Homotopy analysis for the influence of Navier slip flow in a vertical channel with cross diffusion effects

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Abstract This research is to examine the laminar, incompressible free convective Navier slip flow between vertical plates with cross diffusion effects. This investigation includes the first order chemical reaction also. The resulting equations with boundary conditions are reduced into dimensionless form using suitable transformations. Homotopy analysis method has been applied to solve the system. The influence of emerging parameters on fluid flow quantities have been presented graphically. In addition, the nature of physical quantities are shown in tabular form.

Keywords Natural convection · Soret effect · Dufour effect · Chemical reaction · Navier slip · HAM

Introduction

Combined heat and mass transfer in free convection flow between vertical parallel plates has significant importance in many applications. Kairi and Murthy [1] presented the applications and past theoretical investigations on free convection flow in a channel. Fattahi et al. [2] applied the lattice Boltzmann technique to study the nature of free convection flow of nano fluids. Huelsz and Rechtman [3] also applied the lattice Boltzmann technique for heat transfer in an inclined square cavity due to natural convection. Most recently, Terekhov et al. [4] studied the laminar free convection heat transfer between vertical isothermal plates.

Generally, the no-slip condition has been accepted for the fluid over a solid surface boundary condition. At the solid boundary, the general boundary conditions for slip flow has been proposed by Navier [5]. Eegunjobi and Makinde [6] presented the early literature and application of the slip flow in various geometries. Recently, Rundora and Makinde [7] analyzed the influence of Navier slip and variable viscosity through a porous medium with asymmetric convective boundary conditions. Most recently, Ng [8] investigated the nature of starting flow in channels with boundary slip.

The Soret and Dufour effects are encountered in the areas of chemical engineering, geosciences, etc. The applications and early literature can be seen in [9]. Recently, Srinivasacharya et al. [10] studied the mixed convection flow along a vertical wavy surface with the effects of cross diffusion and variable properties in a porous medium. Most recently, Umar et al. [11] presented the effects of cross diffusions and chemical reaction flow in converging and diverging channels.

Most of the chemical reactions comprise both homogeneous and heterogeneous reactions. The homogeneous reaction takes place in bulk of the fluid, while heterogeneous reaction occurs on some catalytic surfaces. Generally, the interaction between the homogeneous and heterogeneous reactions are very complex and is involved in the production and consumption of reactant species at various rates, both on the catalytic surfaces and within the fluid. The applications and early literature has been reported by Kothandapani and Prakash [12]. Most recently, RamReddy and Pradeepa [13] used the spectral quasi-linearization method to study the effect of homogeneous–heterogeneous reactions on non-linear convection flow of micropolar fluid in a porous medium with convective boundary condition.

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In this paper, the natural convection flow in a vertical channel saturated with Navier slip condition has been investigated in the presence of cross diffusions and the rate of chemical reaction. The survey clearly shows that the combined effects with slip flow condition through a vertical channel has not been presented elsewhere. In view of applications and importance, the authors are motivated to take this problem. Homotopy analysis method (HAM) [14–23] has been used for the solution of the present problem. Convergence of the resulting series solution is explained. Then the discussion with respect to the pertinent parameters of the present study on the solutions of velocity, temperature and concentration components.

Mathematical modelling

We consider two-dimensional free convection fluid flow with Navier slip boundary in a vertical channel. Flow configuration is presented in Fig. 1. All the fluid properties are considered to be constant with the exception that the density in the buoyancy term of the balance of momentum equation. Since the plates are infinitely long, all physical variables depend on y only. The governing two-dimensional steady flow equations can be put into the form:

$$v_y = 0 \Rightarrow v = v_0 = \text{constant} \quad (1)$$

$$v_0 \rho \frac{\partial u}{\partial y} = \rho g (\beta_T (T - T_1) + \beta_C (C - C_1)) + \mu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$\rho C_P v_0 \frac{\partial T}{\partial y} = K_f \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \frac{DK_T}{C_S C_P} \frac{\partial^2 C}{\partial y^2} \quad (3)$$

$$v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} - K_1 (C - C_1) \quad (4)$$

with

Fig. 1 Physical model and coordinate system.

