Erratum to: Very ample but not normal lattice polytopes

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In the original publication of the article, there was a mistake in the proof of Proposition 1, because the cone $C(P_q) = \mathbb{R}_{\geq 0}(P_q - v_n)$ at the vertex v_n is not unimodular for $n \geq 4$. The true statement is the following proposition.

Proposition 1 *Let P_q be the lattice polytope defined by (5). For $n = 3$, P_q is very ample. When $n \geq 4$, if $q \equiv 0, 1 \pmod{n-2}$, then kP_q is very ample for $k \geq 2$.*

Proof The set of vertices of P_q is $P_q \cap M = \{v_0, \dots, v_n, v_0^+, \dots, v_n^+\}$. The affine monoid $C_v(P_q) \cap M$ of the cone $C_v(P_q) = \mathbb{R}_{\geq 0}(P_q - v)$ at each vertex v except v_n of P_q is generated by $(P_q - v) \cap M$ as in the original publication.

For the vertex v_n , set $C(P_q) := \mathbb{R}_{\geq 0}(P_q - v_n)$ the simplicial cone with vertex v_n at the origin. We have

$$C(P_q) = \sum_{i=1}^{n-1} \mathbb{R}_{\geq 0}(v_i - v_n) + \mathbb{R}_{\geq 0}(v_n^+ - v_n).$$

We see that if $n = 3$ the cone $C(P_q)$ is unimodular, but if $n \geq 4$ it is not unimodular. We treat the case when $n \geq 4$.

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When $q \equiv 0 \pmod{n-2}$, set

$$w' := \frac{1}{n-2} \sum_{i=1}^{n-1} (v_i - v_n) \in (2(P_q - v_n)) \cap M.$$

Then we can decompose the face $\sum_{i=1}^{n-1} \mathbb{R}_{\geq 0}(v_i - v_n)$ into the union of unimodular cones by the barycentric decomposition at w' . Thus the semi group $C(P_q) \cap M$ is generated by $(2(P_q - v_n)) \cap M$.

When $q \equiv 1 \pmod{n-2}$, set

$$w := \frac{1}{n-2} \left(\sum_{i=1}^{n-1} (v_i - v_n) + (v_n^+ - v_n) \right) \in (2(P_q - v_n)) \cap M.$$

Then we can decompose $C(P_q)$ into the union of unimodular cones by the barycentric decomposition at w . Thus the semi group $C(P_q) \cap M$ is generated by $(2(P_q - v_n)) \cap M$.

This implies that kP_q is very ample for $k \geq 2$. $\square$

Remark For any q , we can decompose the cone $C(P_q)$ into a union of unimodular cones by using lattice points in $(n-2)(P_q - v_n)$. From this and Proposition 3 we see that if $q \geq (2n-3)(n-2) + 1$, then $(n-2)P_q$ is very ample but not normal.