Two-phase flow models for thermal behavior interpretation in horizontal wellbores

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Abstract The interpretation of Distributed Temperature Sensing (DTS) real-time temperature data from downhole is essential to understand wellbore production and production operations management. This paper presents a multi-phase wellbore thermal behavior prediction model for the interpretation of wellbore fluid thermal responses. Based on our previous simulation results on single-phase flow in horizontal wellbores, a two-phase flow model (η_s -driven model) is developed for steady-state conditions in the form of homogeneous and drift-flux models applied to both openhole and perforated completion types. Case studies include the examination of water entry thermal effect and gas mixing thermal effect comparing between the two modeling approaches. Results show that the phenomena of water breakthrough and gas blended in oil can be detected from fluids temperature profiles.

Keywords Horizontal wells · Temperature response · Multiphase analysis · Steady state

List of symbols

A Cross-sectional area of wellbore, L^2 , ft^2
 C_o Profile parameter, [–]
 C_h Fluid heat transfer coefficient, m/t^3-T , BTU/ft^2 hr F
 C_L Liquid heat transfer coefficient, m/t^3-T , BTU/ft^2 hr F
 C_p Isobaric heat capacity, L^2/t^2-T , BTU/lbm R
 d Wellbore diameter, L , ft
 F Force per unit volume, m/L^2-t^2 , lbf/ft^3

F_s Shape factor, [–]
 F_p Flow pattern factor, [–]
 f Friction factor, [–]
 g Acceleration factor, L/t^2 , 32.174 ft/s^2 ; apparent mass flux, m/L^2-t , lbm/ft^2 s
 g_c Unit conversion factor, 32.174 lbm ft/lbf s^2
 h Fluid specific enthalpy at wellbore conditions, L^2/t^2 , BTU/lbm
 h^* Fluid specific enthalpy at reservoir conditions, L^2/t^2 , BTU/lbm
 i Phase, [–]
 J_0 Pressure unit conversion, 144 lbf/ft^2 $psia$
 J_1 Energy unit conversion, 778.16 lbf ft $/BTU$
 J_2 Energy unit conversion, 5.4 $psia$ ft^3/BTU
 K Conductivity, m/t^2-T , BTU/hr ft F
 L Length of wellbore, L , ft
 M Mass rate, M/t , lbm/s
 Pr Prandtl number
 p Pressure, $m/L-t^2$, $psia$
 Q Conductive heat flux, m/t^3 , BTU/ft^2 hr
 q Flowrate, L^3/t , ft^3/s
 Re Reynolds number, [–]
 r Wellbore radius, L , ft
 s Fluid specific entropy, L^2/t^2-T , BTU/lbm F
 T Temperature, T , R
 T^* Reservoir temperature, T , R
 U Overall heat transfer coefficient, m/t^3-T , BTU/ft^2 hr F
 u Velocity, L/t , ft/s
 u^* Inflow fluid velocity, L/t , ft/s
 W Mass rate, m/t , lbm/s
 X Apparent volume fraction, [–]
 x Quality, [–]
 y Holdup, [–]; gas void fraction, [–]

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Greek symbols

A	Coefficient of thermal expansion, $1/T$, $1/F$
θ	Inclination angle, degree
τ	Interfacial tension, m/t^2 , dyne/cm
Γ	Radial mass influx, m/L^3-t , $lbm/ft^3 s$
η_s	Isentropic thermal coefficient, $T/(m/L-t^2)$, F/psi
γ	Pipe open ratio, $[-]$
ρ	Density, m/L^3 , lbm/ft^3

Subscript

B	Bulk
c	Continuous
D	Drift
d	Dispersion
ex	Energy exchange
f	Friction
G	Gas
g	Gravity
h	Enthalpy
ke	Kinetic energy
L	Liquid
j	Segment number
m	Mixture
O	Oil
p	Pressure
ref	Reference
s	Isentropic coefficient
so	Superficial velocity of oil
sg	Superficial velocity of gas
T	Temperature
t	Total
W	Water; wall
w	Friction

