Simple transitive 2-representations of some 2-categories of projective functors

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Abstract We show that every simple transitive 2-representation of the 2-category of projective functors for a certain quotient of the quadratic dual of the preprojective algebra associated with a tree is equivalent to a cell 2-representation.

Keywords Representation theory · 2-category · Simple transitive 2-representation · Cell 2-representation

Mathematics Subject Classification 18D05 · 16G10 · 16D90

1 Introduction

Mazorchuk and Miemietz (2011) started a systematic study of 2-representations for certain 2-categories which should be thought of as analogues of finite dimensional algebras. They introduced the notion of cell 2-representations as a possible 2-analogue of the notion of simple modules. This was revised in Mazorchuk and Miemietz (2016b, c) where the notion of a simple transitive 2-representation was introduced. A weak version of the Jordan–Hölder theory was developed in Mazorchuk and Miemietz (2016b) for simple transitive 2-representations which was a convincing argument that simple transitive 2-representations are proper 2-analogues of simple modules. In many important cases, for example for the 2-category of Soergel bimodules in type A , it turns out that every simple transitive 2-representations is equivalent to a cell 2-representation.

Another class of natural 2-categories, for which every simple transitive 2-representations is equivalent to a cell 2-representation, is the class of 2-categories of projective

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bimodules for a finite dimensional self-injective associative algebra, see [Mazorchuk and Miemitz \(2016b, c\)](#). After [Mazorchuk and Miemitz \(2016b, c\)](#) there were several attempts to extend this results to other associative algebras. Two particular algebras were considered in [Mazorchuk and Zhang \(2017a\)](#) and one more in [Mazorchuk et al. \(2017a\)](#). These two papers have rather different approaches: the approach of [Mazorchuk and Zhang \(2017a\)](#) is based on existence of a non-zero projective–injective module while [Mazorchuk et al. \(2017a\)](#) treats the smallest algebra which does not have any projective–injective modules. Recently, [Mazorchuk and Zhang \(2017b\)](#) extended the approach of [Mazorchuk and Zhang \(2017a\)](#) and completely covered the case of directed algebras which have a non-zero projective–injective module. We refer the reader to [Mazorchuk \(2017\)](#) for a general overview of the problem and related results.

In this note we show that the method developed in [Mazorchuk and Zhang \(2017b\)](#) can also be extended to some interesting algebras which are not directed (but which have a non-zero projective–injective module). The algebras we consider are certain quotients of quadratic duals of preprojective algebras associated with trees (cf. [Ringel 1998](#)). These kinds of algebras appear naturally in Lie theory (see [Stroppel 2005](#); [Martirosyan 2014](#)), in diagram algebras (see [Huerfano and Khovanov 2001](#)) and in the theory of Koszul algebras (see [Dubsky 2017](#)). Our main result is that, for our algebras (which are defined in Sect. 2.1), every simple transitive 2-representation of the corresponding 2-category of projective bimodules is equivalent to a cell 2-representation.

The paper is organized as follows. In the next section we define the type of algebras which we want to study, describe some motivating examples and give all the necessary notions needed to formulate the main result. Section 3 is then devoted to stating and proving the main result.

After the first version of the paper appeared on the arxiv, a more general result was proved in [Mazorchuk et al. \(2017b\)](#) by completely different methods. The results of [Mazorchuk et al. \(2017b\)](#) work for arbitrary finite dimensional algebras without any additional assumptions on existence of projective–injective modules. The proofs of [Mazorchuk et al. \(2017b\)](#) are based on two new ideas: a new version of the category of complexes, called the category of pyramids, and on an embedding of the original 2-category into a bigger 2-category which has some partial adjunction morphisms.

2 Preliminaries

Throughout the paper we work over an algebraically closed field $\mathbb{k}$.

2.1 The algebra $A_{T,S}$

Let n be a positive integer. Let $T = (V, E)$ be a tree with vertices labelled by numbers $1, 2, \dots, n$, where $n > 1$. We denote by $L \subseteq V$ the set of all leaves of T . Denote by $Q = Q_T = (V, \hat{E})$ the quiver where we replace every (unoriented) edge $\{i, j\} \in E$, by two arrows (i.e. oriented edges) (i, j) and (j, i) . Let $\mathbb{k}Q$ be the path algebra of Q .

