SERIEs (2016) 7:307–339
DOI 10.1007/s13209-016-0143-8

SERIEs
Journal of the Spanish Economic Association

CrossMark

ORIGINAL ARTICLE

Information in elections: Do third inflexible candidates always promote truthful behavior?

Ascensión Andina-Díaz[1]

Received: 24 June 2015 / Accepted: 17 May 2016 / Published online: 27 May 2016

Abstract We model an election between two Downsian mainstream candidates and a third inflexible politician. There is uncertainty about the state of the world. Candidates receive signals on the state and propose a policy to implement. There are two classes of voters: ideological, who are biased towards the policy proposed by the third candidate; and non-ideological, who want the policy implemented to correspond to the state of the world. We study two cases: (1) one in which the third candidate supports the most popular policy (in terms of the electorate's prior); (2) another one in which he supports the less popular policy. We obtain that the presence of a third candidate facilitates equilibria in which the two mainstream politicians make informative announcements, specially when the third candidate is biased towards the most popular policy. We also obtain that many of the informative equilibria are sustained by a coalition government, however the coalition is never between the two mainstream candidates. Last, we observe that in equilibrium, the third inflexible candidate has significant chances of winning office.

I thank Nezih Guner, Guillermo Caruana and two anonymous referees for helpful comments. I also thank Miguel A. Meléndez-Jiménez, Elena Manzoni, and seminar participants at University of Málaga, Royal Holloway University of London, 13th Journèes LAGV, and SAEe 2014 for comments and discussions. I gratefully acknowledge financial support from the Ministerio de Educación y Ciencia through projects ECO2010-21624 and ECO2011-26996, and the Junta de Andalucía through project SEJ2011-8065. This paper previously circulated with the title "Like biases and information in elections". The usual disclaimer applies.

✉ Ascensión Andina-Díaz
aandina@uma.es

[1] Departamento de Teoría e Historia Económica, Campus de El Ejido, Universidad de Málaga, 29071 Málaga, Spain

Keywords Electoral competition · Heterogeneous voters · Information transmission · Voting · Coalition governments

JEL Classification D72 · D82

1 Introduction

In systems with a representative democracy, the question of whether the electoral process can aggregate the information the politicians have and transmit it to the voters is of primary importance. Relevant contributions that date back to Harrington (1993), Roemer (1994), Schultz (1995, 1996, 2002), and more recently Heidhues and Lagerlöf (2003), Morelli and Weelden (2013), Loertscher (2012), and Kartik et al. (2015) agree that in the presence of asymmetric information between voters and (better informed) political parties, electoral processes may fail to be informative.[1]

This abundant literature on the policy distortions that arise when politicians have superior information, has so far focused on the classic two-party competition model.[2] However, in light of the recent results in the 2015 Spanish Municipal and Regional Elections and the 2014 European Elections, this focus seems a bit narrow and unrealistic. In fact, the 2014 European Elections showed the erosion of majoritarian politics and the irruption of third extremist parties all over Europe, from France (where *Front National* won 25 % of votes), UK (where *UKIP* got the biggest share of votes) and Spain (where *Podemos*, a party only 4 month old got an 8 % of votes), to Austria, Denmark, Greece and Italy. More recently, the 2015 Spanish Municipal and Regional Elections confirmed the end of our bipartisan system, giving Spain's biggest cities, Madrid and Barcelona, to extreme-left parties. A swift that also experienced other important cities, say A Coruña, Cádiz, Valencia and Zaragoza.

Have nontraditional parties come to stay? According to a study conducted by *Pew Research* in May 2015, the answer is yes. In fact, respondents of four out of six European countries said that the rise of nontraditional political parties is a good thing. This is the case in Germany, Italy, UK and Spain, where an overwhelmingly 70 % of respondents answered that the rise of *Podemos* is good for the Spanish democracy.[3]

Based on the recent empirical evidence, we propose a model that studies the incentives of two mainstream office-motivated politicians to reveal their private information in the presence of a third ideological candidate and an electorate that is heterogeneous.

To this aim, we consider an adaptation of the model in Heidhues and Lagerlöf (2003) (hereafter HL), in which two office-seeker candidates receive imperfect and

[1] A different view is that in Martinelli (2001) and Laslier and der Straeten (2004), who also analyze electoral competition in the presence of asymmetries of information. The distinguishing feature of these papers is that, in addition to parties, they also consider voters with private information about the policy-relevant state variable. This assumption proves crucial to their results that transmission of private information is an equilibrium in this case.

[2] The only exception is Felgenhauer (2012), whose paper is discussed below.

[3] The numbers for the other countries are: 66 % of respondents in UK (*UKIP*), 58 % in Italy (*Five Star Movement*), and 50 % in Germany (*Alternative for Germany*). Only France and Poland showed more than 50 % of respondents thinking that the rise of nontraditional parties is a bad thing. See "*Except in France and Poland, Nontraditional Parties Viewed Positively*", May 29, 2015, Pew Research Center.

correlated signals on the state of the world, which is unknown to voters.[4] Following their specification we consider a binary world, with two states and two possible policies. For expositional purposes, and inspired by the current EU economic crisis, we identify the two policies as (1) an expansionary fiscal policy, that aims to stimulate aggregated demand through public spending; and (2) a policy of austerity, that aims to reduce budget deficits in order to foster the return of sustainable growth. Accordingly, in our words, the state of the world will be either pro-stimulus or pro-austerity. We assume that a policy consisting of expansionary measures or fiscal stimulus is the best fit for the economy if the state is pro-stimulus, and that an austerity policy is the appropriate one if the state is pro-austerity.[5] Following HL, we assume that the two mainstream candidates observe the signals and propose the policies so as to win office.

We extend the HL set-up in two directions. (1) First, we consider the existence of a third candidate that runs for office. Based on casual observation, we assume that the third candidate is ideological and always proposes the same policy (in our case, the expansive fiscal policy). This is an assumption that aids considerably in the tractability of the problem and is grounded in the real world observation that third parties are usually more dogmatic than mainstream ones.[6] Laver and Hunt (1992) find strong evidence to support this idea. In a study for 25 countries, they asked people to locate in a scale 1–20 the position of the most important parties in their country regarding the following question: "*forced to make a choice, would party leaders give up policy objectives in order to get into government or would they sacrifice a place in government in order to maintain policy objectives?*". They observed that systematically, citizens of different countries perceive mainstream candidates as more willing to give up policy objectives (then, as more strategic) and third parties as more likely to give up a place in government (then, as more deterministic).[7] (2) Second, we

4 Heidhues and Lagerlöf (2003) analyze a model with two Downsian candidates. They show that because Bayesian voters penalize the candidates that contradict the electorate's priors, if the prior is biased in favor of a policy, full revelation by the two candidates cannot occur. In this case, the most reasonable equilibrium (Pareto superior) is the popular beliefs one.

5 Despite the introduction of a third candidate, we follow HL and consider an economy where the policy decision is binary. The reason is twofold: first, the common formulation facilitates making comparisons with HL. Second, it stays closer to the current debate within the EU about which strategy is best to return economic growth.

6 Note that this assumption does not mean that the third candidate has no access to the superior information that mainstream candidates have. It simply says that independently of what the information is and the third candidate's chances of winning office, his preferences or those of his party are such that he always maximizes utility by proposing the expansive fiscal policy. In a sense, we model a third candidate that because of his strong ideology, he ends up having his hands tied, and so losses all capacity and/or willingness to propose and campaign on a policy that is not his preferred one. For example, because doing so could be interpreted by party-affiliates as going against the core, the main-principles and the foundations of the party. See Besley (2006) for models of information transmission and politics in which agents with access to private information are not strategic players.

7 In the scale 1–20, value 1 corresponds to *Give up place in government*, and 20 to *Give up policy objectives*. The mean scores for some parties and countries are: (1) Britain: 13.45 for *Conservative Party*, 11.48 for *Labor Party*, and 7.21 for *Scottish National Party*; (2) France: 12.36 for *Socialist Party* and 8.13 for *National Front*; (3) Germany: 14.27 for *Christian Democratic Union*, 10.56 for *Social Democratic Party* and 5.53 for *Greens*; Spain: 13.8 for *Socialist Party*, 13 for *Popular Coalition* (now *People's Party*), 4.8 for *United Left*. See page 125 for the question and pages 135–316 for the data.

consider the existence of heterogeneous voters, divided into two groups. A majority consisting of non-ideological voters, who want the implemented policy to correctly match the state of the world; and a minority of ideological voters, who have a fixed preference for the policy that the third candidate stands for (the expansive fiscal policy).

Thus, our election game incorporates three candidates and two groups of voters. Candidates observe a signal about the state of the world and propose a policy to implemented if elected. Voters observe the policy proposals, update beliefs about which policy is best (if they need to) and cast their vote. Because with three candidates it could be that no politician wins an absolute majority of votes, we need to define the government coalition formation game and the policy implemented by the resulting government. In the paper we consider that there is a function that maps electoral outcomes (vote shares) into a probability distribution over the set of government outcomes (single-party governments and coalition governments). That is, for every vector of vote shares, this function determines the probability that each possible government outcome occurs. We also assume that when a coalition government forms, a lottery (between the policies proposed by the candidates in the coalition) yields the final implemented policy. This is known to everyone and taken into account by the voters when deciding their vote. After casting the votes, votes are counted and the electoral outcome determined. A winner of the election results and the corresponding policy is implemented. Candidates and voters receive their payoff.

The aim of this paper is twofold. First, to analyze whether in this new scenario with a third inflexible candidate and a group of ideological voters, the two mainstream candidates can credibly transmit their information to the voters through their choice of policy platforms. Thus, we focus on fully revealing equilibria. This is our information transmission game. Second, to analyze how Bayesian (non-ideological) voters decide their vote in the presence of the new actors; namely a third inflexible candidate that always proposes the expansive fiscal policy, and a group of ideological voters in favor of this policy. This is our voting game.

We solve the game by backward induction. We start with the voting game. We consider a platform profile that corresponds to a particular class of fully revealing equilibria. Given the platform profile, we analyze the voters' optimal voting behavior. This allows us to obtain the vector of electoral outcomes or vote shares. Then, according to a function, we can obtain the probability that each single-party government and coalition government occurs. We obtain that, in equilibrium, all coalitions that receive positive probability are politically-akin coalitions. That is, coalitions of candidates that propose the same policy. Then, we go backwards and analyze whether the set of government outcomes (that receive positive probability) allows for information transmission from the two mainstream candidates. We obtain that the presence of a third inflexible candidate might actually help ease the information transmission game. This result is in line with Felgenhauer (2012), who in a simpler set-up with one voter and a less sophisticated signal technology, obtains that a third uninformed candidate can help restore efficiency.[8] We also show that for the mainstream candidates to credibly communicate their information to the voters, it is best that the third inflexible candidate

[8] Felgenhauer (2012) neither analyzes the voting game nor characterizes the set of government outcomes that can follow a particular platform profile.

be biased towards the most popular policy (in terms of the electorate's prior). That is, that both ideological and non-ideological voters have the same (a-priori) like bias. This adds a new insight to Felgenhauer (2012)'s conclusion about the informational benefit of an inflexible third candidate, namely that as important as the existence of a third candidate it is his bias. Last, we obtain that for the mainstream candidates to honestly reveal their information, we need voters to have a strict preference for one of these two candidates. That is, that whenever mainstream candidates propose the same policy, voters always vote for the same candidate. Otherwise, that is if under indifference they flip a coin, no information transmission can be achieved. In this sense, voters' captivity seems to be good.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the voting game and Sect. 4 studies the information transmission game. Then, Sect. 5 presents a discussion, where we examine the implications that the two main features of the model: third candidate and heterogenous voters, have on the results. Finally, Sect. 6 concludes.

2 The model

An election is to be held. There are three political candidates and two groups of voters. There are two stages of the world, w_E and w_A. Throughout the paper, w_E refers to an economic situation where expansionary or fiscal stimulus policies are required to return economic growth. We will say that in this case, the state is *pro-stimulus*. On the other hand, w_A refers to a situation where austerity measures are to be implemented first if we want to foster long-run sustainable growth. Here, we say that the state is *pro-austerity*. Let q be the prior probability that the state is w_E, i.e., $P(w = w_E) = q \in (0, 1)$.

The candidates: Candidates are labeled 1, 2 and 3. Candidates 1 and 2 are Downsian and want to get into office. We refer to them as the *mainstream candidates*. Candidate 3 is considered to have a preferred policy that he always proposes. This assumption is intended to capture the empirical observation that third parties are usually more dogmatic than the mainstream ones. We refer to the third candidate as the *ideological or inflexible candidate*.[9]

There are two policy alternatives E and A, where E stands for *expansionary fiscal policies* and A for *austerity measures*. We assume E is the correct policy in state w_E and A is the correct one in state w_A. Candidates 1 and 2 choose which policy to propose so as to win office. Let $x_i \in \{E, A\}$ be the policy proposed by candidate $i \in \{1, 2, 3\}$. Without loss of generality, we assume $x_3 = E$. Rents from office are K.

The information structure of the candidates: Each candidate 1 and 2 receives a signal $s_1, s_2 \in \{e, a\}$, on the state of the world.[10] Signals are correlated, with $\rho \in [0, 1)$

[9] In fact, though there are clear examples of mainstream candidates implementing policies that they used to oppose to (for example market privatizations by Mitterand, tax increases by Reagan, and more recently, cutbacks by Hollande and, in Spain, by Zapatero), there is not so clear evidence of third parties doing so.

[10] This is the same as considering that all three candidates observe a signal on the state. The reason being that the third candidate always proposes the exact same policy; hence, the receipt of a signal or not by the candidate and the content of the signal is totally irrelevant.

Table 1 Signal technology

	$P(s_2 = j/w = w_j)$	$P(s_2 = k/w = w_j)$	Σ
$P(s_1 = j/w = w_j)$	$(1-\varepsilon)^2 + \rho\varepsilon(1-\varepsilon)$	$(1-\rho)\varepsilon(1-\varepsilon)$	$1-\varepsilon$
$P(s_1 = k/w = w_j)$	$(1-\rho)\varepsilon(1-\varepsilon)$	$\varepsilon^2 + \rho\varepsilon(1-\varepsilon)$	ε
Σ	$1-\varepsilon$	ε	1

being a measure of the degree of correlation between the signals. Hence, the higher ρ, the greater the correlation. Additionally, signals are not perfectly informative about the state of the world. With probability $(1-\varepsilon)$, a candidate's signal is equal to the state; with probability ε he receives an incorrect signal. We assume $\varepsilon \in (0, 1/2)$. Table 1 summarizes the signal technology.[11]

Upon receiving a signal, candidates choose the policy to campaign on and to implement if elected.

