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Multiobjective optimization of a gas pipeline network: an ant colony approach

Adarsh Kumar Arya[1] · Shrihari Honwad[1]

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Abstract
Optimization problems in gas pipeline transportation involve numerous variables, multiple objectives and many complex linear–nonlinear equality and inequality constraints. The optimization techniques used for solving multiobjective gas transportation problems are essentially different as those for single-objective optimization. Discovering the Pareto front and non-dominated set of solutions for the nonlinear multiobjective pipeline problem obliges noteworthy registering exertion. In the present paper, for solving multiobjective gas pipeline transportation problem, a multiobjective ant colony optimization technique for pipeline optimization has been developed. The multiobjective problem considered is about minimizing fuel consumption in compressors and maximizing throughput. For validation of the technique used, it has been applied on some test problems reported in the literature. After validation, the technique has then been implemented in the gas pipeline transportation problem. An eighteen-node gas pipeline network has been taken for analysis. The result obtained supports the industrial practice of maximizing throughput at the cost of an increase in fuel consumption in compressors. The technique employed and the results obtained may be used by the pipeline operators and managers to develop strategies for improving the operating conditions of gas pipeline network.

Keywords Multiobjective ant colony optimization · Gas transmission system · Pipeline simulation and optimization · Natural gas consumption in compressors

List of symbols

A_c	Compressor arc
d_e	Density of gas (kg/m^3)
C_p	Specific heat at constant pressure
D_i	Inside diameter of pipeline
e	Absolute roughness (mm)
D_t	Outside diameter of pipeline
E_f	Efficiency
F_s	Factor of safety
f	Friction factor
G	Gas gravity
η_{dr}	Driver efficiency
η_{is}	Isentropic efficiency
η_{me}	Mechanical efficiency
η_p	Polytropic exponent
ω	Rotational speed
P	Pressure (bar)
P_b	Base pressure (bar)
P_c	Critical pressure (bar)
P_s	Suction pressure
P_d	Discharge pressure
P_{sd}	Average pressure
q_{NG}	Volumetric flow rate of natural gas (m^3/s)
me	Methane
z	Compressibility factor
h	Isentropic head (kJ/kg)
H_m	Heat content (J/kg)
C_v	Specific heat at constant volume
k	Isentropic exponent
R	Gas constant—0.08314 (m^3 bar/kmol K)
L	Length (m)
m	Mass flow rate of gas in pipe arc (kg/s)
m_f	Fuel consumed (kg/s)
M_{NG}	Molecular weight of natural gas (kg/mol)
MAOP	Maximum allowable operating pressure (bar)

✉ Adarsh Kumar Arya
adarsharya79@gmail.com

Shrihari Honwad
shrihari.honwad@gmail.com

[1] Department of Chemical Engineering, College of Engineering Studies, University of Petroleum and Energy Studies, Energy Acres, Bidholi, Dehradun, Uttarakhand, India

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Q_b	Volumetric flow rate of natural gas at standard temperature and pressure (m^3/s)
R	Gas constant (m^3 kPa/kmol K)
S	Specified minimum yield stress (bar)
T_g	Gas temperature (K)
$T_{NG,C}$	Critical temperature of natural gas
T_{de}	Temperature de-ration factor
T_b	Base temperature
t_i	Thickness of pipeline
$V_{NG,i}$	Velocity of natural gas
V_{er}	Erosional velocity of gas
LHV	Lower heating value (kJ/kg)
et	Ethane
pr	Propane

Introduction

Natural gas is an important non-renewable energy source. It is used as a fuel source in domestic cooking, in industrial processes as raw material, for generating electricity in power plants and in transport vehicles. To transport gas to consumers requires a gas pipeline network that includes several pipes of same or different diameters, valves and compressor stations. When gas is transported through the pipeline network, the pressure of the gas is lost due to friction of gas molecules with pipe wall and irregular elevation terrain. Normally, 3% of the total gas supplied through the pipeline network is consumed in natural gas-run turbine compressors (Carter 1996). Pipelines are used for transporting huge amounts of natural gas, and therefore, this loss is immense. A very small percentage of fuel savings in turbines will result in huge currency savings. Upgrading pipeline transport performance by reducing consumption of natural gas in turbines used for running compressors is henceforth a critical issue (Arya and Honwad 2015). The other extremely intriguing issue is a multiobjective problem of minimizing fuel usage in turbo compressors and maximizing the gas delivery at the delivery station. The paper addresses the multiobjective optimization model for achieving the maximum throughput at minimum fuel consumption.

Pipeline optimization methods

There has been tremendous search in the field of gas pipeline optimization, but still a strong potential exists in this area. Traditional techniques such as gradient search techniques (Rozer 2003; Tabkhi 2007), dynamic programming techniques (Carter 1998), and heuristic methods (Ríos-Mercado et al. 2006; Conrado and Rozer 2005) have been quite popular. In most of these cases, the techniques used have a noteworthy downside of getting easily caught in neighbourhood optima. Hence, they are ineffective at reaching the global optimal solution because of zero-gradient optimality criteria. Also, these methods lack flexibility in handling discrete design variables and are incompetent in optimizing a partial gas pipeline network that is often required by the pipeline operator. To overcome these drawbacks, evolutionary methods are presently getting into the mainstream. Evolutionary algorithm is probabilistic search techniques that emulate the common natural advancement or the communal conduct of natural species. Some of the popular stochastic methods include: differential evolution (Qin et al. 2009), simulated annealing (Wright and Somani 1998), ant colony (Chebouba et al. 2009), genetic algorithm (Goldberg and Kuo 1985) and particle swarm optimization (Xia et al. 2014). These methods have the advantage that they do not require any gradient information and search for an optimal solution is obtained by continuing to evaluate multiple solution vectors simultaneously. However, amongst the various evolutionary methods, the application of ant colony in optimizing gas pipeline operations has been rare. For example, the technique was successfully exploited to find the optimal pipe diameter (Mohajeri et al. 2012), lowering fuel cost in compressors (Chebouba et al. 2009) and reducing fuel expended in compressors for a fixed throughput (Arya and Honwad 2015). However, these applications are limited only to a single-objective optimization. In the recent years, the application of evolutionary methods for multiobjective optimization is becoming popular. For example, global optimization technique that utilizes the concept of interval analysis and constraint propagation has been utilized for solving multiobjective problem of minimizing energy cost and minimizing net revenue (Bonnans and Spiers 2011). Genetic algorithm has been used for minimizing fuel cost and maximizing line pack volume (Chebouba 2015). However, there has been no application of ant colony for solving multiobjective problem of gas pipeline transportation network. The purpose of this paper is to develop and apply multiobjective ant colony optimization technique to solve multiobjective pipeline problem of minimizing fuel consumption and maximizing throughput of the gas pipeline network.

