Journal of Advanced Ceramics
2013, 2(2): 141–148
DOI: 10.1007/s40145-013-0053-1
ISSN 2226-4108

Research Article

Piezoelectric material selection for transducers under fuzzy environment

Gaurav VATS, Rahul VAISH*

School of Engineering, Indian Institute of Technology Mandi, Himachal Pradesh 175001, India

Received: January 18, 2013; Revised: February 27, 2013; Accepted: March 08, 2013

Abstract: Piezoelectric ceramics are extensively investigated materials for transducer application. The selection of optimal piezoelectric material for this particular application is a tedious task. It depends upon various physical properties, including piezoelectric charge coefficient (d_{33}), electromechanical coupling factor (K_p), dielectric constant (ε_r), and dielectric loss (tanδ). The classical multiple attribute decision making (MADM) can be used for decision making if these properties are known precisely. However, these properties cannot be expressed by exact numerical values, since they are dependent upon the microstructure and fabrication process. Fuzzy-based MADM approaches can be helpful in such cases. In this paper, we have determined the ranks and rank indices (for degree of closeness) of important piezoelectric materials using fuzzy VlseKriterijumska Optimisacija I Kompromisno Resenje (VIKOR) technique. PLZT(8/65/35) ($(Pb_{1-x}La_x)(Zr_yTi_{1-y})O_3$) and KNN–LT–LS ($(K_{0.44}Na_{0.52}Li_{0.04})$–$(Nb_{0.84}Ta_{0.10}Sb_{0.06})O_3$) consecutively are found to be the top-rank piezoelectric ceramics. This indicates that KNN–LT–LS can be used on behalf of lead-based piezo-ceramics.

Keywords: piezoceramics; selection; MADM; fuzzy approach; transducer application

1 Introduction

Technology is tremendously advancing every day. Among these advancing fields, growth in the fields of sensors, actuators and energy harvesting devices is very steep and more rapid than the predictions (Moore's law). Innovations in functional materials can be credited for this. Ferroelectric materials belong to the most renowned families of the functional materials. These materials are at the peak of research and have dragged the attention of technologists and researchers because of their excellent piezoelectric, pyroelectric and non-linear optical properties. Over years, many ferroelectric materials are developed, synthesized, fabricated, characterized and exploited for various industrial applications. Continuous studies are going on around the world in order to explore new materials with more suitable properties. A huge number of materials have been reported in this area [1–3]. These are further sub-divided into two categories of lead-based and lead-free piezoelectric ceramics. The lead zirconate titanate (PZT) family in lead-based piezoelectric ceramics [2], and (K,Na)NbO_3 (KNN), $(Bi_{0.5}Na_{0.5})TiO_3$ (BNT) and $(Bi_{0.5}K_{0.5})TiO_3$ (BKT) based materials among lead-free piezoelectric ceramics are the most popular due to their exceptionally good piezoelectric properties as compared to other reported materials till date [4–6]. It is noted that PZT-based ceramics make severe negative impacts on environment; KNN ceramic has some critical issues

---

* Corresponding author.
E-mail: rahul@iitmandi.ac.in

such as volatility of alkali-oxides, compositional inhomogeneity, poor densification and phase stability [7]. On the other hand, the properties of pure BNT and BKT ceramics are not promising but their solid solutions are sufficiently good for the technological applications [7]. The performance of any device is controlled by various physical properties associated with the materials. For example, in the case of piezoelectric transducer applications, piezoelectric charge coefficient (d_{33}), electromechanical coupling factor (K_p), dielectric constant (ε_r) and dielectric loss ($\tan\delta$) are important properties. However, it is noted that all suitable physical properties from the application point of view are rarely observed in one material. Due to this, researchers are left with no other option rather than enhancing the key parameters/properties by playing with fabrication/processing variables or compositional modifications. The improvement of less suitable parameters/properties is a tedious task. Sometimes it is observed that by altering the processing parameters, methods (physical or chemical) or both together for a material, one property may be boosted rapidly on expense of gradual decrease in other properties. Therefore, it becomes essential to find the materials with optimal characteristics using a compromised approach among all distinguished parameters. The selection of an optimal material from pool of alternative materials on the basis of two or more attributes/properties is a multiple attribute decision making (MADM) problem [8].

A variety of methods are reported under MADM category. These methods include simple additive weighting (SAW), analytic hierarchy process (AHP) [9], graph theory and matrix approach (GTMA) [10], VlseKriterijumska Optimisacija I Kompromisno Resenje (VIKOR) [11], technique for order preference by similarity to ideal solution (TOPSIS) [12] and many others. These methods have some advantages and disadvantages over others. MADM models are used to select the best alternative from a large number of alternatives for a set of selection criteria. Moreover, these also tell about the degree of closeness in terms of rank index. These have been successfully applied to various fields such as manufacturing processes, social science decisions, financial decisions and engineering problems. We have found that these methods are also efficient in material selection [13–26]. The above mentioned MADM approaches work on crisp values of attributes. However, in the case of material selection, most of the attributes/properties can be defined in intervals rather than crisp values because of their dependency on various factors, such as purity, microstructure and fabrication techniques. Material selection with interval values of properties can be dealt with fuzzy set theory aided with MADM approachs [17–23]. The aim of the present work is to select optimal piezoelectric materials for transducers under fuzzy environment using fuzzy VIKOR method.