The voters: There are two groups of voters: a group of ideological voters and a group of non-ideological voters. All voters in a group are identical. Non-ideological voters want the policy to be appropriate to the state. The utility of a non-ideological voter, denoted by 4, is $U_4(E, w_E) = U_4(A, w_A) = 1$ and $U_4(E, w_A) = U_4(A, w_E) = 0$. Ideological voters always prefer fiscal stimulus policies to cutbacks and other stiff austerity measures. The utility of an ideological voter, denoted by 5, is $U_5(E, w_i) = 1$ and $U_5(A, w_i) = 0$, for $i \in \{E, A\}$. Total mass of voters is equal to one. Let β and $1-\beta$ be the fraction of non-ideological and ideological voters, respectively. We assume $\beta \in (\frac{1}{2}, 1)$, i.e., we consider a majority of non-ideological voters.[12]

The election: Voters observe the vector of proposed policies $(x_1, x_2, x_3) \in \{E, A\}^3$. Then, voters update beliefs about which policy is best (if they need to) and cast their vote.[13] This determines the vector of vote shares $v = (v_1, v_2, v_3)$, where v_i denotes the fraction of votes of candidate $i \in \{1, 2, 3\}$, with $v_1 + v_2 + v_3 = 1$.

The winner and the implemented policy: Let $V = \{(v_1, v_2, v_3) \in [0, 1]^3 \mid v_1 + v_2 + v_3 = 1\}$ be the set of *electoral outcomes* (or vote shares), and let v be an element of this set. Additionally, let $G = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$ be the set of *government outcomes* (collection of all single-party governments and coalition governments), and let g be an element of this set. Note that G does not include the grand coalition.[14]

Now we introduce a function $f\colon V \to \triangle G$, that maps electoral outcomes into government outcomes. More precisely, for each $v \in V$,

11 The signal technology is the same as in HL.

12 Otherwise, there would be no transmission of information but candidates would bias their messages towards policy E.

13 Note that ideological voters do not need to update beliefs, as they have a strict preference for policy E.

14 When no candidate has a strict majority of votes, every two-candidate coalition is a minimal winning coalition. Based on this, we argue that if in a posterior stage, candidates were given the opportunity to choose the number of partners in the coalition, they would never say 3. Note that in this case the rents from office, K, would have to be divided in three pieces. According to this, in the paper we exclude the possibility of the grand coalition.

$$f(v) = \left(p_{\{1\}}, p_{\{2\}}, p_{\{3\}}, p_{\{1,2\}}, p_{\{1,3\}}, p_{\{2,3\}}\right)$$
$$\text{with } p_g \in [0, 1] \text{ and } \sum_{g \in G} p_g = 1,$$

where component p_g gives the probability that government outcome $g \in G$ occurs.

Our formulation introduces one assumption: a strict majority of votes is required to govern. Thus, given $v \in V$, our definition of f satisfies:

$$\begin{array}{ll} p_{\{i\}} = 1 & \text{if } v_i > 1/2 \\ p_{\{1,2\}} + p_{\{1,3\}} + p_{\{2,3\}} = 1, \text{ with } p_{\{1,2\}}, p_{\{1,3\}}, p_{\{2,3\}} > 0 & \text{if } \max\{v_1, v_2, v_3\} < 1/2 \\ p_{\{i,j\}} + p_{\{i,k\}} = 1, \text{ with } p_{\{i,j\}}, p_{\{i,k\}} > 0 & \text{if } v_i = 1/2, v_j, v_k > 0 \\ p_{\{i,j\}} = 1 & \text{if } v_i = v_j = 1/2, v_k = 0 \end{array}$$

with $(i, j, k) \in \{1, 2, 3\}^3$, $i \neq j \neq k$.

Two important comments:

- When there is a candidate, say $i \in \{1, 2, 3\}$, that wins an absolute majority of votes, our formulation implies that this candidate governs alone. That is when $v_i > 1/2$, $p_{\{i\}} = 1$; thus $g = \{i\}$. In this case, we say there is a *single-party government*. Here, the policy implemented is the one proposed by the winning candidate i, that is x_i. Candidate i gets the entire rents K from holding office.
- When no candidate receives a strict majority of votes, our formulation considers that any pair of candidates $(i, j) \in \{1, 2, 3\}^2$ such that $v_i + v_j > 1/2$ can form a coalition government.[15] That is, voters assign positive probability to any pair (i, j) that satisfies $v_i + v_j > 1/2$ to enter government. In this case, the final government will necessarily be a *coalition government*.[16] Because all coalitions are minimal, we assume that a candidate in a coalition receives half of the rents K from holding office.

  Next, we define what the implemented policy will be in this case. To this, we first have to define what the (expected) implemented policy of a coalition, say $g = \{i, j\}$, will be. We denote by $x_{\{i,j\}}$ the expected implemented policy of this coalition.

[15] Note that when $\max\{v_1, v_2, v_3\} < 1/2$, any two candidates form a winning coalition, as $v_i + v_j = v_i + (1 - v_i - v_k) = 1 - v_k > 1/2$, for $(i, j, k) \in \{1, 2, 3\}^3$. In the other two cases, the vector of vote shares determines which two candidates form the winning coalition/s.

[16] Note that our function f only depends on the vector of vote shares v and not on the candidates' proposed policies, nor on the candidates' labels. In this sense, our formulation of f is as general as possible, as when no candidate receives a strict majority of votes, it allows any coalition of two candidates that together receive more than half of the votes to agree on the formation of a coalition government. Note also that in this case, our formulation of f allows for any coalition being equally likely to form, or for some coalitions being more likely to form than others. A particular case here would be to consider that the coalition between the two mainstream candidates is extremely difficult to form. This possibility is allowed by our formulation. A different approach would be to directly assume that not all coalitions are feasible; for example, by excluding a coalition of two particular candidates or a coalition of candidates proposing different policies. We consider it an alternative interesting approach that however would require the introduction of additional assumptions that, in our view, could sometimes be difficult to justify.

Given $g = \{i, j\}$, with $(i, j) \in \{1, 2, 3\}^2$ and $i \neq j$, the expected implemented policy $x_{\{i,j\}} \in \{E, A\}$ is assumed to be the result of a lottery between the policies proposed by the candidates in the coalition, and probabilities that are strictly positive for each event. More precisely, given the vector of outcomes (x_i, x_j), with $(x_i, x_j) \in \{E, A\}^2$, policy $x_{\{i,j\}}$ is given by the lottery

$$(x_i, x_j; p_i, p_j) \tag{1}$$

that yields policy x_k with probability p_k, with $k \in \{i, j\}$, $(p_i, p_j) \in (0, 1)^2$ and $p_i + p_j = 1$.[17]

Now, because when $\max\{v_1, v_2, v_3\} < 1/2$ any pair of candidates (i, j), with $(i, j) \in \{1, 2, 3\}^2$, can form a coalition government, the expected implemented policy in this case will be a compound lottery between the policies implemented by the winning coalitions (with $x_{\{i,j\}}$ defined by the lottery above), and probabilities that are strictly positive for each event. That is, given the vector of simple lotteries $(x_{\{1,2\}}, x_{\{1,3\}}, x_{\{2,3\}})$, the compound lottery

$$(x_{\{1,2\}}, x_{\{1,3\}}, x_{\{2,3\}}; p_{\{1,2\}}, p_{\{1,3\}}, p_{\{2,3\}}) \tag{2}$$

is the risky alternative that yields the simple lottery $x_{\{i,j\}}$ with probability $p_{\{i,j\}}$, with $p_{\{i,j\}} \in (0, 1)$, $\sum_{\{i,j\}} p_{\{i,j\}} = 1$ and $\{i, j\} \in \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$.[18]

Timing: The sequence of events is as follows. First, candidates receive a private signal on the stage of the world and choose the policy to propose. Voters observe the policy proposals, update beliefs about which policy is best (if they need to), and cast their vote. Votes are counted and according to function f, the government outcome is determined. It also determines the final implemented policy.

Candidates' strategies: Note that for the ideological candidate, $x_3 = E$. Hence, we restrict notation to exclusively account for the behavior of the two mainstream candidates. We define σ_i^j as the probability that candidate $i \in \{1, 2\}$ proposes policy E after observing signal $j \in \{e, a\}$, and $\sigma_c = (\sigma_1^e, \sigma_1^a; \sigma_2^e, \sigma_2^a)$ as the vector of strategies for the (mainstream) candidates. Henceforth, we refer to $(x_1, x_2) \in \{E, A\}^2$ as a platform profile.

Voters' strategies: We assume that all voters in a group use the same voting strategy, which implicitly means that voters in each group can perfectly coordinate in their voting strategies and choose a common strategy (that they will all follow) that is a best response to the other group strategy. This is a simplifying assumption that is of much help in our analysis. In fact, note that because with a continuum of voters no voter is

[17] The probability of an event could reflect the bargaining power of the corresponding candidate, thus his vote share. In the analysis we do not require of any specific formulation. All that we need is that the two events of the lottery have strictly positive probability.

[18] If there is a pair of candidates that does not form a winning coalition, that is, for some $\forall(i, j) \in \{1, 2, 3\}^2$, $v_i + v_j < 1/2$, the compound lottery would not include the simple lottery $x_{\{i,j\}}$ as an outcome of the compound lottery.

pivotal, any voter's strategy would be a best response, so our voting game would be multiplicity of Nash equilibria.[19]

Note that under this assumption, it is sufficient to define the following two strategies. Let $\sigma_{4,i}^{x_1x_2}$ be the probability that each non-ideological voter votes for $i \in \{1, 2, 3\}$, after having observed the policy proposal $(x_1, x_2) \in \{E, A\}^2$. Similarly, let $\sigma_{5,i}^{x_1x_2}$ be the probability that each ideological voter votes for $i \in \{1, 2, 3\}$, after having observed the policy proposal $(x_1, x_2) \in \{E, A\}^2$. Additionally, let $\sigma_v^{x_1x_2} = (\sigma_4^{x_1x_2}; \sigma_5^{x_1x_2})$, with $\sigma_4^{x_1x_2} = (\sigma_{4,1}^{x_1x_2}, \sigma_{4,2}^{x_1x_2}, \sigma_{4,3}^{x_1x_2})$ and $\sigma_5^{x_1x_2} = (\sigma_{5,1}^{x_1x_2}, \sigma_{5,2}^{x_1x_2}, \sigma_{5,3}^{x_1x_2})$, denote the vector of voting strategies for voters of groups 4 and 5, respectively.

Thus, for example, if $\sigma_4^{x_1x_2} = (1/2, 1/2, 0)$ and $\sigma_5^{x_1x_2} = (0, 0, 1)$, we have a situation in which each non-ideological voter votes for each of the two mainstream candidates with probability $1/2$, and each ideological voter votes for the third candidate with probability one. Now, because the mass of non-ideological and ideological voters is β and $1 - \beta$, respectively, we have that in this case, the (expected) vector of vote shares is $v = (\frac{1}{2}\beta, \frac{1}{2}\beta, 1 - \beta)$.[20]

The equilibria: We are interested in studying whether in the presence of a third inflexible candidate, the two mainstream candidates can credibly communicate their information to the voters through their choice of policy platforms. Thus, our analysis focuses on equilibria with full revelation. In the following, we say that an equilibrium is *fully revealing* if the two mainstream candidates perfectly signal their information at all stages of the nature. Note that if we restrict our attention to this class of equilibria, candidate $i \in \{1, 2\}$ has two strategies: (1) a *faithful strategy*, defined by $(\sigma_i^e, \sigma_i^a) = (1, 0)$, and (2) a *reversed strategy*, defined by $(\sigma_i^e, \sigma_i^a) = (0, 1)$. Hence, in the fully revealing class of equilibrium we focus on, the vector of the candidates' strategies is $\sigma_c \in \{(1, 0; 1, 0), (1, 0; 0, 1), (0, 1; 1, 0), (0, 1; 0, 1)\}$.

The equilibrium concept that we use is perfect Bayesian equilibrium.

Note that although our model considers a one-stage election game, the fact that there are three candidates and two groups of voters makes the analysis of the information transmission game and the voting game a bit complex. Thus, in order to be clear in the exposition of the analysis and the results, we take the following approach. Using a backward induction argument, we analyze the election game in two steps: (1) first, we study the voting game. That is, we analyze how a voter that observes the candidates' platform profile (x_1, x_2) and understands the mechanism that transforms votes into government outcomes (function f), casts her vote. We do this for any platform profile coming from a fully revealing class of equilibrium. (2) Then, we go

19 Alternatively, this assumption can be understood as if voters were following the "*voting recommendations*" of a leader of each group, who dictates the vote of each agent with the purpose of maximize the welfare of the group. This is the case in Morelli (2004), where there are three groups of voters and each group has a leader, who makes "*voting recommendations*" that voters follow "*if and only if such a recommendation constitutes a best response to the recommendations made by the other party leaders*". That is, in his perfect coordination environment, the voting recommendations of the parties' leaders need to constitute a Nash equilibrium.

20 Under the alternative interpretation that voters follow the "*voting recommendations*" of a leader (see Morelli 2004), $\sigma_{4,i}^{x_1x_2}$ could also be interpreted as the partition of votes of non-ideological voters that the leader of this group determines. Similarly for $\sigma_{5,i}^{x_1x_2}$.

backwards and analyze the information transmission game. That is, we investigate whether the resulting government outcomes allow the two mainstream candidates to fully reveal their information to the voters during the election campaign.

3 Voting

Consider the following example. Suppose an election campaign in which the two mainstream candidates propose cutbacks and other stiff austerity measures and the third candidate proposes an expansionary fiscal policy. Which candidate/s would voters vote for?

If the voter is ideological the answer is quite straightforward. Because she has a strict preference for expansionary fiscal policies, it is a dominated strategy for her to vote for any of the two mainstream candidates. Thus, let us suppose she (and all the other ideological voters) votes for candidate 3. Now, what about non-ideological voters?

To answer this question we first have to analyze how non-ideological voters would change their belief about which policy is best, if they were able to learn the content of the two candidates' signals. Note that this is always the case in the fully revealing class of equilibria we focus on. There are two cases: (1) the prior distribution of the state is in favor of the expansionary fiscal policy ($q > 1/2$); and (2) the prior distribution of the state is in favor of the austerity policy ($q < 1/2$).[21]

First, suppose $q > 1/2$. In this case, when a voter has no other information than her prior, she thinks that policy E is the best. Now, let us consider that the voter knew the content of the two signals. If both were in favor of policy E, the voter would be reinforced in her opinion that policy E is the best.[22] If one signal indicated e and the other a, the content of the two signals would cancel out and the voter would still prefer E, her prior.[23] Lastly, suppose the two signals indicated a. Applying Bayes rule, we obtain that $P(w = w_A \mid s_1 = a, s_2 = a) > P(w = w_E \mid s_1 = a, s_2 = a)$ if and only if $(1-q)[(1-\varepsilon)^2 + \rho\varepsilon(1-\varepsilon)] > q[\varepsilon^2 + \rho\varepsilon(1-\varepsilon)]$, which can be rewritten as $q < \widetilde{q} = \frac{(1-\varepsilon)[1-\varepsilon(1-\rho)]}{1-2\varepsilon(1-\varepsilon)(1-\rho)}$. In order for the problem to be interesting, when analyzing the case $q > 1/2$, we will assume that after two a signals, the voter changes her mind and prefers A. This requires the prior probability that E is the best policy being not too large, i.e., $q \in (\frac{1}{2}, \widetilde{q})$.[24]

[21] There is a third case, that corresponds to the situation of a balanced prior ($q = 1/2$). Here, note that after two e (a) signals, a non-ideological voter votes for E (A); and after two contradicting signals, any voting behavior is permitted. This is, therefore, a situation where no popular belief exists, which alleviates the pandering problem and makes information transmission easier to sustain. The results for this case will appear as minor comments or footnotes in the text.

