Soft filter

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Received: 4 July 2013 / Accepted: 9 April 2014 / Published online: 1 May 2014
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Abstract We have studied some soft neighbourhood properties in a soft topological space and introduced soft filters which are defined over an initial universe with a fixed set of parameters. We have set up a soft topology with the help of a soft filter. We also have introduced the concepts of the greatest lower bound and the least upper bound of the family of soft filters over an initial universe, soft filter subbase and soft filter base. Also, we have explored some basic properties of these concepts.

Keywords Soft topological space · Soft neighbourhood system · Soft filter

Introduction

Mathematics is based on exact concepts and there is not vagueness for mathematical concepts. Since in many other fields such as medicine, engineering, economics and sociology, the notions are vague, researchers need to define some new concepts for vagueness. To deal with the these problems in real life, researchers proposed several methods

such as fuzzy set theory, rough set theory and soft set theory. Fuzzy set theory [1] proposed by Zadeh in 1965 provides an appropriate framework for representing and processing vague concepts. The basic idea of fuzzy set theory hinges on fuzzy membership function. By fuzzy membership function, we can determine the belonging of an element to set to a degree. Rough set theory [2] which is proposed by Pawlak in 1982 is another mathematical approach to vagueness to catch the granularity induced by vagueness in information systems. It based on equivalence relation. The advantage of rough set method is that it does not need any additional information about data, like membership in fuzzy set theory.

Theory of fuzzy sets and theory of rough sets can be considered as tools for dealing with vagueness but both of these theories have their own difficulties. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory as mentioned by Molodtsov [3] in 1999. Soft set theory [3] was initiated by Molodtsov as a completely new approach for modeling vagueness and uncertainty. According to Molodtsov [3, 4], the soft set theory has been successfully applied to many fields, such as functions smoothness, game theory, Riemann integration, theory of measurement and so on. He also showed how soft set theory is free from the parametrization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. Soft systems provide a very general framework with the involvement of parameters. Research works on soft sets are progressing rapidly in recent years.

Topological structure of soft sets was studied by many authors: Shabir and Naz [5] introduced the soft topological spaces which are defined over an initial universe with a fixed set of parameters. They studied the concepts of soft open set, soft interior point, soft neighbourhood of a

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point, soft separation axioms and subspace of a soft topological space. As a different approach to soft topology Çağman et al. [6] defined the concepts of soft open set, soft interior, soft closure, soft limit point, soft Hausdorff space. Aygünöğlu and Aygün [7] defined soft continuity of soft mapping, soft product topology and studied properties of soft projection mappings, soft compactness and generalized Tychonoff theorem to the soft topological space. Min [8] gave some results on soft topological spaces. Zorlutuna et al. [9] also investigated soft interior point, soft neighbourhood and soft continuity. They introduced the relationships between soft topology and fuzzy topology. Hussain and Ahmad [10] continued and discussed the properties of soft interior, soft exterior and soft boundary on soft topology. Varol and Aygün [11] defined convergence of sequences in soft topological space, diagonal soft set and studied the properties of soft Hausdorff space.

The main purpose of this paper is to introduce soft filters which are defined over an initial universe with a fixed set of parameters. First, we have given some basic ideas about soft sets, soft topological spaces and the results already studied. Then, we have discussed some basic properties of soft neighbourhoods. We have defined soft filters and studied some basic properties of soft filters. We have set up a soft topology with the help of a soft filter. We have introduced some new concepts in soft filters such as the greatest lower bound and the least upper bound of the family of soft filters, soft filter subbase and soft filter base. Also, we have explored some basic properties of these concepts.

Preliminaries

In this section, we present the basic definitions and results of soft set theory which may be found in earlier studies.

Let X be an initial universe set and E be the set of all possible parameters with respect to X . Parameters are often attributes, characteristics or properties of the objects in X . Let $P(X)$ denote the power set of X . Then, a soft set over X is defined as follows.

Definition 1 [3] A pair (F, A) is called a soft set over X where $A \subseteq E$ and $F : A \rightarrow P(X)$ is a set valued mapping. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $\forall \varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε -approximate elements of the soft set (F, A) . It is worth noting that $F(\varepsilon)$ may be arbitrary. Some of them may be empty, and some may have nonempty intersection.