$$u = \gamma_1 \frac{\partial u}{\partial y}, \quad T = T_1, \quad C = C_1 \quad \text{at} \quad y = -d \quad (5a)$$

$$u = \gamma_2 \frac{\partial u}{\partial y}, \quad T = T_2, \quad C = C_2 \quad \text{at} \quad y = d \quad (5b)$$

where the velocity components in x - and y - are u , v , respectively, g is the acceleration due to gravity, ρ is the density, C_p is the specific heat, μ is the coefficient of viscosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansion, α is the thermal diffusivity, D is the mass diffusivity, K_T is the coefficient of thermal conductivity, C_S is the concentration susceptibility, T_m is the mean fluid temperature, K_f is the thermal diffusion ratio, K_1 is the rate of chemical reaction, γ_1 and γ_2 are the slip coefficients at both the plates.

Introducing the following dimensionless variables

$$\eta = \frac{y}{d}, u = \frac{vGr}{d} f, \theta = \frac{T - T_1}{T_2 - T_1}, \phi = \frac{C - C_1}{C_2 - C_1} \quad (6)$$

in Eqs. (2)–(4), we obtain the governing dimensionless equations as

$$f'' + \theta + N\phi - Re f' = 0 \quad (7)$$

$$\theta'' - Re Pr \theta' + Br Gr^2 (f')^2 + Pr D_f \phi'' = 0 \quad (8)$$

$$\phi'' - Re Sc \phi' + Sc Sr \theta'' - K Sc \phi = 0 \quad (9)$$

with

$$\begin{aligned} \eta = -1 : f - \beta_1 f' = \theta = \phi = 0 \\ \eta = 1 : f - \beta_2 f' = 0 \quad \text{and} \quad \theta = \phi = 1 \end{aligned} \quad (10)$$

where the primes represents differentiation with respect to η , $N = \frac{\beta_C (C_2 - C_1)}{\beta_T (T_2 - T_1)}$ is the buoyancy parameter, $Re = \frac{\rho v_0 d}{\mu}$ is the Reynolds number, $Sc = \frac{v}{D}$ is the Schmidt number, $Gr = \frac{g \beta_T (T_2 - T_1) d^3}{\nu^2}$ is the Grashof number, $Pr = \frac{\mu C_p}{K_f}$ is

the Prandtl number, $Br = \frac{\mu \nu^2}{K_f d^2 (T_2 - T_1)}$ is the Brinkman

number, $K = \frac{K_1 d^2}{v}$ is the chemical reaction parameter,

$S_r = \frac{DK_T (T_2 - T_1)}{v T_m (C_2 - C_1)}$ is the thermo-diffusion parameter and

$D_f = \frac{DK_T (C_2 - C_1)}{v C_S C_P (T_2 - T_1)}$ is the Dufour number. $\beta_1 = \frac{\gamma_1}{d}$,

$\beta_2 = \frac{\gamma_2}{d}$ are the slip parameters.

The skin friction coefficients (C_f), heat (Nu) and mass (Sh) fluxes at the vertical walls can be given by

$$\frac{Re^2}{Gr} C_{f1} = 2f'(-1), \quad \frac{Re^2}{Gr} C_{f2} = 2f'(1) \quad (11)$$

$$Nu_{1,2} = -\theta'(\eta)|_{\eta=-1,1}, \quad Sh_{1,2} = -\phi'(\eta)|_{\eta=-1,1}$$

where

$$C_f = \frac{\tau_w}{\rho v_0^2}, Nu = \frac{q_w d}{K_f(T_2 - T_1)}, Sh = \frac{q_m d}{D(C_2 - C_1)} \quad (12)$$

and

$$\tau_w = \mu \frac{\partial u}{\partial y} \Big|_{y=\pm d}; q_w = -K_f \frac{\partial T}{\partial y} \Big|_{y=\pm d} \quad (13)$$

$$q_m = -D \frac{\partial C}{\partial y} \Big|_{y=\pm d}$$

Effect of the various parameters involved in the investigation on physical coefficients are discussed in the following section.