[22] Applying Bayes rule we observe that $P(w = w_E \mid s_1 = e, s_2 = e) > P(w = w_A \mid s_1 = e, s_2 = e)$ if and only if $q[(1-\varepsilon)^2 + \rho\varepsilon(1-\varepsilon)] > (1-q)[\varepsilon^2 + \rho\varepsilon(1-\varepsilon)]$, which always holds as $\varepsilon < 1/2$ and $q > 1/2$.

[23] By Bayes rule, $P(w = w_E \mid s_i = e, s_j = a) > P(w = w_A \mid s_i = e, s_j = a)$, for $i, j \in \{1, 2\}, i \neq j$, if and only if $q(1-\rho)\varepsilon(1-\varepsilon) > (1-q)(1-\rho)\varepsilon(1-\varepsilon)$, which always holds as $q > 1/2$.

[24] The reader can easily check that $\widetilde{q} > \frac{1}{2}$ if and only if $\varepsilon < \frac{1}{2}$, which is always the case. Hence, the region $q \in (\frac{1}{2}, \widetilde{q})$ does exist.

Analogously, if $q < 1/2$, a voter with no other information than her prior would vote for policy A. Now, if the voter knew that at least one signal indicates a, she would prefer A. Otherwise, we assume she changes her mind and prefers E. Similar to the previous case, this requires the prior probability that A is the best policy being not too large, i.e., $q \in (1 - \widetilde{q}, \frac{1}{2})$.

Assumption 1 (Not too large prior) $q \in (1 - \widetilde{q}, \widetilde{q})$, with $\widetilde{q} = \frac{(1-\varepsilon)[1-\varepsilon(1-\rho)]}{1-2\varepsilon(1-\varepsilon)(1-\rho)}$.

Let $\hat{q} = P(w_E \mid s_1, s_2)$ be the (non-ideological) voters' posterior belief that the state is w_E, conditional on candidates 1 and 2 observing signals s_1 and s_2, respectively. Assumption 1 guarantees that, independently of which policy, E or A, the prior distribution of the state is in favor of, a non-ideological voter may change her mind and end up with a belief that differs from her initial one. That is, if $q \in (1 - \widetilde{q}, \widetilde{q})$, $\hat{q}$ has support in all the interval (0, 1). Additionally, note that the voter's updated belief may favor a policy that is not proposed by any of the candidates.[25] In this case, we assume that non-ideological voters are bound to vote for a candidate that supports their non-preferred policy.

Let us now come back to the example. Remember we were considering a scenario in which the two mainstream candidates are proposing austerity measures, this is $(x_1, x_2) = (A, A)$. Also, suppose we are in a situation in which the posterior belief on the state of the world is $\hat{q} < 1/2$.[26] Finally, note that we consider that (non-ideological) voters can coordinate their voting strategies and choose a common strategy (that they will all follow) that is a best response to the other group strategy. In this case, which candidate/s would non-ideological voters vote for?

Because $\hat{q} < 1/2$, voters believe that policy A is the best. In this case, it is natural to think that non-ideological voters should vote for either one of the two mainstream candidates, 1 or 2. In fact, if they all were to vote for 1, that is $\sigma_4^{AA} = (1, 0, 0)$, the electoral outcome would be $v = (\beta, 0, 1 - \beta)$. Since $\beta > 1/2$, the government outcome would be $g = \{1\}$ and the policy implemented A, in which case the utility to each non-ideological voter would be $1 - \hat{q}$. On the other hand, if they all were to vote for candidate 3 (let us say with probability 1), the electoral outcome would be $v = (0, 0, 1)$, the government outcome $g = \{3\}$ and the policy implemented E. Because the utility to each non-ideological voter in this case would be $\hat{q}$ and $\hat{q} < 1/2$, the result follows.

Now, what about a mixed strategy in which each non-ideological voter were to vote for each of the two mainstream candidates with probability 1/2? Is it an optimal strategy? To answer this question, suppose $\sigma_4^{AA} = (\frac{1}{2}, \frac{1}{2}, 0)$. Note that in this case, $v = (\frac{1}{2}\beta, \frac{1}{2}\beta, 1 - \beta)$. Since $\beta \in (1/2, 1)$, $max\{v_1, v_2, v_3\} < 1/2$. That is, we are in a situation in which any pair of two candidates can form a coalition. In this case,

25 Since $x_3 = E$, this could only be the case if voters observe $(x_1, x_2) = (E, E)$ and the posterior belief is $\hat{q} < \frac{1}{2}$, i.e., the posterior advocates austerity measures. This occurs when either $q > 1/2$ and the two candidates use a reversed strategy, or $q < 1/2$ and there is at least one candidate using a reversed strategy.

26 In a fully revealing equilibrium, this occurs when one of the following cases hold: (1) $q > 1/2$ and the two candidates use a faithful strategy; (2) $q < 1/2$ and the two candidates use a faithful strategy; and (3) $q < 1/2$, one candidate uses a faithful strategy and the other a reversed strategy.

because all coalitions are possible, the implemented policy is given by the compound lottery:

$$\left(x_{\{1,2\}}, x_{\{1,3\}}, x_{\{2,3\}}; p_{\{1,2\}}, p_{\{1,3\}}, p_{\{2,3\}}\right)$$

with $p_{\{i,j\}} \in (0,1), \sum_{\{i,j\}} p_{\{i,j\}} = 1$ and $\{i, j\} \in \{\{1,2\}, \{1,3\}, \{2,3\}\}$; with outcome $x_{\{1,2\}}$ being equal to policy A,[27] and outcome $x_{\{i,3\}}$, with $i \in \{1,2\}$, being the simple lottery:

$$\left(A, E; p_i, p_3\right)$$

with $(p_i, p_3) \in (0,1)^2$, $p_i + p_3 = 1$, and $i \in \{1,2\}$.

Now, let us denote by $EU_4(x_{\{i,j\}})$ the expected utility to any non-ideological voter when the policy implemented is the simple lottery $x_{\{i,j\}}$. Then, the expected utility to each of these voters in the present compound lottery is:

$$\begin{aligned} & p_{\{1,2\}} EU_4(x_{\{1,2\}}) + \sum_{i \in \{1,2\}} p_{\{i,3\}} EU_4(x_{\{i,3\}}) \\ & \quad = p_{\{1,2\}} EU_4(A) + \sum_{i \in \{1,2\}} p_{\{i,3\}} \left(p_i EU_4(A) + p_3 EU_4(E)\right) \\ & \quad = p_{\{1,2\}} (1 - \hat{q}) + \sum_{i \in \{1,2\}} p_{\{i,3\}} \left(p_i (1 - \hat{q}) + p_3 \hat{q}\right). \end{aligned}$$

Note that $(p_i(1 - \hat{q}) + p_3\hat{q})$ is a convex combination of $(1 - \hat{q})$ and $\hat{q}$. Also, remember that if $\sigma_{4,1}^{AA} = 1$, the payoff to each of these voters is $1 - \hat{q}$. Then, since $\forall \hat{q} < 1/2$, $1 - \hat{q} > \hat{q}$, the payoff to each non-ideological voter if they all follow $\sigma_4^{AA} = (1, 0, 0)$ is higher than her payoff if they all use $\sigma_4^{AA} = (\frac{1}{2}, \frac{1}{2}, 0)$. Thus, we can conclude that using a mixed strategy that gives no mainstream candidate an absolute majority of votes is not a best common strategy for this group of voters.

From the analysis above we learn that in equilibrium, no pair of candidates proposing different policies will get enough votes to form a coalition government. That is, all possible winning coalitions are going to be politically-akin coalitions.[28]

Lemma 1 characterizes the voters' optimal voting strategies, given a platform profile and a posterior belief about the state of the world. To simplify the characterization, we assume that ideological voters never vote for a candidate proposing policy A.[29]

Lemma 1 *Consider a platform profile* $(x_1, x_2) \in \{E, A\}^2$*. In the voting game, the optimal voting behavior of the voters,* $\sigma_v^{*x_1x_2}$*, satisfies the following conditions*:

[27] Note that $x_{\{1,2\}}$ is the simple lottery $(A, A; p_1, p_2)$.

[28] Note that in the present paper, this result is driven by the voting behavior of non-ideological voters, and not by the candidates' preferences, that we do not need to specify.

[29] Since voters use common strategies, this is a dominated strategy for this group of voters.

1. *Consider* $(x_1, x_2) = (E, E)$. *For any* $\hat{q} \in (0, 1)$:
   (a) $(\sigma_{4,1}^{*EE}, \sigma_{4,2}^{*EE}, \sigma_{4,3}^{*EE}) \in [0, 1]^3$, *is such that* $\sum_{i=\{1,2,3\}} \sigma_{4,i}^{*EE} = 1$,
   (b) $(\sigma_{5,1}^{*EE}, \sigma_{5,2}^{*EE}, \sigma_{5,3}^{*EE}) \in [0, 1]^3$, *is such that* $\sum_{i=\{1,2,3\}} \sigma_{5,i}^{*EE} = 1$.
2. *Consider* $(x_1, x_2) = (E, A)$.
   (a) *When* $\hat{q} < 1/2$:
      (i) $(\sigma_{4,1}^{*EA}, \sigma_{4,2}^{*EA}, \sigma_{4,3}^{*EA})$, *is such that* $\beta\sigma_{4,2}^{*EA} > \beta(\sigma_{4,1}^{*EA} + \sigma_{4,3}^{*EA}) + (1-\beta)$,
      (ii) $(\sigma_{5,1}^{*EA}, \sigma_{5,2}^{*EA}, \sigma_{5,3}^{*EA})$, *is such that* $\sigma_{5,2}^{*EA} = 0$, $\sum_{i=\{1,3\}} \sigma_{5,i}^{*EA} = 1$.
   (b) *When* $\hat{q} > 1/2$:
      (i) $(\sigma_{4,1}^{*EA}, \sigma_{4,2}^{*EA}, \sigma_{4,3}^{*EA})$, *is such that either* $\sigma_{4,2}^{*EA} > 0$ *and* $\beta\sigma_{4,i}^{*EA} + (1-\beta)\sigma_{5,i}^{*EA} > \beta(\sigma_{4,2}^{*EA} + \sigma_{4,j}^{*EA}) + (1-\beta)\sigma_{5,j}^{*EA}$, *for* $(i, j) \in \{1, 3\}^2, i \neq j$; *or* $\sigma_{4,2}^{*EA} = 0$ *and* $\sum_{i\in\{1,3\}} \sigma_{4,i}^{*EA} = 1$,
      (ii) $(\sigma_{5,1}^{*EA}, \sigma_{5,2}^{*EA}, \sigma_{5,3}^{*EA})$, *is such that* $\sigma_{5,2}^{*EA} = 0$, $\sum_{i=\{1,3\}} \sigma_{5,i}^{*EA} = 1$.
3. *Consider* $(x_1, x_2) = (A, E)$.
   (a) *When* $\hat{q} < 1/2$:
      (i) $(\sigma_{4,1}^{*AE}, \sigma_{4,2}^{*AE}, \sigma_{4,3}^{*AE})$, *is such that* $\beta\sigma_{4,1}^{*AE} > \beta(\sigma_{4,2}^{*AE} + \sigma_{4,3}^{*AE}) + (1-\beta)$,
      (ii) $(\sigma_{5,1}^{*AE}, \sigma_{5,2}^{*AE}, \sigma_{5,3}^{*AE})$, *is such that* $\sigma_{5,1}^{*AE} = 0$, $\sum_{i=\{2,3\}} \sigma_{5,i}^{*AE} = 1$.
   (b) *When* $\hat{q} > 1/2$:
      (i) $(\sigma_{4,1}^{*AE}, \sigma_{4,2}^{*AE}, \sigma_{4,3}^{*AE})$, *is such that either* $\sigma_{4,1}^{*AE} > 0$ *and* $\beta\sigma_{4,i}^{*AE} + (1-\beta)\sigma_{5,i}^{*AE} > \beta(\sigma_{4,1}^{*AE} + \sigma_{4,j}^{*AE}) + (1-\beta)\sigma_{5,j}^{*AE}$, *for* $(i, j) \in \{2, 3\}^2, i \neq j$; *or* $\sigma_{4,1}^{*AE} = 0$ *and* $\sum_{i=\{2,3\}} \sigma_{4,i}^{*AE} = 1$,
      (ii) $(\sigma_{5,1}^{*AE}, \sigma_{5,2}^{*AE}, \sigma_{5,3}^{*AE})$, *is such that* $\sigma_{5,1}^{*AE} = 0$, $\sum_{i=\{2,3\}} \sigma_{5,i}^{*AE} = 1$.
4. *Consider* $(x_1, x_2) = (A, A)$.
   (a) *When* $\hat{q} < 1/2$:
      (i) $(\sigma_{4,1}^{*AA}, \sigma_{4,2}^{*AA}, \sigma_{4,3}^{*AA})$, *is such that* $\beta\sigma_{4,i}^{*AA} > \beta\sigma_{4,j}^{*AA} + (1-\beta)$, *for* $(i, j) \in \{1, 2\}^2, i \neq j$,
      (ii) $(\sigma_{5,1}^{*AA}, \sigma_{5,2}^{*AA}, \sigma_{5,3}^{*AA}) = (0, 0, 1)$.
   (b) *When* $\hat{q} > 1/2$:
      (i) $(\sigma_{4,1}^{*AA}, \sigma_{4,2}^{*AA}, \sigma_{4,3}^{*AA})$ *is such that* $\beta\sigma_{4,3}^{*AA} + (1-\beta) > \beta(\sigma_{4,1}^{*AA} + \sigma_{4,2}^{*AA})$,
      (ii) $(\sigma_{5,1}^{*AA}, \sigma_{5,2}^{*AA}, \sigma_{5,3}^{*AA}) = (0, 0, 1)$.

Proof In the Appendix. □

For each platform profile $(x_1, x_2) \in \{E, A\}^2$, and posterior $\hat{q} \in (0, 1)$, let $\Gamma_v^{*x_1x_2}$ be the set of the voters' optimal voting strategies that satisfy conditions in Lemma 1. Let $\sigma_v^{*x_1x_2} \in \Gamma_v^{*x_1x_2}$ be an element of this set.

Having characterized the voters' optimal voting strategies for a platform profile $(x_1, x_2) \in \{E, A\}^2$ and the posterior $\hat{q} \in (0, 1)$, the next steep is to determine, for each platform profile and posterior, what electoral outcomes $v \in V$ will result and then, given function f, what probability distribution over government outcomes we will have. This will allow us to determine, for each platform profile $(x_1, x_2) \in \{E, A\}^2$, what government outcomes (single-party governments and/or coalition governments) will have strictly positive probability to form. Prior to this, we introduce a new definition.