Example 1 [12] Miss Zeynep and Mr. Ahmet are going to marry and they want to hire a wedding room. The soft set

(F, E) describes the “capacity of the wedding room”. Let $X = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be the wedding rooms under consideration, and $E = \{e_1 = \text{big}, e_2 = \text{central}, e_3 = \text{cheap}, e_4 = \text{quality}, e_5 = \text{elegant}\}$ be the parameter set, $F(e_1) = \{u_2, u_4\}$, $F(e_2) = \{u_1, u_3, u_4\}$, $F(e_3) = \emptyset$, $F(e_4) = \{u_1, u_3, u_5\}$, $F(e_5) = \{u_1, u_6\}$. The soft set (F, E) is as follows: $(F, E) = \{e_1 = \{u_2, u_4\}, e_2 = \{u_1, u_3, u_4\}, e_3 = \emptyset, e_4 = \{u_1, u_3, u_5\}, e_5 = \{u_1, u_6\}\}$.

Definition 2 [13] A soft set F_A on the universe X is defined by the set of ordered pairs $F_A = \{(e, f_A(e)) : e \in E, f_A(e) \in P(X)\}$, where $f_A : E \rightarrow P(X)$, such that $f_A(e) \neq \emptyset$ if $e \in A \subseteq E$ and $f_A(e) = \emptyset$ if $e \notin A$. Here, f_A is called an approximate function of the soft set F_A . The value of $f_A(e)$ may be arbitrary.

Note that the set of all soft sets over X will be denoted by $\mathcal{S}(X, E)$.

Definition 3 [14] Let $F_A \in \mathcal{S}(X, E)$. If $f_A(e) = \emptyset$ for all $e \in E$, then F_A is called an empty soft set, denoted by $F_\emptyset$ or Φ . $f_A(e) = \emptyset$ means that there is no element in X related to the parameter $e \in E$. Therefore, we do not display such elements in the soft sets, as it is meaningless to consider such parameters.

Definition 4 [14] Let $F_A \in \mathcal{S}(X, E)$. If $f_A(e) = X$ for all $e \in A$, then F_A is called an A -universal soft set, denoted by $F_A^\sim = \tilde{A}$. If $A = E$, then the A -universal soft set is called a universal soft set, denoted by $F_E^\sim = \tilde{E}$.

Definition 5 [5] Let Y be a nonempty subset of X , then $\tilde{Y}$ denotes the soft set Y_E over X for which $Y(e) = Y$, for all $e \in E$. In particular, X_E will be denoted by $\tilde{X}$.

Definition 6 [14] Let $F_A, G_B \in \mathcal{S}(X, E)$. Then F_A is a soft subset of G_B , denoted by $F_A \sqsubseteq G_B$ if $f_A(e) \subseteq g_B(e)$, for all $e \in E$.

Definition 7 [14] Let $F_A, G_B \in \mathcal{S}(X, E)$. Then F_A and G_B are soft equal, denoted by $F_A = G_B$, if and only if $f_A(e) = g_B(e)$ for all $e \in E$.

Definition 8 [14] Let $F_A, G_B \in \mathcal{S}(X, E)$. Then, the soft union $F_A \sqcup G_B$, the soft intersection $F_A \sqcap G_B$, and the soft difference $F_A \setminus G_B$ of F_A and G_B are defined by the approximate functions $h_{A \cup B}(e) = f_A(e) \cup g_B(e)$, $h_{A \cap B}(e) = f_A(e) \cap g_B(e)$, $h_{A \setminus B}(e) = f_A(e) \setminus g_B(e)$, respectively, and the soft complement F_A^c of F_A is defined by the approximate function $f_{A^c}(e) = f_A^c(e)$, where $f_A^c(e)$ is the complement of the set $f_A(e)$; that is, $f_A^c(e) = X \setminus f_A(e)$ for all $e \in E$.

Definition 9 [9] Let I be an arbitrary index set and $\{(F_A)_i\}_{i \in I}$ be a subfamily of $\mathcal{S}(X, E)$.

- (a) The union of these soft sets is the soft set G_C , where $g_C(e) = \bigcup_{i \in I} (f_A)_i(e)$ for each $e \in E$. We write $\bigcup_{i \in I} (f_A)_i = G_C$.
- (b) The intersection of these soft sets is the soft set H_D , where $h_D(e) = \bigcap_{i \in I} (f_A)_i(e)$ for all $e \in E$. We write $\bigcap_{i \in I} (f_A)_i = H_D$.

Definition 10 [9] The soft set $F_A \in \mathcal{S}(X, E)$ is called a soft point in $\tilde{X}$, denoted by e_F , if for the element $e \in A$, $f_A(e) \neq \emptyset$ and $f_A(e') = \emptyset$ for all $e' \in A - \{e\}$. The soft point e_F is said to be in the soft set G_B , denoted by $e_F \tilde{\in} G_B$, if for the element $e \in A$ and $f_A(e) \subseteq g_B(e)$.

