Models and valid inequalities to asymmetric skill-based routing problems

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Abstract Given a directed network, where a set of skills S_j is required to serve each node j , and given a set of available technicians, each one having abilities described through a set of skills, we study the problem of defining the tour of each technician in such a way that each service requirement is fulfilled by exactly one technician, and proper skill constraints are satisfied. These constraints state that the service requirement at node j can be operated by any technician having, at least, all the skills in S_j . Given travelling costs for the arcs of the network, which are technician dependent, we want to determine a set of tours of minimum cost which satisfy the skill constraints. This problem, named asymmetric skill VRP (A-SKVRP), originates from a real application arising in Field Service. Also, it is strongly related to Home Care applications. Starting from the preliminary results in Cappanera et al. (Network optimization. 5th international conference, INOC 2011, Hamburg, June 2011. Lecture notes in computer science, Springer, Berlin, vol 6701, pp 354–364, 2011), where a special case is addressed, we propose a hierarchy of three ILP flow-based models for A-SKVRP, which is based on increasing levels of disaggregation of the flow variables used to prevent subtours. The computational results of a first pool of experiments show that the LP bounds produced by the most disaggregated model are very tight, but at the expense of very large computational times in most cases. This becomes critical as the number of involved nodes and

This paper expands and improves upon the LNCS version (Cappanera et al. 2011).

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skills increases. Therefore, in the second part of our study, we enhance the LP bounds obtained with the weaker models by adding, in a cutting plane fashion, cut constraints. As a result of this enhancement, the intermediate model proves to be competitive against the most disaggregated model in computing tight lower bounds, especially as the number of the nodes and of the skills increases. This model combined with the proposed valid inequalities thus qualifies as an efficient tool to address skill-based routing problems arising in application contexts such as Field Service and Home Care.

Keywords Asymmetric VRP · Skill · ILP models · Valid inequalities · Cutting plane · Computational experiments

The problem

Let $G = (N, A)$ be a (complete) directed network, with $|N| = n$ and $|A| = m$, where node 1 denotes the depot. Let each node $j, j \neq 1$, represent a service requirement, and S_j denote the set of the skills required by node j for the associated service call. Let T be the set of available technicians, and let S_t denote the set of the skills characterizing the abilities of technician t . The skills constrain how the node requirements can be operated; the service requirement at node j can be operated by any technician t possessing all the abilities required by j , i.e. $S_j \subseteq S_t$.

Given non-negative technician-dependent travelling costs c_{ij}^t for each $(i, j) \in A$ and technician $t \in T$, we study the problem of defining a set of tours for the technicians, each one starting and ending at node 1, in such a way that each service requirement is fulfilled by exactly one technician, and the skill constraints are satisfied. This problem is referred to as the asymmetric skill VRP (A-SKVRP). It has been introduced in Cappanera et al. (2011) for the special case where a skill s_j is associated with each node j , a skill s_t is associated with each technician t , and there is a gradation of the skills such that j can be visited by t only if $s_t \geq s_j$. A-SKVRP originates from a real application concerning the dispatch of technicians to customer service requests in after-sales service management, referred to as *Field Service* (Agnihotri and Mishra 2004; Rapaccini et al. 2008). Also, it is strongly related to Home Care applications like Home Care Scheduling problems, where coordinators have to assign the care of every client to an operator by considering his workload, the particular skills that the operator should have in order to satisfy the client needs and the travel distance of the operator for reaching client home. See Borsani et al. (2006) for an example of an Integer Linear Programming (ILP) model developed to support human resource short-term planning in Home Care Scheduling where, however, routing aspects are not addressed.

Although the asymmetric skill VRP bears some similarities to the site-dependent VRP (SDVRP), introduced by Nag et al. (1988), where a node j can only be served by a subset of the vehicles (or technicians, in our terminology), observe that A-SKVRP is not a special case of SDVRP. In SDVRP the routing costs are vehicle independent and represented by a symmetric edge cost matrix (Baldacci et al. 2010c). See Baldacci et al. (2010a, b) for exact approaches to SDVRP, and Cordeau

and Laporte (2001) for an example of Tabu Search approach for the SDVRP with time windows. On the other hand, in A-SKVRP the costs are technician dependent and represented by an asymmetric cost matrix.

Also observe that, if we restrict the attention to undirected graphs (and so to symmetric edge cost matrices), then the A-SKVRP can be formulated as a special case of the Multi-Depot VRP (MDVRP), which is more general than SDVRP. In fact, MDVRP is an extension of the Capacitated VRP (CVRP) where a node can be served by an unlimited fleet of identical vehicles of capacity Q , located at p depots, inter-depot routes are not allowed, and costs may vary depending on the depot (Baldacci et al. 2010c). To model the symmetric skill VRP in terms of MDVRP, just introduce a depot d_t of capacity $n - 1$ for each technician t [i.e. introduce a copy of the original depot (node 1) for each technician], and associate with d_t the arc costs which depend on t .

In any case it is important to emphasize that A-SKVRP is introduced and studied in the more general scenario of asymmetric graphs, and thus it is a special case of the asymmetric MDVRP which however, to the best of our knowledge, has never been investigated in the literature so far.

Starting from the preliminary results in Cappanera et al. (2011), in this paper, we propose a hierarchy of ILP flow-based models for A-SKVRP, which is based on increasing levels of disaggregation of the flow variables used to prevent subtours. Flow-based models are a well-known modeling tool to formulate VRP problems, as shown in Gavish and Graves (1979) and Letchford and Salazar-Gonzalez (2006). Specifically, in “[ILP models for the Asymmetric Skill VRP](#)” we present three ILP models such that the Linear Programming (LP) bound of the most disaggregated model is greater than or equal to the one associated with the intermediate model, which in turn is greater than or equal to the LP bound of the less disaggregated model. We investigate the behavior of the three models by performing a first pool of experiments on randomly generated instances characterized by a gradation of three skills, such that node j can be visited by technician t only if $s_t \geq s_j$. The computational results, discussed in “[Model comparison](#)”, provide evidence that the LP bounds produced by the most disaggregated model are very tight. This model thus qualifies as a strong model to solve A-SKVRP instances. However, such a model may be difficult to use, since the computational time needed to solve its LP relaxation tends to increase with the number of the nodes. In addition, it also increases with the number of the skills, and this becomes critical in the above-mentioned application contexts, i.e. Field Service and Home Care, where often the number of the skills is larger than 3, which is the case addressed in the first pool of experiments. Indeed, the size of the most disaggregated model increases with the number of skills and technicians, both in terms of number of variables and constraints.

Due to the considerations above, in the second part of our study we investigate the strength of the most disaggregated model in more depth, by observing that its LP relaxation implies an exponential sized set of cut constraints. Then, in order to achieve a compromise between strength of the LP bound and computational effort required to compute it, we select a suitable subset from this large class of cut constraints, and use it to enhance the LP bounds obtained with the weaker models.

This idea of projecting some constraints satisfied by the solutions of an extended formulation into more compact formulations is a well-known modeling technique to improve the LP relaxation of ILP formulations, see for example Letchford and Salazar-Gonzalez (2006). The projected cuts are described in “[Special cut constraints](#)”, whereas the computational results conducted to assess the impact of the proposed valid inequalities are discussed in “[Assessing the impact of valid inequalities](#)”. These results show that the intermediate model becomes a better alternative than the most disaggregated model after the cut addition. We then study an additional set of cut constraints, of exponential size, and further enhance the intermediate model via a cutting plane approach. The proposed approach, presented in “[Two-cycle inequalities and related cutting plane approach](#)”, proves to be very effective as the number of nodes and skills increases, thus promoting the intermediate model and the proposed valid inequalities as an efficient tool to address skill-based routing problems arising in application contexts such as Field Service and Home Care. This is reported in “[Assessing the impact of the cutting plane approach](#)”. Finally, “[Conclusions](#)” propose suggestions for future research.