Review of pipeline optimization parameters

The majority of the gas transmission system improvement issues concentrates on either design or operation phase. A number of different parameters can be optimized to achieve the operational benefits. Some of the key problems handled in the literature are as follows:

1. Maximize the gas flow rate at the delivery station (Amir and Reza 2014).

2. Determine the optimal configuration of the pipelines and the location of the compressor stations (Edgar and Himmelblau 2001).
3. Minimize power consumption per unit amount of gas in compressors (Bakhouya and De Wolf 2007).
4. Minimizing the sum of investment and operating costs (De Wolf and Smeers 2012).
5. Minimize number of operating compressors in gas pipeline network (Uraikul et al. 2004).
6. Minimize electric power consumption (Munoz et al. 2003).
7. Minimize fuel cost of compressors (Summing et al. 2000).
8. Minimize gas consumption in turbines (Arya and Honwad 2015).
9. Maximize gas network net profit (De Wolf and Smeers 2000).
10. Maximize line pack of the gas pipeline (Amir and Reza 2014).
11. Hydrogen injection maximization in gas pipeline network (Tabkhi 2008).
12. Optimal sizing of gas pipeline networks (Boyd et al. 1994).

Some of the above issues can be combined and solved using multiobjective optimization techniques. The ant colony optimization technique has been rarely used for multiobjective pipeline optimization, but can be effectively utilized to solve these multiobjective issues. The paper explains the multiobjective ant colony optimization (MOACO) technique to optimize the pipeline network. The bi-objective chosen is to minimize fuel consumption in compressors and maximize throughput of the gas pipeline network.

Case study: gas pipeline network connects single gas supply source and single delivery station

The multiobjective problem of minimizing fuel consumption in turbines and maximizing throughput while complying with the transport commitments which would be the

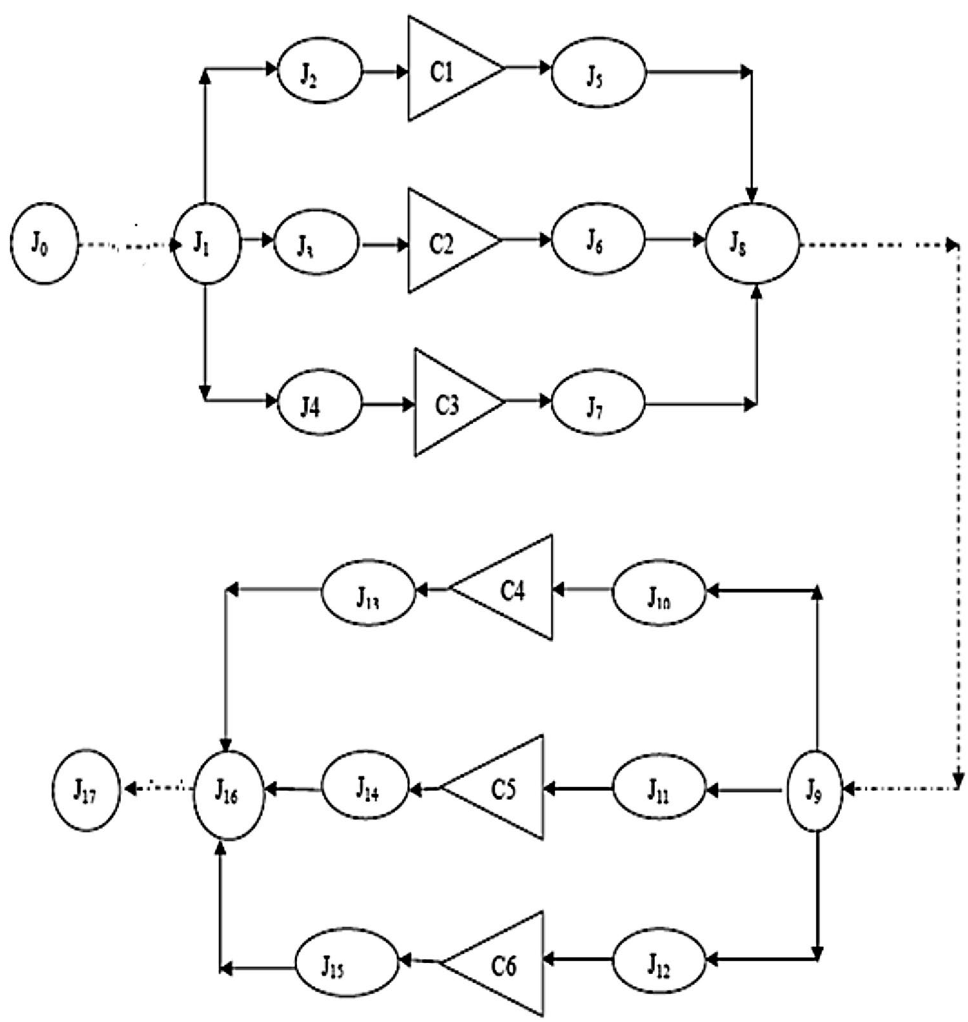

Fig. 1 Gas pipeline network (Tabkhi 2007)

constraints has been considered. The gas pipeline network used as a part of prior work (Arya and Honwad 2015) has been selected for analysis. The network is shown in Fig. 1. The gas pipeline network considered has three long pipelines. These pipelines connect the gas source station to the gas delivery station. Two compressor stations, each including three parallel compressors, are used for providing energy to the gas. Different parameters of the pipeline are reported in Table 1. Source pressure and delivery pressure both vary in a very narrow of 58.8–61.2 bars. In the pipeline network, there are totally forty-five variables that include eighteen for pressure at pipe nodes, fifteen for mass flow rate in pipe arcs, six for compressor speed and six for fuel consumption in compressors. The various variables on the compressor are listed in Table 2.