2 Materials and methods

As discussed above, piezoelectric materials belong to extensively studied families of materials. Their various compositions with different properties are reported in the literature. Only the presence of piezoelectric properties does not make all of these materials viable for technological applications. Many factors simultaneously govern the suitability of a piezoelectric material for different applications. These factors can be sub-divided into two categories, namely primary and secondary factors. Primary factors include physical properties of a material, while secondary factors deal with cost, durability, toxicity, availability, ease and time of fabrication, environmental conditions, etc. Here we are much more concerned about the selection of materials with optimal primary properties. Among the important material properties for piezoelectric transducer applications, electromechanical coupling factor (K_p), dielectric constant (ε_r), dielectric loss ($\tan\delta$) and piezoelectric charge coefficient (d_{33}) are reported to be critical parameters. These are reported to be the key parameters for compositional engineering in order to increase the suitability for transducer applications [27]. High dielectric constant (ε_r), low dielectric loss ($\tan\delta$), high electromechanical coupling factor (K_p), and high piezoelectric charge coefficient (d_{33}) are desirable properties for piezoelectric applications. Vital piezo-ceramics along with their properties are listed in Table 1.

2.1 Modified digital logic

It is a fact that all the properties have different impacts on the performance of devices, and hence cannot be assigned equal weights for any application. So it becomes vital to find out the priority of each property. Modified digital logic (MDL) is one of the well-known techniques to find the weights for the properties [45]. It includes expert opinion to give initial priorities as 1, 2

Table 1 Piezoelectric materials' physical properties and ranks

Fuzzy-VIKOR rank index	Fuzzy-VIKOR rank	VIKOR rank index	VIKOR rank	Material	ε_r	tanδ (%)	K_p	d_{33} (pC/N)
0	1	0	1	PLZT(8/65/35) [2]	3400	0.030	0.65	682
0.174 307	2	0.475 838	2	KNN–LT–LS [44]	1650	0.024	0.48	340
0.255 851	3	0.566 390	3	KNN–LiSbO$_3$(5%) [36]	1288	0.019	0.50	283
0.286 857	4	0.628 284	4	PLZT(12/40/60) [37]	1300	0.013	0.47	235
0.328 695	5	0.688 128	6	0.7BNT–0.2BKT–0.1(Bi$_{0.5}$Li$_{0.5}$)TiO$_3$ [40]	1900	0.044	0.36	231
0.410 394	6	0.671 907	5	BaTiO$_3$ [35]	1700	0.005	0.36	190
0.458 412	7	0.701 702	8	NBT–KBT–LBT [39]	1550	0.034	0.40	216
0.483 007	8	0.743 092	10	KNN–Li(3%);Ta(20%) [41]	920	0.024	0.46	190
0.517 622	9	0.782 174	12	NKN–(Bi$_{0.5}$K$_{0.5}$)TiO$_3$(3%) [29]	850	0.040	0.45	192
0.554 914	10	0.739 268	7	0.92BNT–0.08BT+0.3wt%MnO [43]	1596	0.008	0.36	153
0.566 966	11	0.788 999	13	NBT–KBT–BT(MPB) [24]	730	0.020	0.33	173
0.584 519	12	0.754 449	11	BaTiO$_3$–CaTiO$_3$–Co [38]	1420	0.005	0.31	150
0.620 873	13	0.741 250	9	KNN–LiNbO$_3$(6%) [30]	500	0.040	0.42	235
0.626 584	14	0.796 833	14	KNN–Li(7%) [34]	950	0.080	0.45	240
0.680 084	15	0.798 781	15	KNN–LiTaO$_3$(5%) [31]	570	0.040	0.36	200
0.686 737	16	0.857 628	17	NBT–KBT(50%) [28]	825	0.030	0.22	150
0.701 540	17	0.874 010	18	NBT–KBT–BT [24]	820	0.030	0.16	145
0.755 968	18	0.916 994	19	BBT–KBT90 [1]	827	0.050	0.23	140
0.791 859	19	0.928 853	20	NKN–BaTiO$_3$(2%) [32]	1000	0.040	0.29	104
0.825 774	20	0.851 269	16	Na$_{0.5}$K$_{0.5}$NbO$_3$(HP) [25,26]	496	0.020	0.46	127
0.851 070	21	0.948 485	22	SBT–KBT90 [1]	870	0.040	0.15	110
0.856 080	22	0.947 955	21	SBT–KBT85 [1]	1000	0.050	0.16	120
0.992 947	23	0.995 181	25	Na$_{0.5}$K$_{0.5}$NbO$_3$ [33]	290	0.040	0.35	80
0.998 430	24	0.983 983	24	BBT–KBT80 [1]	630	0.040	0.15	95
0.999 998	25	0.963 870	23	PbNb$_2$O$_6$ [42]	225	0.010	0.07	85