Abusing a bit of notation, for each platform profile $(x_1, x_2) \in \{E, A\}^2$, we denote by $G^{x_1x_2} \subseteq G$ the set of government outcomes such that, for each $g^{x_1x_2} \in G^{x_1x_2}$, there exists optimal strategies for the voters $\sigma_4^{*x_1x_2}$ and $\sigma_5^{*x_1x_2}$ that, given function f, result in $g^{x_1x_2}$ forming a government with (strictly) positive probability when the platform profile (x_1, x_2) is proposed. This definition will be very useful in the analysis of the next section.[30]

To clarify this concept, consider the example above, where $(x_1, x_2) = (A, A)$ and $\hat{q} < 1/2$. According to Lemma 1, in this case $(v_1, v_2, v_3) = (\beta\sigma_{4,1}^{*AA}, \beta\sigma_{4,2}^{*AA}, 1-\beta)$, with $\beta\sigma_{4,i}^{*AA} > \beta\sigma_{4,j}^{*AA} + (1-\beta)$, for $(i, j) \in \{1, 2\}^2, i \neq j$. Thus, in the equilibrium of the voting game, either $v_1 > 1/2$ or $v_2 > 1/2$ occurs. Now, from the definition of function f, we know that if $v_1 > 1/2$, then $p_{\{1\}} = 1$, and if $v_2 > 1/2$, then $p_{\{2\}} = 1$. As a consequence, $G^{AA} = \{\{1\}, \{2\}\}$. That is, given the platform profile $(x_1, x_2) = (A, A)$, when $\hat{q} < 1/2$, there are two government outcomes that, a priori, have (strictly) positive probability: one in which candidate 1 governs alone, $g^{AA} = \{1\}$, and another one in which candidate 2 does it, $g^{AA} = \{2\}$.

Next Corollary characterizes, for every platform profile $(x_1, x_2) \in \{E, A\}^2$, the set $G^{x_1x_2} \subseteq G$.[31]

Corollary 1 *Given a platform profile* $(x_1, x_2) \in \{E, A\}^2$*, the set of voters' optimal voting strategies,* $\Gamma_v^{*x_1x_2}$*, and the function* f*, the set* $G^{x_1x_2}$ *is:*

1. *Consider* $(x_1, x_2) = (E, E)$.
   Then, $\forall\hat{q} \in (0, 1)$, $G^{EE} = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$.
2. *Consider* $(x_1, x_2) = (E, A)$.
   (a) *Then* $\forall\hat{q} < 1/2$, $G^{EA} = \{\{2\}\}$,
   (b) *Then* $\forall\hat{q} > 1/2$, $G^{EA} = \{\{1\}, \{3\}, \{1, 3\}\}$.
3. *Consider* $(x_1, x_2) = (A, E)$.
   (a) *Then* $\forall\hat{q} < 1/2$*, then* $G^{AE} = \{\{1\}\}$,
   (b) *Then* $\forall\hat{q} > 1/2$, $G^{AE} = \{\{2\}, \{3\}, \{2, 3\}\}$.
3. *Consider* $(x_1, x_2) = (A, A)$.
   (a) *Then* $\forall\hat{q} < 1/2$*, then* $G^{AA} = \{\{1\}, \{2\}\}$,
   (b) *Then* $\forall\hat{q} > 1/2$*, then* $G^{AA} = \{\{3\}\}$.

Corollary 1 shows that in the equilibrium of the voting game, only the candidates that propose the policies that, according to the posterior belief $\hat{q}$, are more likely to match the state of the world, have chances of entering a government. Thus, as already anticipated, no coalition of candidates proposing different policies will ever form. Additionally, we observe that except for the case $(x_1, x_2) = (E, E)$, the two mainstream candidates will never get enough votes to form a winning coalition together.

To have an intuition for the proof, consider $(x_1, x_2) = (E, E)$. In this case, any voting strategy for the voters constitute a best response. Then, any $v \in V$ is possible and thus, any government outcome has strictly positive probability. Accordingly, $G^{EE} =$

30 Given a platform profile $(x_1, x_2) \in \{E, A\}^2$, since $G^{x_1x_2}$ depends on the set of the voters' optimal voting strategies, $\Gamma_v^{*x_1x_2}$, we will write $G^{x_1x_2}(\Gamma_v^{*x_1x_2})$ when it clarifies the analysis.

31 The characterization is done for $q \neq 1/2$. As for the case $q = 1/2$, the posterior on the state can be either $\hat{q} > 1/2$, $\hat{q} < 1/2$ or $\hat{q} = 1/2$. Regarding the last case, an analogous argument to the one used in the text shows that for any platform profile $(x_1, x_2) \in \{E, A\}^2$, $G^{x_1x_2} = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$. Hence, under this assumption, in equilibrium any coalition can form.

$\{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$. Now suppose $(x_1, x_2) = (E, A)$ and $\hat{q} < 1/2$. In this case, non-ideological voters prefer policy A. According to Lemma 1, any vector of voting strategies σ_v^{EA} such that the resulting electoral outcome is $v_2 > v_1 + v_3$ is optimal. Because in this case $v_2 > 1/2$, applying function f, we have $p_{\{2\}} = 1$; thus $G^{EA} = \{\{2\}\}$. Suppose now $(x_1, x_2) = (E, A)$ and $\hat{q} > 1/2$. Here, voters agree that the best policy is E. Then, any voting strategy σ_4^{EA} that guarantees that policy E is implemented with probability 1 is a best response. This occurs when the vector of voting strategies σ_v^{EA} results in either $v_i > 1/2$, for $i \in \{1, 3\}$; or $v_2 = 0$ and $v_1 = v_3 = 1/2$. Thus, $G^{EA} = \{\{1\}, \{3\}, \{1, 3\}\}$. Last, suppose $(x_1, x_2) = (A, A)$. When $\hat{q} > 1/2$, voters agree that the best policy is E. In this case, any vector of voting strategies σ_v^{AA} that results in $v_3 > 1/2$ is optimal. Then, $G^{AA} = \{\{3\}\}$.[32]

4 Information transmission

We now return to the question of whether the two mainstream candidates can credibly communicate their information to the voters through their choice of policy platforms. To this aim, we go backwards and analyze whether for a given platform profile $(x_1, x_2) \in \{E, A\}^2$, the optimal voting behavior and the resulting government outcomes $g^{x_1 x_2} \subseteq G^{x_1 x_2}$, allow to sustain platform profile $(x_1, x_2) \in \{E, A\}^2$ as part of a fully revealing equilibrium.

We restrict attention to fully revealing equilibria and differentiate two cases, according to which state of the world the prior is in favor of.[33]

Like biases

Let us start considering the case in which politicians know that expansionary fiscal policies are more popular with the electorate than cutbacks and other stiff austerity measures. In terms of our analysis, this means $P(w_E) = q > 1/2$. Here, note that non-ideological voters with no other information than their prior prefer policy E. Since ideological voters are in our model biased towards policy E, we say we have in this case a scenario with *like biases*.

Perhaps somewhat surprisingly, our results show that candidates can, in this scenario, credibly communicate their information to the voters. More interestingly, we obtain that the number of government outcomes that allows for full revelation is greater here than in the case of opposing biases, which is analyzed next.[34]

How can it be that the number of equilibria in which the two mainstream candidates behave honestly is higher when the electorate is homogeneous than when voters have different points of view? To see the intuition for this result, first note that in both scenarios, the mainstream candidates have an incentive to bias their message towards

32 The case $(x_1, x_2) = (A, A)$ and $\hat{q} < 1/2$ is the situation discussed previously in the text. As for the case $(x_1, x_2) = (A, E)$, the reasoning is analogous to that when $(x_1, x_2) = (E, A)$.

33 The analysis of this section is done for the cases in which the electorates' prior is biased in favor of one of the states. As for the case $q = 1/2$, after two contradicting signals, the posterior will be $\hat{q} = 1/2$, which does not impose any restriction on the government composition. That is, if $q = 1/2$, information transmission is easily attainable.

34 This case will refer to the situation in which the prior is in favor of policy A.

the electorate's preferred policy, as in HL.[35] However, the presence of a third candidate who dogmatically supports fiscal stimulus policies introduces a second effect, that reduces the attractiveness of proposing E.[36] In the case of like biases, this second effect counterbalances the first one, hence facilitating equilibria in which the two mainstream candidates behave honestly. In contrast, in the case of opposing biases, the first and the second effects go in the same direction, therefore making information transmission more difficult to hold.

Prior to the results, given a platform profile $(x_1, x_2) \in \{E, A\}^2$, we define $\hat{\Gamma}_v^{*x_1x_2} \subset \Gamma_v^{*x_1x_2}$ as the set of voters' optimal voting strategies that satisfy conditions (1)–(3) of anonymity:

1. If when $(x_1, x_2) = (E, A)$, $p_{\{1\}} = 1$; then when $(x_1, x_2) = (A, E)$, $p_{\{2\}} = 1$.
2. If when $(x_1, x_2) = (E, A)$, $p_{\{3\}} = 1$, then when $(x_1, x_2) = (A, E)$, $p_{\{3\}} = 1$.
3. If when $(x_1, x_2) = (E, A)$, $p_{\{1,3\}} = 1$, then when $(x_1, x_2) = (A, E)$, $p_{\{2,3\}} = 1$.

Note that the conditions on $\hat{\Gamma}_v^{*x_1x_2}$ only restrict the voting behavior when the two mainstream candidates support different policies (in which case, the voters' preferred policy is E). That is, when the non-ideological voters' choice is between the inflexible candidate and the mainstream candidate that proposes E. With respect to these situations, we say that any $\sigma_v^{*x_1x_2} \in \hat{\Gamma}_v^{*x_1x_2}$ holds the anonymity condition, which means that the identity of the mainstream politician (whether he is 1 or 2) does not affect the voter's optimal voting behavior. The result in the Proposition 1 considers $\sigma_v^{*x_1x_2} \in \hat{\Gamma}_v^{*x_1x_2}$, which allows us to simplify the analysis.[37]

Last, for $i, j \in \{e, a\}$, let us define $P_{i|j} = P(s_2 = i \mid s_1 = j)$. Also, we will write $g^{x_1x_2}(\hat{\Gamma}_v^{*x_1x_2}) = \{g\}$ when we want to refer to a situation in which given a platform profile $(x_1, x_2) \in \{E, A\}^2$, and the set of the voters' optimal voting strategies $\hat{\Gamma}_v^{*x_1x_2}$, applying f we obtain $p_{\{g\}} = 1$, with $g \in G$. That is, given $(x_1, x_2) \in \{E, A\}^2$, $g^{x_1x_2}(\hat{\Gamma}_v^{*x_1x_2}) = \{g\}$ is equivalent to $p_{\{g\}} = 1$.

Proposition 1 (*Like biases*) *Consider* $q \in (\frac{1}{2}, \widetilde{q})$. *A configuration* $(\sigma_c^*, \sigma_v^{*x_1x_2}; g^{x_1x_2}(\hat{\Gamma}_v^{*x_1x_2}))_{(x_1,x_2)\in\{E,A\}^2, \sigma_v^{*x_1x_2}\in\hat{\Gamma}_v^{*x_1x_2}}$ *constitutes a fully revealing equilibrium if and only if*:

(i) $\sigma_c^* = (1, 0; 1, 0)$ *and for any* $(x_1, x_2) \in \{E, A\}^2$, $\sigma_v^{*x_1x_2} \in \hat{\Gamma}_v^{*x_1x_2}$ *is such that either*:

(a) $g^{EA}(\hat{\Gamma}_v^{*EA}) = g^{AE}(\hat{\Gamma}_v^{*AE}) = \{3\}$, $g^{AA}(\hat{\Gamma}_v^{*AA}) = g^{EE}(\hat{\Gamma}_v^{*EE}) = \{1\}$; *or*

(b) $g^{EA}(\hat{\Gamma}_v^{*EA}) = g^{AE}(\hat{\Gamma}_v^{*AE}) = \{3\}$, $g^{AA}(\hat{\Gamma}_v^{*AA}) = \{1\}$, $g^{EE}(\hat{\Gamma}_v^{*EE}) = \{1, 3\}$, *with* $P_{e|e} \geq 2P_{a|e}$; *or*

(c) $g^{EA}(\hat{\Gamma}_v^{*EA}) = g^{AE}(\hat{\Gamma}_v^{*AE}) = \{3\}$, $g^{AA}(\hat{\Gamma}_v^{*AA}) = g^{EE}(\hat{\Gamma}_v^{*EE}) = \{2\}$; *or*

[35] Given that signals are of the same quality, any voter observing two contradicting signals would cancel out the content of the two signals and so, would vote for the candidate that proposes the policy the prior is in favor of.

[36] Any mainstream candidate proposing E has to take into account that some of the voters who prefer E may elect the inflexible candidate.

[37] Apart form Proposition 1, the restriction to $\sigma_v^{*x_1x_2} \in \hat{\Gamma}_v^{*x_1x_2}$ will also be used in Proposition 4. The rest of the results in the paper consider the unrestricted set $\Gamma_v^{*x_1x_2}$.

(d) $g^{EA}(\hat{\Gamma}_v^{*EA}) = g^{AE}(\hat{\Gamma}_v^{*AE}) = \{3\}$, $g^{AA}(\hat{\Gamma}_v^{*AA}) = \{2\}$, $g^{EE}(\hat{\Gamma}_v^{*EE}) = \{2, 3\}$, *with* $P_{e|e} \geq 2P_{a|e}$; *or*

(ii) $\sigma_c^* = (1, 0; 0, 1)$ *and for any* $(x_1, x_2) \in \{E, A\}^2$, $\sigma_v^{*x_1x_2} \in \hat{\Gamma}_v^{*x_1x_2}$ *is such that* $g^{AE}(\hat{\Gamma}_v^{*AE}) = \{1\}$, $g^{AA}(\hat{\Gamma}_v^{*AA}) = \{3\}$ *and either*:

(a) $g^{EE}(\hat{\Gamma}_v^{*EE}) = \{1\}$, $g^{EA}(\hat{\Gamma}_v^{*EA}) = \{3\}$; *or*
(b) $g^{EE}(\hat{\Gamma}_v^{*EE}) = \{3\}$, $g^{EA}(\hat{\Gamma}_v^{*EA}) = \{1\}$; *or*
(c) $g^{EE}(\hat{\Gamma}_v^{*EE}) = \{3\}$, $g^{EA}(\hat{\Gamma}_v^{*EA}) = \{1, 3\}$, $P_{e|e} > 2P_{a|e}$; *or*
(d) $g^{EE}(\hat{\Gamma}_v^{*EE}) = \{1, 3\}$, $g^{EA}(\hat{\Gamma}_v^{*EA}) = \{1, 3\}$; *or*
(e) $g^{EE}(\hat{\Gamma}_v^{*EE}) = \{1, 3\}$, $g^{EA}(\hat{\Gamma}_v^{*EA}) = \{1\}$, $P_{a|a} > 2P_{e|a}$; *or*

(iii) $\sigma_c^* = (0, 1; 1, 0)$ *and for any* $(x_1, x_2) \in \{E, A\}^2$, $\sigma_v^{*x_1x_2} \in \hat{\Gamma}_v^{*x_1x_2}$ *is such that* $g^{EA}(\hat{\Gamma}_v^{*EA}) = \{2\}$, $g^{AA}(\hat{\Gamma}_v^{*AA}) = \{3\}$ *and either*:

(a) $g^{EE}(\hat{\Gamma}_v^{*EE}) = \{2\}$, $g^{AE}(\hat{\Gamma}_v^{*AE}) = \{3\}$; *or*
(b) $g^{EE}(\hat{\Gamma}_v^{*EE}) = \{3\}$, $g^{AE}(\hat{\Gamma}_v^{*AE}) = \{2\}$; *or*
(c) $g^{EE}(\hat{\Gamma}_v^{*EE}) = \{3\}$, $g^{AE}(\hat{\Gamma}_v^{*AE}) = \{2, 3\}$, $P_{e|e} > 2P_{a|e}$; *or*
(d) $g^{EE}(\hat{\Gamma}_v^{*EE}) = \{2, 3\}$, $g^{AE}(\hat{\Gamma}_v^{*AE}) = \{2, 3\}$; *or*
(e) $g^{EE}(\hat{\Gamma}_v^{*EE}) = \{2, 3\}$, $g^{AE}(\hat{\Gamma}_v^{*AE}) = \{2\}$, $P_{a|a} > 2P_{e|a}$; *or*

(iv) $\sigma_c^* = (0, 1; 0, 1)$ *and for any* $(x_1, x_2) \in \{E, A\}^2$, $\sigma_v^{*x_1x_2} \in \hat{\Gamma}_v^{*x_1x_2}$ *is such that* $g^{AA}(\hat{\Gamma}_v^{*AA}) = g^{EA}(\hat{\Gamma}_v^{*EA}) = g^{AE}(\hat{\Gamma}_v^{*AE}) = g^{EE}(\hat{\Gamma}_v^{*EE}) = \{3\}$.