Proposition 1 [9] Let $e_F \tilde{\in} \tilde{X}$ and $G_B \sqsubseteq \tilde{X}$. If $e_F \tilde{\in} G_B$ then $e_F \notin G_B^c$.

Definition 11 [5] Let $\tilde{\tau} \sqsubseteq \mathcal{S}(X, E)$. A soft topology on X , denoted by $\tilde{\tau}$, is a collection of soft subsets of X having the following properties:

- (1) $\Phi, \tilde{X} \in \tilde{\tau}$
- (2) If $F_A, G_B \in \tilde{\tau}$, then $F_A \sqcap G_B \in \tilde{\tau}$
- (3) If $(F_A)_\lambda \in \tilde{\tau}$, $\forall \lambda \in \Delta$, then $\bigcup_{\lambda \in \Delta} (F_A)_\lambda \in \tilde{\tau}$

The pair $(X, \tilde{\tau})$ is called a soft topological space. Every member of $\tilde{\tau}$ is called a soft open set. A soft set F_A is called soft closed in $(X, \tilde{\tau})$ if $F_A^c \in \tilde{\tau}$.

Definition 12 [9] A soft set G_B in a soft topological space $(X, \tilde{\tau})$ is called a soft neighbourhood of the soft point $e_F \tilde{\in} \tilde{X}$ if there exists a soft open set H_C such that $e_F \tilde{\in} H_C \sqsubseteq G_B$.

Remark 1 A soft set G_B in a soft topological space $(X, \tilde{\tau})$ is not only a soft neighbourhood of the soft point e_F , it is also a soft neighbourhood for all soft points of the soft set H_C by Definition 12.

The neighbourhood system of a soft point e_F , denoted by $\mathcal{N}_{\tilde{\tau}}(e_F)$, is the family of all its soft neighbourhoods.

Theorem 1 [9] The neighbourhood system $\mathcal{N}_{\tilde{\tau}}(e_F)$ at e_F in a soft topological space $(X, \tilde{\tau})$ has the following properties:

- V_1) If $G_B \in \mathcal{N}_{\tilde{\tau}}(e_F)$, then $e_F \tilde{\in} G_B$,
- V_2) If $G_B \in \mathcal{N}_{\tilde{\tau}}(e_F)$ and $G_B \sqsubseteq H_C$, then $H_C \in \mathcal{N}_{\tilde{\tau}}(e_F)$,
- V_3) If $G_B, H_C \in \mathcal{N}_{\tilde{\tau}}(e_F)$, then $G_B \sqcap H_C \in \mathcal{N}_{\tilde{\tau}}(e_F)$,
- V_4) If $G_B \in \mathcal{N}_{\tilde{\tau}}(e_F)$, then there is a $M_D \in \mathcal{N}_{\tilde{\tau}}(e_F)$ such that $G_B \in \mathcal{N}_{\tilde{\tau}}(e'_H)$ for each $e'_H \tilde{\in} M_D$.

Definition 13 Let $(X, \tilde{\tau})$ be a soft topological space and let $\mathcal{G}_{\tilde{\tau}}(e_F)$ be a family of some soft neighbourhoods of soft point e_F . If, for each soft neighbourhood G_B of e_F , there exists a $H_C \in \mathcal{G}_{\tilde{\tau}}(e_F)$ such that $e_F \tilde{\in} H_C \sqsubseteq G_B$, then we say that $\mathcal{G}_{\tilde{\tau}}(e_F)$ is a soft neighbourhood base at e_F .

Theorem 2 Let $(X, \tilde{\tau})$ be a soft topological space and $G_B \in \mathcal{S}(X, E)$. G_B is soft open if and only if it is a soft neighbourhood of each of its soft points.

Proof $\Rightarrow$ If G_B is a soft open set, then for each soft point $e_F \tilde{\in} G_B$, $e_F \tilde{\in} G_B \sqsubseteq G_B$. Since $G_B \in \tilde{\tau}$, G_B is a soft neighbourhood of each of its soft points by Definition 12.