Problem formulation

This section presents the equations used in modelling of gas pipeline network. The following assumptions are used while modelling a steady-state gas pipeline network (Balachandran 2006):

1. *Isothermal gas flow* The temperature of the gas along the entire pipeline is assumed to be constant.
2. *Steady gas flow* The gas is assumed to be flowing in steady-state conditions, that is, fluid flow properties such as pressure and velocity at a particular location do not vary with time. In a real scenario, the presence of turbulence and eddies can make the flow somewhat unsteady. However, if the flow rate of gas at a particular location is constant, the steady-state assumption deviates only little from real situations.
3. *One-dimensional axial flow* The flow is assumed to be one-dimensional axial flow, that is, the variation in flow properties takes place in axial directions only. In all the other directions, the flow property change takes place very slowly and is therefore neglected.
4. *Gas is thermally and calorically perfect* In the present case, it has been assumed that the gas is thermally perfect, that is, internal energy and enthalpy are functions of temperature only. The gas is also assumed to be calorically perfect, that is, specific heats (C_{p} and C_{v}) are independent of temperature.

Objective function

Objective Function 1 $(O.F)_1$ Minimizing fuel consumption in compressors.

$$(\mathrm{O.F})_1 = \min g(m_i, P_{\mathrm{s}}, P_{\mathrm{d}}) = \min \sum m_{\mathrm{f}} = \sum_{i,j\in A_{\mathrm{c}}} \left(\frac{m_j \times h_{ij}}{\mathrm{LHV}_{\mathrm{NG}}}\right) \times \left(\frac{10^6}{\eta_{\mathrm{is}} \times \eta_{\mathrm{dr}} \times \eta_{\mathrm{me}}}\right) \tag{1a}$$

Objective Function 2 $(O.F)_2$ Maximizing throughput at the delivery station.

$$(\mathrm{O.F})_2 = \max(\mathrm{throughput}) \tag{1b}$$

Both the objective functions, given in Eqs. (1a) and (1b) are liable to transport commitments, defined by *equality and inequality constraints.* The following section first looks at the equations used for calculating the natural gas properties (Eqs. 2–11) and further the constraints (Eqs. 14–18 and Table 3) imposed on the objective functions.

Equations for natural gas property calculation

The natural gas moving in the pipeline is considered to be a mixture of methane (70%), ethane (25%) and propane (5%). The property of natural gas depends on the property of its constituent gases. Kay's rule has been used for calculating the natural gas properties (Smith and Van Ness 1998). Equation (2) is used for calculating the molecular weight of the gas mixture, Eqs. (3, 4) for calculating the critical temperature and pressure of the gas, Eq. (5) for calculating heat content of the natural gas mixture and Eq. (6) for calculating the isentropic exponent

Table 1 Dimensions of gas pipeline network

Pipe arcs (nodes adjoined)	Corresponding length of pipe segment (m)	Corresponding diameter (m)
$A_1(J_0 - J_1), A_2(J_1 - J_2), A_3(J_1 - J_3), A_4(J_1 - J_4)$ $A_5(J_5 - J_8), A_6(J_6 - J_8), A_7(J_7 - J_8), A_8(J_8 - J_9),$ $A_9(J_9 - J_{10}), A_{10}(J_9 - J_{11}), A_{11}(J_9 - J_{12}), A_{12}(J_{13} - J_{16}),$ $A_{13}(J_{14} - J_{16}), A_{14}(J_{15} - J_{16}), A_{15}(J_{16} - J_{17})$	10^5, 200, 300, 100, 200, 100, 200, 10^5, 10^2, 10^2, 10^2, 10^2, 400, 10^2, 10^5	0.787, 0.330, 0.381, 0.330, 0.330, 0.330, 0.330, 0.838, 0.381, 0.330, 0.432, 0.330, 0.330, 0.330, 0.889

Table 2 Compressor notation

Compressor no.	Corresponding compression ratio	Variables at the compressor
$C_{01}, C_{02}, C_{03}, C_{04}, C_{05}, C_{06}$	${P_5}/{P_2}, {P_6}/{P_3}, {P_7}/{P_4}, {P_{13}}/{P_{10}}, {P_{14}}/{P_{11}}, {P_{15}}/{P_{12}}$	$\omega_{i=1,\ldots,6}, m_{f_{i=1,\ldots,6}}$

Table 3 Mass balance equations

Junction	Mass equalization equations applied on junction points
J_1, C_{01}, C_{02}	$m_{n01} - (m_{n02} + m_{n03} + m_{n04}) = 0; m_{n02} - (m_{nc_{01}} + m_{n05}) = 0; m_{n03} - (m_{nc_{02}} + m_{n06}) = 0$
C_{03}, J_8	$m_{n04} - (m_{nc_{03}} + m_{n07}) = 0; m_{n08} - (m_{n05} + m_{06} + m_{n07}) = 0$
J_9, C_{04}, C_{05}	$m_{n08} - (m_{n09} + m_{10} + m_{n11}) = 0; m_{n09} - (m_{nc_{04}} + m_{n12}) = 0; m_{n10} - (m_{nc_{05}} + m_{n13}) = 0; m_{n09} - (m_{nc_{04}} + m_{n12}) = 0$
C_{06}, J_{16}	$m_{n11} - (m_{nc_{06}} + m_{n14}) = 0; m_{n15} - (m_{n12} + m_{n13} + m_{n14}) = 0$

$$M_{\mathrm{NG}} = M_{\mathrm{me}} \times y_{\mathrm{me}} + M_{\mathrm{et}} \times y_{\mathrm{et}} + M_{\mathrm{pr}} \times y_{\mathrm{pr}} \tag{2}$$

$$T_{\mathrm{NG,C}} = T_{\mathrm{me,C}} \times y_{\mathrm{me}} + T_{\mathrm{et,C}} \times y_{\mathrm{et}} + T_{\mathrm{pr,C}} \times y_{\mathrm{pr}} \tag{3}$$

$$P_{\mathrm{NG,C}} = P_{\mathrm{me,C}} \times y_{\mathrm{me}} + P_{\mathrm{et,C}} \times y_{\mathrm{et}} + P_{\mathrm{pr,C}} \times y_{\mathrm{pr}} \tag{4}$$

$$\begin{aligned}(\mathrm{LHV})_{\mathrm{NG}} =& \left(\mathrm{LHV}_{\mathrm{me}} \times y_{\mathrm{me}} \times M_{\mathrm{me}}\right) \\ &+ \left(\mathrm{LHV}_{\mathrm{et}} \times y_{\mathrm{et}} \times M_{\mathrm{et}}\right) \\ &+ \left(\mathrm{LHV}_{\mathrm{pr}} \times y_{\mathrm{pr}} \times M_{\mathrm{pr}}\right)\end{aligned} \tag{5}$$

$$k = \frac{(C_{\mathrm{p,me}} \times y_{\mathrm{me}} + C_{\mathrm{p,et}} \times y_{\mathrm{et}} + C_{\mathrm{p,pr}} \times y_{\mathrm{pr}})}{(C_{\mathrm{p,me}} \times y_{\mathrm{me}} + C_{\mathrm{p,et}} \times y_{\mathrm{et}} + C_{\mathrm{p,pr}} \times y_{\mathrm{pr}}) - R}. \tag{6}$$

Pipeline and compressor modelling

The equations presented in this section are applicable to each of the fifteen pipe arcs as well as to each of the six compressors.