Now we define a certain quotient $A_{T,S}$ of $\mathbb{k}Q$. For this, fix a (possibly empty) subset S of L , and consider the ideal $\mathcal{I}$ of $\mathbb{k}Q$ generated by the following relations:

- For all pairwise distinct $v_1, v_2, v_3 \in V$ such that there are arrows $a_1, a_2 \in \hat{E}$ with

$$v_1 \xrightarrow{a_1} v_2 \xrightarrow{a_2} v_3,$$

we set $a_2 a_1 = 0$.

- For all pairwise distinct vertices $v_1, v_2, v_3 \in V$ such that there exist arrows $a_1, a_2, b_1, b_2 \in \hat{E}$ with

$$\begin{array}{ccccc} & a_1 & & a_2 & \\ v_1 & \xrightarrow{\quad} & v_2 & \xrightarrow{\quad} & v_3, \\ & b_1 & & b_2 & \end{array}$$

we set $a_1 b_1 = b_2 a_2$.

- For $v \in V$ and $s \in S$ such that there are arrows $a, b \in \hat{E}$ with

$$\begin{array}{ccc} & a & \\ v & \xrightarrow{\quad} & s, \\ & b & \end{array}$$

we set $ab = 0$.

The algebra $A_{T,S}$, which we will denote simply by A , is now defined as the quotient of $\mathbb{k}Q$ by the ideal $\mathcal{I}$. We denote the idempotents of A by e_i , for each $i \in V$. For $i \in V$, we set $P_i := Ae_i$ and denote by L_i the simple top of P_i .

The structure of projective A -modules follows directly from the defining relations:

- If $i \in V \setminus S$, then P_i is projective–injective of Loewy length three with isomorphic top and socle. The module $\text{Rad}(P_i)/\text{Soc}(P_i)$ is multiplicity-free and contains all simple L_j such that $\{i, j\} \in E$.
- If $i \in S = V$, then $n = 2$ and P_i is projective–injective of Loewy length two with non-isomorphic top and socle.
- If $i \in S \neq V$, then P_i is not injective, it has Loewy length two and its socle is isomorphic to L_j , where $j \in V$ is the unique vertex such that $\{i, j\} \in E$.

From the above description we see that the algebra A is self-injective if and only if $S = \emptyset$ or $S = V$ (in the latter case we have $n = 2$).

The motivation for the above definition stems from the following examples.

Example 2.1 Let T be the following Dynkin diagram of type A_n :

$$1 \text{ --- } 2 \text{ --- } \cdots \text{ --- } n-1 \text{ --- } n.$$

We have $L = \{1, n\}$. Set $S := \{n\}$. Then Q_T is the following quiver:

$$\begin{array}{ccccccc} & a_1 & & a_2 & & a_{n-2} & & a_{n-1} \\ 1 & \xrightarrow{\quad} & 2 & \xrightarrow{\quad} & \cdots & \xrightarrow{\quad} & n-1 & \xrightarrow{\quad} & \boxed{n} \\ & b_1 & & b_2 & & b_{n-2} & & b_{n-1} \end{array}$$

The distinguished leaf n is the one which is in S . The relations in $A = A_{T,S}$ are given by

$$\begin{aligned} a_{i+1}a_i &= b_{i+1}b_i = 0, & i &= 1, \dots, n-2; \\ a_ib_i &= b_{i+1}a_{i+1}, & i &= 1, \dots, n-2; \\ a_{n-1}b_{n-1} &= 0. \end{aligned}$$

The module category over this algebra is equivalent to the principal block of parabolic category $\mathcal{O}$ associated to the complex Lie algebra $\mathfrak{sl}_n$ and a parabolic subalgebra of $\mathfrak{sl}_n$ for which the semi-simple part of the Levi quotient is isomorphic to $\mathfrak{sl}_{n-1}$, see e.g. [Stroppel \(2005\)](#).

A second example is:

Example 2.2 Let T be the following tree on 4 vertices

We have $L = \{1, 3, 4\}$. Set $S := \{3, 4\}$. Then Q_T looks as follows:

The relations in $A_{T,S}$ are given by

$$\begin{aligned} a_2a_1 &= a_3a_1 = b_1b_2 = b_1b_3 = a_3b_2 = a_2b_3 = a_2b_2 = a_3b_3 = 0, \\ a_1b_1 &= b_2a_2 = b_3a_3. \end{aligned}$$

These kinds of quivers appear as parts of infinite quivers in e.g. [Martirosyan \(2014\)](#).