$\Leftarrow$ For each soft point $e_F \tilde{\in} G_B$, there exists a soft set $(H_C)_{e_F} \in \tilde{\tau}$ such that $e_F \tilde{\in} (H_C)_{e_F} \sqsubseteq G_B$. For each $e_F \tilde{\in} G_B$, since $e_F \tilde{\in} (H_C)_{e_F}$ we obtain $e_F \tilde{\in} \bigcup_{e_F \tilde{\in} G_B} (H_C)_{e_F}$ and

$G_B \sqsubseteq \bigcup_{e_F \tilde{\in} G_B} (H_C)_{e_F}$. Conversely, for each $e_F \tilde{\in} G_B$, since $(H_C)_{e_F} \sqsubseteq G_B$, we obtain $\bigcup_{e_F \tilde{\in} G_B} (H_C)_{e_F} \sqsubseteq G_B$. Consequently, we get $G_B = \bigcup_{e_F \tilde{\in} G_B} (H_C)_{e_F}$. Hence, G_B is a soft open set by Definition 11. $\square$

Theorem 3 If for each soft point $e_F \tilde{\in} \tilde{X}$ there corresponds a family $\mathcal{N}(e_F)$ such that the properties V_1 , V_2 , V_3 and V_4 in Theorem 1 are satisfied, then there is a unique $\tilde{\tau}$ soft topological structure over X such that for each $e_F \tilde{\in} \tilde{X}$, $\mathcal{N}(e_F)$ is the family of $\tilde{\tau}$ -soft neighbourhoods of e_F .

Proof Let $\tilde{\tau} = \{G_B \in \mathcal{S}(X, E) : e_F \tilde{\in} G_B \Rightarrow G_B \in \mathcal{N}(e_F)\}$. It is clear that, $\tilde{\tau}$ is a soft topology over X . The family $\tilde{\tau}$ certainly satisfies axioms (3) and (2) in Definition 11: for (3), this follows immediately from V_2 in Theorem 1, and for (2), from V_3 in Theorem 1. The axiom (1) in Definition 11 is a result of V_2 and V_3 in Theorem 1. It remains to show that, in the soft topology defined by $\tilde{\tau}$, $\mathcal{N}(e_F)$ is the set of $\tilde{\tau}$ -soft neighbourhoods of e_F for each $e_F \tilde{\in} \tilde{X}$. It follows from V_2 in Theorem 1 that every $\tilde{\tau}$ -soft neighbourhood of e_F belongs to $\mathcal{N}(e_F)$. Conversely, let $(G_B)_1$ be a soft set belonging to $\mathcal{N}(e_F)$, and let $(G_B)_2$ be the soft set of soft points $e_M \tilde{\in} \tilde{X}$ such that $(G_B)_1 \in \mathcal{N}(e_M)$. If we can show that $e_F \tilde{\in} (G_B)_2$, $(G_B)_2 \sqsubseteq (G_B)_1$ and $(G_B)_2 \in \tilde{\tau}$, then the proof will be complete. Since for every soft point $e_M \tilde{\in} (G_B)_2$ belongs to $(G_B)_1$ by reason of V_1 in Theorem 1 and the hypothesis $(G_B)_1 \in \mathcal{N}(e_M)$, we obtain $(G_B)_2 \sqsubseteq (G_B)_1$. Since $(G_B)_1 \in \mathcal{N}(e_F)$ and $(G_B)_2 \sqsubseteq (G_B)_1$, we have $e_F \tilde{\in} (G_B)_2$. It remains to show that

$(G_B)_2 \in \tilde{\tau}$, i.e. that $(G_B)_2 \in \mathcal{N}(e_M)$ for each $e_M \in (G_B)_2$. If $e_M \in (G_B)_2$ then by V_4) in Theorem 1 there is a soft set $(G_B)_3$ such that for each $e_H \in (G_B)_3$ we have $(G_B)_1 \in \mathcal{N}(e_H)$. Since $(G_B)_1 \in \mathcal{N}(e_H)$ means that $e_H \in (G_B)_2$, it follows that $(G_B)_3 \subseteq (G_B)_2$ and, therefore, by V_2) in Theorem 1, that $(G_B)_2 \in \mathcal{N}(e_M)$. $\square$

Soft filter

Definition 14 Let $\tilde{\mathcal{F}} \subseteq \mathcal{S}(X, E)$, then $\tilde{\mathcal{F}}$ is called a soft filter on X if $\tilde{\mathcal{F}}$ satisfies the following properties:

$$\begin{aligned} f_1) & \Phi \notin \tilde{\mathcal{F}} \\ f_2) & \forall F_A, G_B \in \tilde{\mathcal{F}} \implies F_A \sqcap G_B \in \tilde{\mathcal{F}} \\ f_3) & \forall F_A \in \tilde{\mathcal{F}} \text{ and } F_A \subseteq G_B \implies G_B \in \tilde{\mathcal{F}} \end{aligned}$$

Remark 2 $\tilde{\mathcal{F}}$ satisfies the following property from f_1) and f_2): “The all finite intersections of soft sets of $\tilde{\mathcal{F}}$ are not soft empty”.