We conclude this section by pointing out that, to the best of our knowledge, no formulations and exact approaches have been proposed in the literature to the asymmetric MDVRP. Therefore, no comparison can be performed between the formulations proposed in this paper for A-SKVRP (and the related computational results) and alternative approaches from the literature for the more general asymmetric MDVRP. A comparison might be performed only between the symmetric skill VRP and MDVRP. Anyway, although very efficient exact approaches have been proposed for the symmetric VRP, and in particular to MDVRP, all based on set partitioning formulations, indeed the best state-of-the-art exact code for symmetric VRPs, described in Baldacci et al. (2010c), may not be so efficient to address the symmetric skill VRP instances, due to the intrinsic uncapped nature of skill VRP.¹

ILP models for the asymmetric skill VRP

In Cappanera et al. (2011) three ILP models have been proposed in order to formulate the asymmetric skill VRP when a gradation of the skills is defined. Here, the models are generalized to the case where arbitrary subsets of skills characterize nodes as well as technicians. The three models share the design variables and some sets of constraints which, as classical in the VRP literature, are used to define the tours of the technicians. The objective function, to be minimized, is also common to the three models. For each $(i, j) \in A$ and $t \in T$ such that $(S_i \cup S_j) \subseteq S_t$, let:

$$x_{ij}^t = \begin{cases} 1 & \text{if } (i, j) \text{ belongs to the tour of technician } t \\ 0 & \text{otherwise} \end{cases}$$

Note that condition $(S_i \cup S_j) \subseteq S_t$, for each arc (i, j) and technician t , allows one to incorporate the skill constraints directly into the variable definition. The design

¹ Based on personal communication with the authors.

variables must satisfy constraints (1), (2) and (3). In particular, the first set of constraints guarantees that each client is visited once, while the second set guarantees the skill consistency along the tour of technician t , i.e. if a properly skilled technician t enters a node j , then t leaves j .

$$\sum_{i \in N} \sum_{t: (S_i \cup S_j) \subseteq S_t} x_{ij}^t = 1 \quad j = 2, \dots, n \quad (1)$$

$$\sum_{i \in N: S_i \subseteq S_t} x_{ij}^t = \sum_{i \in N: S_i \subseteq S_t} x_{ji}^t \quad j = 2, \dots, n, t : S_j \subseteq S_t \quad (2)$$

$$x_{ij}^t \in \{0, 1\} \quad (i, j) \in A, t : (S_i \cup S_j) \subseteq S_t \quad (3)$$

The objective function, to be minimized, defines the overall routing cost:

$$\sum_{(i,j) \in A} \sum_{t: (S_i \cup S_j) \subseteq S_t} c_{ij}^t x_{ij}^t \quad (4)$$

The ILP models differ in the way they prevent subtours, that is tours which are disconnected from the depot. The proposed models use auxiliary flow variables and standard flow conservation constraints, plus linking constraints to relate the flow variables to the design variables, on increasing levels of variable and constraint disaggregation. Skill information is suitably added to the linking constraints, depending on the considered level of disaggregation. Let us discuss these flow model components in more detail.

The aggregated model

The first model, called *Agg*, prevents subtours using an auxiliary flow variable y_{ij} for each $(i, j) \in A$, that indicates the amount of flow pushed along (i, j) [or, equivalently, the number of clients to be visited after traversing the arc (i, j) , included node j]. This is typical in aggregated flow (or single commodity flow) formulations for vehicle routing problems. See Gavish and Graves (1979) for pioneering flow-based formulations in the context of VRP.

Using these flow variables and the design variables x_{ij}^t , model *Agg* comprises the following constraints in addition to (1)–(3):

$$\begin{aligned} \sum_{(i,j) \in A} y_{ij} &= n - 1 \\ \sum_{(i,j) \in A} y_{ij} - \sum_{(j,i) \in A} y_{ji} &= 1 \quad j \in N \setminus \{1\} \\ y_{ij} &\leq (n - 1) \sum_{t: (S_i \cup S_j) \subseteq S_t} x_{ij}^t \quad (i, j) \in A \\ y_{ij} &\geq 0 \quad (i, j) \in A \end{aligned}$$

As specified before, the skill-based routing is determined by minimizing function (4). The first group of constraints states that $n - 1$ units of flow leave the depot, while the second group guarantees that each client receives one unit of flow. The third group is formed by the constraints linking the flow variables to the design variables.

The destination disaggregated model

The second model is obtained from *Agg* by performing a disaggregation of the flow variables y_{ij} by destination. This way of strengthening the linear programming relaxation of single commodity flow models is a well-known technique (to the best of our knowledge, this technique was first suggested by Rardin and Choe (1979)). Specifically, replace the flow variables y_{ij} by the disaggregated flow variables $y_{ij}^k, k \in N \setminus \{1\}$, that specify the amount of flow traversing the arc (i, j) that is destined to node k . In particular, the variables y_{ij}^k are defined for $j \neq 1$ and $i \neq k$.

Using these disaggregated flow variables and the design variables x_{ij}^t , we obtain a new model, referred to as *DestDis*, which includes the following constraints in addition to the common block (1)–(3):

$$\begin{aligned} \sum_{(i,j) \in A: i \neq k, j \neq 1} y_{ij}^k - \sum_{(j,i) \in A: i \neq 1, j \neq k} y_{ji}^k &= \begin{cases} 1 & \text{if } i = 1 \\ 0 & \text{if } i \neq 1, k \\ -1 & \text{if } i = k \end{cases} \quad i \in N, k = 2, \dots, n \\ y_{ij}^k &\leq \sum_{t: (S_i \cup S_j \cup S_k) \subseteq S_t} x_{ij}^t \quad (i, j) \in A, k \in N : k \neq 1, i, j \neq 1. \\ y_{ij}^k &\geq 0 \quad (i, j) \in A : j \neq 1, k \neq 1, i \end{aligned} \quad (5)$$

Again, the objective function is given by (4).

In model *DestDis*, the flow-based constraints state that one unit of flow must be sent from the depot to each client or, equivalently, that the solution must contain a path from the depot to node k , for each $k \in N \setminus \{1\}$. Thus, the flow variables y_{ij}^k can be reinterpreted as indicating whether the arc (i, j) is in the path from the depot to node k .

Observe that model *DestDis* is more than a straightforward multicommodity reformulation of model *Agg*. In fact, a straightforward disaggregation would lead to the weaker linking constraints $y_{ij}^k \leq \sum_{t: (S_i \cup S_j) \subseteq S_t} x_{ij}^t, (i, j) \in A, k \in V, k \neq 1, i, j \neq 1$. On the other hand, the flow variable disaggregation, together with the skill definition, allows one to enhance the linking constraints, leading to the strengthened form (5). This suggests that the LP bound of *DestDis* should be substantially tighter than the one returned by *Agg* (it is easy to see that it is always greater than or equal to the LP bound of model *Agg*).

The destination-technician disaggregated model

The third model combines disaggregation by destination with disaggregation by technician. In fact, by performing a further step of disaggregation, the flow variables $y_{ij}^k, k \in N \setminus \{1\}$, which characterize model *DestDis*, are replaced by the flow variables y_{ij}^{kt} , with $j \neq 1, i \neq k$ and $t: (S_i \cup S_j \cup S_k) \subseteq S_t$. Each variable y_{ij}^{kt} indicates whether the arc (i, j) belongs to the path from the depot to node k , and whether it is traversed by technician t . The model uses additional auxiliary variables $w^{kt}, k = 2, \dots, n, t: S_k \subseteq S_t$, indicating whether node k is served by technician t .

Using the variables y_{ij}^{kt} and w^{kt} , the model, hereafter named *DestTechDis*, comprises the following constraints in addition to (1)–(3), under the minimization stated by (4):

$$\sum_{\substack{(i,j) \in A: i \neq k, j \neq 1, \\ (S_i \cup S_j) \subseteq S_t}} y_{ij}^{kt} - \sum_{\substack{(j,i) \in A: i \neq 1, j \neq k, \\ (S_i \cup S_j) \subseteq S_t}} y_{ji}^{kt} = \begin{cases} w^{kt} & \text{if } i = 1 \\ 0 & \text{if } i \neq 1, k \\ -w^{kt} & \text{if } i = k \end{cases} \quad i \in N, k = 2, \dots, n, t : S_k \subseteq S_t \quad (6)$$

$$\sum_{t: S_k \subseteq S_t} w^{kt} = 1 \quad k = 2, \dots, n \quad (7)$$

$$y_{ij}^{kt} \leq x_{ij}^t \quad (i, j) \in A, \quad k \in N : j \neq 1, k \neq 1, i, \\ t : (S_i \cup S_j \cup S_k) \subseteq S_t \quad (8)$$

$$w^{kt} = \sum_{j \in N: S_j \subseteq S_t} x_{jk}^t \quad k = 2, \dots, n, t : S_k \subseteq S_t \quad (9)$$

$$w^{kt} \in \{0, 1\} \quad k = 2, \dots, n, t : S_k \subseteq S_t$$

$$y_{ij}^{kt} \geq 0 \quad (i, j) \in A : j \neq 1, k \neq 1, i, \quad t : (S_i \cup S_j \cup S_k) \subseteq S_t$$

The flow conservation constraints (6) state that the variables y_{ij}^{kt} define a path from the depot to node k , if k is served by technician t (i.e. $w^{kt} = 1$). Note that, if we relate these flow variables to the flow variables of model *DestDis*, according to $\sum_t y_{ij}^{kt} = y_{ij}^k$, $(i, j) \in A : j \neq 1, k \neq 1, i$, then the linking constraints (8) can be viewed as a disaggregated form of the linking constraints in model *DestDis*. This shows that the LP bound of *DestTechDis* is never worse than the LP bound of *DestDis*, and also suggests that the LP bound of *DestTechDis* can be tighter than the one returned by *DestDis*.