and 3 for less, equally and more important properties, respectively. Based on the expert opinion, decision table is formed under pair-wise comparison. Prior to the formation of MDL table, one needs to estimate the number of possible positive decisions as $N=n(n-1)/n$, where n is the number of attributes/properties. Further summation of all positive decisions (P) for a particular property on normalization leads to final weight (W) as

$$W_j = \frac{P_j}{\sum_{j=1}^{n} P_j} \tag{1}$$

2. 2 VIKOR method

VIKOR method is a compromising-approach MADM model [11]. The analysis of VIKOR is highly accurate and provides closeness to real solution. It makes use of the utility weight, thus enabling different users to apply expert opinion. The normalization norms used in VIKOR are linear. Calculation of VIKOR index involves the following steps.

Step 1 Determination of ideal and negative ideal solutions.

The ideal solution f^* and the negative ideal solution f^- are determined as

$$f^* = \{(\max f_{ij}, j \in J) \text{ or } (\min f_{ij}, j \in J')\} \tag{2}$$

$$f^- = \{(\min f_{ij}, j \in J) \text{ or } (\max f_{ij}, j \in J')\} \tag{3}$$

where f_{ij} is the jth property of the ith material; J corresponds to benefit criteria and J' corresponds to cost criteria.

Step 2 Calculation of utility measure and regret measure.

$$S_i = \sum_{j=1}^{n} W_j \frac{f_j^* - f_{ij}}{f_j^* - f_j^-}; \forall i \tag{4}$$

$$R_i = \max_j \left(W_j \frac{f_j^* - f_{ij}}{f_j^* - f_j^-} \right); \forall i \tag{5}$$

where S_i and R_i represent the utility measure and regret measure, respectively; W_j is the relative weight assigned to the jth property.

Step 3 Determination of VIKOR index.

$$Q_i = v \frac{S_i - S^*}{S^- - S^*} + (1 - v) \frac{R_i - R^*}{R^- - R^*}; \forall i \tag{6}$$

where Q_i represents VIKOR value of the ith material; v is the group utility weight, generally considered as 0.5 (unsupervised).

$$S^* = \min_i(S_i) \tag{7}$$

$$S^- = \max_i(S_i) \tag{8}$$

$$R^* = \min_i(R_i) \tag{9}$$

$$R^- = \max_i(R_i) \tag{10}$$

The material with the least value of VIKOR index Q_i is preferred.

2.3 Fuzzy logic method

It includes a set of numbers within the interval [0, 1], which describe the smallest possible, most promising and largest possible values [18] as illustrated in Fig. 1. In this method, initially all comparisons are done using linguistic variables. Further, these linguistic variables are assigned to fuzzy values in order to have comparable numerical values without any ambiguity. For this, here we have used trapezoidal fuzzy numbers (a_1, a_2, a_3, a_4) for $\{a_1, a_2, a_3, a_4 \in \mathbf{R};\ a_1 \leqslant a_2 \leqslant a_3 \leqslant a_4\}$. It is one of the most common and simplest kinds of division used for fuzzy numbers. The membership function $\mu_a(x)$ of trapezoidal fuzzy number is defined as

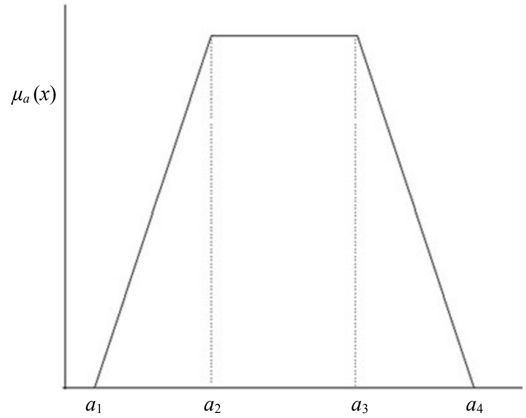

Fig. 1 Trapezoidal fuzzy number.

$$\mu_a(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & x \in [a_1,\ a_2] \\ 1, & x \in [a_2,\ a_3] \\ \dfrac{a_4 - x}{a_4 - a_3}, & x \in [a_3,\ a_4] \\ 0, & \text{Otherwise} \end{cases} \tag{11}$$

The linguistic variables and corresponding fuzzy numbers are shown in Table 2.

The pre-assigned fuzzy numbers are aggregated using following Eqs. (12)–(16) [19]:

$$x_{ij} = \{x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}\} \tag{12}$$

where x_{ij} is the fuzzy aggregated rating for M materials.