Remark 3 It is clear that $\tilde{X} \in \tilde{\mathcal{F}}$ by f_3).

Example 2 The family $\tilde{\mathcal{F}} = \{\tilde{X}\}$ is a soft filter over X .

Example 3 Let $\Phi \neq F_A \in \mathcal{S}(X, E)$. Then, the family $\tilde{\mathcal{F}}_{F_A} = \{G_B : F_A \subseteq G_B \in \mathcal{S}(X, E)\}$ is a soft filter over X and it is called atomic soft filter.

Example 4 Let $\mathbb{N}$ be the natural numbers and E be a finite set. Consider the family $\tilde{\mathcal{F}} = \{F_A : \bigcup_{e \in E} \mathbb{N} \setminus f_A(e) \text{ is finite}\}$, then $\tilde{\mathcal{F}}$ is a soft filter and it is called soft Frechet filter.

Example 5 Let X be an infinite set and E be a finite set. Consider the family $\tilde{\mathcal{F}} = \{F_A : \bigcup_{e \in E} X \setminus f_A(e) \text{ is finite}\}$, then $\tilde{\mathcal{F}}$ is a soft filter and it is called soft cofinite filter.

Example 6 Let X be an uncountable set and E be a countable set. Consider the family $\tilde{\mathcal{F}} = \{F_A : \bigcup_{e \in E} X \setminus f_A(e) \text{ is countable}\}$, then $\tilde{\mathcal{F}}$ is a soft filter and it is called soft cocountable filter.

Proposition 2 Let $\tilde{\mathcal{F}}$ be a soft filter over X . Then the collection

$$\tilde{\mathcal{F}}_e = \{f_A(e) : F_A \in \tilde{\mathcal{F}}\}$$

for each $e \in E$, does not define a filter over X .

Proof If $f_A(e) = \emptyset$ for any $e \in E$ then $\emptyset \in \tilde{\mathcal{F}}_e$. Hence $\tilde{\mathcal{F}}_e$ is not a filter over X in general topology. $\square$

Example 7 Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2, e_3\}$ and $\tilde{\mathcal{F}} = \{\tilde{X}, (F_A)_1, (F_A)_2, (F_A)_3, (F_A)_4\}$ be a soft filter defined over X where $(F_A)_1 = \{(e_1, \emptyset), (e_2, \emptyset), (e_3, \{x_1, x_3\})\}$, $(F_A)_2 = \{(e_1, \emptyset), (e_2, \{x_2, x_3\}), (e_3, \{x_1, x_3\})\}$, $(F_A)_3 = \{(e_1, \{x_1, x_2\}), (e_2, \emptyset), (e_3, \{x_1, x_3\})\}$, $(F_A)_4 = \{(e_1, \{x_1, x_2\}), (e_2, \{x_2, x_3\}), (e_3, \{x_1, x_3\})\}$. Since $\emptyset \in \tilde{\mathcal{F}}_{e_1}$, then $\tilde{\mathcal{F}}_{e_1}$ is not a filter over X .

Theorem 4 Let $(X, \tilde{\tau})$ be a soft topological space. The neighbourhood system $\mathcal{N}_{\tilde{\tau}}(e_F)$ is a soft filter for every $e_F \in \tilde{X}$. Also, it is called soft neighbourhoods filter of the soft point e_F .

Proof

- f_1) By V_1) in Theorem 1, since $e_F \in G_B$, we obtain $\Phi \notin \mathcal{N}_{\tilde{\tau}}(e_F)$.
- f_2) This is clearly seen by V_3) in Theorem 1.
- f_3) This is clearly seen by V_2) in Theorem 1.

$\square$

Now, we set up a soft topology with the help of a soft filter.

Theorem 5 If, for every $e_F \in \tilde{X}$, there exists a soft filter $\tilde{\mathcal{F}}(e_F) = \mathcal{N}_{\tilde{\tau}}(e_F)$ which satisfies the following two properties, then there exists a unique soft topology $\tilde{\tau}$ such that $\tilde{\mathcal{F}}(e_F)$ consists of the $\tilde{\tau}$ -soft neighbourhoods of the soft point e_F .

V_{f_1}) Every soft set in the soft filter $\tilde{\mathcal{F}}(e_F)$ contains the soft point e_F ,

V_{f_2}) For every $G_B \in \tilde{\mathcal{F}}(e_F)$ there exists a $H_C \in \tilde{\mathcal{F}}(e_F)$ such that for every $e_M \in H_C$, $G_B \in \tilde{\mathcal{F}}(e_M)$.