Proof In the Appendix. □

Proposition 1 yields a number of comments. First and foremost, that the most robust equilibria are the ones in which candidates use a faithful strategy (case (i)). Nonetheless, these are the only equilibria in which the two mainstream candidates have a chance of winning office (although not together in a coalition, as discussed next). Apart from this scenario, one of the mainstream candidates (the one using the reversed strategy) never gets elected, which is the reason why we can sustain these equilibria. In this sense, the equilibria in (ii)–(iv) are more fragile than those in (i).

Let us therefore comment in more detail the results in (i). An important conclusion is that for the mainstream candidates to use a faithful strategy, voters must penalize any disagreement between the policies proposed by these two candidates. That is, information transmission requires that the ideological candidate forms a single-party government whenever the mainstream candidates announce different policies. But not only this, we also obtain that one of the two mainstream candidates is always excluded from the government. A direct implication of this result is that no coalition between the two mainstream candidates can ever form.[38] To have an intuition for this result, note that when $(x_1, x_2) = (A, A)$ and $\widehat{q} < \frac{1}{2}$, we showed that non-ideological voters had to vote for one of the mainstream candidates with probability one.[39] We now observe that for information transmission to occur, after observing $(x_1, x_2) = (E, E)$, non-ideological voters have to vote again, with probability one, for the same mainstream

[38] Note that this also occurs for the other class of fully revealing equilibria.

[39] To prevent an eventual coalition between the inflexible candidate (proposing E) and a mainstream candidate (proposing A), where the implemented policy could differ from the non-ideological voters' preferred policy, i.e. *A*.

candidate. Otherwise, the candidate who previously received no votes would find it profitable to deviate to E.

Thus, from the analysis we learn that information transmission requires of voters having a strict preference for one of the two mainstream candidates, that they use whenever are indifferent between them. Otherwise, that is if under indifference voters flip a coin (thus they may swing from candidate 1 to 2 and viceversa), no information transmission can be achieved.

There is a last remark to make. It refers to the effect of signal quality and the correlation between signals on the probability that transmission of information occurs. To this respect, we observe that $P_{i|i} \geq 2P_{i|j}$ is a necessary condition for some of the government outcomes to sustain information transmission. Since $P_{i|i}$ is increasing in ρ and decreasing in ε, and $P_{i|j}$ is decreasing in ρ and increasing in ε, we obtain that the higher the correlation between the candidates' signals (ρ) and the lower the signal's error (ε), the higher the number of government outcomes that sustain information transmission. Corollary 2 formalizes this idea.

Corollary 2 *The number of government outcomes that allow for information transmission increases in*:

(i) *the correlation between the candidates' signals,*
(ii) *the quality of the signals.*

Proof From the signal technology described in Table 1, we obtain $P_{i|i} = (1-\varepsilon)^2 + \varepsilon^2 + 2\rho\varepsilon(1-\varepsilon)$ and $P_{i|j} = 2(1-\rho)\varepsilon(1-\varepsilon)$. Since $\partial P_{i|i}/\partial\rho > 0$, $\partial P_{i|i}/\partial\varepsilon = 2(2\varepsilon-1)(1-\rho) < 0$, $\partial P_{i|j}/\partial\rho < 0$ and $\partial P_{i|j}/\partial\varepsilon = 2(1-\rho)(1-2\varepsilon) > 0$, the proof follows. □

Opposing biases

Let us now consider the case of non-ideological voters ex ante thinking that austerity measures are more appropriate than expansionary fiscal policies to return sustainable economic growth. In terms of our analysis, this means $P(w_E) = q < 1/2$. Because ideological and non-ideological voters differ in this case in their (a priori) preferred policy, we say we have here an scenario of *opposing biases*.

As already pointed out, our result here shows that having an electorate with opposing views does not facilitate information transmission, but instead makes it harder. We base this conclusion on two facts. First, the number of equilibria in which candidates can credibly communicate their information to voters is smaller than in the case of like biases. The reason being that with opposing biases, the incentive to go for the electorate's prior (now A) is no longer counterbalanced, but even reinforced, by the existence of a third inflexible candidate that dogmatically supports policy E. Second, the equilibria in the present case are quite fragile, in that they always embed one candidate losing the election (the one using a reversed strategy).[40]

Proposition 2 (*Opposing biases*) *Consider* $q \in (1-\widetilde{q}, \frac{1}{2})$. *A configuration* $(\sigma_c^*, \sigma_v^{*x_1x_2}; g^{x_1x_2}(\Gamma_v^{*x_1x_2}))_{(x_1,x_2)\in\{E,A\}^2, \sigma_v^{*x_1x_2}\in\Gamma_v^{*x_1x_2}}$ *constitutes a fully revealing equilibrium if and only if:*

[40] Similarly to cases (ii)–(iv) in Proposition 1.

(i) $\sigma_c^* = (1,0;0,1)$ *and for any* $(x_1,x_2) \in \{E,A\}^2, \sigma_v^{*x_1x_2}$ *is such that* $g^{AA}(\Gamma_v^{*AA}) = g^{AE}(\Gamma_v^{*AE}) = g^{EA}(\Gamma_v^{*EA}) = g^{EE}(\Gamma_v^{*EE}) = \{1\}$; *or*
(ii) $\sigma_c^* = (0,1;1,0)$ *and for any* $(x_1,x_2) \in \{E,A\}^2, \sigma_v^{*x_1x_2}$ *is such that* $g^{AA}(\Gamma_v^{*AA}) = g^{AE}(\Gamma_v^{*AE}) = g^{EA}(\Gamma_v^{*EA}) = g^{EE}(\Gamma_v^{*EE}) = \{2\}$.

Proof In the Appendix. □

The comparison between the results in Propositions 1 and 2 draws an important conclusion. It is neither the fact that voters are homogeneous, nor even the existence of an inflexible third politician, that facilitates credible transmission of information by mainstream candidates. The crucial feature is the combination of both. That is, having an inflexible third candidate that caters to the (ex-ante) most popular policy. In this sense, our results suggest that for third candidates to help indeed restore the informativeness of electoral processes, the electorate should be like biased.

5 Discussion

We now discuss how the two main assumptions in the model affect our results.

5.1 Two-candidate competition

We first relax the assumption of three candidates competing for office and analyze a scenario with, exclusively, two office-motivated candidates. Note that in this case, there is no room for coalitions, as winning office is simply a matter of getting more votes than your opponent. Given $(x_1, x_2) \in \{E, A\}^2$, let $\sigma_v^{x_1x_2} = (\sigma_4^{x_1x_2}; \sigma_5^{x_1x_2})$, with $\sigma_i^{x_1x_2} = (\sigma_{i,1}^{x_1x_2}, \sigma_{i,2}^{x_1x_2})$ and $i \in \{4, 5\}$ be the vector of voting strategies for voters 4 and 5 in this case.

As in the main body of the paper, we posit two scenarios: like biases and opposing biases.[41] Interestingly, our results now show that there is no information transmission in the case of like biases, but that there is in the case of opposing biases. Note this is in contrast to the results in the previous section, where the presence of an electorate with a like bias facilitated equilibria in which the two mainstream candidates behaved honestly. This means that the result in the present set-up follows much in line with the stark intuition that heterogeneity of views within the voters reduces the incentives to go for the electorate's prior. In fact, this reason explains the next result.

Proposition 3 (*Two-candidate election*) *Suppose* $q \neq 1/2$. *A configuration* $(\sigma_c^*, \sigma_v^{*x_1x_2})_{(x_1,x_2)\in\{E,A\}^2}$ *constitutes a fully revealing equilibrium if and only if* $q < 1/2, \beta = 1/2, \sigma_v^{*AE} = (1,0;0,1), \sigma_v^{*EA} = (0,1;1,0), \sigma_v^{*AA} = \sigma_v^{*EE} = (\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2})$ *and either* $\sigma_c^* = (1,0;1,0)$ *or* $\sigma_c^* = (0,1;0,1)$.

Proof In the Appendix. □

[41] As for the case $q = 1/2$, it can be shown that there is an equilibrium in which the two candidates use a faithful strategy, i.e., $\sigma_c = (1,0;1,0)$. A necessary condition is required for this equilibrium to hold: $\sigma_{4,1}^{*AE} + \sigma_{4,2}^{*EA} = \frac{1}{\beta}$, hence $\sigma_{4,2}^{*AE} + \sigma_{4,1}^{*EA} = \frac{2\beta-1}{\beta}$, which further requires $\beta \geq 1/2$.

Proposition 3 shows that with two candidates competing for office, the electoral process can be informative, but only if voters have opposite views. Otherwise, the incentive to go for the popular belief prevents any information flow (as in HL). Additionally, it also requires that ideological and non-ideological voters are equal in size, and that when the two candidates choose the same platform, they share votes equally. Under these conditions, our result shows that introducing heterogeneity between voters restores the possibility of an informative equilibrium in a two-candidate context, thus breaking with HL's result.

5.2 One type of voter

Let us now consider that voters are only of one type: non-ideological. That is, there is an unanimous consensus in the electorate that the best policy is that fitting the state of the world.

To be consistent with the main body of the paper, we consider that the third candidate is biased in favor of a policy of fiscal stimulus. Additionally, we assume $q > 1/2$, i.e., with no other information, voters think that expansionary fiscal policies are the most appropriate ones to return economic growth.[42]

Proceeding as in the main body of the paper, we can analyze the voters' optimal voting strategy. Note that in this case the analysis is simpler, as non-ideological voters no longer have to worry about what a second group of voters vote. We obtain the next result:

Lemma 2 *Consider a platform profile* $(x_1, x_2) \in \{E, A\}^2$. *In the voting game with one group of voters, the optimal voting behavior of the voters,* $\sigma_4^{*x_1x_2}$, *satisfies the following conditions*:

1. *Consider* $(x_1, x_2) = (E, E)$. *For any* $\hat{q} \in (0, 1)$, $(\sigma_{4,1}^{*EE}, \sigma_{4,2}^{*EE}, \sigma_{4,3}^{*EE}) \in [0, 1]^3$, *is such that* $\sum_{i=\{1,2,3\}} \sigma_{4,i}^{*EE} = 1$.
2. *Consider* $(x_1, x_2) = (E, A)$.
   (a) *When* $\hat{q} < 1/2$, $(\sigma_{4,1}^{*EA}, \sigma_{4,2}^{*EA}, \sigma_{4,3}^{*EA})$, *is such that* $\sigma_{4,2}^{*EA} > \sigma_{4,1}^{*EA} + \sigma_{4,3}^{*EA}$.
   (b) *When* $\hat{q} > 1/2$, $(\sigma_{4,1}^{*EA}, \sigma_{4,2}^{*EA}, \sigma_{4,3}^{*EA})$, *is such that either* $\sigma_{4,2}^{*EA} > 0$ *and* $\sigma_{4,i}^{*EA} > \sigma_{4,2}^{*EA} + \sigma_{4,j}^{*EA}$, *for* $(i, j) \in \{1, 3\}^2, i \neq j$; *or* $\sigma_{4,2}^{*EA} = 0$ *and* $\sum_{i\in\{1,3\}} \sigma_{4,i}^{*EA} = 1$.
3. *Consider* $(x_1, x_2) = (A, E)$.
   (a) *When* $\hat{q} < 1/2$, $(\sigma_{4,1}^{*AE}, \sigma_{4,2}^{*AE}, \sigma_{4,3}^{*AE})$, *is such that* $\sigma_{4,1}^{*AE} > \sigma_{4,2}^{*AE} + \sigma_{4,3}^{*AE}$.
   (b) *When* $\hat{q} > 1/2$, $(\sigma_{4,1}^{*AE}, \sigma_{4,2}^{*AE}, \sigma_{4,3}^{*AE})$, *is such that either* $\sigma_{4,1}^{*AE} > 0$ *and* $\sigma_{4,i}^{*AE} > \sigma_{4,1}^{*AE} + \sigma_{4,j}^{*AE}$, *for* $(i, j) \in \{2, 3\}^2, i \neq j$; *or* $\sigma_{4,1}^{*AE} = 0$ *and* $\sum_{i=\{2,3\}} \sigma_{4,i}^{*AE} = 1$.
4. *Consider* $(x_1, x_2) = (A, A)$.

[42] In the case $q < 1/2$, we predict no equilibrium with full revelation. The reason is that the incentive to go for the electorates' prior (now A) is not counterbalanced by the existence of a third dogmatic candidate proposing that policy, but it is even reinforced by the fact that this candidate always proposes E.

(a) *When* $\hat{q} < 1/2$, $(\sigma_{4,1}^{*AA}, \sigma_{4,2}^{*AA}, \sigma_{4,3}^{*AA})$, *is such that either* $\sigma_{4,3}^{*AA} > 0$ *and* $\sigma_{4,i}^{*AA} > \sigma_{4,j}^{*AA} + \sigma_{4,3}^{*AA}$, *for* $(i, j) \in \{1, 2\}^2, i \neq j$; *or* $\sigma_{4,3}^{*AA} = 0$ *and* $\sum_{i=\{1,2\}} \sigma_{4,i}^{*AE} = 1$.
(b) *When* $\hat{q} > 1/2$, $(\sigma_{4,1}^{*AA}, \sigma_{4,2}^{*AA}, \sigma_{4,3}^{*AA})$ *is such that* $\sigma_{4,3}^{*AA} > \sigma_{4,1}^{*AA} + \sigma_{4,2}^{*AA}$.

Analogously to the case with two groups of voters, for each platform profile $(x_1, x_2) \in \{E, A\}^2$, let $\Gamma_4^{*x_1x_2}$ be the set of the voters' optimal voting strategies that satisfy conditions in Lemma 1. Let $\sigma_4^{*x_1x_2} \in \Gamma_4^{*x_1x_2}$ be an element of this set. We next write the electoral outcomes in terms of government outcomes.