Pressure drop in pipeline

The general flow equation (Eq. 7) obtained by applying energy balance on the pipe segment is used to obtain a pressure drop of gas in the pipeline (Menon 2005)

$$\begin{aligned}P_{\mathrm{s}}^2 - P_{\mathrm{d}}^2 =& \left(\frac{16 \times m_i^2 \times z_i \times R \times T}{\pi^2 \times D_i^4 \times M}\right) \\ &\times \left[\left(2 \times \log_{10}\left(\frac{P_{\mathrm{s}}}{P_{\mathrm{d}}}\right) - \left(\frac{f_i \times L_i}{D_i}\right)\right)\right].\end{aligned} \tag{7}$$

Average pressure in pipeline segment

Equation 8 is used to calculate the average pressure of gas in the pipe segment (Mohring et al. 2004).

$$P_{\mathrm{sd}} = \left(\frac{2}{3}\right) \times \left[P_{\mathrm{s}} + P_{\mathrm{d}} - \frac{P_{\mathrm{s}} \times P_{\mathrm{d}}}{P_{\mathrm{s}} + P_{\mathrm{d}}}\right]. \tag{8}$$

Friction factor

Colebrook White, Modified Colebrook White and AGA (American Gas Association) equations are used to calculate the friction factor in gas pipelines (Menon 2005; Abdolahia et al. 2007). However, for fully rough pipelines, where Reynolds number is very high, Colebrook equation, given in Eq. (9), is the most recommended equation

$$f_i = -2 \log_{10}\left(\frac{e}{3.71 \times D_i}\right)^{-2}. \tag{9}$$

Gas velocity

The velocity of gas moving in the pipeline is obtained from Eq. (10) (Menon 2005)

$$\begin{aligned}v_{\mathrm{NG},i} =& 14.7359 \times \left(\frac{q_{\mathrm{NG,base}} \times 24 \times 3600}{\left(D_o \times 10^3 - 2 \times t_i \times 10^3\right)^2}\right) \\ &\times \left(\frac{P_{\mathrm{base}}}{T_{\mathrm{base}}}\right) \times \left(\frac{z_i \times T_{\mathrm{g}}}{P_{\mathrm{sd}} \times 10^2}\right).\end{aligned} \tag{10}$$

Compressibility factor

Numerous equations such as Standing–Katz method, AGA and CNGA (California Natural Gas Association) method (Menon 2005), Dranchuk method (Dranchuk and Abou-Kassem 1975) are used to calculate the compressibility factor of gas. However, due to the preciseness and quick estimation, CNGA is the most preferred equation and is used in the present work (Eq. 11)

$$z_i = 1 + \left(0.257 - 0.533 \times \frac{T_{\mathrm{NG,C}}}{T_{\mathrm{g}}}\right) \times \frac{P_{\mathrm{sd}}}{P_{\mathrm{C,NG}}}. \tag{11}$$

Isentropic head

Head is the amount of energy added per unit amount of gas compressed in compressors. Equation 12 is used for calculating isentropic head (Smith and Van Ness 1998)

$$h_{ij} = \left(\frac{z_i \times R \times T}{M}\right) \times \left(\frac{k}{k-1}\right) \times \left[\left(\frac{P_{\mathrm{d}}}{P_{\mathrm{s}}}\right)^{\frac{k-1}{k}} - 1\right]. \tag{12}$$

Isentropic efficiency

Isentropic efficiency is used to compare the actual performance of a turbine and the performance that would have been achieved under ideal conditions for the same inlet and exit conditions. Equation 13 is used for calculating the isentropic efficiency of compressors (Smith and Van Ness 1998)

$$\eta = \frac{\left(\frac{P_d}{P_s}\right)^{\frac{k-1}{k}} - 1}{\left(\frac{P_d}{P_s}\right)^{\frac{n_p-1}{n_p}} - 1}. \tag{13}$$

Pipeline and compressor constraints

The constraints define the bounds on the pipeline as well as compressors. These constraints are now discussed.

Pipeline constraints

Erosional velocity

When gas moves in the pipeline, its pressure and density decrease. According to continuity equation, the mass of gas remains conserved; hence, the decrease in density causes an increase in gas velocity. However, this increase in gas velocity has to be kept lower than the erosional velocity (upper limit of velocity) to allow for corrosion inhibition, minimize noise and vibration of pipelines (Eq. 14). Velocity of gas is generally kept less than 50% of erosional velocity (Eq. 15) (Mohitpour et al. 2003).

$$v_{er} = 122\sqrt{\frac{z_i \times R \times T}{P_{sd} \times M}} \tag{14}$$

$$v \leq 0.5 * v_{er} \tag{15}$$

A minimum gas velocity of 3–4 m/s (lower limit of velocity) is to be kept in pipeline to minimize the liquid fall out (Petrowiki 2015).

Maximum allowable operating pressure (MAOP)

MAOP refers to the highest pressure under which a piece of equipment or pipeline can be operated safely. The case study taken is of a 'high-pressure cross-country pipeline network'. In a high-pressure gas pipeline network, the pressure always remains above 1 bar (lower limit of pressure) and always below the MAOP (upper limit of pressure). Equation 16 is used for fixing up the lower and upper limits of pressures (Menon 2005).

$$1 \prec P_i \prec \text{MAOP} \tag{16}$$

where

$$(\text{MOP}) = \left(\frac{2 * t_i * S * E_f * F_s * T_{de}}{D_i}\right)$$

Table 4 shows the numerical values of the lower limit (LL) and upper limit (UL) of velocity and pressure.