$$x_{ij1} = \min\{a_{ijk1}\} \tag{13}$$

$$x_{ij2} = \frac{1}{M}\sum a_{ijk2} \tag{14}$$

$$x_{ij3} = \frac{1}{M}\sum a_{ijk3} \tag{15}$$

$$x_{ij4} = \max\{a_{ijk4}\} \tag{16}$$

The first basic necessity of any comparison is that it should be on the same scale and quantities being compared must be of the same dimension. Therefore, our next step is the normalization of aggregated fuzzy rating. Similar to VIKOR, here we can also have two situations. One is properties with higher desired values and the other is properties with lower desired values. Mathematically, normalization is done as [18]

$$\mu_{ij} = \left(\frac{x_{ij1}}{x_{ij1}^+}, \frac{x_{ij2}}{x_{ij2}^+}, \frac{x_{ij3}}{x_{ij3}^+}, \frac{x_{ij4}}{x_{ij4}^+}\right); j \in J \tag{17}$$

$$\mu_{ij} = \left(\frac{x_{ij1}^-}{x_{ij1}}, \frac{x_{ij2}^-}{x_{ij2}}, \frac{x_{ij3}^-}{x_{ij3}}, \frac{x_{ij4}^-}{x_{ij4}}\right); j \in J' \tag{18}$$

where $x_{ij4}^+ = \max(x_{ij4}),\ j \in J$; $x_{ij1}^- = \min(x_{ij1}),\ j \in J'$; J corresponds to benefit criteria and J' corresponds to cost criteria. Thereafter, defuzzification (Eq. (19)) [18] is done to have the crisp values for each property corresponding to each material.

Table 2 Intervals, linguistic terms and corresponding fuzzy numbers for each material

ε_r	$\tan\delta$ (%)	K_p	d_{33}	Linguistic variable	Fuzzy number
>2500	<0.014	>0.6	>350	Exceptionally high (EH)	(0.8, 0.9, 1.0, 1.0)
1950–2500	0.014–0.028	0.5–0.6	240–350	Very high (VH)	(0.7, 0.8, 0.8, 0.9)
1450–1950	0.029–0.042	0.4–0.49	221–239	High (H)	(0.5, 0.6, 0.7, 0.8)
961–1450	0.043–0.056	0.3–0.39	161–220	Above average (AA)	(0.4, 0.5, 0.5, 0.6)
831–960	0.057–0.070	0.2–0.29	135–160	Average (A)	(0.2, 0.3, 0.4, 0.5)
570–830	0.071–0.084	0.1–0.19	100–134	Very low (VL)	(0.1, 0.2, 0.2, 0.3)
<570	>0.084	<0.1	<100	Extremely low (EL) (undesirable)	(0.0, 0.0, 0.1, 0.2)

$$f_{ij}=\text{Defuzz}(x_{ij})=\frac{\int \mu(x)x\mathrm{d}x}{\int \mu(x)\mathrm{d}x}$$

$$=\frac{\int_{x_{ij1}}^{x_{ij2}}\left(\frac{x-x_{ij1}}{x_{ij2}-x_{ij1}}\right)x\mathrm{d}x+\int_{x_{ij2}}^{x_{ij3}}x\mathrm{d}x+\int_{x_{ij3}}^{x_{ij4}}\left(\frac{x_{ij4}-x}{x_{ij4}-x_{ij3}}\right)x\mathrm{d}x}{\int_{x_{ij1}}^{x_{ij2}}\left(\frac{x-x_{ij1}}{x_{ij2}-x_{ij1}}\right)\mathrm{d}x+\int_{x_{ij2}}^{x_{ij3}}\mathrm{d}x+\int_{x_{ij3}}^{x_{ij4}}\left(\frac{x_{ij4}-x}{x_{ij4}-x_{ij3}}\right)\mathrm{d}x}$$

$$=\frac{-x_{ij1}x_{ij2}+x_{ij3}x_{ij4}+\frac{1}{3}(x_{ij4}-x_{ij3})^2+\frac{1}{3}(x_{ij2}-x_{ij1})^2}{-x_{ij1}-x_{ij2}+x_{ij3}+x_{ij4}} \tag{19}$$

Thus the crisp values, obtained corresponding to material understudy, are used in VIKOR method to calculate the final ranking (Eqs. (2)–(10)).

3 Results and discussion

The properties, such as K_p, ε_r, tanδ and d_{33}, have their own importance for various piezoelectric applications and have different priorities. Piezoelectric constant shows an ability of material to produce electrical signal on application of mechanical strain or vice versa, which is solemnly a key parameter in deciding material for actuator and sensor applications. Therefore, it is always given priority over the other properties. Similarly, dielectric constant is the essence of material to store the electrical energy, and tanδ shows the inherent dissipation of stored electrical energy. K_p is the conversion efficiency of the material. In order to assign relative weights to the above mentioned properties, we have made pair-wise comparison and allocated numbers 1, 2 and 3 to less, equally or more important properties, respectively. The relative decision matrix is formed based on pair-wise comparison (MDL approach) as illustrated in Table 3. Table 4 summarizes the calculation for weights for all the properties under study. It is clearly depicted in Fig. 2 that piezoelectric constant (d_{33}) is the most important parameter with maximum weight assigned followed by dielectric loss and dielectric constant, which are found to be equally important. K_p is found to be the least important property among the considered properties in this study.