Proof Since the axioms $f_1), f_2), f_3), V_{f_1})$ and $V_{f_2})$ are equivalent to the neighbourhood axioms $V_1) - V_4)$, by Theorem 3, there exists a soft topology $\tilde{\tau}$ such that $\tilde{\mathcal{F}}(e_F)$ consists of the $\tilde{\tau}$ -soft neighbourhoods of the soft point e_F . $\square$

Example 8 Let $(X, \tilde{\tau})$ be a soft topological space and $e_F \in \tilde{X}$. Since G_B cannot be an element of $\mathcal{G}_{\tilde{\tau}}(e_F)$ for every $H_C \in \mathcal{G}_{\tilde{\tau}}(e_F)$ and $H_C \subseteq G_B$, then the soft neighbourhood base $\mathcal{G}_{\tilde{\tau}}(e_F)$ is not a soft filter over X .

Definition 15 Let $\tilde{\mathcal{F}}_1$ and $\tilde{\mathcal{F}}_2$ be soft filters over X . If $\tilde{\mathcal{F}}_1 \sqsubseteq \tilde{\mathcal{F}}_2$, then $\tilde{\mathcal{F}}_2$ is soft finer than $\tilde{\mathcal{F}}_1$. If $\tilde{\mathcal{F}}_1 \sqsubset \tilde{\mathcal{F}}_2$, then $\tilde{\mathcal{F}}_2$ is soft strictly finer than $\tilde{\mathcal{F}}_1$. If either $\tilde{\mathcal{F}}_1 \sqsubseteq \tilde{\mathcal{F}}_2$ or $\tilde{\mathcal{F}}_2 \sqsubseteq \tilde{\mathcal{F}}_1$, then $\tilde{\mathcal{F}}_1$ is comparable with $\tilde{\mathcal{F}}_2$.

Proposition 3 Let $(\tilde{\mathcal{F}}_i)_{i \in I}$ be a family of soft filters. Then, the family $(\tilde{\mathcal{F}}_i)_{i \in I}$ is partially ordered according to " $\sqsubseteq$ ".

Theorem 6 Let $(\tilde{\mathcal{F}}_i)_{i \in I}$ be a family of soft filters over X . Then $\tilde{\mathcal{F}} = \bigcap_{i \in I} \tilde{\mathcal{F}}_i$ is a soft filter over X .

Proof

f_1) Since $\Phi \notin \tilde{\mathcal{F}}_i$ for each $i \in I$, then Φ does not belong to $\tilde{\mathcal{F}} = \bigcap_{i \in I} \tilde{\mathcal{F}}_i$.

f_2) Let $F_A, G_B \in \tilde{\mathcal{F}} = \bigcap_{i \in I} \tilde{\mathcal{F}}_i$. Then, $F_A, G_B \in \tilde{\mathcal{F}}_i$ for each $i \in I$. Since $F_A \sqcap G_B \in \tilde{\mathcal{F}}_i$ for each $i \in I$, so we obtain $F_A \sqcap G_B \in \tilde{\mathcal{F}} = \bigcap_{i \in I} \tilde{\mathcal{F}}_i$.

f_3) Let $F_A \in \tilde{\mathcal{F}} = \bigcap_{i \in I} \tilde{\mathcal{F}}_i$ and $F_A \sqsubseteq G_B$. Since $F_A \in \tilde{\mathcal{F}}_i$ for each $i \in I$ and $F_A \sqsubseteq G_B$, we get $G_B \in \tilde{\mathcal{F}}_i$ for each $i \in I$. Hence $G_B \in \tilde{\mathcal{F}} = \bigcap_{i \in I} \tilde{\mathcal{F}}_i$. $\square$

Remark 4 The soft filter $\tilde{\mathcal{F}}$ in Theorem 6 is called the greatest lower bound of the family $(\tilde{\mathcal{F}}_i)_{i \in I}$.

Remark 5 The union of soft filters over X is generally not a soft filter over X .