Mass balance equations

The general mass balance equation is given by the following equation:

Input = Output + Consumption + Accumulation

For the process to be at steady state, the following equation is considered:

Input = Output + Consumption

Table 4 Upper and lower limits of pressure and velocity in pipe arc

S. no.	D (m)	t (m)	S (bar)	P (UL)-MAOP	P (LL)	M	V_{er} (UL)	V (LL)
1	0.787	0.00635	6205.284	57.67839	40	20.95	17.4399198	3
2	0.33	0.004775	3861.0656	64.36045	40	20.95	16.5097883	3
3	0.381	0.004191	3861.0656	48.92742	40	20.95	18.9354173	3
4	0.33	0.004775	3861.0656	64.36045	40	20.95	16.5097883	3
5	0.33	0.004775	3861.0656	64.36045	40	20.95	16.5097883	3
6	0.33	0.004775	3861.0656	64.36045	40	20.95	16.5097883	3
7	0.33	0.004775	3861.0656	64.36045	40	20.95	16.5097883	3
8	0.838	0.00635	6205.2812	54.16811	40	20.95	17.9961343	3
9	0.381	0.004775	3861.0656	55.74527	40	20.95	17.7397307	3
10	0.33	0.004775	3861.0656	64.36045	40	20.95	16.5097883	3
11	0.432	0.005334	3861.0656	54.9198	40	20.95	17.8725528	3
12	0.33	0.004775	3861.0656	64.36045	40	20.95	16.5097883	3
13	0.33	0.004775	3861.0656	64.36045	40	20.95	16.5097883	3
14	0.33	0.004775	3861.0656	64.36045	40	20.95	16.5097883	3
15	0.889	0.00635	6205.284	51.06062	40	20.95	18.5356576	3

The equation has been applied on each of the ten junction points. The mass balance equations are presented in Table 3. The upper and lower limits for flow rate were taken as 200 and 100 kg/s. These values were initially taken randomly since the throughput of the gas varies in a much narrower range than these bounds. This is also shown in Fig. 6. A wider range of flow rate can be also taken, but it will unnecessarily take more time to obtain Pareto points. (This is because a wider search space takes more time to converge.)

Compressor constraints

Apart from constraints applied to the pipeline, the constraints have been also imposed on compressors. Two constraints have been applied to each of the compressors. One is the constraint on compressor speed and the other one on the mass of gas achieved at the entrance node of the compressor. Equations 17 (Tabkhi 2007) and 18 (Odom 1990) are used for calculating these constraints. These constraints have to be respected to avoid choking in the pipeline.

$$166.7 < \omega_i < 450 \tag{17}$$

$$q_{i,\mathrm{Co}} \le \left(\frac{\pi}{4} \times D_i^2\right) \times \left(\sqrt{\frac{k \times z_i \times R \times T}{M}}\right) \times \left(\frac{2}{k+1}\right)^{\frac{k+1}{2\times(k-1)}} \tag{18}$$

Constraints defined by Eqs. 17 and 18 are applied to each of the compressors.

The multiobjective problem taken is to minimize the fuel consumption (Eq. 1a) and maximize the throughput (Eq. 1b).

Both of the objectives are liable to various equality and inequality constraints (Eqs. 12–18 and Table 3). Equations 2–11 are used for evaluating the properties of natural gas. The multiobjective ant colony optimization (MOACO) technique has been utilized to obtain the optimal solutions.

Multiobjective ant colony optimization for solving multiobjective pipeline problems

Dorigo (1992), observing the conduct of the natural ants, created a meta-heuristic strategy to solve real-world optimization problems that are prominently known as an 'ant colony optimization strategy'. Numerous ant species such as 'Linepithema Humile' lay down a chemical pheromone trail while moving from its food source to the nest. As ants are visually impaired, the pheromone trail laid by the past ants goes about as a motivation for the new ants, to locate the same food source once more. Pheromone tends to vanish with time; consequently, the shorter path has more stigmergy when compared with the more drawn out paths. So an extensive number of ants are roused to pick the shortest path. This fundamental theme of natural ants has been used as a motivation to create counterfeit artificial ants that tackle the real-life optimization issues, which are exceptionally difficult to comprehend and are made in a similar way. Much the same as the dissipation procedure of pheromone, artificial ants locate the global solutions by constantly upgrading the target objective. Taking inspiration from the original technique, many analysts adapted the algorithm and presented an extended version of the ant colony technique (Dorigo and Stützle 2004; Schlueter 2012). The present work concentrates on one such recently developed technique (Schlueter 2012).

The Multiobjective ACO technique works in the following five main steps:

1. Convert multiobjective problem to a single-objective problem.
2. Create solution matrix.
3. Initialize the matrix with a random guess of solutions in the solution matrix.
4. Probabilistically search for new solutions.
5. Update the solutions using local or gradient search methods.

Convert multiobjective problem to a single-objective problem

The two objectives are to be combined and converted to a single-objective function using adaptive weighted sum methods (Kim and De Weck 2004). The following equation gives the methodology of combining the two functions.

$$F_{\mathrm{Total}}^{f} = \alpha_{f-1} F_{\mathrm{Total}}^{f-1} + (1 - \alpha_{f-1}) F_f \tag{19}$$

Here $F_{\mathrm{Total}}^{1} = F_1$

Here f is the number of objectives, F is the objective function and α is the relative weight of the objective function.

Create solution matrix

Artificial ants keep a history of the past solutions found in the solution matrix. For an optimization problem having 'n' number of variables and k solutions, the solution matrix (SM) keeps 'n', variables, (v_1^1, v_1^2, … , v_1^n) and associated k objective function values ($f(v_1)$, $f(v_2)$, … ,$f(v_k)$). Figure 2 shows a solution matrix.

Initialize the matrix with a random guess of solutions in the solution matrix

In the first step, the matrix is initialized with k random solutions. In each iteration, l solutions are generated. Then from

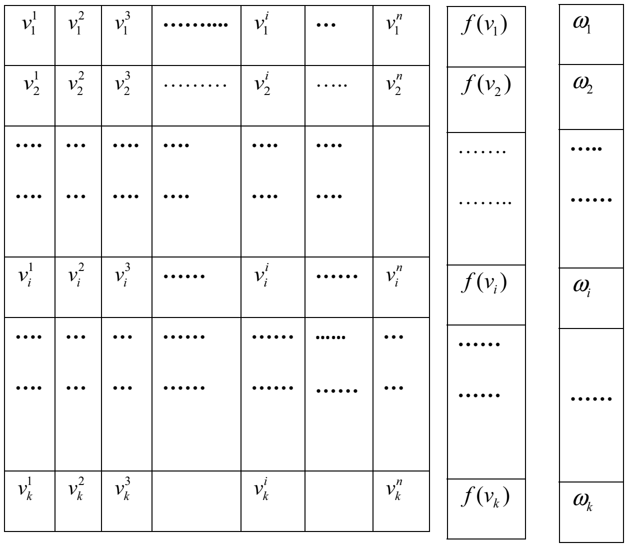

Fig. 2 Solution matrix created by ACO. The solutions are to be kept in the order of their weights ($\omega_1 \geq \omega_2 \geq \cdots \omega_i \geq \cdots \omega_i$), from top to bottom. For minimization problem, $f(v_1) \leq f(v_2) \leq \cdots f(v_2) \cdots \leq f(v_k)$

$(k + l)$ set of solutions, the fittest k solutions are kept in the matrix, while l solutions are discarded. These k solutions are sorted and kept in the matrix depending on their weight w_k. Comparatively, better solutions are kept at higher positions, while the bad ones are at kept at the lowest position in the solution matrix. For minimization problems, the best solutions are those that have lower objective function values.