Table 3 Decision matrix for calculation of weights (pair-wise comparison)

	ε_r	tanδ	d_{33}	K_p
ε_r	2	2	1	3
tanδ	2	2	1	3
d_{33}	3	3	2	3
K_p	1	1	1	2

Table 4 Number of possible decisions for properties understudy

Goal	Number of possible decisions						Positive decision	Weighted factor
	1	2	3	4	5	6	—	—
ε_r	2	1	3	—	—	—	6	6/24
tanδ	2	—	—	1	3	—	6	6/24
d_{33}	—	3	—	3	—	3	9	9/24
K_p	—	—	1	—	1	1	3	3/24

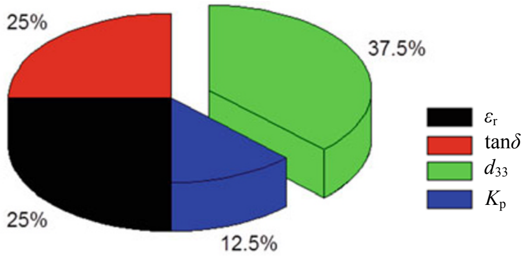

Fig. 2 Subjective weights assigned using MDL method.

Initially we made ranking using VIKOR method with weights for each criterion (property) obtained by MDL. The MDL weights add-ups a subjective reasoning part to VIKOR approach by material point of view. PLZT(8/65/35) and KNN–LT–LS are found to be at the top among lead-based and lead-free families consecutively. KNN–LT–LS is also studied experimentally and reported to have significant potential for transducer applications [46]. Thus it can be termed as the most suitable candidate for these applications under lead-free piezoceramic families. The ranks and respective rank indices for all materials understudy are shown in Table 1. The difference in rank index indicates about the extent of closeness among any two materials. For the first two materials, this difference is found to be 0.475 838 (almost 50%). On the other hand, it can be visually analyzed that the values for ε_r and d_{33} are nearly double for PLZT as compared to KNN–LT–LS, while the values of K_p and tanδ are even less than 1.5 times of KNN–LT–LS. Similar observations can also be seen for other materials. MDL has improved the ranking by providing the weightage of properties based on reasoning by material experts. But in order to have exact comparison, here rises a need of such system which can perform well for inter-criteria (property) comparison for each attribute (material). Comparing the values for a particular property for all materials is quite easier than comparing the values of different properties for a single material. This is because of the fact that there is no clear boundary between all criteria (properties). It is difficult to determine the equivalence of intervals for different properties for a single material. It becomes much more cumbersome as the number of materials and associate range increases.

Fuzzy logic approach works well with such kind of problems. It utilizes linguistic variables for pre-decided (as illustrated in Table 2 for the present study) comparative ranges of different properties. These ranges are selected very carefully. Any alteration in these ranges can affect the ranking of the system. The worst range is termed as extremely low (EL) (undesirable), and the best is termed as exceptionally high (EH). We have chosen these ranges as suggested by various material experts. It is clear in Table 2 that lower value of tanδ is kept in exceptionally high (most desirable) range for assigning linguistic variable, so it is no more a cost criteria. It is used as a benefit criterion in both normalization and implementation of VIKOR. It is used to reduce any possibility of ambiguity. Further these variables are replaced by fuzzy numbers as displayed in Table 5 according to the terms assigned in Table 2. Fuzzy values are normalized and crisp values are obtained (Table 6) using Eq. (19) as discussed in the previous section. These values are inter-comparisons between all properties for each material. Later these values are used to calculate the rank indices of the piezoelectric ceramics understudy using VIKOR method which is shown in Table 1. Top four ranks obtained by fuzzy and conventional VIKOR are the same but thereafter the ranking is changed. The most important observation is the variation in the rank index obtained by both the methods. The rank index thus obtained not only provides us fair ranking but also forms the clusters of materials which show closeness in values of the properties. The more is the closeness, the more is the possibility of interchangeability for a technological application. The main advantage of this approach over conventional VIKOR is that it is entirely based on verbal reasoning. VIKOR after fuzzy logic advancement is no more merely a data-dependent technique; rather it has become a comprehensive decision making technique.

Table 5 Importance of materials with respect to properties in terms of fuzzy numbers