Corollary 3 *Given a platform profile* $(x_1, x_2) \in \{E, A\}^2$, *the set of voters' optimal voting strategies,* $\Gamma_4^{*x_1x_2}$, *and the function* f, *in the game with one group of voters the set* $G^{x_1x_2}$ *is:*

1. *Consider* $(x_1, x_2) = (E, E)$.
   Then, $\forall \hat{q} \in (0, 1)$, $G^{EE} = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$.
2. *Consider* $(x_1, x_2) = (E, A)$.
   (a) *Then* $\forall \hat{q} < 1/2$, $G^{EA} = \{\{2\}\}$,
   (b) *Then* $\forall \hat{q} > 1/2$, $G^{EA} = \{\{1\}, \{3\}, \{1, 3\}\}$.
3. *Consider* $(x_1, x_2) = (A, E)$.
   (a) *Then* $\forall \hat{q} < 1/2$, $G^{AE} = \{\{1\}\}$,
   (b) *Then* $\forall \hat{q} > 1/2$, $G^{AE} = \{\{2\}, \{3\}, \{2, 3\}\}$.
4. *Consider* $(x_1, x_2) = (A, A)$.
   (a) *Then* $\forall \hat{q} < 1/2$, $G^{AA} = \{\{1\}, \{2\}, \{1, 2\}\}$,
   (b) *Then* $\forall \hat{q} > 1/2$, $G^{AA} = \{\{3\}\}$.

A comparison between the scenario with two groups of voters (Lemma 1 and Corollary 1) and the present case (Lemma 2 and Corollary 2), shows that because in the current situation the non-ideological voters no longer have to worry about the ideological ones, those cases in which the existence of the latter class of agents obliged the former to carefully cast their vote, are now altered. We observe that this is the case when $\hat{q} < 1/2$. Otherwise, all voters had a preference for the expansionary fiscal policy, so whether there is one or two groups of voters is irrelevant for the results. Additionally, we observe that for the number of groups of voters to make a difference, we need that more than one candidate proposes the austerity policy. Otherwise, it is clear which candidate should non-ideological voters vote for. This means that the difference is to be found in the case $(x_1, x_2) = (A, A)$ and $\hat{q} < 1/2$. Here, we obtain that a coalition between the two mainstream candidates can now form. The reason is that with no votes for the third candidate, any voters' voting strategy such that $\sigma_{4,2}^{AA} = 0$ is a best response. This is in contrast to the scenario with two groups of voters, where the coalition between the two mainstream candidates was not possible.

Taking into account this new electoral outcome, we go backwards and analyze the information transmission game.

Proposition 4 (*One type of voter*) *A configuration* $(\sigma_c^*, \sigma_4^{*x_1x_2};\ g^{x_1x_2}(\hat{\Gamma}_4^{*x_1x_2}))_{(x_1,x_2)\in\{E,A\}^2, \sigma_4^{*x_1x_2}\in\hat{\Gamma}_4^{*x_1x_2}}$ *constitutes a fully revealing equilibrium if and only if it satisfies one of the conditions* (i)–(iv) *of Proposition* 1, *or condition*

(v) $\sigma_c^* = (1, 0; 1, 0)$ *and for any* $(x_1, x_2) \in \{E, A\}^2$, $\sigma_v^{*x_1x_2} \in \hat{\Gamma}_4^{*x_1x_2}$ *is such that either*:
  (a) $g^{EA}(\hat{\Gamma}_4^{*EA}) = \{1, 3\}$, $g^{AE}(\hat{\Gamma}_4^{*AE}) = \{2, 3\}$, $g^{AA}(\hat{\Gamma}_4^{*AA}) = \{1, 2\}$, $g^{EE}(\hat{\Gamma}_4^{*EE}) = \{3\}$; *or*
  (b) $g^{EA}(\hat{\Gamma}_4^{*EA}) = g^{AE}(\hat{\Gamma}_4^{*AE}) = \{3\}$, $g^{AA}(\hat{\Gamma}_4^{*AA}) = \{1, 2\}$, $g^{EE}(\hat{\Gamma}_4^{*EE}) = \{1, 2\}$.

Proof In the Appendix. □

We obtain that a scenario with just one group of voters facilitates information transmission by the two mainstream candidates. The reason is that in addition to the equilibria characterized in Proposition 1, we now have two new government configurations that sustain honest behavior. Interestingly, we obtain that in these new government configurations, mainstream candidates are better off (compared to the benchmark scenario), while the third inflexible candidate is worst off. In fact, any disagreement between the two mainstream candidates does no longer result in these candidates being always excluded from office. We also observe that under this new scenario, if the mainstream candidates happen to announce the same policy, they can now form a coalition government.

6 Conclusion

It is little wonder that the past 2014 European Elections will be remembered as the *rejection election*.[43] Indeed, poll results all over Europe showed the end of the seemingly settled two-party political system of the mid-20th century. From Britain to Denmark, France, Italy or Spain, "*the center-left and center-right groups that form the core of national and European Union politics have seen their power eroded by the rise of extremist parties very different from one another, united only by their rejection of the way things are, both at home and at the European Union*" ("As goes Greece, so goes Europe?", *The New York Times*, 28th May 2014).

The rise of third parties presents an unprecedented challenge to mainstream political parties and to society as a whole. What consequences will this threat have for the economic model that will govern us in the near future is something that we will learn in due course. Working within a small area of this topic, this paper presents results that move towards showing the consequences that this threat can have on the resulting electoral outcomes and the informativeness of the electoral processes.

In this work we argue that the presence of a third inflexible candidate might help ease the information transmission game, specially, when all voters are biased in the same direction and so there are no conflicting views in the electorate. This result, which a priori may look a bit contradicting, is robust. We observe it when in a posterior stage we consider the existence of only one group of voters, and obtain that the number of government outcomes that sustain information transmission is greater here than in the previous case. Another interesting result is that for the mainstream candidates to honestly reveal their information, we need voters to have a strict preference for one

[43] "If this was the rejection election, where does mainstream politics go?", *The Guardian*, 28th May 2014.

of these two candidates. That is, that whenever mainstream candidates propose the same policy, voters always vote for the same candidate. Otherwise, that is if under indifference they flip a coin, no information transmission can be achieved. In this sense, voters' captivity seems to be good.

Appendix

Proof of Lemma 1:

Proof In the analysis that follows, we consider that ideological voters never vote for a candidate proposing policy A. Next, we differentiate four cases:

(i) Consider $(x_1, x_2) = (EE)$. In this case only expansionary fiscal policies are being proposed, therefore the final policy will always be $x = E$. This means that any voting strategy is optimal for voters. That is, the optimal strategies of the non-ideological and ideological voters satisfy, respectively, $(\sigma_{4,1}^{*EE}, \sigma_{4,2}^{*EE}, \sigma_{4,3}^{*EE}) \in [0, 1]^3$, such that $\sum_{i=\{1,2,3\}} \sigma_{4,i}^{*EE} = 1$; and $(\sigma_{5,1}^{*EE}, \sigma_{5,2}^{*EE}, \sigma_{5,3}^{*EE}) \in [0, 1]^3$, such that $\sum_{i=\{1,2,3\}} \sigma_{5,i}^{*EE} = 1$.

(ii) Consider $(x_1, x_2) = (EA)$. In this case $(\sigma_{5,1}^{*EA}, \sigma_{5,2}^{*EA}, \sigma_{5,3}^{*EA})$ is such that $\sigma_{5,2}^{*EA} = 0$, with $\sum_{i\in\{1,3\}} \sigma_{5,i}^{*EA} = 1$.
First, consider $\hat{q} < 1/2$. In this case, if non-ideological voters were to vote for candidate 2 with probability 1, $\sigma_{4,2}^{EA} = 1$, the electoral outcome would be $(v_1, v_2, v_3) = ((1-\beta)\sigma_{5,1}^{*EA}, \beta, (1-\beta)\sigma_{5,3}^{*EA})$. Since $\beta > 1/2$, $g = \{2\}$, in which case the payoff to each non-ideological voter would be $1 - \hat{q}$. Now, if all non-ideological voters where to vote in such a way that candidate 2 did not get a strict majority of votes, their individual payoff would be smaller, as the policy implemented by the resulting government would be E with positive probability. Then, the optimal strategy of non-ideological voters, $(\sigma_{4,1}^{*EA}, \sigma_{4,2}^{*EA}, \sigma_{4,3}^{*EA})$, satisfies $\beta\sigma_{4,2}^{*EA} > \beta(\sigma_{4,1}^{*EA} + \sigma_{4,3}^{*EA}) + (1-\beta)$.
Now, consider $\hat{q} > 1/2$. Here, if $G \subseteq \{\{1\}, \{3\}, \{1, 3\}\}$, the payoff to each non-ideological voter would be $\hat{q}$, whereas it would be smaller otherwise.[44] Hence, non-ideological voters must prevent candidate 2 from entering the government. This means that their voting strategy must satisfy that if neither candidate 1 nor 3 gets an absolute majority of votes, candidate 2 cannot form a government. Mathematically, $v_2 + v_i < 1/2$, for $i \in \{1, 3\}$. But this is a contradiction, as whenever $\max\{v_1, v_2, v_3\} < 1/2$, any pair of two candidates gets an absolute majority of votes. Thus, the optimal strategy of non-ideological voters,

[44] In these other cases candidate 2 would be in the government, in which case there would be a positive probability that policy A were implemented.

$(\sigma_{4,1}^{*EA}, \sigma_{4,2}^{*EA}, \sigma_{4,3}^{*EA})$, is such that either $\sigma_{4,2}^{*EA} > 0$ and $\beta\sigma_{4,i}^{*EA} + (1-\beta)\sigma_{5,i}^{*EA} > \beta(\sigma_{4,2}^{*EA} + \sigma_{4,j}^{*EA}) + (1-\beta)\sigma_{5,j}^{*EA}$, for $(i, j) \in \{1, 3\}^2, i \neq j$; or $\sigma_{4,2}^{*EA} = 0$ and $\sum_{i\in\{1,3\}} \sigma_{4,i}^{*EA} = 1$.

(iii) Consider $(x_1, x_2) = (AE)$. In this case $(\sigma_{5,1}^{*AE}, \sigma_{5,2}^{*AE}, \sigma_{5,3}^{*AE})$, such that $\sigma_{5,1}^{*AE} = 0$, with $\sum_{i\in\{2,3\}} \sigma_{5,i}^{*AE} = 1$. Now, analogously to the previous case, we obtain:
When $\hat{q} < 1/2$, $(\sigma_{4,1}^{*AE}, \sigma_{4,2}^{*AE}, \sigma_{4,3}^{*AE})$, such that $\beta\sigma_{4,1}^{*AE} > \beta(\sigma_{4,2}^{*AE} + \sigma_{4,3}^{*AE}) + (1-\beta)$.
When $\hat{q} > 1/2$, $(\sigma_{4,1}^{*AE}, \sigma_{4,2}^{*AE}, \sigma_{4,3}^{*AE})$, such that either $\sigma_{4,1}^{*AE} > 0$ and $\beta\sigma_{4,i}^{*AE} + (1-\beta)\sigma_{5,i}^{*AE} > \beta(\sigma_{4,1}^{*AE} + \sigma_{4,j}^{*AE}) + (1-\beta)\sigma_{5,j}^{*AE}$, for $(i, j) \in \{2, 3\}^2, i \neq j$; or $\sigma_{4,1}^{*AE} = 0$ and $\sum_{i\in\{2,3\}} \sigma_{4,i}^{*AE} = 1$.

(iv) Last, consider $(x_1, x_2) = (AA)$. In this case $(\sigma_{5,1}^{*AA}, \sigma_{5,2}^{*AA}, \sigma_{5,3}^{*AA}) = (0, 0, 1)$.
First, consider $\hat{q} < 1/2$. In this case, if non-ideological voters were to vote for one of the two mainstream candidates (let us say candidate 1) with probability 1, $\sigma_{4,1}^{AA} = 1$, the electoral outcome would be $(v_1, v_2, v_3) = (\beta, 0, 1-\beta)$, the government outcome $g = \{1\}$, and the payoff to each of these voters $1 - \hat{q}$. In contrast, if they all voted in such a way that candidate 1 (nor 2) no longer obtained a majority of votes, their individual expected utility would be smaller, as policy E would be implemented with positive probability. Then, the optimal strategy of these voters, $(\sigma_{4,1}^{*AA}, \sigma_{4,2}^{*AA}, \sigma_{4,3}^{*AA})$, satisfies that either one of the two mainstream candidates receive a strict majority of votes. This is, $\beta\sigma_{4,i}^{*AA} > \beta\sigma_{4,j}^{*AA} + (1-\beta)$, for $(i, j) \in \{1, 2\}^2, i \neq j$.
Last, consider $\hat{q} > 1/2$. In this case, if non-ideological voters were to vote for candidate 3 with probability 1, $\sigma_{4,3}^{AA} = 1$, the electoral outcome would be $(v_1, v_2, v_3) = (0, 0, 1)$, the government outcome $g = \{3\}$, and the payoff to each of these voters $\hat{q}$. In contrast, if they all voted in such a way that the third inflexible candidate no longer obtained a majority of votes and one of the two mainstream candidates were to enter the government, their individual expected utility would be smaller, as policy A would be implemented with positive probability. Then, the optimal strategy of these voters, $(\sigma_{4,1}^{*AA}, \sigma_{4,2}^{*AA}, \sigma_{4,3}^{*AA})$, satisfies $\beta\sigma_{4,3}^{*AA} + (1-\beta) > \beta(\sigma_{4,1}^{*AA} + \sigma_{4,2}^{*AA})$. This completes the proof. □

Proof of Proposition 1:

Proof We have to analyze four cases:

(i) Suppose $\sigma_c = (1, 0; 1, 0)$ is part of an equilibrium.
In equilibrium, choosing E when having observed e must be a best response for candidates 1 (Eq. 3) and 2 (Eq. 4), respectively:

$$P_{e|e}\begin{cases} K & \text{if} \quad g^{EE} = \{1\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\}, \{1,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{2\}, \{3\}, \{2,3\}\} \end{cases} + P_{a|e}\begin{cases} K & \text{if} \quad g^{EA} = \{1\} \\ K/2 & \text{if} \quad g^{EA} = \{1,3\} \\ 0 & \text{if} \quad g^{EA} = \{3\} \end{cases}$$
$$\geq P_{a|e}\begin{cases} K & \text{if} \quad g^{AA} = \{1\} \\ 0 & \text{if} \quad g^{AA} = \{2\} \end{cases} \tag{3}$$

$$P_{e|e}\begin{cases}K & \text{if} \quad g^{EE}=\{2\}\\ K/2 & \text{if} \quad g^{EE}\in\{\{1,2\},\{2,3\}\}\\ 0 & \text{if} \quad g^{EE}\in\{\{1\},\{3\},\{1,3\}\}\end{cases} + P_{a|e}\begin{cases}K & \text{if} \quad g^{AE}=\{2\}\\ K/2 & \text{if} \quad g^{AE}=\{2,3\}\\ 0 & \text{if} \quad g^{AE}=\{3\}\end{cases}$$
$$\geq P_{a|e}\begin{cases}K & \text{if} \quad g^{AA}=\{2\}\\ 0 & \text{if} \quad g^{AA}=\{1\}\end{cases} \tag{4}$$

Analogously, choosing A when having observed a must be a best response for candidates 1 (Eq. 5) and 2 (Eq. 6), respectively:

$$P_{a|a}\begin{cases}K & \text{if} \quad g^{AA}=\{1\}\\ 0 & \text{if} \quad g^{AA}=\{2\}\end{cases} \geq P_{a|a}\begin{cases}K & \text{if} \quad g^{EA}=\{1\}\\ K/2 & \text{if} \quad g^{EA}=\{1,3\}\\ 0 & \text{if} \quad g^{EA}=\{3\}\end{cases}$$
$$+P_{e|a}\begin{cases}K & \text{if} \quad g^{EE}=\{1\}\\ K/2 & \text{if} \quad g^{EE}\in\{\{1,2\},\{1,3\}\}\\ 0 & \text{if} \quad g^{EE}\in\{\{2\},\{3\},\{2,3\}\}\end{cases} \tag{5}$$

$$P_{a|a}\begin{cases}K & \text{if} \quad g^{AA}=\{2\}\\ 0 & \text{if} \quad g^{AA}=\{1\}\end{cases} \geq P_{a|a}\begin{cases}K & \text{if} \quad g^{AE}=\{2\}\\ K/2 & \text{if} \quad g^{AE}=\{2,3\}\\ 0 & \text{if} \quad g^{AE}=\{3\}\end{cases}$$
$$+P_{e|a}\begin{cases}K & \text{if} \quad g^{EE}=\{2\}\\ K/2 & \text{if} \quad g^{EE}\in\{\{1,2\},\{2,3\}\}\\ 0 & \text{if} \quad g^{EE}\in\{\{1\},\{3\},\{1,3\}\}\end{cases} \tag{6}$$

Now, let us first suppose $g^{EA}=\{1\}$. Then, we have $g^{AE}=\{2\}$. In this case, a necessary condition for (5) to hold is $g^{AA}=\{1\}$, but then (6) cannot hold. Similarly, let us suppose $g^{EA}=\{1,3\}$. Then, we have $g^{AE}=\{2,3\}$. Again, $g^{AA}=\{1\}$ is necessary for (5) to hold, but then (6) cannot hold. Hence, this is not an equilibrium.