Homotopy solution

For HAM solutions, we choose the initial approximations of $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ as follows:

$$f_0(\eta) = 0, \quad \theta_0(\eta) = \frac{1+\eta}{2}, \quad \phi_0(\eta) = \frac{1+\eta}{2} \quad (14)$$

with the auxiliary linear operator

$$L = \frac{\partial^2}{\partial \eta^2} \quad \text{such that} \quad L_1(c_1\eta + c_2) = 0 \quad (15)$$

in which c_1 and c_2 are the arbitrary constants. h_1 , h_2 and h_3 (the convergence control parameters) are introduced in zeroth-order deformations as

$$(1-p)L[f(\eta;p) - f_0(\eta)] = ph_1N_1[f(\eta;p)] \quad (16)$$

$$(1-p)L[\theta(\eta;p) - \theta_0(\eta)] = ph_2N_2[\theta(\eta;p)] \quad (17)$$

$$(1-p)L[\phi(\eta;p) - \phi_0(\eta)] = ph_3N_3[\phi(\eta;p)] \quad (18)$$

subject to the boundary conditions

$$f(-1;p) - \beta_1 f'(-1;p) = 0, f(1;p) - \beta_2 f'(1;p) = 0$$

$$\theta(-1;p) = 0, \theta(1;p) = 1, \phi(-1;p) = 0, \phi(1;p) = 1 \quad (19)$$

where $p \in [0, 1]$ is the embedding parameter and the non-linear operators N_1 , N_2 and N_3 are defined as:

$$N_1[f(\eta, p), \theta(\eta, p), \phi(\eta, p)] = f''(\eta, p) + \theta(\eta, p) + N\phi(\eta, p) - Re f'(\eta, p) \quad (20)$$

$$N_2[f(\eta, p), \theta(\eta, p), \phi(\eta, p)] = \theta''(\eta, p) - Re Pr \theta'(\eta, p) + Br Gr^2 (f'(\eta, p))^2 + Pr D_f \phi''(\eta, p) \quad (21)$$

$$N_3[f(\eta, p), \theta(\eta, p), \phi(\eta, p)] = f''(\eta, p) - Re Sc \phi'(\eta, p) + Sc Sr \theta''(\eta, p) - K Sc \phi(\eta, p) \quad (22)$$

For $p = 0$, we have the initial guess approximations

$$f(\eta; 0) = f_0(\eta), \theta(\eta; 0) = \theta_0(\eta), \phi(\eta; 0) = \phi_0(\eta) \quad (23)$$

When $p = 1$, Eqs. (16)–(18) are same as (7)–(9), respectively, therefore, at $p = 1$, we get the final solutions

$$f(\eta; 1) = f(\eta), \theta(\eta; 1) = \theta(\eta), \phi(\eta; 1) = \phi(\eta) \quad (24)$$

Hence, the process of giving an increment to p from 0 to 1 is the process of $f(\eta; p)$ varying continuously from the initial guess $f_0(\eta)$ to the final solution $f(\eta)$ (similar for $\theta(\eta, p)$ and $\phi(\eta, p)$). This kind of continuous variation is called deformation in topology so that we call system of Eqs. (16)–(19), the zeroth-order deformation equation. Next, the m th-order deformation equations follow as

$$L_1[f_m(\eta) - \chi_m f_{m-1}(\eta)] = h_1 R_m^f(\eta) \quad (25)$$

$$L[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = h_2 R_m^\theta(\eta) \quad (26)$$

$$L[\phi_m(\eta) - \chi_m \phi_{m-1}(\eta)] = h_3 R_m^\phi(\eta) \quad (27)$$

with the boundary conditions

$$f_m(-1) = 0, \quad f_m(1) = 0, \quad \theta_m(-1) = 0$$

$$\theta_m(1) = 0, \quad \phi_m(-1) = 0, \quad \phi_m(1) = 0, \quad (28)$$

where

$$R_m^f(\eta) = f'' + \theta + N\phi - Re f' \quad (29)$$

$$R_m^\theta(\eta) = \theta'' - Re Pr \theta' + Br Gr^2 \sum_{n=0}^{m-1} f'_{m-1-n} f'_n + Pr D_f \phi'' \quad (30)$$