2.2 The 2-category $\mathcal{C}_A$

Recall, from [Mazorchuk and Miemietz \(2011\)](#), that a 2-category $\mathcal{C}$ is called *finitary* provided that

- it has finitely many objects;

- each $\mathcal{C}(i, j)$ is equivalent to the category of projective modules over some finite dimensional $\mathbb{k}$ -algebra;
- all compositions are additive and $\mathbb{k}$ -linear, when applicable;
- all identity 1-morphisms are indecomposable.

Finitary 2-categories are natural 2-analogues of finite dimensional $\mathbb{k}$ -algebras. For more details, we refer the reader to Mazorchuk and Miemietz (2011).

From now on we fix a tree T and a subset S of its leaves. Let $A = A_{T,S}$. Following Mazorchuk and Miemietz (2011, Subsection 7.3), we define the finitary 2-category $\mathcal{C}_A$ of projective endofunctors of $A\text{-mod}$. Fix a small category $\mathcal{C}$ equivalent to $A\text{-mod}$. The 2-category $\mathcal{C}_A$ has one object i , which we identify with $\mathcal{C}$. Indecomposable 1-morphisms are endofunctors of $\mathcal{C}$ isomorphic (after equivalence with $A\text{-mod}$) to tensoring with:

- the regular A - A -bimodule ${}_A A_A$ (this corresponds to the identity 1-morphism $\mathbb{1}_i$);
- the indecomposable A - A -bimodule $Ae_i \otimes_{\mathbb{k}} e_j A$, for some $i, j \in \{1, 2, \dots, n\}$, we denote such a 1-morphism by F_{ij} .

Lastly, 2-morphisms are homomorphisms of A - A -bimodules.

A *finitary 2-representation* of $\mathcal{C}_A$ is a (strict) 2-functor from $\mathcal{C}_A$ to the 2-category of small finitary additive $\mathbb{k}$ -linear categories. In other words, $\mathbf{M}$ is given by an additive and $\mathbb{k}$ -linear functorial action on a category $\mathbf{M}(i)$ which is equivalent to the category $B\text{-proj}$ of projective modules over some finite dimensional associative $\mathbb{k}$ -algebra B . For $M \in \mathbf{M}(i)$, we will often write FM instead of $\mathbf{M}(F)(M)$. All finitary 2-representation of $\mathcal{C}_A$ form a 2-category $\mathcal{C}_A\text{-afmod}$ where 1-morphisms are strong 2-natural transformations and 2-morphisms are modifications, see Mazorchuk and Miemietz (2016a, Section 2.3) for details.

We call a finitary 2-representation $\mathbf{M}$ of $\mathcal{C}_A$ *transitive* if, for every non-zero object $X \in \mathbf{M}(i)$, the additive closure of $\{FX\}$, where F runs through all 1-morphisms in $\mathcal{C}_A$, equals $\mathbf{M}(i)$. We call a transitive 2-representation $\mathbf{M}$ *simple* if $\mathbf{M}(i)$ has no proper $\mathcal{C}_A$ -invariant ideals. For more details on this, we refer the reader to Mazorchuk and Miemietz (2016b, c).

One class of examples of simple transitive 2-representations are so-called cell 2-representations. For details about these we refer the reader to Mazorchuk and Miemietz (2011); Mazorchuk and Miemietz (2014). The 2-category $\mathcal{C}_A$ has two two-sided cells: the first one consisting of $\mathbb{1}_i$ and the second one containing all F_{ij} . The first two-sided cell is a left cell. The second two-sided cell contains n different left cells, namely, $\mathcal{L}_j := \{F_{ij} : i = 1, 2, \dots, n\}$, where $j = 1, 2, \dots, n$. Up to equivalence, we have two cell 2-representations:

- The cell 2-representations $\mathbf{C}_{\mathbb{1}_i}$ which is given as the quotient of the left regular action of $\mathcal{C}_A$ on $\mathcal{C}_A(i, i)$ by the unique maximal $\mathcal{C}_A$ -invariant ideal (cf. Mazorchuk and Miemietz 2014, Section 6).
- Each cell 2-representations $\mathbf{C}_{\mathcal{L}_j}$, where $j = 1, 2, \dots, n$, is equivalent to the defining 2-representation (the defining action on $\mathcal{C}$).

2.3 Positive idempotent matrices

One of the ingredients in our proofs is the following classification of non-negative idempotent matrices, see [Flor \(1969\)](#).