Introduction

The DTS technology has been increasingly accepted for the permanent downhole monitoring of horizontal wells given their increasing ability to register even the smallest temperature changes (less than 0.1°C) in fluid wellbore behavior (Yoshioka et al. 2005b). The temperature log performed by DTS tool helps reservoir operators to get real-time temperature data and determine how to optimize reservoir productions. The real-time temperature profiles given by DTS tools is hoped to be able to aid in the identification of different producing fluids (oil, water and gas), locations of undesired water breakthrough, gas entry, tubing leakage and productive or non-productive zones along a horizontal wellbore (Wang et al. 2010). In order to better interpret wellbore temperature data, wellbore fluid thermal behavior (temperature, pressure and velocity) reliable prediction models are critical and the analysis of the fundamental factors that lead to such behaviors

are essential to explain and identify different production conditions.

Several temperature prediction models for both vertical and horizontal flow have been proposed over the years (Ayala and Dong 2015) mainly for pipeline and tubing conditions. As for multi-phase fluids thermal models, three types of models are commonly used by the petroleum industry to model the impact of multiphase fluid on wellbores—homogeneous, drift-flux and mechanistic. The fundamental assumption in the homogeneous model is that fluids in the system are perfectly mixed so that there is no slip between each phase, hence forming a homogeneous mixture (Hasan and Kabir 2002). Treated as a single-phase fluid, the two-phase fluid is considered to have one velocity-mixture velocity. Fluid properties can be represented by mixture properties. In previous work (Yoshioka et al. 2005b), a homogeneous model has been successfully applied in industry to interpret wellbore fluid thermal behavior. The homogeneous model with slip considered between two phases is named the drift-flux model. To allow for the slip between two phases, empirical parameters are needed to estimate the volume fraction of each phase. Compared to the homogeneous model, a drift-flux model is not only capable of capturing the two-phase in situ volume fraction, but also capable of considering different flow patterns, which gives more reliable and realistic wellbore velocity, pressure and temperature profiles. Therefore, drift-flux model is used in many reservoir simulators (Shi et al. 2005). Mechanistic models are the most accurate among these three models for consideration of the detailed physics of each flow pattern. However, at some flow-pattern transitions, the mechanistic model can cause discontinuities of pressure drop or holdup, resulting in a convergence problem (Shi et al. 2005). Zuber and Findlay (1965) proposed a general drift-flux method to predict average volumetric concentration in vertical two-phase flow systems. Velocity and concentration profiles are generated by considering the effects of the relative velocity between two phases and a non-uniform flow. França and Lahey (1992) applied drift-flux techniques in horizontal air/water two-phase flows. Their model is able to predict and correct experiment data in various flow regimes. Hasan and Kabir (1999) applied a drift-flux approach and developed a simplified model for an oil–water flow in vertical and deviated wellbores. After investigating three flow patterns, a single expression for calculating drift-flux velocity was developed. Shi et al. (2005) proposed a drift-flux model of water/gas and oil/water two-phase flow in wellbores ranging from vertical to near-horizontal. The model used experimental data from a large diameter pipe to determine drift-flux parameters. Hapanowicz (2008) tested the accuracy of available drift-flux models in evaluating the slip between water and oil phases in a horizontal pipe. A

flow pattern determination method was proposed for implementing the drift-flux model. Based on previous work, Choi et al. (2012) developed a drift-flux closure relationship to estimate phase holdup in gas–liquid pipe flow. The correlation gave satisfactory prediction of phase holdup over a wide range of flow patterns and wellbore inclination conditions. Hasan and Kabir (1988b) presented a mechanistic model for multi-phase flow in vertical wells. The model is able to predict flow patterns, void fractions and pressure drops of vertical wellbore fluid. They also presented a similar mechanistic model that can be applied in deviated wells (Hasan and Kabir 1988a). Petalas and Aziz (1998) proposed a mechanistic model for multi-phase flow in pipe. New empirical correlations were developed for liquid–gas flow in different flow patterns, along with solution procedures. Ouyang and Aziz (2000) published a homogeneous model for gas–liquid flow in horizontal wells. In their pressure gradient equation, an accelerational pressure gradient caused by wall influx/outflux and fluid expansion is considered. Yoshioka et al. (2005a, b) proposed a homogeneous model for oil–water flow and drift-flux model regarding oil–gas flow in wellbore temperature prediction models.