$$R_m^\phi(\eta) = \phi'' - Re Sc \phi' + Sc Sr \theta'' - K Sc \phi \quad (31)$$

for m being integer

$$\chi_m = 0 \quad \text{for} \quad m \leq 1$$

$$= 1 \quad \text{for} \quad m > 1 \quad (32)$$

The initial guess approximations $f_0(\eta)$, $\theta_0(\eta)$ and $\phi_0(\eta)$, the linear operators L and the auxiliary parameters h_1 , h_2 and h_3 are assumed to be selected such that Eqs. (16)–(19) have solution at each point $p \in [0, 1]$ and also with the help of Taylor's series and due to Eq. (23); $f(\eta; p)$, $\theta(\eta; p)$ and $\phi(\eta; p)$ can be expressed as

$$f(\eta; p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m \quad (33)$$

$$\theta(\eta; p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) p^m \quad (34)$$

$$\phi(\eta; p) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta) p^m \quad (35)$$

in which h_1 , h_2 and h_3 are chosen in such a way that the series (33)–(35) are convergent [15] at $p = 1$. Therefore, we have from (24) that

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \quad (36)$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) \quad (37)$$

$$\phi(\eta) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta) \quad (38)$$

for which we presume that the initial guesses to f , θ and ϕ , the auxiliary linear operators L and the non-zero auxiliary parameters h_1 , h_2 and h_3 are so properly selected that the deformation $f(\eta, p)$, $\theta(\eta, p)$ and $\phi(\eta, p)$ are smooth enough and their m th-order derivatives with respect to p in Eqs. (36)–(38) exist and are given, respectively, by

$$f_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m f(\eta; p)}{\partial p^m} \right|_{p=0}, \quad \theta_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \theta(\eta; p)}{\partial p^m} \right|_{p=0} \quad \text{and} \\ \phi_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \phi(\eta; p)}{\partial p^m} \right|_{p=0}. \quad \text{It is clear that the convergence of}$$

Taylor series at $p = 1$ is a prior assumption, whose justification is provided via a theorem [25], so that the system in (36)–(38) holds true. The formulae in (36)–(38) provide us with a direct relationship between the initial guesses and the exact solutions. All the effects of interaction of the magnetic field as well as of the heat transfer, Hall and Ion effects and couple stress flow field can be studied from the exact formulas (36)–(38). Moreover, a special emphasize should be placed here that the m th-order deformation system (25)–(28) is a linear differential equation system with the auxiliary linear operators L whose fundamental solution is known.

Convergence of the HAM solution

The expressions for f , θ and ϕ contain the auxiliary parameters h_1 , h_2 and h_3 . As pointed out by Liao [14], the convergence and the rate of approximation for the HAM solution strongly depend on the values of auxiliary parameter h . For this purpose, h -curves are plotted to choose h_1 , h_2 and h_3 in such a manner that the solutions (33)–(35) ensure convergence [14]. Here to see the admissible values of h_1 , h_2 and h_3 , the h -curves are plotted for 15th-order of approximation in Figs. 2, 3 and 4 by

Fig. 2 The h curve of $f(\eta)$

Fig. 3 The h curve of $\theta(\eta)$

Fig. 4 The h curve of $\phi(\eta)$

taking the values of the parameters $N = 2, Re = 2, Pr = 0.71, Sc = 0.22, Br = 0.5, Gr = 0.5, Sr = 0.5, Df = 0.5, K = 1.0$. It is clearly noted from Fig. 2 that the range for the admissible values of h_1 is $-1.25 < h_1 < -0.6$. From Fig. 3, it can be seen that the h -curve has a parallel line segment that corresponds to a region $-1.15 < h_2 < -0.65$. Figure 4 depicts that the admissible value of h_3 are $-1.2 < h_3 < -0.6$. A wide valid zone is evident in these figures ensuring convergence of the series. To choose optimal value of auxiliary parameters, the average residual errors (see Ref. [15] for more details) are defined as

$$E_{f,m} = \frac{1}{2K} \sum_{i=-K}^K \left(N_1 \left[\sum_{j=0}^m f_j(i\Delta t) \right] \right)^2 \quad (39)$$