Equalities (9) indicate that the variables w^{kt} can be eliminated from the model. However, they are maintained to make the flow conservation constraints easier to interpret. Note also that of the three sets of equalities, (1), (7) and (9), two sets imply the other, and so one set can be removed. This is easily seen in the case of (1) or (7) being redundant. For the other case, consider the following result:

Lemma 1 *The constraints $w^{kt} = \sum_{j \in N: S_j \subseteq S_t} x_{jk}^t$, $k = 2, \dots, n, t : S_k \subseteq S_t$, are redundant in the LP relaxation of model *DestTechDis*.*

Proof Remove constraints (9) from model *DestTechDis*. Then observe that the LP of the resulting, reduced, model implies the constraints $w^{kt} \leq \sum_{j \in N: S_j \subseteq S_t} x_{jk}^t$, $k = 2, \dots, n, t : S_k \subseteq S_t$. This follows from (6) and (8).

Then, by adding these inequalities over t , and combining with (1) and (7), we conclude that these inequalities cannot be strict. Therefore, constraints (9) are implied by the LP relaxation of the reduced model or, equivalently, they are redundant in the LP relaxation of *DestTechDis*. The thesis follows.

A hierarchy of ILP flow-based models for A-SKVRP has thus been presented, which is based on increasing levels of disaggregation of the flow variables. As already observed, the LP bound of the most disaggregated model is greater than or

equal to the one associated with the intermediate model *DestDis*, which, in turn, is greater than or equal to the LP bound of *Agg*. We have investigated this relationship in more detail through a first pool of experiments reported in “[Model comparison](#)”. The main conclusion is that model *DestTechDis* produces very tight LP bounds. However, it may be difficult to use in practice, since the computational time needed to solve its LP relaxation is very large in most cases. Therefore, as already outlined, we have analyzed more closely the relation between the LP relaxation of the models *DestTechDis* and *DestDis* and this is the topic of the next section.

Valid inequalities and cutting plane approach

In this section, we discuss the strength of the LP relaxation of the model *DestTechDis*. We present a large subset of cut-set inequalities, on the variables x_{ij}^t , which are implied by the LP relaxation of this model. Then, based on the analysis of the LP solutions of the weaker models, we select a subset of the new inequalities in order to improve the LP relaxation of the weaker models, as reported in “[Experimental campaign](#)”.

Strength of model *DestTechDis*

From the *maximum flow–minimum cut* theorem (see, e.g. Ahuja et al. 1993), it is easy to show that the LP relaxation of *DestTechDis* implies the following exponential sized set of cut constraints:

$$\sum_{i \in N \setminus S, j \in S: (S_i \cup S_j) \subseteq S_t} x_{ij}^t \geq \sum_{j \in N: S_j \subseteq S_t} x_{jk}^t \quad S \subseteq N \setminus \{1\}, k \in S, t \in T: S_k \subseteq S_t. \quad (10)$$

Based on this observation, we have selected a suitable subset from the large class of cut constraints (10), and use it to enhance the weaker models *Agg* and *DestDis*.

Specifically, by looking at the subgraphs induced by the variables x_{ij}^t having a positive value in the LP solution of models *Agg* and *DestDis*, for each t , we observed the following facts:

1. the LP solution contains many two cycles (i.e. cycle of type $(i - j - i)$) or composition of two cycles in the subgraphs associated with technicians t not used in the solution (i.e. $x_{ij}^t = 0$ for all nodes j).
2. the LP solution contains two cycles (or again composition of two cycles) also in the subgraphs of technicians used, i.e. connected to the depot.

Special cases of constraints (10) have therefore been investigated in order to eliminate these two-cycle structures, and to enhance the weaker models going toward the quality of the most disaggregated model.

Special cut constraints

In the first part of the analysis devoted to the enhancement of the weaker models, the following special cut constraints, of polynomial size, have been added to the LP relaxation of *Agg* and *DestDis*:

$$\sum_{j \in N \setminus \{1\} : S_j \subseteq S_t} x_{1j}^t \geq \sum_{(j,i) : S_j \subseteq S_t} x_{ji}^t \quad t \in T, i \neq 1 : S_i \subseteq S_t \quad (11)$$

$$x_{ik}^t \leq \sum_{(j,i) : j \neq k, S_j \subseteq S_t} x_{ji}^t \quad t \in T, i, k \neq 1 : (S_i \cup S_k) \subseteq S_t. \quad (12)$$

Constraints (11) guarantee that, if technician t is not used, then none of the design variables relative to t can be greater than zero. Constraints (12) state that, if the arc (i, k) belongs to the solution, then the predecessor of node i must be other than k . Hereafter these two sets of valid inequalities will be referred to as TC1 and TC2, respectively, where TC stands for two cycles. *Agg* and *DestDis* augmented with TC1 and TC2 will be referred to as *StrongAgg* and *StrongDestDis*, respectively.

Observe that (11) are special cut constraints of type (10) obtained by choosing $S = \{1\}$. These valid inequalities allow one to eliminate the two cycles of the first type. Also, (12) are special cut constraints, as shown below.

Lemma 2 (12) are special cut constraints of type (10).

Proof Consider the following cut constraints of type (10), obtained by choosing $S = i, k$:

$$\sum_{(j,k) : j \neq i, S_j \subseteq S_t} x_{jk}^t + \sum_{(j,i) : j \neq k, S_j \subseteq S_t} x_{ji}^t \geq \sum_{(j,k) : S_j \subseteq S_t} x_{jk}^t. \quad (13)$$

(13) are equivalent to:

$$\sum_{(j,i) : j \neq k, S_j \subseteq S_t} x_{ji}^t \geq x_{ik}^t. \quad (14)$$

The thesis follows.

Two-cycle inequalities and related cutting plane approach

To further enhance model *DestDis*, which proved to be very competitive after the additions of cuts TC1 and TC2 (see “Assessing the impact of valid inequalities”), another set of cut constraints has been addressed in order to eliminate the two cycles which still arise in the LP solutions. These are cut constraints of type (10) where S is a set of nodes which induces a two-cycle [i.e. a cycle of type $(i - j - i)$] or a composition of two cycles in the solution of the LP relaxation of *StrongDestDis*. These cut constraints will be referred to as *two-cycle inequalities*. Since the cardinality of this set may be exponential with respect to the input size, a heuristic separation procedure has been implemented to separate inequalities from this subset, and a cutting plane approach has been designed based on this separation procedure. For each technician t , the procedure visits the graph starting from the depot. It stores in S a potential candidate node k , and a subset of nodes i and j , reachable from k , such that both $x_{ij}^t > 0$ and $x_{ji}^t > 0$. The key idea is to identify a composition of two cycles, that includes k , in the subgraph associated with technician t . The constructed set S and the corresponding k are returned by the separation procedure only if $|S| \geq 3$ (since the valid inequalities TC1 and TC2 are already included into the model), and if the total sum of the variables x_{ij}^t within

S , that is for i and j belonging to S , is “sufficiently high”, i.e. greater than or equal to $|S|/2$. The last condition guarantees that, in the subsets S returned by the heuristic, the majority of the nodes are connected through links which are inner to the subset. Our testing showed that this heuristic proved to be very efficient in separating violated two-cycle inequalities. Each iteration of the cutting plane approach consists in performing the separation procedure described above, and adding to the model the packet of violated two-cycle inequalities discovered by this procedure. In other words, several violated inequalities may be added all in each iteration of the proposed approach.

Observe that in the skill VRP scenario where a single skill s_j is associated with each node j , a single skill s_t is associated with each technician t , and a gradation of the skills is defined such that a node j can be visited by a technician t only if $s_t \geq s_j$, then the A-SKVRP models, and the related valid inequalities, can be derived from the ILP formulations and the valid inequalities presented above by replacing the operator $\subseteq$ by $\leq$ in the stated skill constraints. For example, $j, t : S_j \subseteq S_t$ becomes $j, t : s_j \leq s_t$. This is the scenario addressed in the experimental campaign.

Experimental campaign

The main achievements that are shown, respectively, in the next three subsections are the following: (a) model *DestTechDis* has a very strong LP relaxation; unfortunately, the computational time required to solve the LP relaxation becomes unacceptable on large instances that arise from real application contexts; (b) the weaker models *Agg* and *DestDis* greatly benefit from the introduction of a subset of valid inequalities that are implied by model *DestTechDis*; in particular, the gap of the intermediate model *DestDis* becomes very tight after the introduction of some of these constraints, suggesting that *DestDis* is a good tradeoff between quality of the bounds and computational effort required to compute them; (c) a cutting plane approach based on model *DestDis* with an additional set of cut inequalities, when applied to large instances, is able to provide solutions very close (within 2 % in most cases) to the ones obtained by the stronger model *DestTechDis*, in much shorter computational times.

All the models and the cutting plane approach have been implemented via AMPL using the solver Cplex 12.1 on a AMD Athlon (tm) 64×2 Dual Core Processor 4200+ (CPU MHz 2211.186). In the following the computational times are expressed in seconds of CPU time.