Probabilistically searching for new solutions

Initially the solutions of the objective function are randomly initiated. But from the next iteration, the solutions are sampled through the incremental constructional methodology using a probabilistic function. In principle, any function $P(x) \geq 0$ is called a probability density function if it satisfies the following property:

$$\int_{-\infty}^{+\infty} P(x)\mathrm{d}x = 1 \tag{20}$$

The present paper uses a recently developed weighted Gaussian probability distribution function given in Eq. (21).

$$P^i(x) = \sum_{l=1}^{k} w_l p_l^i(x) = \sum_{l=1}^{k} w_l \left(\frac{1}{\sigma_l^i \sqrt{2\pi}} \right) \exp\left(\frac{-\left(x - \mu_l^i\right)^2}{2\sigma_l^{i^2}} \right) \tag{21}$$

The above equation is characterized by three parameters: (w, μ, σ). These three parameters are referred to as the weight of the solution, mean of the corresponding Gaussian function and standard deviation. These three parameters similar to the pheromone laid by the real ants are used to guide the ants to find the new solution in the search domain.

The weight of a solution is used for determining the attractiveness of a solution to be chosen in the next evolution and is given by Eq. (22) (Schlueter 2012)

$$w_j = \frac{1}{qk\sqrt{2\pi}} \exp\left(\frac{-(\mathrm{rank}(j) - 1)^2}{2q^2k^2} \right) \tag{22}$$

In the equation, the term rank(j) is given the rank of the solution. The term $(\mathrm{rank}(j) - 1)$ corresponds to the mean obtained by setting Gaussian function that is equal to 1. q is a parameter in the ant colony algorithm.

The probability of choosing a particular solution j amongst various available solutions is obtained from the equation:

$$p_j = \frac{w_j}{\sum_{l=1}^{k} w_l} \tag{23}$$

The mean μ is given by the solutions stored in the solution matrix.

$$\mu_l^i = v_l^i \tag{24}$$

The standard deviation σ is obtained from the following relation:

$$\sigma = \xi \sum_{l=1}^{k} \frac{\left| d_l^i - d_j^i \right|}{k - 1} \tag{25}$$

ξ factor decides the convergence speed of the algorithm. The higher the value of ξ, the lower is the convergence speed.

Updating the solution

Depending on the new values of weight, mean and standard deviation, the solution matrix table is continuously updated.

Multiobjective test functions

For checking the validity of the proposed technique, three popular test functions, ZDT1, ZDT2 and ZDT3, are chosen from the literature (Deb 2010; Schlueter 2012; Zitzler et al. 2000). ZDT1 function is given in Eqs. 26–28, ZDT2 function in Eqs. 29–31 and ZDT3 function in Eqs. 32–34.

1. ZDT1 function

   Number of variables $n = 4$

$$f_1(x) = x_1; \tag{26}$$

$$f_2(x) = y * t; \tag{27}$$

$$y = 1 + \frac{9 * \sum_{i=2}^{n} x_i}{(n-1)};$$

$$t = 1 - \sqrt{\frac{f_1(x)}{y}}; \tag{28}$$

$$x_i = [0, 1], i = 1, \dots, 30$$

2. ZDT2 function
   Number of variables $n = 4$

$$f_1(x) = x_1; \tag{29}$$

$$f_2(x) = y * \left(1 - \frac{f_1(x)}{y}\right)^2 \tag{30}$$

$$y = 1 + \frac{9 * \sum_{i=2}^{n} x_i}{(n-1)}; \tag{31}$$

3. ZDT3 function
   Number of variable $n = 4$

$$f_1(x) = x_1; \tag{32}$$

$$f_2(x) = y * \left(1 - \sqrt{\frac{f_1}{y}} - \left(\frac{f_1}{y}\right) \times \sin\left(10\pi f_1\right)\right) \tag{33}$$

$$y = 1 + \frac{9 * \sum_{i=2}^{n} x_i}{(n-1)}. \tag{34}$$

Result and discussion of multiobjective optimization

Multiobjective ant colony optimization technique generates numerous solutions. Here we have chosen to keep three best solutions generated by the ants and update them in succeeding iterations. The multiobjective ant colony algorithm has been first tested on standard test functions ZDT1, ZDT2 and ZDT3 (Deb 2010). The Pareto obtained utilizing MOACO has then been compared with standard Pareto plots. The three test functions were discussed in Eqs. (26–34). The standard Pareto for all the three test functions is shown in Figs. 3a, 4a and 5a. Pareto optimal solutions obtained using multiobjective ant colony are shown in Figs. 3b, 4b and 5b.

The curve shown in Figs. 2, 3 and 4 shows the same nature of the curves for ZDT1, ZDT2 and ZDT3. The validity of multiobjective ant colony technique on ZDT functions verifies the correctness of the technique used. Table 5 shows the computational load comparison for MOACO Pareto front and standard Pareto front in terms of CPU time as well as number of iterations.

Now, after validating the technique, it has been utilized to obtain Pareto front of gas pipeline network optimization problem (Tabkhi 2007). The two objectives were combined using adaptive weighted sum method. Eleven different weights were chosen in a way that the sum of the two weights always remains equal to one. MOACO gives Pareto front in 52 h on a processor Intel Core Duo 2, 3 GHz, 2 GB of RAM.