Theorem 2.3 *Let I be a non-negative idempotent matrix of rank k . Then there exists a permutation matrix P such that*

$$P^{-1}IP = \begin{pmatrix} 0 & AJ & AJB \\ 0 & J & JB \\ 0 & 0 & 0 \end{pmatrix} \quad \text{with} \quad J = \begin{pmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & J_k \end{pmatrix}.$$

Here, each J_i is a non-negative idempotent matrix of rank one and A, B are non-negative matrices of the appropriate size.

Remark 2.4 This theorem can be applied to quasi-idempotent (but not nilpotent) matrices as well. If $I^2 = \lambda I$ and $\lambda \neq 0$, then $(\frac{1}{\lambda}I)^2 = \frac{1}{\lambda^2}I^2 = \frac{1}{\lambda}I$. Hence $\frac{1}{\lambda}I$ is an idempotent and thus can be described by the above theorem.

3 Main result

Fix a tree T and a subset S of its leaves and set $A = A_{T,S}$. Then our main result can be stated as follows:

Theorem 3.1 *Let $\mathbf{M}$ be a simple transitive 2-representation of $\mathcal{C}_A$, then $\mathbf{M}$ is equivalent to a cell 2-representation.*

Note that, if $S = \emptyset$ or $S = V$, then the algebra A is self-injective and hence $\mathcal{C}_A$ is a weakly fiat 2-category. In this case the statement follows from Mazorchuk and Miemietz (2016b, Theorem 15) and Mazorchuk and Miemietz (2016c, Theorem 33). Therefore, in what follows, we assume that $S \neq \emptyset, V$.

3.1 Some notation

For a simple transitive 2-representation $\mathbf{M}$ of $\mathcal{C}_A$, we denote by B a basic $\mathbb{k}$ -algebra such that $\mathbf{M}(\mathbf{i})$ is equivalent to $B\text{-proj}$. Moreover, let $1 = \epsilon_1 + \epsilon_2 + \cdots + \epsilon_r$ be a decomposition of the identity in B into a sum of pairwise orthogonal primitive idempotents. Similarly to the situation in A , we denote, for $1 \leq i, j \leq r$, by G_{ij} the endofunctor of $B\text{-mod}$ given by tensoring with the indecomposable projective B - B -bimodule $B\epsilon_i \otimes \epsilon_j B$. Note that, a priori, there is no reason why we should have $r = n$.

For $i = 1, 2, \dots, r$, we denote by Q_i the projective B -module $B\epsilon_i$.

We may, without loss of generality, assume that $\mathbf{M}$ is faithful since $\mathcal{C}_A$ is simple which was shown in Mazorchuk et al. (2017a, Subsection 3.2). Indeed, if we assume

that $\mathbf{M}$ is not faithful, then $M(F_{ij}) = 0$, for all i, j . However, then the quotient of $\mathcal{C}_A$ by the ideal generated by all F_{ij} satisfies all the assumptions of Mazorchuk and Miemietz (2016b, Theorem 18) and therefore $\mathbf{M}$ is equivalent to the cell 2-representation $\mathbf{C}_{1_1}$ in this case.

So let us from now on assume that $\mathbf{M}$ is faithful and, in particular, that all $\mathbf{M}(F_{ij})$ are non-zero. As we have seen above, A has a non-zero projective–injective module and thus, combining Mazorchuk and Zhang (2017a, Section 3) and Kildetoft et al. (2016, Theorem 2), we deduce that each $\mathbf{M}(F_{ij})$ is a projective endofunctor of $B\text{-mod}$ and, as such, is isomorphic to a non-empty direct sum of G_{st} , for some $1 \leq s, t \leq r$, possibly with multiplicities.

3.2 The sets X_i and Y_i

Following Mazorchuk and Zhang (2017b), for $1 \leq i, j \leq n$, we define

- $X_{ij} := \{s \mid G_{st} \text{ is isomorphic to a direct summand of } \mathbf{M}(F_{ij}), \text{ for some } 1 \leq t \leq r\}$,
- $Y_{ij} := \{t \mid G_{st} \text{ is isomorphic to a direct summand of } \mathbf{M}(F_{ij}), \text{ for some } 1 \leq s \leq r\}$.

First of all, note that X_{ij} and Y_{ij} are non-empty as each $\mathbf{M}(F_{ij})$ is non-zero due to faithfulness of $\mathbf{M}$.