Model comparison

Here, we present the results of the first pool of experiments on a suite of randomly generated instances characterized by three skills. As indicated before, a skill s_j is associated with each node j , a skill s_t is associated with each technician t , and a gradation of the skills is defined such that a node j can be visited by a technician t only if $s_t \geq s_j$. Symmetric as well as asymmetric instances compose this suite of

A-SKVRP instances. The experiments aim at comparing the ILP models presented in “[ILP models for the Asymmetric Skill VRP](#)” both in terms of quality of the returned LP bounds and also in terms of the required computational times.

A-SKVRP instances: scenario with three skills

We used two test sets originally suggested for the VRP, with the data relative to the skills. Specifically, the VRP instances of the first test set are characterized by the number of nodes (either 20 or 40) and by the type (4 different choices). There are two types of Euclidean instances referred to as *tc* and *te* according to the position of the depot in the grid. In the *tc* instances the depot is located on a corner of the grid, while in the *te* instances the depot is located in the centre of the grid. In addition, there are two types of random instances: *tra* denotes asymmetric instances and *trs* denotes the symmetric ones. Five instances are available for each given number of nodes and type, thus summing up to a total of 40 ($2 \times 4 \times 5$) instances. The VRP instances *tc* and *te* were created for the computational testing reported in Gouveia (1995), and since then they have been used in the computational testing reported in many other papers concerning trees, for instance Gouveia et al. (2011). These instances have also been used for the results reported in the paper on VRP (Godinho et al. 2009). The VRP instances *tra* and *trs* have been generated for the experiments in Godinho et al. (2009).

In order to generate the first group of A-SKVRP instances we used instance number one (over the five available) for each type and for each given number of nodes, thus summing up to 8 original instances over 40.

The VRP instances of the second test set are asymmetric CVRP instances proposed in Fischetti et al. (1994) and available at <http://www.or.deis.unibo.it/research.html>. They are characterized by a number of nodes in {34, 36, 39, 45, 48, 56, 65, 71} and by three vehicles. In order to generate the second group of A-SKVRP instances we used the larger instances in this test set, i.e. A045, A048, A056, A065 and A071.

In the generated A-SKVRP instances there is a one-to-one correspondence between skills and technicians, i.e. we have a technician for each skill. In this pool of experiments we set both the number of technicians and the number of skills equal to 3 (observe that this is the number of vehicles characterizing the asymmetric instances in Fischetti et al. 1994), where skill 1 denotes the lowest skill and skill 3 is the highest (w.l.o.g. the depot is assumed to have skill 1).

For each original instance of the first test set we generated 18 A-SKVRP instances according to the pattern used to assign a skill to the nodes. The patterns used are the following:

- 50-10-40 (for a total of 6 permutations)
- 50-20-30 (for a total of 6 permutations)
- 40-40-20 (for a total of 3 permutations)
- 30-30-40 (for a total of 3 permutations)

where the numbers indicate the percentage of nodes requiring a specific skill. As an example, pattern 50-10-40 is used to denote possible scenarios where 50 % of nodes

requires a certain skill, which can be skill 1, 2 or 3, 40 % of nodes requires another skill, and 10 % of nodes requires a third skill, thus summing up to 6 combinations.

For each original instance of the second test set we generated six A-SKVRP instances according to the following patterns:

50-40-10
 40-50-10
 50-30-20
 30-50-20
 40-40-20
 40-30-30

In contrast to the original VRP instances used, in the A-SKVRP instances the costs depend on the arc as well as on the technician. We assumed that the cost of an arc (i, j) is increasing with the technician skills, i.e. the more a technician is qualified higher the cost. The formula used to generate the costs is the following:

$$c_{ij}^t = c_{ij}\delta t, \quad (15)$$

where c_{ij} represents the cost of arc (i, j) in the original instances, while the factor δ has been set equal to 2. Observe that these costs satisfy the triangular property for each technician when the original costs do. Trivially if $c_{ij} \leq c_{ih} + c_{hj}$, the inequality $c_{ij}\delta k \leq (c_{ih} + c_{hj})\delta k$ holds true. Under the hypothesis of costs not satisfying the triangular property, the proposed models do not assure that, for a given technician, the schedule is composed of a single tour, i.e. it might be cheaper for the technician come back to the depot more than once.

Computational results

We have performed our computational experiments with the instances of the two groups. In Tables 1, 2 and 4, a comparison of the models *Agg*, *DestDis* and *DestTechDis* on the larger instances of the first group, i.e. the ones with 40 nodes, is given in terms of average time required to solve the continuous relaxation and in terms of percentage gap of the LP bound with respect either to the optimum value when available or to the best integer solution value. Model *Agg* is used to get the optimum value or an upper bound to it. For the aggregated model the computational time required to get the optimum is also reported: a time limit of 6 h is fixed and when such a limit is exceeded the value of the best integer solution is returned. Since the computational time was not an issue for the instances with 20 nodes, which were solved quite easily via model *Agg*, the computational results related to such a subset of instances are omitted. Similarly, in Tables 3 and 5 a comparison of the models *Agg*, *DestDis* and *DestTechDis* is given for the instances of the second group. Again, model *Agg* is used to get the optimum value or an upper bound to it. Due to their very large size, a time limit of 12 h has been fixed for the instances belonging to the families *A065* and *A071*, i.e. the A-SKVRP instances with 65 and 71 nodes.

Table 1 Comparison in terms of optimality gap: first group (40 nodes-*tc-te*)

Instance	AggIPTime	Agg	DestDis	DestTechDis
tc40-1-20-4-16.dat	21,741.87	30.24	6.03	1.92
tc40-1-20-16-4.dat	21,699.26	39.77	11.49	3.23
tc40-1-4-20-16.dat	2,865.86	22.73	2.52	0.30
tc40-1-16-20-4.dat	21,691.04	39.73	10.64	4.41
tc40-1-4-16-20.dat	742.10	20.34	1.94	0.00
tc40-1-16-4-20.dat	4,907.78	26.43	4.24	0.73
tc40-1-20-12-8.dat	21,718.93	35.23	11.83	3.52
tc40-1-20-8-12.dat	19,093.48	33.86	4.00	0.18
tc40-1-8-20-12.dat	21,653.45	30.04	4.55	0.59
tc40-1-12-20-8.dat	21,694.86	31.22	8.42	1.79
tc40-1-8-12-20.dat	175.17	21.10	1.85	0.00
tc40-1-12-8-20.dat	4,476.51	29.63	5.70	1.22
tc40-1-16-16-8.dat	21,695.64	35.99	10.17	4.36
tc40-1-8-16-16.dat	985.21	24.32	1.91	0.00
tc40-1-16-8-16.dat	3,992.99	28.66	3.48	0.82
tc40-1-16-12-12.dat	3,469.94	29.41	4.37	0.13
tc40-1-12-16-12.dat	21,742.12	30.76	6.01	0.49
tc40-1-12-12-16.dat	3,184.70	24.14	2.08	0.00
Avg tc40-1	12,085.05	29.64	5.62	1.32
te40-1-20-4-16.dat	1,192.86	23.93	5.24	0.38
te40-1-20-16-4.dat	21,709.62	38.92	16.87	8.85
te40-1-4-20-16.dat	20.46	17.67	5.30	0.00
te40-1-16-20-4.dat	21,709.06	39.44	15.10	8.78
te40-1-4-16-20.dat	7.31	15.09	2.66	0.00
te40-1-16-4-20.dat	2,611.68	20.07	4.40	0.28
te40-1-20-12-8.dat	21,701.03	32.50	11.66	4.33
te40-1-20-8-12.dat	21,721.15	28.63	6.52	0.00
te40-1-8-20-12.dat	21,704.39	33.11	15.77	9.44
te40-1-12-20-8.dat	21,697.92	32.52	12.88	3.79
te40-1-8-12-20.dat	181.85	15.72	5.44	0.00
te40-1-12-8-20.dat	154.84	16.49	3.91	0.00
te40-1-16-16-8.dat	21,739.62	29.04	11.26	2.66
te40-1-8-16-16.dat	2,729.12	20.53	6.23	0.42
te40-1-16-8-16.dat	21,757.85	25.56	11.89	2.09
te40-1-16-12-12.dat	6,681.37	24.02	4.04	0.00
te40-1-12-16-12.dat	21,731.71	31.78	11.54	2.49
te40-1-12-12-16.dat	9,919.81	23.55	9.28	0.30
Avg te40-1	12,165.09	26.03	8.89	2.43

It appears that, in the first group of instances, computing the optimum integer solution is computational expensive for the instances of type *tc* and *te* (see the average computational times reported under *AggIP* in Table 4), whereas the ones of

type *tra* are efficiently solved to optimality via model *Agg*. The instances of type *trs* show an intermediate behavior. Also note that for *tc* and *te* instances the time limit is reached, respectively, 8 and 9 times over 18, whereas for *tra* and *trs* instances the optimum is always returned significantly earlier than the time limit. In all cases, the enhancement produced by increasing the level of disaggregation leads to significant improvements of the LP bounds (see the average gaps in bold in Tables 1, 2). This trend is particularly evident for the most difficult instances of the group, i.e. the ones of type *tc* and *te*, where the most disaggregated model *DestTechDis* provides an average gap of 1.32 for *tc* family and of 2.43 for *te* family, while the average gaps provided by model *Agg* are, respectively, 29.64 and 26.03 for the two families. Note that *DestTechDis* is able to find a solution of optimal value in 20 cases over 72 with a bigger frequency on the *te* and the *trs* instances (respectively, 6 and 7 times over 18). Clearly, the required computational times become higher when the level of disaggregation increases, reaching an average time of 855.29 s with *DestTechDis* for the *te* instances. Model *DestDis* shows an intermediate behavior for all the tested instances, in terms of gap and often also in terms of computational time. In fact, the gaps provided by *DestDis* are much better than the ones related to model *Agg*, although not tight as the ones computed by *DestTechDis*.