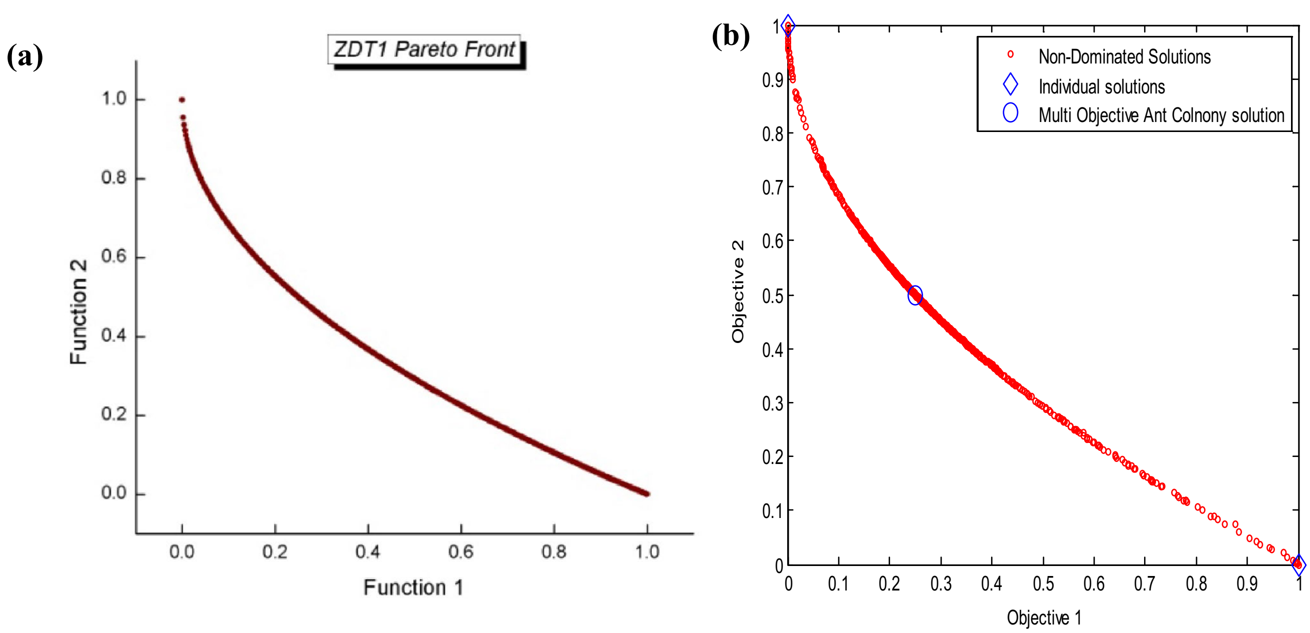

Fig. 3 Pareto front of ZDT1 **a** standard Pareto front (Deb 2010) and **b** obtained by multiobjective ant colony optimization

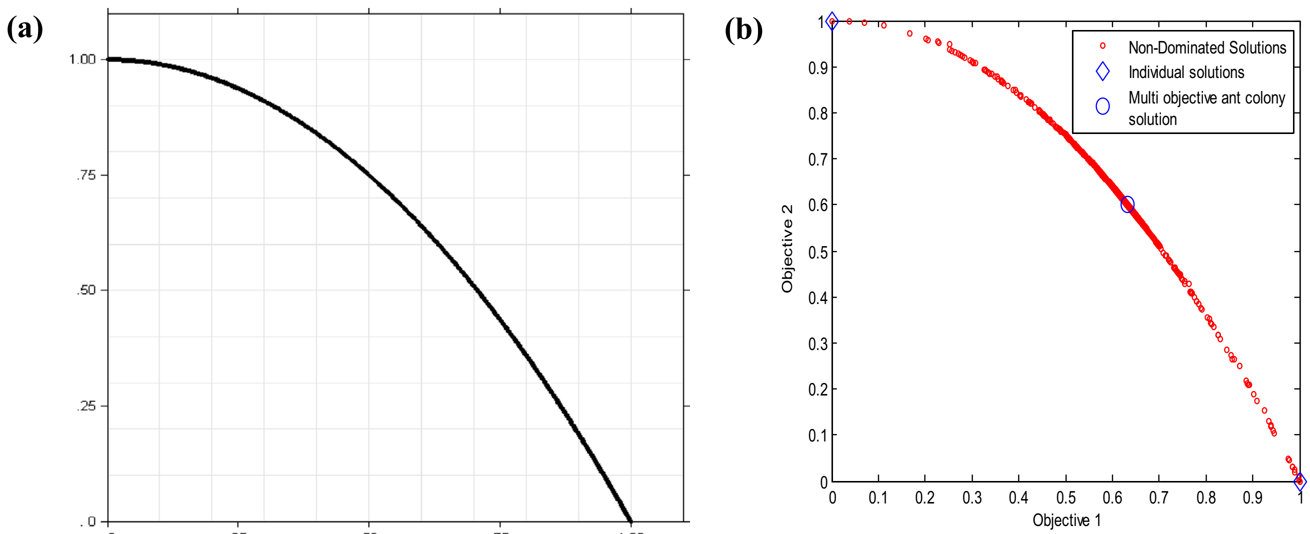

Fig. 4 Pareto front of ZDT2 **a** standard Pareto front (Deb 2010) and **b** obtained by multiobjective ant colony optimization