In Mazorchuk and Zhang (2017b, Lemma 20), it is shown that $X_{ij_1} = X_{ij_2}$, for all $j_1, j_2 \in \{1, \dots, n\}$, and thus we may denote by X_i the common value of all X_{ij} . Similarly, the sets Y_{ij} only depend on j , hence we may denote by Y_j the common value of the Y_{ij} , for all i . In Mazorchuk and Zhang (2017b, Lemmas 21, 22), it is shown that $X_q = Y_q$, for all q and moreover that $X_1 \cup X_2 \cup \dots \cup X_n = \{1, 2, \dots, r\}$.

3.3 Analysis of the sets X_i

For a 1-morphism H in $\mathcal{C}_A$, we will denote by $[H]$ the $r \times r$ matrix with coefficients h_{st} , where $s, t \in \{1, 2, \dots, r\}$, such that h_{st} gives the multiplicity of Q_s in HQ_t .

Lemma 3.2 *For each $i \in \{1, \dots, n\}$, we have $|X_i| = 1$,*

Proof First we note that, for all $1 \leq i \leq n$, we have $F_{ii} \circ F_{ii} \cong F_{ii}^{\oplus \dim(e_i A e_i)}$ and hence $[F_{ii}]^2 = k_i [F_{ii}]$, where

$$k_i = \begin{cases} 2, & i \notin S; \\ 1, & i \in S. \end{cases}$$

Hence we can apply Theorem 2.3 to $\frac{1}{k_i} [F_{ii}]$. This yields that there exists an ordering of the basis vectors such that

$$\frac{1}{k_i} [F_{ii}] = \begin{pmatrix} 0 & AJ & AJB \\ 0 & J & JB \\ 0 & 0 & 0 \end{pmatrix}. \quad (1)$$

However, we have seen that $X_i = Y_i$, for any ordering of the basis. This implies that if the l -th row of $\frac{1}{k_i}[F_{ii}]$ is zero, then so is the l -th column. Thus we get that $A = B = 0$ and, in particular, that

$$\frac{1}{k_i}[F_{ii}] = \begin{pmatrix} 0 & 0 \\ 0 & J \end{pmatrix}.$$

If $i \in S$, we are done as $k_i = 1$ and thus the trace of the corresponding matrix is 1 which yields that $[F_{ii}]$ has to contain exactly one non-zero diagonal element and thus $|X_i| = 1$.

Let now $i \notin S$. We may restrict the action of $\mathcal{C}_A$ to the 2-full finitary 2-subcategory $\mathcal{D}$ of $\mathcal{C}_A$ whose indecomposable 1-morphisms are the ones which are isomorphic to either $\mathbb{1}_i$ or F_{ii} . This 2-category, clearly, has only strongly regular two-sided cells. As $i \notin S$, the projective module P_i is also injective and hence F_{ii} is a self-adjoint functor (see Mazorchuk and Miemietz 2011, Subsection 7.3). Therefore $\mathcal{D}$ satisfies all assumptions of Mazorchuk and Miemietz (2016b, Theorem 18) and hence every simple transitive 2-representation of $\mathcal{D}$ is equivalent to a cell 2-representation.

The 2-category $\mathcal{D}$ has two left cells (both are also two-sided cells) and each left cell contains a unique indecomposable 1-morphism. The matrix of F_{ii} in these 2-representations is either (0) or (2). This implies that, for $i \notin S$, all diagonal elements in $[F_{ii}]$ are either equal to 0 or to 2. As $[F_{ii}]$ has trace 2, it follows again that $[F_{ii}]$ contains a unique non-zero diagonal element and thus $|X_i| = 1$. $\square$

Next we are going to prove that the X_i 's are mutually disjoint.