In this study, on the basis of the single-phase η_s -driven model developed in our previous work (Ayala and Dong 2015), two-phase flow wellbore thermal models (oil–water and oil–gas) are presented in both homogeneous and drift-flux forms. A perforated wellbore completion type is implemented in the models by considering heat conduction between reservoir and wellbore fluids. Case studies including oil–water and oil–gas flow in horizontal wellbore systems are conducted, together with detailed interpretation of fluids thermal behaviors.

Prediction models

Single-phase and two-phase homogeneous model

The single-phase thermal response of horizontal wellbores for steady-state conditions is given by (Ayala and Dong 2015):

$$\frac{dT}{dx} = \frac{\Gamma}{\rho u C_p} \left[(h^* - h) + \frac{(u^2 + u^{*2})}{2} \right] + \frac{2(1-\gamma)}{r \rho u C_p} U(T^* - T) - \frac{1}{\rho C_p} F_w + \eta_s \frac{dp}{dx}, \quad (1)$$

where Γ is radio mass influx $\left(\frac{\text{lbm}}{\text{ft}^2 \text{s}}\right)$, γ is pipe open ratio (Yoshioka et al. 2005b) and U is overall heat transfer coefficient (BTU/ft² hr F). This equation shows that wellbore temperature behavior is prescribed by the combination effect of energy exchange, friction and fluid's isentropic expansion.

For two-phase flow, the homogeneous model can be used if the no-slip between phases assumption is valid. Because of the no-slip assumption of the homogeneous model, the volume fraction of each phase can be directly evaluated by the ratio of the flowrate of one phase to the total volumetric flowrate. For such two-phase homogeneous conditions, Eq. (1) would still apply by treating the two phases as a pseudo-single phase with average properties. Appendix A provides the definition of applicable two-phase variables. In homogeneous oil–gas flow, the liquid holdup is estimated as

$$y_O = \frac{q_O}{q_O + q_G} \quad (2)$$

The fluid of each phase is transported at same velocity, which is mixture velocity:

$$u_m = u_{SO} + u_{SG} \quad (3)$$

The steady-state momentum balance equation for the homogeneous model also uses the single-phase version evaluated using mixture properties:

$$\frac{d(\rho_m u_m^2)}{dx} + \frac{dp}{dx} = F_w + F_g, \quad (4)$$

where

$$F_w = \frac{f_m \rho_m u_m^2}{2d} \quad (5)$$

$$F_g = \rho_m g \sin \theta \quad (6)$$

In the homogeneous model, mixture density ρ_m is given by

$$\rho_m = y_O \rho_O + y_G \rho_G, \quad (7)$$

where f_m is the mixture Moody friction factor. For its calculation, the mixture Reynolds number is calculated as

$$Re_m = \frac{\rho_m u_m d}{\mu_m}, \quad (8)$$

in which the mixture viscosity is given as

$$\mu_m = y_O \mu_O + y_G \mu_G \quad (9)$$

For oil–water flow, the equations to calculate volume fraction of each phase and the mixture properties become similar to Eqs. (2) to (9) by replacing the subscript 'G' (gas) to 'W' (water).

For homogeneous two-phase flow, Eq. (1) can be written for each phase as

$$y_i \rho_i u_m C_{p_i} \frac{dT_i}{dx} = \Gamma_i \left[(h_i^* - h_i) + \frac{(u_m^2 + u_i^{*2})}{2} \right] + \frac{2(1-\gamma)}{r} Q_i - y_i u_m F_w + y_i \rho_i u_m C_{p_i} \eta_{s_i} \frac{dp_i}{dx} \quad (10)$$