Example 9 Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tilde{\mathcal{F}}_1 = \{\tilde{X}, (F_A)_1, (F_A)_2, (F_A)_3\}$, $\tilde{\mathcal{F}}_2 = \{\tilde{X}, (G_B)_1, (G_B)_2, (G_B)_3\}$ be two soft filters defined over X where $(F_A)_1 = \{(e_1, \{x_1\}), (e_2, \{x_2, x_3\})\}$, $(F_A)_2 = \{(e_1, \{x_1, x_2\}), (e_2, X)\}$, $(F_A)_3 = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$ and $(G_B)_1 = \{(e_1, \{x_3\}), (e_2, \{x_1, x_2\})\}$, $(G_B)_2 = \{(e_1, \{x_1, x_3\}), (e_2, X)\}$, $(G_B)_3 = \{(e_1, \{x_1, x_3\}), (e_2, \{x_1, x_2\})\}$. Now, we define $\tilde{\mathcal{F}} = \tilde{\mathcal{F}}_1 \cup \tilde{\mathcal{F}}_2 = \{\tilde{X}, (F_A)_1, (F_A)_2, (F_A)_3, (G_B)_1, (G_B)_2, (G_B)_3\}$. If we take $(F_A)_1 \sqcap (G_B)_1 = H_C$ then $h_C(e_1) = (f_A)_1(e_1) \cap (g_B)_1(e_1) = \{x_1\} \cap \{x_3\} = \emptyset$ and $h_C(e_2) = (f_A)_1(e_2) \cap (g_B)_1(e_2) = \{x_2, x_3\} \cap \{x_1, x_2\} = \{x_2\}$ but $H_C \notin \tilde{\mathcal{F}}$. Hence $\tilde{\mathcal{F}}$ is not a soft filter over X .

Now, we investigate the least upper bound of the family of soft filters over X .

Proposition 4 Let $\tilde{S} \sqsubseteq \mathcal{S}(X, E)$. Then there exists a soft filter $\tilde{\mathcal{F}}$ which contains the soft family $\tilde{S}$, if $\tilde{S}$ has the following property: "The all finite intersections of soft sets of $\tilde{S}$ are not soft empty".

Proof Let

$$\tilde{S} = \{(F_A)_i : \forall i \in J (J \text{ is finite}), \bigcap_{i \in J} (F_A)_i \neq \Phi\}.$$

Then we give the family which consists of finite intersections of elements of $\tilde{S}$; $\tilde{\beta} = \{G_B : \forall i \in J (J \text{ is finite}), (F_A)_i \in \tilde{S} \text{ and } G_B = \bigcap_{i \in J} (F_A)_i\}$.

Then, the family $\tilde{\mathcal{F}}(\tilde{S}) = \{H_C : G_B \in \tilde{\beta} \text{ and } G_B \sqsubseteq H_C\}$ is a soft filter over X .

f_1) Since $\Phi \notin \tilde{\beta}$, for every $H_C \in \tilde{\mathcal{F}}(\tilde{S})$, $H_C \neq \Phi$ and so $\Phi \notin \tilde{\mathcal{F}}(\tilde{S})$.

f_2) Let $(H_C)_1, (H_C)_2 \in \tilde{\mathcal{F}}(\tilde{S})$. There exist soft sets $(G_B)_1, (G_B)_2 \in \tilde{\beta}$ such that $(G_B)_1 \sqsubseteq (H_C)_1$ and $(G_B)_2 \sqsubseteq (H_C)_2$. From the definition of $\tilde{\beta}$, $\Phi \neq (G_B)_1 \sqcap (G_B)_2 \in \tilde{\beta}$. Since $(G_B)_1 \sqcap (G_B)_2 \sqsubseteq (H_C)_1 \sqcap (H_C)_2$, we obtain $(H_C)_1 \sqcap (H_C)_2 \in \tilde{\mathcal{F}}(\tilde{S})$.

f_3) Let $(H_C)_1 \in \tilde{\mathcal{F}}(\tilde{S})$ and $(H_C)_1 \sqsubseteq (H_C)_2$. Then, there exists a soft set $G_B \in \tilde{\beta}$ such that $G_B \sqsubseteq (H_C)_1$. Since $(H_C)_1 \sqsubseteq (H_C)_2$, we obtain $(H_C)_2 \in \tilde{\mathcal{F}}(\tilde{S})$. $\square$

Remark 6 The soft filter $\tilde{\mathcal{F}}(\tilde{S})$ in Proposition 4 is said to be generated by $\tilde{S}$ and $\tilde{S}$ is said to be soft filter subbase of $\tilde{\mathcal{F}}(\tilde{S})$. It is clear that, $\tilde{S} \sqsubseteq \tilde{\mathcal{F}}(\tilde{S})$.

Proposition 5 The soft filter $\tilde{\mathcal{F}}(\tilde{S})$ which is generated by $\tilde{S}$ is the coarsest soft filter which contains $\tilde{S}$.

