107)$$

$$\psi(t, y, z) = \frac{\int_{\mathbb{R}} F(t, x, y) (e^{ix\psi(t,z)} - 1) dx}{\int_{\mathbb{R}} F(t, x, y) dx} \quad (6.108)$$

where

$$\begin{aligned} F(t, x, y) = & \int_{\mathbb{R}} \exp\left[\int_0^t \int_{\mathbb{R}} ix\psi(s, \zeta) \tilde{N}(ds, d\zeta) + \int_0^t ix\beta(s) dB(s) \right. \\ & \left. + \int_t^{T_0} \int_{\mathbb{R}} (e^{ix\psi(s, \zeta)} - 1 - ix\psi(s, \zeta)) \nu(d\zeta) ds \right. \\ & \left. - \int_t^{T_0} \frac{1}{2} x^2 \beta^2(s) ds - ixy\right] dx. \end{aligned} \quad (6.109)$$