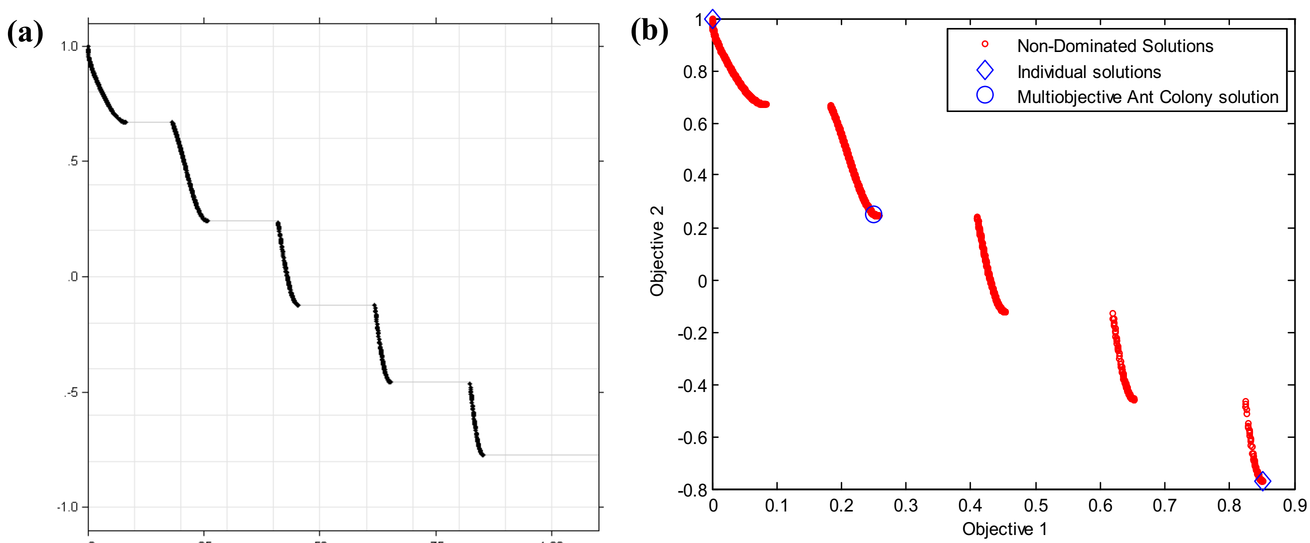

Fig. 5 Pareto front of ZDT3 **a** standard Pareto front (Deb 2010) and **b** obtained by multiobjective ant colony optimization

Table 5 Computational load comparison between the generation of standard Pareto front and MOACO approach

Type of function	Computational load in terms of CPU time (s)		Computational load in terms of iterations	
	Standard Pareto (Chase et al. 2015)	ACO	Standard Pareto (Chase et al. 2015)	ACO
ZDT1	3.85	3	10,000	9000
ZDT3	5.25	4.28	10,000	8500
ZDT3	3.93	3	10,000	9200

The interaction between the two objectives gives rise to a set of compromised Pareto optimal solutions. It can be seen that each solution on the Pareto optimal curve of Fig. 6 is not dominated by any other solution, i.e., an improvement in one objective can be obtained only by worsening the second objective. In our case, the improvement in minimizing the fuel consumptions decreases the throughput, thus worsening the second objective. This leads to a trade-off relationship between fuel consumption and throughput in which a pipeline operator can choose a preferred solution. The Pareto front obtained reveals two more important things: first, it gives the bounds on the network capacity in terms of mass flow delivery. Second, for an imposed mass flow delivery

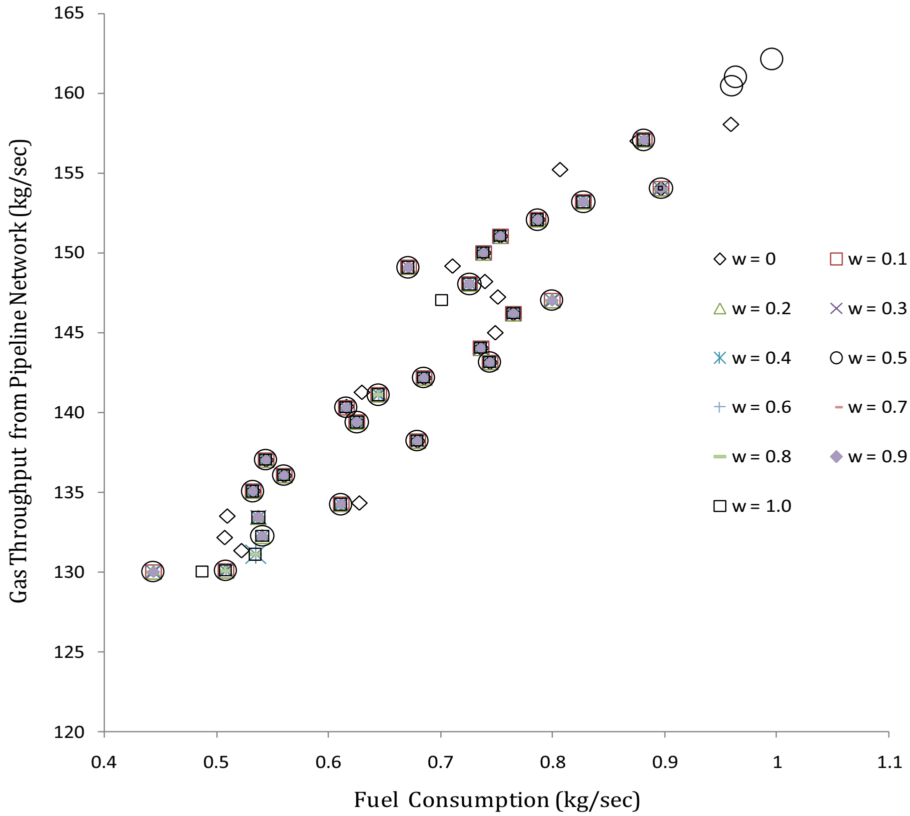

Fig. 6 Pareto front for different weights obtained using multiobjective ant colony optimization technique

that corresponds to a practical case of Natural Gas Company, the minimal fuel consumption can be obtained by tuning compressor stations (particularly output pressure and input pressure) at values provided by the optimizer.

Figure 6 shows that Pareto solutions of each weight for a fixed throughput are quite similar, since most of the points are converging. The comparison of the Pareto front of different weights reveals a crucial information, i.e., the solution obtained does not preserve one's initial preferences of choosing weights no matter how the weights were set. The solution obtained by combining the multiple objective functions to a single-objective function depends on the relative importance of the objective functions $f_1(x)$ and $f_2(x)$. So, while setting the weights for combining the objectives, only the relative importance of the objectives should be considered and not the relative magnitude of the function values. This very useful idea has been often overlooked in most of the literature available.

Conclusion

In the present paper, an analytical steady-state model that quantifies the objectives of fuel consumption in compressors and throughput of the gas pipeline network was developed. The objectives were incorporated in an MOACO Algorithm. The optimization approach seeks to optimize, the multiobjective function of minimizing fuel consumption in compressors and maximizing throughput of the gas pipeline network. Both the objectives are balanced through a proper selection of pressures at nodes and mass rate in pipe arcs. The MOACO algorithm was applied to a fifteen pipe arc pipeline system to generate Pareto optimal solutions for multiobjective function. A multicriteria approach on gas pipeline network problem (as opposed to a single-criterion optimization problem) was used that makes possible to generate several solutions from which the most appropriate one can be chosen based on additional analysis, such as involvement of decision-maker to improve the acceptance of managers and practitioners. The Pareto optimal solutions obtained for multiobjective optimization support the industrial practice of maximizing gas throughput at the cost of an increase in fuel consumption. The technique and the result obtained can be used to develop strategies in improving the operating conditions and hence performance of a gas pipeline network. The usefulness of the MOACO technique for a typical multisupply and multidelivery looped gas pipeline network needs further investigation.

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