If we now consider the second group of instances, we notice that these large asymmetric instances are very hard to solve to optimality. In fact, in only 10 cases over 30 the time limit is not exceeded. Observe, in particular, that the time limit of 12 h set to solve to optimality the A-SKVRP instances with 65 and 71 nodes is always over. Also for this group of instances the enhancement of the level of disaggregation improves the LP bounds in a significant way (see the average gaps in bold in Table 3). In particular, the most disaggregated model *DestTechDis* provides a very good average gap that ranges from the minimum 1.92 (A045 family) to the maximum 6.10 (A065 family), while the average gaps provided by model *Agg* range from 17.60 (A045 family) to 23.30 (A056 family). However, also in this case the required computational times become prohibitive when the level of disaggregation increases, reaching an average time of 15537.47 s with *DestTechDis* for the larger instances with 71 nodes. As before, model *DestDis* has an intermediate behavior for all the tested instances of the second group, in terms of gap and also in terms of computational time. In particular, the gaps provided by *DestDis* are much better than the ones related to model *Agg*, although not tight as the ones computed by *DestTechDis*. Observe, however, that for the larger instances, i.e. the ones with 56, 65 and 71 nodes, the average gaps provided by *DestDis* are at most twice the gaps returned via model *DestTechDis* (see the average gaps reported in bold in Table 3).

As a summary, in both groups of instances the LP bounds produced by *DestTechDis* are very tight. However, *DestTechDis* may be difficult to use, since the computational time needed to solve its LP relaxation is very large in most cases, especially for the large asymmetric instances of the second group. This may be an

Table 2 Comparison in terms of optimality gap: first group (40 nodes-tra-trs)

Instance	AggIPTime	Agg	DestDis	DestTechDis
tra40-1-20-4-16.dat	1.89	4.47	0.36	0.00
tra40-1-20-16-4.dat	71.87	16.69	7.99	2.50
tra40-1-4-20-16.dat	1.51	3.76	1.65	0.05
tra40-1-16-20-4.dat	106.65	12.52	7.57	1.68
tra40-1-4-16-20.dat	2.32	7.34	3.99	1.07
tra40-1-16-4-20.dat	1.01	7.63	1.01	0.18
tra40-1-20-12-8.dat	197.02	16.39	7.55	4.20
tra40-1-20-8-12.dat	11.98	8.43	5.93	0.60
tra40-1-8-20-12.dat	57.98	9.84	5.37	1.60
tra40-1-12-20-8.dat	11.16	9.97	6.67	1.37
tra40-1-8-12-20.dat	0.70	4.53	2.51	0.00
tra40-1-12-8-20.dat	21.58	6.15	1.85	0.91
tra40-1-16-16-8.dat	142.26	13.88	8.02	2.91
tra40-1-8-16-16.dat	5.03	8.40	4.15	0.48
tra40-1-16-8-16.dat	3.32	7.20	4.44	0.37
tra40-1-16-12-12.dat	19.12	8.67	3.02	0.55
tra40-1-12-16-12.dat	36.17	11.08	7.09	2.51
tra40-1-12-12-16.dat	5.78	7.97	3.40	0.00
Avg tra40-1	38.74	9.16	4.59	1.17
trs40-1-20-4-16.dat	184.28	29.94	5.61	0.12
trs40-1-20-16-4.dat	359.20	38.00	10.30	1.61
trs40-1-4-20-16.dat	1.59	21.26	1.95	0.00
trs40-1-16-20-4.dat	130.64	30.35	5.36	0.63
trs40-1-4-16-20.dat	0.87	18.79	1.55	0.00
trs40-1-16-4-20.dat	12.56	28.35	3.05	0.00
trs40-1-20-12-8.dat	1,930.65	32.66	10.42	1.82
trs40-1-20-8-12.dat	56.90	33.34	6.35	1.31
trs40-1-8-20-12.dat	115.75	26.14	6.09	0.14
trs40-1-12-20-8.dat	761.10	35.89	7.93	1.91
trs40-1-8-12-20.dat	2.10	20.40	4.28	0.00
trs40-1-12-8-20.dat	65.60	22.66	3.58	0.00
trs40-1-16-16-8.dat	2,618.03	35.05	8.30	0.85
trs40-1-8-16-16.dat	0.98	24.99	3.73	0.00
trs40-1-16-8-16.dat	15.64	28.70	5.22	0.71
trs40-1-16-12-12.dat	167.81	29.89	8.90	1.65
trs40-1-12-16-12.dat	241.72	29.05	6.50	0.70
trs40-1-12-12-16.dat	250.82	31.98	8.06	0.00
Avg trs40-1	384.24	28.75	5.95	0.64

issue in real applications where the number of the nodes is usually large and where, in addition, the number of involved skills and technicians can be larger than 3, which is the case addressed in the first pool of experiments.

Table 3 Comparison in terms of optimality gap: second group

Instance	AggIPTime	Agg	DestDis	DestTechDis
A045-03f-d2-23-18-4.dat	21,732.59	19.68	8.45	3.25
A045-03f-d2-18-23-4.dat	10,982.91	18.40	6.79	2.01
A045-03f-d2-23-13-9.dat	1,154.37	17.75	6.18	0.00
A045-03f-d2-13-23-9.dat	5,563.51	17.54	5.90	2.75
A045-03f-d2-18-18-9.dat	5,247.59	18.82	7.36	3.40
A045-03f-d2-19-13-13.dat	1,487.40	13.43	4.35	0.10
Avg A045	7,694.73	17.60	6.51	1.92
A048-03f-d2-24-19-5.dat	21,730.29	21.87	7.14	1.63
A048-03f-d2-19-24-5.dat	18,952.51	18.95	5.91	1.55
A048-03f-d2-24-15-9.dat	21,743.53	22.59	10.65	6.93
A048-03f-d2-15-24-9.dat	2,464.10	14.34	3.55	0.00
A048-03f-d2-19-19-10.dat	16,649.30	21.01	7.08	3.21
A048-03f-d2-20-14-14.dat	21,648.84	17.54	6.50	3.35
Avg A048	17,198.09	19.38	6.81	2.78
A056-03f-d2-28-22-6.dat	21,727.61	31.10	13.59	10.18
A056-03f-d2-22-28-6.dat	21,698.00	21.92	5.45	1.64
A056-03f-d2-28-17-11.dat	21,716.66	29.23	15.50	6.65
A056-03f-d2-17-28-11.dat	16,131.78	16.40	5.52	1.93
A056-03f-d2-22-22-12.dat	20,915.98	20.92	6.19	2.28
A056-03f-d2-22-17-17.dat	21,697.77	20.25	7.68	2.74
Avg A056	20,647.97	23.30	8.99	4.24
A065-03f-d2-33-26-6.dat	43,200.00	30.55	17.62	12.04
A065-03f-d2-26-33-6.dat	43,200.00	29.67	15.69	9.87
A065-03f-d2-33-19-13.dat	43,200.00	17.21	6.98	3.53
A065-03f-d2-19-33-13.dat	43,200.00	17.08	9.20	3.36
A065-03f-d2-26-26-13.dat	43,200.00	20.21	10.25	4.75
A065-03f-d2-25-20-20.dat	43,200.00	17.26	7.48	3.02
Avg A065	43,200.00	22.00	11.20	6.10
A071-03f-d2-36-28-7.dat	43,200.00	24.59	13.60	7.70
A071-03f-d2-28-36-7.dat	43,200.00	16.37	3.93	0.44
A071-03f-d2-36-21-14.dat	43,200.00	20.22	12.36	7.10
A071-03f-d2-21-36-14.dat	43,200.00	20.28	9.94	3.71
A071-03f-d2-28-28-15.dat	43,200.00	21.44	9.83	4.32
A071-03f-d2-29-21-21.dat	43,200.00	15.60	9.02	4.66
Avg A071	43,200.00	19.75	9.78	4.66

In order to achieve a compromise between strength of the LP bound and computational effort required to compute it, in the successive phase of our computational analysis we have also evaluated the weaker models *Agg* and *DestDis* enhanced with the addition of the valid inequalities TC1 and TC2, presented in “[Special cut constraints](#)”. This will be described in the next section.