Last, let us suppose $g^{EA}=\{3\}$. Then, we have $g^{AE}=\{3\}$. Here we have two cases: (a) Consider $g^{AA}=\{1\}$. A necessary condition for (6) to hold is $g^{EE}\in\{\{1\},\{3\},\{1,3\}\}$, and a necessary condition for (3) to hold is either $g^{EE}\in\{\{1,2\},\{1,3\}\}$ and $P_{e|e}\geq 2P_{a|e}$, or $g^{EE}=\{1\}$. Note that if $g^{EE}\in\{\{1\},\{1,3\}\}$, conditions (4) and (5) hold. Hence, if either $g^{EE}=\{13\}$ and $P_{e|e}\geq 2P_{a|e}$, or $g^{EE}=\{1\}$, there is an equilibrium in this case. (b) Analogously to the previous case, there is an equilibrium in which $g^{AA}=\{2\}$ and either $g^{EE}=\{2,3\}$ and $P_{e|e}\geq 2P_{a|e}$, or $g^{EE}=\{2\}$.

(ii) Suppose $\sigma_c=(1,0;0,1)$ is part of an equilibrium.
In equilibrium, choosing E when having observed e must be a best response for candidate 1 (Eq. 7), and choosing A when having observed e must be a best response for candidate 2 (Eq. 8):

$$P_{e|e}\begin{cases}K & \text{if} \quad g^{EA}=\{1\}\\ K/2 & \text{if} \quad g^{EA}=\{1,3\}\\ 0 & \text{if} \quad g^{EA}=\{3\}\end{cases} + P_{a|e}\begin{cases}K & \text{if} \quad g^{EE}=\{1\}\\ K/2 & \text{if} \quad g^{EE}\in\{\{1,2\},\{1,3\}\}\\ 0 & \text{if} \quad g^{EE}\in\{\{2\},\{3\},\{2,3\}\}\end{cases}$$
$$\geq P_{a|e}K. \tag{7}$$

$$0 \geq P_{e|e} \begin{cases} K & \text{if} \quad g^{EE} = \{2\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{2,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{1\},\{3\},\{1,3\}\} \end{cases} \tag{8}$$

Analogously, choosing A when having observed a must be a best response for candidate 1 (Eq. 9), and choosing E when having observed a must be a best response for candidate 2 (Eq. 10):

$$P_{a|a}K \geq P_{a|a} \begin{cases} K & \text{if} \quad g^{EE} = \{1\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{1,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{2\},\{3\},\{2,3\}\} \end{cases} + P_{e|a} \begin{cases} K & \text{if} \quad g^{EA} = \{1\} \\ K/2 & \text{if} \quad g^{EA} = \{1,3\} \\ 0 & \text{if} \quad g^{EA} = \{3\} \end{cases} \tag{9}$$

$$P_{e|a} \begin{cases} K & \text{if} \quad g^{EE} = \{2\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{2,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{1\},\{3\},\{2,3\}\} \end{cases} \geq 0 \tag{10}$$

First, note that condition (10) always hold, and that a necessary condition for (8) to hold is $g^{EE} \in \{\{1\},\{3\},\{1,3\}\}$.

Then, let us first consider $g^{EE} = \{1\}$. In this case, a necessary condition for (9) to hold is $g^{EA} = \{3\}$, in which case (7) holds. Hence, there is an equilibrium in this case.

Second, let us consider $g^{EE} = \{3\}$. A necessary condition for (7) to hold is either $g^{EA} = \{1,3\}$ and $P_{e|e} > 2P_{a|e}$, or $g^{EA} = \{1\}$. Note that in both cases condition (9) holds. Hence, there is an equilibrium in this case.

Last, let us consider $g^{EE} = \{1,3\}$. Again, a necessary condition for (7) to hold is either $g^{EA} = \{1,3\}$ or $g^{EA} = \{1\}$. (a) If $g^{EA} = \{1\}$, a necessary condition for (9) to hold is $P_{a|a} > 2P_{e|a}$. Hence, there is an equilibrium in this case. (b) If $g^{EA} = \{1,3\}$, (9) always hold. Hence, there is an equilibrium in this case.

(iii) The case $\sigma_c = (0, 1; 1, 0)$ is analogous to the previous one.

(iv) Last, suppose $\sigma_c = (0, 1; 0, 1)$ is part of an equilibrium.
In equilibrium, choosing A when having observed e must be a best response for candidates 1 (Eq. 11) and 2 (Eq. 12), respectively:

$$0 \geq P_{e|e} \begin{cases} K & \text{if} \quad g^{EA} = \{1\} \\ K/2 & \text{if} \quad g^{EA} = \{1,3\} \\ 0 & \text{if} \quad g^{EA} = \{3\} \end{cases} + P_{a|e} \begin{cases} K & \text{if} \quad g^{EE} = \{1\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{1,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{2\},\{3\},\{2,3\}\} \end{cases} \tag{11}$$

$$0 \geq P_{e|e} \begin{cases} K & \text{if} \quad g^{AE} = \{2\} \\ K/2 & \text{if} \quad g^{AE} = \{2,3\} \\ 0 & \text{if} \quad g^{AE} = \{3\} \end{cases} + P_{a|e} \begin{cases} K & \text{if} \quad g^{EE} = \{2\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{2,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{1\},\{3\},\{1,3\}\} \end{cases} \tag{12}$$

For (11) and (12) to hold, a necessary condition is $g^{EA} = g^{AE} = g^{EE} = \{3\}$. Additionally, note that if $(x_1, x_2) = (AA)$, the non-ideological voter thinks policy E is the best one, then $g^{AA} = \{3\}$. Hence, candidates 1 and 2 never get votes, and so this is an equilibrium. This completes the proof. □

Proof of Proposition 2:

Proof We have to analyze four cases:

(i) Suppose $\sigma_c = (1, 0; 1, 0)$ is part of an equilibrium.
In equilibrium, choosing E when having observed e must be a best response for candidates 1 (Eq. 13) and 2 (Eq. 14), respectively:

$$P_{e|e} \begin{cases} K & \text{if} \quad g^{EE} = \{1\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{1,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{2\},\{3\},\{2,3\}\} \end{cases} \geq P_{e|e}K + P_{a|e} \begin{cases} K & \text{if} \quad g^{AA} = \{1\} \\ 0 & \text{if} \quad g^{AA} = \{2\} \end{cases} \tag{13}$$

$$P_{e|e} \begin{cases} K & \text{if} \quad g^{EE} = \{2\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{2,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{1\},\{3\},\{1,3\}\} \end{cases} \geq P_{e|e}K + P_{a|e} \begin{cases} K & \text{if} \quad g^{AA} = \{2\} \\ 0 & \text{if} \quad g^{AA} = \{1\} \end{cases} \tag{14}$$

A necessary condition for (13) to hold is $g^{EE} = \{1\}$. But then (14) cannot hold. Hence, this is not an equilibrium.

(ii) Suppose $\sigma_c = (1, 0; 0, 1)$ is part of an equilibrium.
In equilibrium, choosing E when having observed e must be a best response for candidate 1 (Eq. 15), and choosing A when having observed e must be a best response for candidate 2 (Eq. 16):

$$P_{e|e} \begin{cases} K & \text{if} \quad g^{EA} = \{1\} \\ K/2 & \text{if} \quad g^{EA} = \{1,3\} \\ 0 & \text{if} \quad g^{EA} = \{3\} \end{cases} + P_{a|e} \begin{cases} K & \text{if} \quad g^{EE} = \{1\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{1,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{2\},\{3\},\{2,3\}\} \end{cases}$$
$$\geq P_{e|e} \begin{cases} K & \text{if} \quad g^{AA} = \{1\} \\ 0 & \text{if} \quad g^{AA} = \{2\} \end{cases} + P_{a|e}K \tag{15}$$

$$P_{a|e} \begin{cases} K & \text{if} \quad g^{AA} = \{2\} \\ 0 & \text{if} \quad g^{AA} = \{1\} \end{cases} \geq P_{e|e} \begin{cases} K & \text{if} \quad g^{EE} = \{2\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{2,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{1\},\{3\},\{1,3\}\} \end{cases} \tag{16}$$

Analogously, choosing A when having observed a must be a best response for candidate 1 (Eq. 17), and choosing E when having observed a must be a best response for candidate 2 (Eq. 18):

$$P_{a|a}K + P_{e|a}\begin{cases} K & \text{if} \quad g^{AA} = \{1\} \\ 0 & \text{if} \quad g^{AA} = \{2\} \end{cases}$$
$$\geq P_{a|a}\begin{cases} K & \text{if} \quad g^{EE} = \{1\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{1,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{2\},\{3\},\{2,3\}\} \end{cases}$$
$$+P_{e|a}\begin{cases} K & \text{if} \quad g^{EA} = \{1\} \\ K/2 & \text{if} \quad g^{EA} = \{1,3\} \\ 0 & \text{if} \quad g^{EA} = \{3\} \end{cases} \tag{17}$$

$$P_{e|a}\begin{cases} K & \text{if} \quad g^{EE} = \{2\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{2,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{1\},\{3\},\{2,3\}\} \end{cases} \geq P_{a|a}\begin{cases} K & \text{if} \quad g^{AA} = \{2\} \\ 0 & \text{if} \quad g^{AA} = \{1\} \end{cases} \tag{18}$$

First, note that if $g^{AA} = \{2\}$, condition (18) cannot hold. Hence, this is not an equilibrium.
Now, consider $g^{AA} = \{1\}$. A necessary condition for (16) to hold is $g^{EE} \in \{\{1\},\{3\},\{1,3\}\}$, and the necessary conditions for (15) to hold are $g^{EA} = g^{EE} = \{1\}$. Note that in this case, both (17) and (18) hold. Hence, there is an equilibrium in this case.

(iii) The case $\sigma_c = (0, 1; 1, 0)$ is analogous to the previous one.

(iv) Last, suppose $\sigma_c = (0, 1; 0, 1)$ is part of an equilibrium.
In equilibrium, choosing E when having observed a must be a best response for candidates 1 (Eq. 19) and 2 (Eq. 20), respectively:

$$P_{a|a}\begin{cases} K & \text{if} \quad g^{EE} = \{1\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{1,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{2\},\{3\},\{2,3\}\} \end{cases} \geq P_{a|a}K \tag{19}$$

$$P_{a|a}\begin{cases} K & \text{if} \quad g^{EE} = \{2\} \\ K/2 & \text{if} \quad g^{EE} \in \{\{1,2\},\{2,3\}\} \\ 0 & \text{if} \quad g^{EE} \in \{\{1\},\{3\},\{1,3\}\} \end{cases} \geq P_{a|a}K \tag{20}$$

For (19) to hold, a necessary condition is $g^{EE} = \{1\}$. But then (20) cannot hold. Hence, this is not an equilibrium. This completes the proof. □

Proof of Proposition 3:

Proof We prove it in two steps: like biases and opposing biases.

1. Let us first consider the case of like biases, i.e. $q > \frac{1}{2}$. We have to analyze four cases:
   1.i) Suppose $\sigma_c = (1, 0; 1, 0)$ is part of an equilibrium.

Here, choosing A when having observed a must be a best response for candidates 1 and 2:

$$P_{a|a}[\beta\sigma_{4,1}^{AA} + (1-\beta)\sigma_{5,1}^{AA}] \geq P_{a|a} + P_{e|a}[\beta\sigma_{4,1}^{EE} + (1-\beta)\sigma_{5,1}^{EE}]$$
$$P_{a|a}[\beta\sigma_{4,2}^{AA} + (1-\beta)\sigma_{5,2}^{AA}] \geq P_{a|a} + P_{e|a}[\beta\sigma_{4,2}^{EE} + (1-\beta)\sigma_{5,2}^{EE}]$$

Adding inequalities we obtain $0 \geq P_{a|a} + P_{e|a} = 1$, which is impossible. Hence, there is no equilibrium in this case.

1.ii) Suppose $\sigma_c = (1, 0; 0, 1)$ is part of an equilibrium.
Here, choosing A when having observed a must be a best response for candidate 1:

$$P_{a|a}\beta + P_{e|a}[\beta\sigma_{4,1}^{AA} + (1-\beta)\sigma_{5,1}^{AA}] \geq P_{a|a}[\beta\sigma_{4,1}^{EE} + (1-\beta)\sigma_{5,1}^{EE}] + P_{e|a}$$

Analogously, choosing A when having observed e must be a best response for candidate 2:

$$P_{a|e}[\beta\sigma_{4,2}^{AA} + (1-\beta)\sigma_{5,2}^{AA}] \geq P_{e|e}[\beta\sigma_{4,2}^{EE} + (1-\beta)\sigma_{5,2}^{EE}] + P_{a|e}(1-\beta)$$

Since, for $i, j \in \{E, A\}$ and $k \in \{4, 5\}$, $P_{i|j} = P_{j|i}$ and $\sigma_{k,1}^{ii} + \sigma_{k,2}^{ii} = 1$, adding inequalities and rearranging, we obtain $0 \geq P_{a|a}(1-\beta) + P_{e|a}(1-\beta)$. Since for $i, j \in \{E, A\}$, $P_{j|i} + P_{i|i} = 1$, there is no equilibrium in this case.

1.iii) The case $\sigma_c = (0, 1; 1, 0)$ is analogous to the previous one.