Lemma 3.3 *For $i, j \in \{1, \dots, n\}$ such that $i \neq j$, we have $X_i \cap X_j = \emptyset$.*

Proof Let $i, j \in \{1, \dots, n\}$ be such that $i \neq j$ and assume that $X_i \cap X_j \neq \emptyset$. This implies, by Lemma 3.2, that $X_i = X_j$ is a singleton, call it $X_i = \{s\}$. By the above, we have that $Y_i = X_i = X_j = Y_j = \{s\}$. This implies that

$$\mathbf{M}(F_{ii}) = \mathbf{M}(F_{jj}) = \mathbf{M}(F_{ij}) = \mathbf{M}(F_{ji}) = G_{ss}.$$

We have, for any $1 \leq k, l \leq n$,

$$\dim e_l A e_k = \begin{cases} 2, & k = l \notin S, \\ 1, & \{k, l\} \in E \text{ or } k = l \in S, \\ 0, & \text{else.} \end{cases} \quad (2)$$

On the one hand, we know that $\dim \epsilon_s B \epsilon_s \geq 1$ and thus $G_{ss} \circ G_{ss} \neq 0$. On the other hand, we have that $F_{ij} \circ F_{ij} = F_{ij}^{\oplus \dim(e_j A e_i)}$. This implies that $\dim(e_j A e_i) \neq 0$ and hence $\{i, j\} \in E$, because of (2). Further, as we assume that $S \neq V$, we also have $\{i, j\} \not\subset S$. Let us assume that $i \notin S$.

As $i \notin S$, (2) yields the following:

$$\begin{aligned} G_{ss}^{\oplus 2} &= \mathbf{M}(F_{ii}^{\oplus 2}) = \mathbf{M}(F_{ii} \circ F_{ii}) = \mathbf{M}(F_{ii})\mathbf{M}(F_{ii}) \\ &= \mathbf{M}(F_{ji})\mathbf{M}(F_{ji}) = \mathbf{M}(F_{ji} \circ F_{ji}) = \mathbf{M}(F_{ji}) = G_{ss}. \end{aligned}$$

As $G_{ss} \neq 0$, this equality is impossible. The obtained contradiction proves our claim. $\square$

From the above, we have $n = r$ and, without loss of generality, we may assume $X_i = \{i\}$, for all $i = 1, 2, \dots, n$.

Corollary 3.4 *For $i, j = 1, 2, \dots, n$, we have $\dim e_i A e_j = \dim \epsilon_i B \epsilon_j$.*

Proof This follows immediately since every F_{st} acts via G_{st} , by comparing

$$F_{si} \circ F_{jt} \cong F_{st}^{\oplus \dim e_i A e_j} \quad \text{with} \quad G_{si} \circ G_{jt} \cong G_{st}^{\oplus \dim \epsilon_i B \epsilon_j}$$

$\square$

3.4 Proof of Theorem 3.1

With the results of Sect. 3.3 at hand, the proof of Theorem 3.1 can now be done using similar arguments as used in Mazorchuk and Zhang (2017b, Section 5) or Mackaay and Mazorchuk (2017, Subsection 4.9). Consider the principal 2-representation $\mathbf{P}_1 := \mathcal{C}_A(\mathbf{i}, _)$ of $\mathcal{C}_A$, that is the regular action of $\mathcal{C}_A$ on $\mathcal{C}_A(\mathbf{i}, \mathbf{i})$. Set $\mathbf{N} := \text{add}(F_{i1})$, where $i = 1, 2, \dots, k$, be the additive closure of all F_{i1} . Now, $\mathbf{N}$ is $\mathcal{C}_A$ -stable and thus gives rise to a 2-representation of $\mathcal{C}_A$. By Mazorchuk and Miemietz (2014, Subsection 6.5), we have that there exists a unique $\mathcal{C}_A$ -stable left ideal $\mathbf{I}$ in $\mathbf{N}$ and the corresponding quotient is exactly the cell 2-representation $\mathbf{C}_{\mathcal{L}_1}$.

Now, mapping $\mathbb{1}_i$ to the simple object corresponding to Q_1 in the abelianization of $\mathbf{M}$, induces a 2-natural transformation $\Phi : \mathbf{N} \rightarrow \mathbf{M}$. Due to the results of the previous subsection, we know that Φ maps indecomposable 1-morphisms in $\mathcal{L}_1$ to indecomposable objects in $\mathbf{M}$ inducing a bijection on the corresponding isomorphism classes. By uniqueness of the maximal ideal, the kernel of Φ is contained in $\mathbf{I}$. However, by Corollary 3.4, the Cartan matrices of A and B are the same. This implies that, on the one hand, the kernel of Φ cannot be smaller than $\mathbf{I}$ and, on the other hand, that Φ must be full. Therefore Φ induces an equivalence between $\mathbf{N}/\mathbf{I} \cong \mathbf{C}_{\mathcal{L}_1}$ and $\mathbf{M}$. The claim of the theorem follows.

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