$$E_{\theta,m} = \frac{1}{2K} \sum_{i=-K}^K \left(N_2 \left[\sum_{j=0}^m \theta_j(i\Delta t) \right] \right)^2 \quad (40)$$

$$E_{\phi,m} = \frac{1}{2K} \sum_{i=-K}^K \left(N_3 \left[\sum_{j=0}^m \phi_j(i\Delta t) \right] \right)^2 \quad (41)$$

where $\Delta t = 1/K$ and $K = 5$. At different order of approximations (m), minimum of average residual errors are shown in Tables 1, 2 and 3. It is clear from Table 1 that the average residual error for f is minimum at $h_1 = -0.93$. It can be seen from Table 2 that the minimum of average residual error for θ attains at $h_2 = -0.93$. Table 3 depicts that at $h_3 = -0.93$, E_{ϕ} attains minimum. Therefore, the optimum values of convergence control parameters are taken as $h_1 = -0.93, h_2 = -0.93, h_3 = -0.93$.

To see the accuracy of the solutions, the residual errors are defined for the system as

$$RE_f = f_n''(\eta) + \theta_n(\eta) + N\phi_n(\eta) - Ref_n'(\eta) \quad (42)$$

Table 1 Optimal value of h_1 at different order of approximations

Order	Optimal of h_1	Minimum of E_m
10	-0.93	1.30×10^{-10}
15	-0.93	5.45×10^{-16}
20	-0.93	1.36×10^{-18}

Table 2 Optimal value of h_2 at different order of approximations

Order	Optimal of h_2	Minimum of E_m
10	-0.94	2.01×10^{-10}
15	-0.93	4.33×10^{-14}
20	-0.93	1.44×10^{-16}

Table 3 Optimal value of h_3 at different order of approximations

Order	Optimal of h_3	Minimum of E_m
10	-0.93	1.50×10^{-11}
15	-0.93	5.63×10^{-15}
20	-0.93	2.10×10^{-17}

Fig. 5 Residual error of $f(\eta)$ when $\beta_h = 2, \beta_i = 2, \alpha = 0.5, Ha = 5$

Fig. 6 Residual error of $\theta(\eta)$ when $\beta_h = 2, \beta_i = 2, \alpha = 0.5, Ha = 5$

$$RE_{\theta} = \theta_n''(\eta) - RePr\theta_n'(\eta) + BrGr^2(f_n'(\eta))^2 + PrD_f\phi_n''(\eta) \quad (43)$$

$$RE_{\theta} = \phi_n''(\eta) - ReSc\phi_n'(\eta) + ScSr\theta_n''(\eta) - KSc\phi_n(\eta) \quad (44)$$

where $f_n(\eta)$, $\theta_n(\eta)$ and $\phi_n(\eta)$ are the HAM solutions for $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$, respectively. For optimality of the

Fig. 7 Residual error of $\phi(\eta)$ when $\beta_h = 2, \beta_i = 2, \alpha = 0.5, Ha = 5$

convergence control parameters, residual error [24] for different values of h in the convergence region displayed in Figs. 5, 6 and 7. We examine that $h_1 = -0.93, h_2 = -0.93, h_3 = -0.93$ gives a better solution. Table 4 establishes the convergence of the obtained series solution. It is found from the above observations that the series given by (33)–(35) converge in the whole region of η when $h_1 = -0.93, h_2 = -0.93, h_3 = -0.93$.

To pursue the convergence of the HAM solutions to the exact ones, the graphs for the ratio (following the recent work of [25])

$$\beta_f = \left| \frac{f_m(h)}{f_{m-1}(h)} \right|, \beta_\theta = \left| \frac{\theta_m(h)}{\theta_{m-1}(h)} \right|, \beta_\phi = \left| \frac{\phi_m(h)}{\phi_{m-1}(h)} \right| \quad (45)$$

against the number of terms m in the homotopy series is presented in Figs. 8, 9 and 10. Figures strongly indicate that a finite limit of β will be attained in the limit of $m \rightarrow \infty$, which will remain less than or equals to unity. The velocity and temperature solutions seem to converge in an oscillatory manner requiring more terms in the homotopy series. Thus, the convergence to the exact solution is assured by the HAM method.