Proof Suppose that $\tilde{S} \sqsubseteq \tilde{\mathcal{F}}_1$. By Proposition 4, $\tilde{S} \sqsubseteq \tilde{\beta} \sqsubseteq \tilde{\mathcal{F}}_1$. By Remark 6, for every $H_C \in \tilde{\mathcal{F}}(\tilde{S})$, there exists a $G_B \in \tilde{\beta}$ such that $G_B \sqsubseteq H_C$. Since $\tilde{\beta} \sqsubseteq \tilde{\mathcal{F}}_1$ then $G_B \in \tilde{\mathcal{F}}_1$. Since $\tilde{\mathcal{F}}_1$ is a soft filter, $H_C \in \tilde{\mathcal{F}}_1$ by f_3) in Definition 14. Hence we obtain $\tilde{\mathcal{F}}(\tilde{S}) \sqsubseteq \tilde{\mathcal{F}}_1$. $\square$

Theorem 7 The family $(\tilde{\mathcal{F}}_i)_{i \in I}$ of soft filters over X has a least upper bound if and only if for all finite subfamilies $(\tilde{\mathcal{F}}_i)_{1 \leq i \leq n}$ of $(\tilde{\mathcal{F}}_i)_{i \in I}$ and all $(G_B)_i \in \tilde{\mathcal{F}}_i (1 \leq i \leq n)$, the intersection $(G_B)_1 \sqcap \dots \sqcap (G_B)_n$ is not soft empty.

Proof $\implies$ If there exists a least upper bound of the family $(\tilde{\mathcal{F}}_i)_{i \in I}$, by f_1 and f_2 in Definition 14, for all finite subfamilies $(\tilde{\mathcal{F}}_i)_{1 \leq i \leq n}$ of $(\tilde{\mathcal{F}}_i)_{i \in I}$ and all $(G_B)_i \in \tilde{\mathcal{F}}_i$ ($1 \leq i \leq n$), the intersection $(G_B)_1 \cap \dots \cap (G_B)_n \neq \Phi$.

$\Leftarrow$ Let $(G_B)_1 \cap \dots \cap (G_B)_n \neq \Phi$ for all finite subfamilies $(\tilde{\mathcal{F}}_i)_{1 \leq i \leq n}$ of $(\tilde{\mathcal{F}}_i)_{i \in I}$ and all $(G_B)_i \in \tilde{\mathcal{F}}_i$ ($1 \leq i \leq n$). Then, the soft filter $\tilde{\mathcal{F}}(\tilde{S})$ generated by

$$\tilde{S} = \sqcup_{i \in I} \tilde{\mathcal{F}}_i = \{F_A : (\exists i \in I) F_A \in \tilde{\mathcal{F}}_i\}$$

is the least upper bound of the family $(\tilde{\mathcal{F}}_i)_{i \in I}$ by Proposition 5. $\square$

Definition 16 Let $\tilde{\beta} \sqsubseteq \mathcal{S}(X, E)$, then $\tilde{\beta}$ is said to be a soft filter base on X if

- $b_1) \tilde{\beta} \neq \Phi$ and $\Phi \notin \tilde{\beta}$,
- $b_2) \forall F_A, G_B \in \tilde{\beta}, \exists H_C \in \tilde{\beta} \ni H_C \sqsubseteq F_A \sqcap G_B$.

Remark 7 $\tilde{\beta}$ which is in Proposition 4 is a soft filter base.

Remark 8 It is clear that, every soft filter is a soft filter base.

Example 10 Let $\Phi \neq F_A \in \mathcal{S}(X, E)$. Then, the family $\tilde{\beta} = \{F_A\}$ is a soft filter base over X .

Example 11 Let (X, τ) be a soft topological space and $e_F \in \tilde{X}$. The soft neighbourhood base $\mathcal{G}_\tau(e_F)$ is a soft filter base over X .

- $b_1)$ Clearly, $\mathcal{G}_\tau(e_F) \neq \emptyset$. For every $H_C \in \mathcal{G}_\tau(e_F)$, $e_F \in H_C$. Then, $H_C \neq \Phi$. Hence, we obtain $\Phi \notin \mathcal{G}_\tau(e_F)$.
- $b_2)$ Let $G_B, H_C \in \mathcal{G}_\tau(e_F)$. Since $G_B, H_C \in \mathcal{N}_\tau(e_F)$, we get $G_B \sqcap H_C \in \mathcal{N}_\tau(e_F)$. By Definition 13, there exists a $K_D \in \mathcal{G}_\tau(e_F)$ such that $K_D \sqsubseteq G_B \sqcap H_C$. Hence we get $\mathcal{G}_\tau(e_F)$ is a soft filter base of soft neighbourhoods filter $\mathcal{N}_\tau(e_F)$ by Definition 16.

Author contribution All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

Conflict of interest The authors declare that they have no competing interests.

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