Table 4 Comparison in terms of average time: first group (40 nodes)

Instance	AggIP	Agg	DestDis	DestTechDis
tc40-1	12,085.05	0.24	598.25	721.08
te40-1	12,165.09	0.26	459.85	855.29
tra40-1	38.74	0.37	206.61	168.02
trs40-1	384.24	0.29	752.12	805.23

Table 5 Comparison in terms of average time: second group

Instance	AggIP	Agg	DestDis	DestTechDis
A045	7,694.73	0.42	306.21	533.41
A048	17,198.09	0.48	598.61	1,000.25
A056	20,647.97	1.40	1,314.40	3,031.66
A065	43,200.00	3.72	3,039.72	9,266.48
A071	43,200.00	5.71	3,902.66	15,537.47

Assessing the impact of valid inequalities

We have conducted some tests to evaluate the strength of the valid inequalities TC1 and TC2 when added to models *Agg* and *DestDis* on the two groups of A-SKVRP instances described in “[A-SKVRP instances: scenario with 3 skills](#)”. In Tables 6, 7 and 8 a comparison of the models *StrongAgg*, *StrongDestDis* (i.e. *Agg* and *DestDis* equipped with TC1 and TC2) and *DestTechDis* is given for the 40 nodes instances of the first group in terms of percentage gap of the LP bound with respect to the optimum value (which has been computed via model *Agg*, as remarked above) and of average time required to solve the continuous relaxation. Analogous results are provided in Tables 9 and 10 for the asymmetric instances of the second group.

The results show that TC1 and TC2 are indeed very effective. The introduction of these inequalities in model *Agg* leads to reductions of the average percentage gap from 29.64 to 13.33 for the instances of type *tc*, and from 26.03 to 9.29 for the instances of type *te*. Drastic gap improvements are achieved also for the other sets of instances, including the asymmetric instances of the second group. Substantial reductions are obtained also by *StrongDestDis*. These results are summarized in Tables 11 and 12.

It is worth observing that the intermediate model is in most cases much faster after the addition of cuts TC1 and TC2, i.e. *StrongDestDis* is faster than *DestDis*, as the average times in Tables 8 and 10 show (this occurs for the instances of type *tc*, *te* and *tra*, and even more for all the instances in the second group).

Also note that model *DestDis*, when enhanced with TC1 and TC2, is able to find a solution of optimal value in 9 over 72 cases in the first group of instances; on the contrary, model *DestDis* without any cuts is unable to find the optimum within the same time limit.

As already observed, the instances of type *tra* and *trs* of the first group of instances appear to be easy to solve to optimality in most cases (see Table 7).

Table 6 TC1 and TC2—comparison in terms of optimality gap: first group (40 nodes-tc-te)

Instance	AggIPTime	StrongAgg	StrongDestDis	DestTechDis
tc40-1-20-4-16.dat	21,741.87	12.32	4.04	1.92
tc40-1-20-16-4.dat	21,699.26	21.61	7.78	3.23
tc40-1-4-20-16.dat	2,865.86	11.05	1.85	0.30
tc40-1-16-20-4.dat	21,691.04	21.88	7.42	4.41
tc40-1-4-16-20.dat	742.10	7.14	0.85	0.00
tc40-1-16-4-20.dat	4,907.78	9.15	2.75	0.73
tc40-1-20-12-8.dat	21,718.93	18.15	6.71	3.52
tc40-1-20-8-12.dat	19,093.48	15.04	1.09	0.18
tc40-1-8-20-12.dat	21,653.45	13.02	2.03	0.59
tc40-1-12-20-8.dat	21,694.86	17.61	4.21	1.79
tc40-1-8-12-20.dat	175.17	6.93	0.12	0.00
tc40-1-12-8-20.dat	4,476.51	12.42	2.30	1.22
tc40-1-16-16-8.dat	21,695.64	20.51	6.45	4.36
tc40-1-8-16-16.dat	985.21	6.60	0.00	0.00
tc40-1-16-8-16.dat	3,992.99	10.86	1.92	0.82
tc40-1-16-12-12.dat	3,469.94	12.08	1.21	0.13
tc40-1-12-16-12.dat	21,742.12	15.60	2.96	0.49
tc40-1-12-12-16.dat	3,184.70	7.92	0.00	0.00
Avg tc40-1	12,085.05	13.33	2.98	1.32
te40-1-20-4-16.dat	1,192.86	5.59	2.39	0.38
te40-1-20-16-4.dat	21,709.62	21.66	11.62	8.85
te40-1-4-20-16.dat	20.46	2.32	1.45	0.00
te40-1-16-20-4.dat	21,709.06	21.19	11.17	8.78
te40-1-4-16-20.dat	7.31	0.90	0.21	0.00
te40-1-16-4-20.dat	2,611.68	4.82	1.39	0.28
te40-1-20-12-8.dat	21,701.03	15.89	5.86	4.33
te40-1-20-8-12.dat	21,721.15	9.49	3.18	0.00
te40-1-8-20-12.dat	21,704.39	17.99	11.02	9.44
te40-1-12-20-8.dat	21,697.92	14.64	5.85	3.79
te40-1-8-12-20.dat	181.85	1.33	0.83	0.00
te40-1-12-8-20.dat	154.84	2.02	1.57	0.00
te40-1-16-16-8.dat	21,739.62	12.56	7.02	2.66
te40-1-8-16-16.dat	2,729.12	3.85	1.96	0.42
te40-1-16-8-16.dat	21,757.85	8.33	4.36	2.09
te40-1-16-12-12.dat	6,681.37	6.66	0.26	0.00
te40-1-12-16-12.dat	21,731.71	11.57	5.94	2.49
te40-1-12-12-16.dat	9,919.81	6.44	3.99	0.30
Avg te40-1	12,165.09	9.29	4.45	2.43

Therefore, the instances of type *tra* and *trs* have not been considered anymore in the rest of the experimentation. On the other hand, the instances of type *tc* and *te* appear to be more difficult to solve. The same observation holds for the instances of the

Table 7 TC1 and TC2—comparison in terms of optimality gap: first group (40 nodes-tra-trs)

Instance	AggIPTime	StrongAgg	StrongDestDis	DestTechDis
tra40-1-20-4-16.dat	1.89	0.19	0.00	0.00
tra40-1-20-16-4.dat	71.87	2.54	2.54	2.50
tra40-1-4-20-16.dat	1.51	0.16	0.11	0.05
tra40-1-16-20-4.dat	106.65	1.71	1.71	1.68
tra40-1-4-16-20.dat	2.32	1.22	1.07	1.07
tra40-1-16-4-20.dat	1.01	0.70	0.22	0.18
tra40-1-20-12-8.dat	197.02	4.63	4.49	4.20
tra40-1-20-8-12.dat	11.98	1.01	0.75	0.60
tra40-1-8-20-12.dat	57.98	2.00	2.00	1.60
tra40-1-12-20-8.dat	11.16	1.37	1.37	1.37
tra40-1-8-12-20.dat	0.70	0.00	0.00	0.00
tra40-1-12-8-20.dat	21.58	1.60	1.17	0.91
tra40-1-16-16-8.dat	142.26	3.75	3.39	2.91
tra40-1-8-16-16.dat	5.03	0.86	0.48	0.48
tra40-1-16-8-16.dat	3.32	0.74	0.38	0.37
tra40-1-16-12-12.dat	19.12	1.40	0.80	0.55
tra40-1-12-16-12.dat	36.17	3.73	3.73	2.51
tra40-1-12-12-16.dat	5.78	0.34	0.13	0.00
Avg tra40-1	38.74	1.55	1.35	1.17
trs40-1-20-4-16.dat	184.28	1.95	1.33	0.12
trs40-1-20-16-4.dat	359.20	3.04	2.73	1.61
trs40-1-4-20-16.dat	1.59	0.14	0.00	0.00
trs40-1-16-20-4.dat	130.64	3.06	1.53	0.63
trs40-1-4-16-20.dat	0.87	0.00	0.00	0.00
trs40-1-16-4-20.dat	12.56	1.68	1.15	0.00
trs40-1-20-12-8.dat	1,930.65	4.02	3.18	1.82
trs40-1-20-8-12.dat	56.90	2.31	1.84	1.31
trs40-1-8-20-12.dat	115.75	0.62	0.32	0.14
trs40-1-12-20-8.dat	761.10	3.90	3.40	1.91
trs40-1-8-12-20.dat	2.10	0.01	0.00	0.00
trs40-1-12-8-20.dat	65.60	1.04	0.00	0.00
trs40-1-16-16-8.dat	2,618.03	3.44	1.93	0.85
trs40-1-8-16-16.dat	0.98	0.00	0.00	0.00
trs40-1-16-8-16.dat	15.64	2.11	1.83	0.71
trs40-1-16-12-12.dat	167.81	3.98	3.72	1.65
trs40-1-12-16-12.dat	241.72	2.78	2.57	0.70
trs40-1-12-12-16.dat	250.82	4.22	2.42	0.00
Avg trs40-1	384.24	2.13	1.55	0.64

second group, of larger size. On these more difficult instances, the intermediate model *StrongDestDis* behaves much better than *StrongAgg* in terms of percentage gaps. Furthermore, after the cut addition the gaps of the intermediate model are

Table 8 TC1 and TC2—comparison in terms of average time: first group (40 nodes)

Instance	AggIP	StrongAgg	StrongDestDis	DestTechDis
tc40-1	12,085.05	0.77	317.83	721.08
te40-1	12,165.09	0.83	263.03	855.29
tra40-1	38.74	0.69	82.17	168.02
trs40-1	384.24	1.17	218.30	805.23

quite small (indeed, they are close to the ones produced by the strongest model *DestTechDis*, see Tables 6, 9), and they are often still computed in a reasonable computational time (see Tables 8, 10). These facts have motivated us to spend more time in enhancing the intermediate model since, among the proposed models, it appears to be a good compromise between efficacy and efficiency. This will be the subject of the next section, devoted to the further enhancement of model *StrongDestDis* via the two-cycle inequalities presented in “[Two-cycle inequalities and related cutting plane approach](#)”.