Materials	ε_r	tanδ	K_p	d_{33}
PLZT(8/65/35)	(0.8, 0.9, 1.0, 1.0)	(0.7, 0.8, 0.8, 0.9)	(0.8, 0.9, 1.0, 1.0)	(0.8, 0.9, 1.0, 1.0)
KNN–LT–LS	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)
$BaTiO_3$	(0.5, 0.6, 0.7, 0.8)	(0.8, 0.9, 1.0, 1.0)	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)
KNN–$LiSbO_3$(5%)	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
PLZT(12/40/60)	(0.4, 0.5, 0.5, 0.6)	(0.8, 0.9, 1.0, 1.0)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
0.7BNT–0.2BKT–0.1$(Bi_{0.5}Li_{0.5})TiO_3$	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)	(0.5, 0.6, 0.7, 0.8)
0.92BNT–0.08BT+0.3wt%MnO	(0.5, 0.6, 0.7, 0.8)	(0.8, 0.9, 1.0, 1.0)	(0.4, 0.5, 0.5, 0.6)	(0.2, 0.3, 0.4, 0.5)
$BaTiO_3$–$CaTiO_3$–Co	(0.4, 0.5, 0.5, 0.6)	(0.8, 0.9, 1.0, 1.0)	(0.4, 0.5, 0.5, 0.6)	(0.2, 0.3, 0.4, 0.5)
NBT–KBT–LBT	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)
KNN–Li(3%); Ta(20%)	(0.2, 0.3, 0.4, 0.5)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)
NBT–KBT–BT(MPB)	(0.1, 0.2, 0.2, 0.3)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)
$Na_{0.5}K_{0.5}NbO_3$(HP)	(0.0, 0.0, 0.1, 0.2)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.1, 0.2, 0.2, 0.3)
$PbNb_2O_6$	(0.0, 0.0, 0.1, 0.2)	(0.7, 0.8, 0.8, 0.9)	(0.0, 0.0, 0.1, 0.2)	(0.0, 0.0, 0.1, 0.2)
NBT–KBT(50%)	(0.1, 0.2, 0.2, 0.3)	(0.7, 0.8, 0.8, 0.9)	(0.2, 0.3, 0.4, 0.5)	(0.2, 0.3, 0.4, 0.5)
NBT–KBT–BT	(0.1, 0.2, 0.2, 0.3)	(0.7, 0.8, 0.8, 0.9)	(0.1, 0.2, 0.2, 0.3)	(0.2, 0.3, 0.4, 0.5)
NKN–$(Bi_{0.5}K_{0.5})TiO_3$(3%)	(0.2, 0.3, 0.4, 0.5)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)
KNN–$LiNbO_3$(6%)	(0.0, 0.0, 0.1, 0.2)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
KNN–$LiTaO_3$(5%)	(0.0, 0.0, 0.1, 0.2)	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)
NKN–$BaTiO_3$(2%)	(0.4, 0.5, 0.5, 0.6)	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)	(0.1, 0.2, 0.2, 0.3)
SBT–KBT90	(0.2, 0.3, 0.4, 0.5)	(0.5, 0.6, 0.7, 0.8)	(0.1, 0.2, 0.2, 0.3)	(0.1, 0.2, 0.2, 0.3)
BBT–KBT80	(0.1, 0.2, 0.2, 0.3)	(0.5, 0.6, 0.7, 0.8)	(0.1, 0.2, 0.2, 0.3)	(0.0, 0.0, 0.1, 0.2)
$Na_{0.5}K_{0.5}NbO_3$	(0.0, 0.0, 0.1, 0.2)	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)	(0.0, 0.0, 0.1, 0.2)
SBT–KBT85	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)	(0.1, 0.2, 0.2, 0.3)	(0.1, 0.2, 0.2, 0.3)
BBT–KBT90	(0.1, 0.2, 0.2, 0.3)	(0.4, 0.5, 0.5, 0.6)	(0.2, 0.3, 0.4, 0.5)	(0.2, 0.3, 0.4, 0.5)
KNN–Li(7%)	(0.2, 0.3, 0.4, 0.5)	(0.1, 0.2, 0.2, 0.3)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)

Table 6 Crisp values of material ratings

Materials	ε_r	$\tan\delta$	K_p	d_{33}
PLZT(8/65/35)	0.922	0.800	0.922	0.922
KNN–LT–LS	0.650	0.800	0.650	0.800
$BaTiO_3$	0.650	0.922	0.500	0.500
KNN–$LiSbO_3$(5%)	0.500	0.800	0.800	0.800
PLZT(12/40/60)	0.500	0.922	0.650	0.650
0.7BNT–0.2BKT–0.1$(Bi_{0.5}Li_{0.5})TiO_3$	0.650	0.650	0.500	0.650
0.92BNT–0.08BT+0.3wt%MnO	0.650	0.922	0.500	0.350
$BaTiO_3$–$CaTiO_3$–Co	0.500	0.922	0.500	0.350
NBT–KBT–LBT	0.650	0.650	0.650	0.500
KNN–Li(3%); Ta(20%)	0.350	0.800	0.650	0.500
NBT–KBT–BT(MPB)	0.200	0.800	0.500	0.500
$Na_{0.5}K_{0.5}NbO_3$(HP)	0.078	0.800	0.650	0.200
$PbNb_2O_6$	0.078	0.800	0.078	0.078
NBT–KBT(50%)	0.200	0.800	0.350	0.350
NBT–KBT–BT	0.200	0.800	0.200	0.350
NKN–$(Bi_{0.5}K_{0.5})TiO_3$(3%)	0.350	0.650	0.650	0.500
KNN–$LiNbO_3$(6%)	0.078	0.650	0.650	0.650
KNN–$LiTaO_3$(5%)	0.078	0.650	0.500	0.500
NKN–$BaTiO_3$(2%)	0.500	0.650	0.500	0.200
SBT–KBT90	0.350	0.650	0.200	0.200
BBT–KBT80	0.200	0.650	0.200	0.078
$Na_{0.5}K_{0.5}NbO_3$	0.078	0.650	0.500	0.078
SBT–KBT85	0.500	0.500	0.200	0.200
BBT–KBT90	0.200	0.500	0.350	0.350
KNN–Li(7%)	0.350	0.200	0.650	0.800

4 Conclusions

MADM methods are employed for selection of piezoelectric ceramics for transducer applications. MDL method is used to calculate the weightage of physical properties for these materials and are weighted as $d_{33}>\varepsilon_r=\tan\delta>K_p$. Further priority order of materials is determined using conventional and fuzzy VIKOR incorporation with MDL weights. PLZT (8/65/35) (lead-based) and KNN–LT–LS (lead-free) are found to be at the first and second positions, respectively. The present study proposes the feasibility of fuzzy VIKOR method in material selection when the properties are not exact numerical values.