Fig. 8 The ratio β_f from the theorem to reveal the convergence of the HAM solutions

Discussion of results

In the absence of Brinkman ($Br = 0$) number, the system of Eqs. (7)–(9) reduces to the linear system of equations for which exact solution exist. Comparison between analytical solution and HAM solution is shown in Table 5. The comparisons are found to be in a very good agreement. Therefore, the HAM code can be used with great confidence to study the problem considered in this paper.

The profiles of velocity ($f(\eta)$), temperature ($\theta(\eta)$) and concentration ($\phi(\eta)$) are computed and presented through plots in Figs. 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 and 21 with different values of $\beta_1, \beta_2, Sr, D_f, K$. Computations were carried out by fixing the parameters $Pr = 0.71, N = 2, Br = 0.5, Da = 0.5, Re = 2, Sc = 0.22, Gr = 0.5$ to analyze the effects of the emerging parameters $\beta_1, \beta_2, Sr, D_f$ and K .

Figure 11 presents the influence of the slip flow parameter β_1 on $f(\eta)$. It is clear that increase in the parameter β_1 leads to increase the flow velocity $f(\eta)$ at the injection wall and there is slight reverse trend can be found at the terminal wall. Figure 12 shows that the effect of β_2

Table 4 Convergence of HAM solutions for different order of approximations

Order	$f(0)$	$\theta(0)$	$\phi(0)$
4	0.4910169474433473893	0.387920461619234213657136	0.428633469889216020427361
8	0.4947894544255234236	0.378481745766847297706702	0.424271364200980527767338
12	0.4948135285227568114	0.378404924235389153211848	0.424337509824766393328840
16	0.4948137463089264307	0.378403051384050246849151	0.424339108852924445123161
20	0.4948137528096351068	0.378403073998202584356433	0.424339130178531704167978
24	0.4948137530282150863	0.378403078729212125985981	0.424339129132763810471734
28	0.4948137530306036798	0.378403079015738519907694	0.424339129022152340152007
30	0.4948137530306036798	0.378403079015738519907694	0.424339129022152340152007

Fig. 9 The ratio β_θ from the theorem to reveal the convergence of the HAM solutions

Fig. 11 Variation $f(\eta)$ with β_1 when $\beta_2 = 0.1, Sr = 0.5, D_f = 0.5, K = 1.0$

Fig. 10 The ratio β_ϕ from the theorem to reveal the convergence of the HAM solutions

Fig. 12 Variation $f(\eta)$ with β_2 when $\beta_1 = 0.1, Sr = 0.5, D_f = 0.5, K = 1.0$

on the flow velocity $f(\eta)$. It can be seen from this figure that as β_2 increases the flow velocity decreases noticeably at the initial wall but appreciable decrease in the velocity is noticed at the suction wall.

The effect of Soret parameter on velocity, temperature and concentration profiles are presented in Figs. 13, 14 and 15. It is clear from Fig. 13 that the higher values of Soret

Table 5 Comparison of exact solution ($Br = 0$) with HAM solution

η	f		θ		ϕ	
	Analytical	HAM	Analytical	HAM	Analytical	HAM
-1	0.026479	0.026479	0	0	0	0
-0.6	0.148563	0.148563	0.112667	0.112667	0.173056	0.173056
-0.2	0.275758	0.275758	0.257042	0.257042	0.359029	0.359029
0.2	0.345735	0.345735	0.443246	0.443246	0.558629	0.558629
0.6	0.254633	0.254633	0.684836	0.684836	0.772288	0.772288
1	-0.16612	-0.16612	1	1	1	1

Fig. 13 Soret effect on $f(\eta)$ when $\beta_1 = 0.1, \beta_2 = 0.1, D_f = 0.5, K = 1.0$

Fig. 15 Soret effect on $\phi(\eta)$ when $\beta_1 = 0.1, \beta_2 = 0.1, D_f = 0.5, K = 1.0$