Assessing the impact of the cutting plane approach

Scenario with 40 nodes and 3 skills

We firstly tested the cutting plane approach on the *tc* instances with 40 nodes and 3 skills, described in “[A-SKVRP instances: scenario with 3 skills](#)”. As observed before, among the A-SKVRP instances with 40 nodes, these *tc* instances proved to be difficult to solve with the three models presented in “[ILP models for the Asymmetric Skill VRP](#)”. Thus these instances define a good initial set for testing the strength of the two-cycle inequalities. In Table 13, we report the optimality gap related to the first iteration of the cutting plane approach, as well as the one obtained after four iterations (experimentally we observed that, for the considered instances, the improvement is negligible after the fourth iteration). Also the optimality gap obtained by the models *StrongDestDis* and *DestTechDis* is given. We observe that the addition of the two-cycle inequalities improves on the gap returned by model *StrongDestDis* in all cases. In some cases the improvement is remarkable allowing us to conclude that these inequalities qualify as an interesting tool to approximate the very good LP bound provided by *DestTechDis*. The computational times required by the cutting plane approach on the considered instances were, however, not competitive with the ones required by *DestTechDis*, essentially due to the number of nodes, which is high but still tractable, and to the limited number of skills and technicians characterizing the considered instances. In fact, model *DestTechDis* is solvable in less than 1,000 s (see Table 8).

As already observed, in application contexts such as Field Service and Home Care, the number of nodes may be larger than 40 and the number of involved skills may be larger than 3. Increasing the number of the nodes becomes critical for the strongest model *DestTechDis*, as shown by the results in the previous sections concerning the large instances of the second group. Also, increasing the number of

Table 9 TC1 and TC2—comparison in terms of optimality gap: second group

Instance	AggIPTime	StrongAgg	StrongDestDis	DestTechDis
A045-03f-d2-23-18-4.dat	21,732.59	11.65	5.27	3.25
A045-03f-d2-18-23-4.dat	10,982.91	9.15	3.75	2.01
A045-03f-d2-23-13-9.dat	1,154.37	4.95	2.99	0.00
A045-03f-d2-13-23-9.dat	5,563.51	10.03	3.55	2.75
A045-03f-d2-18-18-9.dat	5,247.59	6.86	4.26	3.40
A045-03f-d2-19-13-13.dat	1,487.40	6.31	1.46	0.10
Avg A045	7,694.73	8.16	3.55	1.92
A048-03f-d2-24-19-5.dat	21,730.29	11.27	4.22	1.63
A048-03f-d2-19-24-5.dat	18,952.51	6.59	3.24	1.55
A048-03f-d2-24-15-9.dat	21,743.53	12.05	7.18	6.93
A048-03f-d2-15-24-9.dat	2,464.10	3.34	0.04	0.00
A048-03f-d2-19-19-10.dat	16,649.30	9.34	4.70	3.21
A048-03f-d2-20-14-14.dat	21,648.84	6.85	4.07	3.35
Avg A048	17,198.09	8.24	3.91	2.78
A056-03f-d2-28-22-6.dat	21,727.61	22.05	11.17	10.18
A056-03f-d2-22-28-6.dat	21,698.00	14.99	3.06	1.64
A056-03f-d2-28-17-11.dat	21,716.66	21.10	12.10	6.65
A056-03f-d2-17-28-11.dat	16,131.78	8.09	3.58	1.93
A056-03f-d2-22-22-12.dat	20,915.98	10.51	3.70	2.28
A056-03f-d2-22-17-17.dat	21,697.77	11.65	4.88	2.74
Avg A056	20,647.97	14.73	6.41	4.24
A065-03f-d2-33-26-6.dat	43,200.00	23.22	14.52	12.04
A065-03f-d2-26-33-6.dat	43,200.00	20.79	12.59	9.87
A065-03f-d2-33-19-13.dat	43,200.00	10.26	5.61	3.53
A065-03f-d2-19-33-13.dat	43,200.00	8.40	4.86	3.36
A065-03f-d2-26-26-13.dat	43,200.00	12.66	6.94	4.75
A065-03f-d2-25-20-20.dat	43,200.00	7.35	4.51	3.02
Avg A065	43,200.00	13.78	8.17	6.10
A071-03f-d2-36-28-7.dat	43,200.00	17.32	11.06	7.70
A071-03f-d2-28-36-7.dat	43,200.00	6.80	1.57	0.44
A071-03f-d2-36-21-14.dat	43,200.00	12.97	9.84	7.10
A071-03f-d2-21-36-14.dat	43,200.00	12.10	7.22	3.71
A071-03f-d2-28-28-15.dat	43,200.00	11.00	7.96	4.32
A071-03f-d2-29-21-21.dat	43,200.00	10.30	6.36	4.66
Avg A071	43,200.00	11.75	7.33	4.66

the skills becomes critical for *DestTechDis*, whose size increases with the number of skills and technicians. Therefore, in the last phase of the computational experience we have investigated the effectiveness of the cutting plane approach as the number of nodes and skills increases.

Table 10 TC1 and TC2—comparison in terms of average time: second group

Instance	AggIP	StrongAgg	StrongDestDis	DestTechDis
A045	7,694.73	1.03	218.58	533.41
A048	17,198.09	1.65	303.81	1,000.25
A056	20,647.97	2.81	539.09	3,031.66
A065	43,200.00	5.87	1,929.58	9,266.48
A071	43,200.00	7.22	2,454.96	15,537.47

Table 11 TC1 and TC2—comparison in terms of average gap: first group (40 nodes)

Instance	Agg	StrongAgg	DestDis	StrongDestDis	DestTechDis
tc40-1	29.64	13.33	5.62	2.98	1.32
te40-1	26.03	9.29	8.89	4.45	2.43
tra40-1	9.16	1.55	4.59	1.35	1.17
trs40-1	28.75	2.13	5.95	1.55	0.64

Table 12 TC1 and TC2—comparison in terms of average gap: second group

Instance	Agg	StrongAgg	DestDis	StrongDestDis	DestTechDis
A045	17.60	8.16	6.51	3.55	1.92
A048	19.38	8.24	6.81	3.91	2.78
A056	23.30	14.73	8.99	6.41	4.24
A065	22.00	13.78	11.20	8.17	6.10
A071	19.75	11.75	9.78	7.33	4.66

Trend for increasing number of nodes

We tested the effectiveness of the cutting plane approach as the number of the nodes increases by addressing the largest instances of the second group, i.e. the ones with 56, 65 and 71 nodes. A comparison of the models *StrongDestDis*, *StrongDestDis* enhanced with the cutting plane approach and *DestTechDis* is given in terms of percentage optimality gap in Table 14, and in terms of average time required to solve the continuous relaxation in Table 15. Specifically, we provide the optimality gap and the computational time related to the first iteration of the cutting plane approach (StrongDestDis+CP).

As observed, the time of *DestTechDis* grows very fast, requiring more than 4 h on average on the largest instances with 71 nodes (see Table 15). Therefore, the use of model *DestTechDis* is not suitable for instances with a large number of nodes. On the other hand, the cutting plane approach improves on the gap returned by model *StrongDestDis* (except only 2 cases) within a time which is about one half the time required by the strongest model. Concerning the optimality gap, *StrongDestDis*

Table 13 Optimality gap via cutting plane (40 nodes-tc-3 skills)

Instance	StrongDestDis	1 iteration	4 iterations	DestTechDis
tc40-1-20-4-16.dat	4.04	3.28	2.55	1.92
tc40-1-20-16-4.dat	7.78	6.63	5.46	3.23
tc40-1-4-20-16.dat	1.85	1.36	0.54	0.30
tc40-1-16-20-4.dat	7.42	6.71	6.27	4.41
tc40-1-4-16-20.dat	0.85	0.00	0.00	0.00
tc40-1-16-4-20.dat	2.75	1.99	1.77	0.73
tc40-1-20-12-8.dat	6.71	6.08	6.08	3.52
tc40-1-20-8-12.dat	1.09	0.45	0.44	0.18
tc40-1-8-20-12.dat	2.03	1.85	1.85	0.59
tc40-1-12-20-8.dat	4.21	3.58	2.45	1.79
tc40-1-8-12-20.dat	0.12	0.00	0.00	0.00
tc40-1-12-8-20.dat	2.30	2.30	2.17	1.22
tc40-1-16-16-8.dat	6.45	5.83	5.73	4.36
tc40-1-8-16-16.dat	0.00	0.00	0.00	0.00
tc40-1-16-8-16.dat	1.92	1.01	1.01	0.82
tc40-1-16-12-12.dat	1.21	0.76	0.19	0.13
tc40-1-12-16-12.dat	2.96	2.33	1.28	0.49
tc40-1-12-12-16.dat	0.00	0.00	0.00	0.00
Avg tc40-1	2.98	2.45	2.10	1.32

Average values are reported in bold

enhanced with the cutting plane provides gaps whose difference, with respect to the ones found by *DestTechDis*, is less than 2 % in most cases, while the average improvement with respect to the gap provided by *StrongDestDis* is at least 0.38 % (see Table 14).