Acknowledgements

Rahul Vaish gratefully acknowledges financial support from Department of Science and Technology (DST), New Delhi, India under INSPIRE Faculty Award (ENG-01)-2011.

References

[1] Shrout TR, Zhang SJ. Lead-free piezoelectric ceramics: Alternatives for PZT? *J Electroceram* 2007, **19**: 111–124.

[2] Haertling GH. Ferroelectric ceramics: History and technology. *J Am Ceram Soc* 1999, **82**: 797–818.

[3] Rödel J, Klaus WJ, Seifert TP, *et al.* Perspective on the development of lead-free piezoceramics. *J Am Ceram Soc* 2009, **92**: 1153–1177.

[4] Xiao DQ, Lin DM, Zhu JG, *et al.* Studies on new systems of BNT-based lead-free piezoelectric ceramics. *J Electroceram* 2008, **21**: 34–38.

[5] Lau ST, Cheng CH, Choy SH, *et al.* Lead-free ceramics for pyroelectric applications. *J Appl Phys* 2008, **103**: 104105.

[6] Saito Y, Takao H, Tani T, *et al.* Lead-free piezoceramics. *Nature* 2004, **432**: 84–87.

[7] Damjanovic D, Klein N, Li J, *et al.* What can be expected from lead-free piezoelectric materials? *Funct Mater Lett* 2010, **3**: 5–13.

[8] Sen P, Yang JB. *Multiple Criteria Decision Support in Engineering Design*. Berlin: Springer Verlag, 1998.

[9] Saaty TL. How to make a decision: The analytic hierarchy process. *Eur J Oper Res* 1990, **48**: 9–26.

[10] Rao RV. A material selection model using graph theory and matrix approach. *Mat Sci Eng A* 2006, **431**: 248–255.

[11] Opricovic S, Tzeng GH. Extended VIKOR method in comparison with outranking methods. *Eur J Oper Res* 2007, **178**: 514–529.

[12] Deng H, Yeh CH, Willis RJ. Inter-company comparison using TOPSIS with objective weights. *Comput Oper Res* 2000, **27**: 963–973.

[13] Vaish R. Piezoelectric and pyroelectric materials selection. *Int J Appl Ceram Technol* 2012, DOI: 10.1111/j.1744-7402.2012.02765.x.

[14] Chauhan A, Vaish R. Magnetic material selection using multiple attribute decision making approach. *Mater Design* 2012, **36**: 1–5.

[15] Chauhan A, Vaish R. A comparative study on material selection for micro-electromechanical systems. *Mater Design* 2012, **41**: 177–181.

[16] Chauhan A, Vaish R. Hard coating material selection using multi-criteria decision making. *Mater Design* 2013, **44**: 240–245.

[17] Girubha RJ, Vinodh S. Application of fuzzy VIKOR and environmental impact analysis for material selection of an automotive component. *Mater Design*

2012, **37**: 478–486.

[18] Shemshadi A, Shirazi H, Toreihi M, *et al.* A fuzzy VIKOR method for supplier selection based on entropy measure for objective weighting. *Expert Syst Appl* 2011, **38**: 12160–12167.

[19] Opricovic S. Fuzzy VIKOR with an application to water resources planning. *Expert Syst Appl* 2011, **38**: 12983–12990.

[20] Devi K. Extension of VIKOR method in intuitionistic fuzzy environment for robot selection. *Expert Syst Appl* 2011, **38**: 14163–14168.

[21] Kaya T, Kahraman C. Fuzzy multiple criteria forestry decision making based on an integrated VIKOR and AHP approach. *Expert Syst Appl* 2011, **38**: 7326–7333.

[22] Chatterjee P, Athawale VM, Chakraborty S. Selection of materials using compromise ranking and outranking methods. *Mater Design* 2009, **30**: 4043–4053.

[23] Kuo MS, Liang GS. Combining VIKOR with GRA techniques to evaluate service quality of airports under fuzzy environment. *Expert Syst Appl* 2011, **38**: 1304–1312.

[24] Chen W, Li YM, Xu Q, *et al.* Electromechanical properties and morphotropic phase boundary of $Na_{0.5}Bi_{0.5}TiO_3$–$K_{0.5}Bi_{0.5}TiO_3$–$BaTiO_3$ lead-free piezoelectric ceramics. *J Electroceram* 2005, **15**: 229–235.