Fig. 14 Soret effect on $\theta(\eta)$ when $\beta_1 = 0.1, \beta_2 = 0.1, D_f = 0.5, K = 1.0$

Fig. 16 Dufour effect on $f(\eta)$ when $\beta_1 = 0.1, \beta_2 = 0.1, Sr = 0.5, K = 1.0$

number Sr increases the velocity of the flow. Since the values of Sr increases due to either an decrease in the temperature difference or increase in the concentration difference, with this lowest peak of the flow velocity compatible with the lowest Soret number. Figure 14 shows the influence of Sr on θ . It is observed that the temperature decreases as Sr increases. The nature of ϕ with the influence of Sr is shown in Fig. 15. It is noted that the dimensionless concentration increases as an increase in the Soret number. The results clearly exhibits that the flow field is significantly influenced by the Soret number.

The effect of Dufour number (D_f) on dimensionless velocity can be seen in Fig. 16. It is clear that the velocity

of the fluid increases with an increase in diffusion thermo parameter. Since the values of D_f increases due to either an increase in the temperature difference or decrease in the concentration difference, i.e., the lowest peak of the flow velocity compatible with the lowest Dufour number. The effect of diffusion thermo parameter on dimensionless temperature is presented in Fig. 17. It is noted that the temperature profile increases with an increase in Dufour number. Figure 18 demonstrates that the concentration of the fluid decreases as Dufour effect increases. This is because of temperature gradients contribution to diffusion of the species.

Fig. 17 Dufour effect on $\theta(\eta)$ when $\beta_1 = 0.1, \beta_2 = 0.1, Sr = 0.5, K = 1.0$

Fig. 18 Dufour effect on $\phi(\eta)$ when $\beta_1 = 0.1, \beta_2 = 0.1, Sr = 0.5, K = 1.0$

The chemical reaction parameter K effect on velocity, temperature and concentration profiles are shown in Figs. 19, 20 and 21. It is observed from these figures that the fluid flow velocity decreases as K increases. The temperature increases and concentration of the fluid decreases when the chemical reaction parameter increases. Since the chemical molecular diffusivity drop down when chemical reaction is high, i.e., lesser diffusion. Hence, they are procured by the transfer of species. The higher K will decrease the concentration species. Therefore, with the increase in K suppresses the distribution of the concentration at all the points of the fluid flow. With this, it can be

Fig. 19 Chemical reaction effect on $f(\eta)$ when $\beta_1 = 0.1, \beta_2 = 0.1, Sr = 0.5, D_f = 0.5$

Fig. 20 Chemical reaction effect on $\theta(\eta)$ when $\beta_1 = 0.1, \beta_2 = 0.1, Sr = 0.5, D_f = 0.5$

claimed that on the distribution of the concentration, massive effect is higher with heavier diffusing species.

Variation of slip parameters (β_1 and β_2), thermal diffusion parameter (Sr), diffusion thermo parameter (D_f), together with the chemical reaction parameter K are shown in Table 6 by fixing the remaining parameters. It is seen that the friction factor decreases at both the walls with an increase in the slip parameters β_1 and β_2 . It is clear that increase in β_1 leads to increase in heat transfer coefficient at the plates where as the mass transfer rate increases at the initial wall and decreases at the terminal wall. It can be

observed that at the initial plate the heat transfer rate decreases and at the second plate it is found to be increasing while the reverse trend is perceived on mass transfer rate with an increase in β_2 . It is observed from this table that the skin friction coefficient, heat transfer rate increases at the injection wall and decreases at the terminal wall with the increase of Soret parameter (Sr), where as mass transfer rate have reverse trend at both the plates. It is noticed that higher values of Dufour parameter (D_f) increases the friction factor and mass transfer rate at the initial plate and decreases at the suction wall, while heat transfer rate shows the reverse trend at two walls. The effect of chemical reaction parameter K on skin friction, heat and mass transfer rates are presented in Table 6. It is

Fig. 21 Chemical reaction effect on $\phi(\eta)$ when $\beta_1 = 0.1, \beta_2 = 0.1, Sr = 0.5, D_f = 0.5$

seen from the table that all the physical parameters are decreasing at the injection wall and those are increasing at the suction wall with the enhance in K . It is also seen from this table that the higher values of K increases the mass transfer rate at $\eta = -1$ and decreases at $\eta = 1$. The influence of the emerging parameters are patently obvious from the Table 6, and hence are not discussed for conciseness.