Trend for increasing number of skills

We generated two additional sets of A-SKVRP instances, with, respectively, 6 and 9 skills and technicians, starting from the *tc* instance number one with 40 nodes. In the case of 6 skills, we generated 8 A-SKVRP instances according to the following patterns used to assign a skill to the nodes:

25-15-15-15-15-15 (for a total of 6 permutations)

50-10-10-10-10-10 (only the 2 extreme permutations 50-10-10-10-10-10 and 10-10-10-10-10-50)

where the numbers indicate the percentage of nodes requiring a specific skill. The last two extreme permutations have been generated in order to evaluate the impact of the pattern assignment on the difficulty of A-SKVRP instances (the intuition is that pattern 50-10-10-10-10-10 should be more difficult than 10-10-10-10-10-50, since there are more choices in the assignment of technicians to nodes requiring the

Table 14 Comparison in terms of optimality gap: 56, 65 and 71 nodes

Instance	StrongDestDis	StrongDestDis+CP	DestTechDis
A056-03f-d2-28-22-6.dat	11.17	10.86	10.18
A056-03f-d2-22-28-6.dat	3.06	2.99	1.64
A056-03f-d2-28-17-11.dat	12.10	10.62	6.65
A056-03f-d2-17-28-11.dat	3.58	3.30	1.93
A056-03f-d2-22-22-12.dat	3.70	3.60	2.28
A056-03f-d2-22-17-17.dat	4.88	4.55	2.74
Avg A056	6.41	5.99	4.24
A065-03f-d2-33-26-6.dat	14.52	13.75	12.04
A065-03f-d2-26-33-6.dat	12.59	11.75	9.87
A065-03f-d2-33-19-13.dat	5.61	5.28	3.53
A065-03f-d2-19-33-13.dat	4.86	4.68	3.36
A065-03f-d2-26-26-13.dat	6.94	6.94	4.75
A065-03f-d2-25-20-20.dat	4.51	4.35	3.02
Avg A065	8.17	7.79	6.10
A071-03f-d2-36-28-7.dat	11.06	10.67	7.70
A071-03f-d2-28-36-7.dat	1.57	1.57	0.44
A071-03f-d2-36-21-14.dat	9.84	9.58	7.10
A071-03f-d2-21-36-14.dat	7.22	6.01	3.71
A071-03f-d2-28-28-15.dat	7.96	7.53	4.32
A071-03f-d2-29-21-21.dat	6.36	6.34	4.66
Avg A071	7.33	6.95	4.66

Average values are reported in bold

Table 15 Comparison in terms of average time: 56, 65 and 71 nodes

Instance	StrongDestDis	StrongDestDis+CP	DestTechDis
A056	539.09	1,669.61	3,031.66
A065	1,929.58	4,371.94	9,266.48
A071	2,454.96	6,082.60	15,537.47

lowest ability). In the case of nine skills, we used the pattern 20-10-10-10-10-10-10-10-10, for a total of 9 permutations.

The formula used to generate the costs is the one described in “[A-SKVRP instances: scenario with 3 skills](#)”.

Also in this case a comparison of the models *StrongDestDis*, *StrongDestDis* enhanced via the cutting plane approach and *DestTechDis* is given in terms of percentage gap of the LP bound with respect to the best integer solution computed via model *Agg* with a time limit of 6 h in Tables 16 and 17, and in terms of average time required to solve the continuous relaxation in Table 18. Specifically, we provide the optimality gap and the computational time related to the first iteration of the cutting plane approach (*StrongDestDis+CP*).

Instance	StrongDestDis	StrongDestDis+CP	DestTechDis
tc40-1-10-6-6-6-6-6.dat	7.55	7.10	5.47
tc40-1-6-10-6-6-6-6.dat	8.12	6.99	4.99
tc40-1-6-6-10-6-6-6.dat	2.73	1.54	1.31
tc40-1-6-6-6-10-6-6.dat	1.05	0.69	0.00
tc40-1-6-6-6-6-10-6.dat	7.87	6.45	5.17
tc40-1-6-6-6-6-6-10.dat	0.06	0.00	0.00
tc40-1-20-4-4-4-4-4-4.dat	4.56	3.72	1.15
tc40-1-4-4-4-4-4-4-20.dat	0.00	0.00	0.00
Avg tc40-1	3.99	3.31	2.26
Avg tc40-1 Critical	6.57	5.52	4.23

Table 17 Comparison in terms of optimality gap: 9 skills

Instance	StrongDestDis	StrongDestDis+CP	DestTechDis
tc40-1-8-4-4-4-4-4-4-4-4.dat	8.22	7.81	6.26
tc40-1-4-8-4-4-4-4-4-4-4.dat	9.30	8.58	6.42
tc40-1-4-4-8-4-4-4-4-4-4.dat	2.01	1.74	0.63
tc40-1-4-4-4-8-4-4-4-4-4.dat	11.74	11.54	10.07
tc40-1-4-4-4-4-8-4-4-4-4.dat	7.49	6.64	3.14
tc40-1-4-4-4-4-4-8-4-4-4.dat	2.93	2.16	0.80
tc40-1-4-4-4-4-4-4-8-4-4.dat	2.76	2.36	1.71
tc40-1-4-4-4-4-4-4-4-8-4.dat	7.77	7.48	6.92
tc40-1-4-4-4-4-4-4-4-4-8.dat	1.49	1.22	0.04
Avg tc40-1	5.97	5.50	4.00
Avg tc40-1critical	5.69	5.22	3.72

Table 18 Comparison in terms of average time: 6 and 9 skills

3Instance	StrongDestDis	StrongDestDis+CP	DestTechDis
tc40-1: 6 skills	469.17	2,006.83	1,880.79
tc40-1: 6 skills critical	545.98	1,516.62	2,230.97
tc40-1: 9 skills	620.87	2,309.82	4,691.05
tc40-1: 9 skills critical	638.21	1,750.94	4,817.53

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when the number of the skills increases, whereas the time required by *DestTechDis* grows very fast; the average time for the case of 6 skills is more than twice the average time for three skills, and the same trend can be observed going from 6 to 9 skills (see Tables 8, 18). This confirms that the use of model *DestTechDis* is not suitable for instances with a high number of skills and technicians. On the other hand, the cutting plane approach associated with the model *StrongDestDis* appears to be a good choice, especially for the critical instances in the two datasets. Concerning the optimality gap, *StrongDestDis* enhanced with the cutting plane provides gaps whose difference, with respect to the ones found by *DestTechDis*, is on average below 1.55 % (see Tables 16, 17).

The intermediate model enhanced with the cut addition and strengthened by means of the cutting plane approach appears to be able to provide a lower bound of very good quality also on large A-SKVRP instances.

Conclusions

The problem of defining a set of tours, each one operated by a skilled technician, so as to fulfill service requirements asking for particular skills, has been defined and studied. Three models, characterized by an increasing level of variables disaggregation, have been formulated and tested on a large suite of randomly generated instances. The basic model is an adaptation of the single-commodity flow model proposed by Gavish and Graves (1979). The second model is an enhancement of the multicommodity version of the first model, while the third one is a “skilled-based” disaggregation of the second model. Computational results support a series of interesting observations. First, the third model produces very tight LP bounds. Second, the computational times required to solve the models increase with the complexity of the model, by indicating that the model producing the best LP bounds is difficult to use when attempting to solve instances with a large number of nodes and more than three levels of skill. Third, valid inequalities that are implied by the model with the strongest LP relaxation can be added to the weaker models leading to substantial improvements on the lower bounds provided by these models. In particular, the intermediate model, equipped with the proposed set of valid inequalities, appears to represent a good trade-off between quality of the LP bound and computational burden. Additionally, the behavior of the intermediate model can be improved further by a cutting plane approach up to the extent that it can be used successfully to address large scale A-SKVRP problems arising in challenging real application contexts. A study of the proposed models and valid inequalities in Home Care settings is ongoing.

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