[25] Jaeger RE, Egerton L. Hot pressing of potassium–sodium niobates. *J Am Ceram Soc* 1962, **45**: 209–213.

[26] Haertling GH. Properties of hot-pressed ferroelectric alkali niobate ceramics. *J Am Ceram Soc* 1967, **50**: 329–330.

[27] Park SE, Shrout TR. Characteristics of relaxor-based piezoelectric single crystals for ultrasonic transducers. *IEEE T Ultrason Ferr* 1997, **44**: 1140–1147

[28] Elkechai O, Manier M, Mercurio JP. $Na_{0.5}Bi_{0.5}TiO_3$–$K_{0.5}Bi_{0.5}TiO_3$ (NBT–KBT) system: A structural and electrical study. *Phys Status Solidi a* 1996, **157**: 499–506.

[29] Zuo RZ, Fang XS, Ye C, *et al.* Phase transitional behavior and piezoelectric properties of lead-free $(Na_{0.5}K_{0.5})NbO_3$–$(Bi_{0.5}K_{0.5})TiO_3$ ceramics. *J Am Ceram Soc* 2007, **90**: 2424–2428.

[30] Guo YP, Kakimoto K, Ohsato H. Phase transitional behavior and piezoelectric properties of $(Na_{0.5}K_{0.5})NbO_3$–$LiNbO_3$ ceramics. *Appl Phys Lett* 2004, **85**: 4121.

[31] Guo Y, Kakimoto K, Ohsato H. $(Na_{0.5}K_{0.5})NbO_3$ –$LiTaO_3$ lead-free piezoelectric ceramics. *Mater Lett* 2005, **59**: 241–244.

[32] Ahn CW, Park HY, Nahm S, *et al.* Structural variation and piezoelectric properties of $0.95(Na_{0.5}K_{0.5})NbO_3$–$0.05BaTiO_3$ ceramics. *Sensor Actuat A*: *Phys* 2007, **136**: 255–260.

[33] Egerton L, Dillon DM. Piezoelectric and dielectric properties of ceramics in the system potassium–sodium niobate. *J Am Ceram Soc* 1959, **42**: 438–442.

[34] Hollenstein E, Davis M, Damjanovic D, *et al.* Piezoelectric properties of Li- and Ta-modified $(K_{0.5}Na_{0.5})NbO_3$ ceramics. *Appl Phys Lett* 2005, **87**: 182905.

[35] Jaffe H. Piezoelectric ceramics. *J Am Ceram Soc* 1958, **41**: 494–498.

[36] Zang GZ, Wang JF, Chen HC, *et al.* Perovskite $(Na_{0.5}K_{0.5})_{1-x}(LiSb)_xNb_{1-x}O_3$ lead-free piezoceramics. *Appl Phys Lett* 2006, **88**: 212908.

[37] Jaffe H, Berlincourt DA. Piezoelectric transducer materials. *Proc IEEE* 1965, **53**: 1372–1386.

[38] Schofield D, Brown RF. An investigation of some barium titanate compositions for transducer applications. *Can J Phys* 1957, **35**: 594–607.

[39] Yuan Y, Zhang SR, Zhou XH, *et al.* Phase transition and temperature dependences of electrical properties of $[Bi_{0.5}(Na_{1-x-y}K_xLi_y)_{0.5}]TiO_3$ ceramics. *Jpn J Appl Phys* 2006, **45**: 831–834.

[40] Lin DM, Xiao DQ, Zhu JG, *et al.* Piezoelectric and ferroelectric properties of $[Bi_{0.5}(Na_{1-x-y}K_xLi_y)_{0.5}]TiO_3$ lead-free piezoelectric ceramics. *Appl Phys Lett* 2006, **88**: 062901.

[41] Saito Y, Takao H. High performance lead-free piezoelectric ceramics in the $(K,Na)NbO_3$–$LiTaO_3$ solid solution system. *Ferroelectrics* 2006, **338**: 17–32.

[42] Tressler JF, Alkoy S, Newnham RE. Piezoelectric sensors and sensor materials. *J Electroceram* 1998, **2**: 257–272.

[43] Zhu MK, Liu LY, Hou YD, *et al.* Microstructure and electrical properties of MnO-doped $(Na_{0.5}Bi_{0.5})_{0.92}Ba_{0.08}TiO_3$ lead-free piezoceramics. *J Am Ceram Soc* 2007, **90**: 120–124.

[44] Hagh NM, Jadidian B, Safari A. Property-processing relationship in lead-free $(K,Na,Li)NbO_3$-solid solution system. *J Electroceram* 2007, **18**: 339–346.

[45] Dehgan-Manshadi B, Mahmudi H, Abedian A, *et al.* A novel method for material selection in mechanical design: Combination of non-linear normalization and a modified digital logic method. *Mater Design* 2007, **28**: 8–15.

[46] Hagh NM, Jadidian B, Ashbahian E, *et al.* Lead-free piezoelectric ceramic transducer in the donor-doped $K_{1/2}Na_{1/2}NbO_3$ solid solution system. *IEEE T Ultrason Ferr* 2008, **55**: 214–224.