1.iv) Last, suppose $\sigma_c = (0, 1; 0, 1)$ is part of an equilibrium.
Here, choosing A when having observed e must be a best response for any of the two mainstream candidates, let us say, candidate 1. That is to say:

$$P_{e|e}[\beta\sigma_{4,1}^{AA} + (1-\beta)\sigma_{5,1}^{AA}] \geq P_{e|e} + P_{a|e}[\beta\sigma_{4,1}^{EE} + (1-\beta)\sigma_{5,1}^{EE}]$$

Since $\beta\sigma_{4,1}^{AA} + (1-\beta)\sigma_{5,1}^{AA} \leq 1$, there is no equilibrium in this case.

2. Let us now consider the case of opposing biases, i.e., $q < \frac{1}{2}$. Again, there are four cases to analyze:

2.i) Suppose $\sigma_c = (1, 0; 1, 0)$ is part of an equilibrium.
Here, choosing E when having observed e must be a best response for candidates 1 and 2:

$$P_{e|e}[\beta\sigma_{4,1}^{EE} + (1-\beta)\sigma_{5,1}^{EE}] + P_{a|e}(1-\beta) \geq P_{e|e}\beta + P_{a|e}[\beta\sigma_{4,1}^{AA} + (1-\beta)\sigma_{5,1}^{AA}] \tag{21}$$

$$P_{e|e}[\beta\sigma_{4,2}^{EE} + (1-\beta)\sigma_{5,2}^{EE}] + P_{a|e}(1-\beta) \geq P_{e|e}\beta + P_{a|e}[\beta\sigma_{4,2}^{AA} + (1-\beta)\sigma_{5,2}^{AA}] \tag{22}$$

Adding inequalities we obtain $P_{e|e}(1-2\beta) + P_{a|e}(1-2\beta) \geq 0 \Leftrightarrow \beta \leq 1/2$.

Similarly, in equilibrium, choosing A when having observed a must be a best response for candidates 1 and 2:

$$P_{a|a}[\beta\sigma_{4,1}^{AA} + (1-\beta)\sigma_{5,1}^{AA}] + P_{e|a}\beta \geq P_{a|a}(1-\beta)$$
$$+P_{e|a}[\beta\sigma_{4,1}^{EE} + (1-\beta)\sigma_{5,1}^{EE}] \quad (23)$$

$$P_{a|a}[\beta\sigma_{4,2}^{AA} + (1-\beta)\sigma_{5,2}^{AA}] + P_{e|a}\beta \geq P_{a|a}(1-\beta)$$
$$+P_{e|a}[\beta\sigma_{4,2}^{EE} + (1-\beta)\sigma_{5,2}^{EE}] \quad (24)$$

Adding inequalities we obtain $0 \geq P_{a|a}(1-2\beta) + P_{e|a}(1-2\beta) \Leftrightarrow \beta \geq 1/2$. Hence, both requirements can only meet if $\beta = 1/2$. Now, if $\beta = 1/2$, and taking into account that for $k \in \{4, 5\}$ and $i \in \{E, A\}$, $\sigma_{k,1}^{ii} + \sigma_{k,2}^{ii} = 1$, inequalities (21)–(24) can be rewritten as:

$$P_{a|e}(1 - \sigma_{4,1}^{AA} - \sigma_{5,1}^{AA}) = P_{e|e}(1 - \sigma_{4,1}^{EE} - \sigma_{5,1}^{EE})$$
$$P_{a|a}(1 - \sigma_{4,1}^{AA} - \sigma_{5,1}^{AA}) = P_{e|a}(1 - \sigma_{4,1}^{EE} - \sigma_{5,1}^{EE})$$

Now, since $P_{i|i} > P_{j|i}$, there is a unique solution for the system above, which is $\sigma_{4,1}^{AA} = \sigma_{5,1}^{AA} = \sigma_{4,1}^{EE} = \sigma_{5,1}^{EE} = 1/2$. Hence, in this case, we have an equilibrium.

2.ii) Suppose $\sigma_c = (1, 0; 0, 1)$ is part of an equilibrium. In this case, the payoffs coincide with those in case 1.ii). Hence, this is not an equilibrium.

2.iii) Analogously, there is neither an equilibrium in the case $\sigma_c = (0, 1; 1, 0)$.

2.iv) Last, suppose $\sigma_c = (0, 1; 0, 1)$ is part of an equilibrium.
Here, choosing A when having observed e must be a best response for candidates 1 and 2. That is to say:

$$P_{e|e}[\beta\sigma_{4,1}^{AA} + (1-\beta)\sigma_{5,1}^{AA}] + P_{a|e}\beta \geq P_{e|e}(1-\beta)$$
$$+P_{a|e}[\beta\sigma_{4,1}^{EE} + (1-\beta)\sigma_{5,1}^{EE}] \quad (25)$$

$$P_{e|e}[\beta\sigma_{4,2}^{AA} + (1-\beta)\sigma_{5,2}^{AA}] + P_{a|e}\beta \geq P_{e|e}(1-\beta)$$
$$+P_{a|e}[\beta\sigma_{4,2}^{EE} + (1-\beta)\sigma_{5,2}^{EE}] \quad (26)$$

Adding inequalities we obtain $0 \geq P_{e|e}(1-2\beta) + P_{a|e}(1-2\beta) \Leftrightarrow \beta \geq 1/2$. Similarly, in equilibrium, choosing E when having observed a must be a best response for candidates 1 and 2. That is to say:

$$P_{a|a}[\beta\sigma_{4,1}^{EE} + (1-\beta)\sigma_{5,1}^{EE}] + P_{e|a}(1-\beta) \geq P_{a|a}\beta$$
$$+P_{e|a}[\beta\sigma_{4,1}^{AA} + (1-\beta)\sigma_{5,1}^{AA}] \quad (27)$$

$$P_{a|a}[\beta\sigma_{4,2}^{EE} + (1-\beta)\sigma_{5,2}^{EE}] + P_{e|a}(1-\beta) \geq P_{a|a}\beta$$
$$+P_{e|a}[\beta\sigma_{4,2}^{AA} + (1-\beta)\sigma_{5,2}^{AA}] \quad (28)$$

Adding inequalities we obtain $P_{a|a}(1-2\beta)+P_{e|a}(1-2\beta)\geq 0 \Leftrightarrow \beta\leq 1/2$. Hence, both requirements can only meet if $\beta = 1/2$. Now, if $\beta = 1/2$, and taking into account that for $k \in \{4, 5\}$ and $i \in \{E, A\}$, $\sigma^{ii}_{k,1} + \sigma^{ii}_{k,2} = 1$, inequalities (25)–(28) can be rewritten as:

$$P_{a|e}(1-\sigma^{EE}_{4,1}-\sigma^{EE}_{5,1}) = P_{e|e}(1-\sigma^{AA}_{4,1}-\sigma^{AA}_{5,1})$$
$$P_{a|a}(1-\sigma^{EE}_{4,1}-\sigma^{EE}_{5,1}) = P_{e|a}(1-\sigma^{AA}_{4,1}-\sigma^{AA}_{5,1})$$

Now, since $P_{i|i} > P_{j|i}$, there is a unique solution for the system above, which is $\sigma^{AA}_{4,1} = \sigma^{EE}_{4,1} = \sigma^{AA}_{5,1} = \sigma^{EE}_{5,1} = 1/2$. Hence, in this case, there is an equilibrium of the type conjectured. □

Proof of Proposition 4:

Proof First, suppose an equilibrium in which either $\sigma_c = (1, 0; 0, 1)$, $\sigma_c = (1, 0; 0, 1)$ or $\sigma_c = (0, 1; 1, 0)$. In these cases, note that when the platforms profile observed is $(x_1, x_2) = (AA)$, applying Bayes' rule we obtain $\hat{q} > 1/2$. Hence, $g^{AA} = \{3\}$, as it was the case in the analogous situations of Proposition 1. The analysis of these cases are therefore identical to those in Proposition 1, thus omitted.

Let us now suppose $\sigma_c = (1, 0; 1, 0)$ is part of an equilibrium.

In equilibrium, choosing E when having observed e must be a best response for candidates 1 (Eq. 29) and 2 (Eq. 30), respectively:

$$P_{e|e}\begin{cases} K & \text{if} \quad g^{EE}=\{1\} \\ K/2 & \text{if} \quad g^{EE}\in\{\{1,2\},\{1,3\}\} \\ 0 & \text{if} \quad g^{EE}\in\{\{2\},\{3\},\{2,3\}\} \end{cases} + P_{a|e}\begin{cases} K & \text{if} \quad g^{EA}=\{1\} \\ K/2 & \text{if} \quad g^{EA}=\{1,3\} \\ 0 & \text{if} \quad g^{EA}=\{3\} \end{cases}$$
$$\geq P_{a|e}\begin{cases} K & \text{if} \quad g^{AA}=\{1\} \\ K/2 & \text{if} \quad g^{AA}=\{1,2\} \\ 0 & \text{if} \quad g^{AA}=\{2\} \end{cases} \tag{29}$$

$$P_{e|e}\begin{cases} K & \text{if} \quad g^{EE}=\{2\} \\ K/2 & \text{if} \quad g^{EE}\in\{\{1,2\},\{2,3\}\} \\ 0 & \text{if} \quad g^{EE}\in\{\{1\},\{3\},\{1,3\}\} \end{cases} + P_{a|e}\begin{cases} K & \text{if} \quad g^{AE}=\{2\} \\ K/2 & \text{if} \quad g^{AE}=\{2,3\} \\ 0 & \text{if} \quad g^{AE}=\{3\} \end{cases}$$
$$\geq P_{a|e}\begin{cases} K & \text{if} \quad g^{AA}=\{2\} \\ K/2 & \text{if} \quad g^{AA}=\{1,2\} \\ 0 & \text{if} \quad g^{AA}=\{1\} \end{cases} \tag{30}$$

Analogously, choosing A when having observed a must be a best response for candidates 1 (Eq. 31) and 2 (Eq. 32), respectively:

$$P_{a|a}\begin{cases} K & \text{if} \quad g^{AA}=\{1\} \\ K/2 & \text{if} \quad g^{AA}=\{1,2\} \\ 0 & \text{if} \quad g^{AA}=\{2\} \end{cases} \geq P_{a|a}\begin{cases} K & \text{if} \quad g^{EA}=\{1\} \\ K/2 & \text{if} \quad g^{EA}=\{1,3\} \\ 0 & \text{if} \quad g^{EA}=\{3\} \end{cases}$$
$$+P_{e|a}\begin{cases} K & \text{if} \quad g^{EE}=\{1\} \\ K/2 & \text{if} \quad g^{EE}\in\{\{1,2\},\{1,3\}\} \\ 0 & \text{if} \quad g^{EE}\in\{\{2\},\{3\},\{2,3\}\} \end{cases} \tag{31}$$

$$P_{a|a}\begin{cases} K & \text{if} \quad g^{AA}=\{2\} \\ K/2 & \text{if} \quad g^{AA}=\{1,2\} \\ 0 & \text{if} \quad g^{AA}=\{1\} \end{cases} \geq P_{a|a}\begin{cases} K & \text{if} \quad g^{AE}=\{2\} \\ K/2 & \text{if} \quad g^{AE}=\{2,3\} \\ 0 & \text{if} \quad g^{AE}=\{3\} \end{cases}$$
$$+P_{e|a}\begin{cases} K & \text{if} \quad g^{EE}=\{2\} \\ K/2 & \text{if} \quad g^{EE}\in\{\{1,2\},\{2,3\}\} \\ 0 & \text{if} \quad g^{EE}\in\{\{1\},\{3\},\{1,3\}\} \end{cases} \tag{32}$$

Now, let us first suppose $g^{EA}=\{1\}$. Then, we have $g^{AE}=\{2\}$. In this case, a necessary condition for (31) to hold is $G^{AAF}=\{1\}$, but then (32) cannot hold.

Similarly, let us suppose $g^{EA}=\{1,3\}$. Then, we have $g^{AE}=\{2,3\}$. Now, $g^{AA}\in\{\{1\},\{1,2\}\}$ is necessary for (31) to hold. If $g^{AA}=\{1\}$ we are in the previous situation, therefore no equilibrium exists. Let us consider $g^{AA}=\{1,2\}$. Then, a necessary condition for (31) to hold is $g^{EE}\in\{\{2\},\{3\},\{2,3\}\}$, and a necessary condition for (32) to hold is $g^{EE}\in\{\{1\},\{3\},\{1,3\}\}$. Note that if $g^{EE}=\{3\}$, conditions (29) and (30) hold. Hence, there is an equilibrium in this case.

Last, let us suppose $g^{EA}=\{3\}$. Then, we have $g^{AE}=\{3\}$. Here we have three cases: (a) consider $g^{AA}=\{1\}$. A necessary condition for (32) to hold is $g^{EE}\in\{\{1\},\{3\},\{1,3\}\}$, and a necessary condition for (29) to hold is either $g^{EE}\in\{\{1,2\},\{1,3\}\}$ and $P_{e|e}\geq 2P_{a|e}$, or $g^{EE}=\{1\}$. Note that if $g^{EE}\in\{\{1\},\{1,3\}\}$, conditions (30) and (31) hold. Hence, if either $g^{EE}=\{1,3\}$ and $P_{e|e}\geq 2P_{a|e}$, or $g^{EE}=\{1\}$, there is an equilibrium of the type conjectured. (b) Analogously to the previous case, there is an equilibrium in which $g^{AA}=\{2\}$ and either $g^{EE}=\{2,3\}$ and $P_{e|e}\geq 2P_{a|e}$, or $g^{EE}=\{2\}$. (c) Last, consider $g^{AA}=\{1,2\}$. A necessary condition for (29) to hold is $g^{EE}\in\{\{1\},\{1,2\},\{1,3\}\}$, and a necessary condition for (30) to hold $g^{EE}\in\{\{2\},\{1,2\},\{2,3\}\}$. Note that if $g^{EE}=\{1,2\}$, conditions (31) and (32) hold. Hence, there is an equilibrium in this case. This completes the proof. □

References

Besley T (2006) Principled agents? Political economy of good goverment. Oxford University Press, Oxford

Felgenhauer M (2012) Revealing information in electoral competition. Public Choice 153(1–2):55–68

Harrington JE (1993) Economic policy, economic performance, and elections. Am Econ Rev 83(1):27–42

Heidhues P, Lagerlöf J (2003) Hiding information in electoral competition. Games Econ Behav 42(1):48–74

Kartik N, Squintani F, Tinn K (2015) Information revelation and pandering in elections. Columbia University, New York

Laslier J-F, der Straeten KV (2004) Electoral competition under imperfect information. Econ Theory 24(2):419–446

Laver M, Hunt WB (1992) Policy and party competition. Routledge, London

Loertscher S (2012) Location choice and information transmission. University of Melbourne, Melbourne 1137

Martinelli C (2001) Elections with privately informed parties and voters. Public Choice 108(1–2):147–167

Morelli M (2004) Party formation and policy outcomes under different electoral systems. Rev Econ Stud 71(3):829–853

Morelli M, Weelden RV (2013) Ideology and information in policymaking. J Theor Politics 25(3):412–439

Roemer JE (1994) The strategic role of party ideology when voters are uncertain about how the economy works. Am Political Sci Rev 88(2):327–335

Schultz C (1995) The politics of persuasion when voters are rational. Scand J Econ 97(3):357–368

Schultz C (1996) Polarization and inefficient policies. Rev Econ Stud 63(2):331–343

Schultz C (2002) Policy biases with voters' uncertainty about the economy and the government. Eur Econ Rev 46(3):487–506