Conclusions

Two-dimensional natural convection flow in a vertical channel in the presence of chemical reaction and the cross diffusion effects with Navier slip. Homotopy analysis method has been employed to solve the final system of equations. The markable findings are summarized as:

- Flow velocity and the rate of mass transfer increases at the injection wall, decreases slightly at the suction wall and the rate of heat transfer increases at both the walls with an enhance in slip parameter β_1 .
- As β_2 increases, the fluid flow velocity and friction factors decrease (slightly at the injection wall and marginally at the suction wall). Where as heat transfer rate disintegrates at the injection wall and increases at the end wall, while the opposite nature is noticed on the rate of mass transfer with an enhance in β_2 .
- It is noticed that the velocity and concentration of the fluid increases and temperature profile decreases with an increase in Soret parameter. In addition, it is identified that the friction factor, Nusselt number increases at the injection plate and decreases at the end wall as Sr increases, where as mass transfer rate shows the reverse trend at both the walls.

Table 6 Effects of skin friction coefficient, heat and mass transfer rates for various values of $Sr, D_f, \beta_1, \beta_2$ and K

β_1	β_2	Sr	D_f	K	$f'(-1)$	$f'(1)$	Nu_1	Nu_1	Sh_1	Sh_2
0.1	0.1	0.5	0.5	0.5	0.1829	-2.7919	-0.1154	-1.2373	-0.3733	-0.7861
0.2	0.1	0.5	0.5	0.5	0.1820	-2.8385	-0.1153	-1.2317	-0.3732	-0.7866
0.3	0.1	0.5	0.5	0.5	0.1811	-2.8846	-0.1152	-1.2261	-0.3731	-0.7871
0.1	0.1	0.5	0.5	0.5	0.1829	-2.7919	-0.1154	-1.2373	-0.3733	-0.7861
0.1	0.2	0.5	0.5	0.5	0.1651	-3.7536	-0.1162	-1.0957	-0.3709	-0.7990
0.1	0.3	0.5	0.5	0.5	0.1287	-5.7585	-0.1252	-0.6181	-0.3600	-0.8402
0.1	0.1	0.2	0.5	0.5	0.1762	-2.7462	-0.1191	-1.2214	-0.3476	-0.8326
0.1	0.1	0.4	0.5	0.5	0.1806	-2.7765	-0.1166	-1.2319	-0.3646	-0.8018
0.1	0.1	0.6	0.5	0.5	0.1852	-2.8076	-0.1140	-1.2427	-0.3819	-0.7701
0.1	0.1	0.5	0.2	0.5	0.1817	-2.7756	-0.1028	-1.3161	-0.3792	-0.7824
0.1	0.1	0.5	0.4	0.5	0.1825	-2.7863	-0.1110	-1.2639	-0.3753	-0.7849
0.1	0.1	0.5	0.6	0.5	0.1833	-2.7977	-0.1198	-1.2104	-0.3711	-0.7873
0.1	0.1	0.5	0.5	0.2	0.1882	-2.8287	-0.1127	-1.2507	-0.3924	-0.7466
0.1	0.1	0.5	0.5	0.4	0.1846	-2.8040	-0.1145	-1.2417	-0.3795	-0.7731
0.1	0.1	0.5	0.5	0.6	0.1812	-2.7801	-0.1161	-1.2329	-0.3671	-0.7989

- Fluid flow velocity, θ increases and the dimensionless concentration profile decreases as D_f increases. It is seen that as Dufour parameter increases, the skin friction and mass transfer rate at $\eta = -1$ and decreases at $\eta = 1$ wall, while the reverse trend is noticed in case of heat transfer rate at both the walls.
- As reaction parameter increases, the velocity and concentration of the fluid decreases and the temperature profile decreases. It is also observed that the values of $f'(0)$, Nusselt and Sherwood numbers decrease at the initial wall and those increase at the terminal plate with the increase in K .

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