Adding the thermal equations for the each phase, and assuming that each phase has same pressure and temperature at the same segment of the wellbore, the final form of the temperature equation becomes

$$\frac{dT}{dx} = \frac{\sum_i \Gamma_i}{u_m \sum_i y_i \rho_i C_{p_i}} \left[(h_i^* - h_i) + \frac{(u_m^2 + u_i^2)}{2} \right] + \frac{2(1-\gamma)}{ru_m \sum_i y_i \rho_i C_{p_i}} U(T^* - T) - \frac{1}{\sum_i y_i \rho_i C_{p_i}} F_w + \frac{\sum_i y_i \rho_i C_{p_i} \eta_{s_i} dp}{\sum_i y_i \rho_i C_{p_i} dx} \quad (11)$$

Two-phase model: drift-flux model

In a drift-flux model, slip between phases is considered. Because of the non-uniform velocity profiles, one phase of two-phase flow is transported at a higher velocity than the other phase. For oil–gas two-phase flow, gas tends to have a higher velocity than oil; while for the water–oil flow, it depends on whether the flow pattern is O/W (oil phase dispersed in water phase) or W/O (water phase dispersed in oil phase). A dispersed phase has a higher velocity than the continuous phase. Compared to the homogeneous model, the evaluation of holdup (in situ volume fraction) of each phase in drift-flux model comes from an empirical correlation based on experiments.

Two mechanisms are considered in the oil–gas two-phase flow drift-flux model. First, there are non-uniform velocity and phase distribution profiles over the cross section of the wellbore. In the center of a wellbore, gas tends to have the highest concentration, with the highest local mixture velocity, so the average gas velocity is higher than that of oil. Second, due to a buoyancy effect in vertical wells, gas has the tendency to rise vertically through oil (Shi et al. 2005). The drift-flux model for the oil–gas phase can be expressed as

$$u_G = C_o u_m + u_D, \quad (12)$$

where C_o is the profile parameter (distribution coefficient) that describes the velocity effect and concentration profiles; u_D is the drift-flux velocity, which represents the buoyancy effect and C_o varies between 1.0 and 1.2 and is estimated by Choi et al. (2012) in their proposed model as

$$C_o = \frac{2}{1 + (Re_m/1000)^2} + \frac{1.2 - 0.2 \sqrt{\rho_G/\rho_O} (1 - \exp(-18y_G))}{1 + (1000/Re_m)^2} \quad (13)$$

Choi et al. (2012) also presented a modified model to calculate the drift velocity, including the inclination effect:

$$u_D = A \cos \theta + B \left(\frac{g \tau_{O-G} |\Delta \rho|}{\rho_O^2} \right)^{0.25} \sin \theta. \quad (14)$$

$A = 0.0246$, $B = 1.606$ where $|\Delta \rho|$ is the absolute value of the density difference between the oil and gas phases, τ_{O-G} is surface tension between oil and gas phases. With the gas velocity calculated by drift-flux model, volume fractions of gas phase and liquid phase can be evaluated as

$$y_G = \frac{u_{SG}}{u_G} \quad (15)$$

$$y_O = \frac{u_{SO}}{u_O} = 1 - y_G \quad (16)$$

In water–oil flow, two types of flow system are considered: W/O and O/W. The determination of flow pattern is based on the boundary line in generalized flow patterns mapped by Hapanowicz (2008), written as

$$g_O = 1.3525 g_W^{0.812}, \quad (17)$$

where g_O , g_W are the apparent mass flux of oil and water, expressed as

$$g = \frac{\dot{m}}{A}, \quad (18)$$

where $\dot{m}$ is the mass flowrate, written as

$$\dot{m} = q \times \rho \quad (19)$$

To determine the flow pattern of oil–water flow system, g_O and g_W are calculated, respectively. If $g_O > 1.3525 g_W^{0.812}$, the flow pattern of the system is considered as W/O; otherwise, the flow pattern of the system is considered as O/W.

The drift-flux model of the liquid–liquid flow system given by Hapanowicz (2008) is

$$u_d = C_d u_m + u_D, \quad (20)$$

where subscript d denotes dispersion phase.

The determination of the profile parameter C_d and the drift velocity of the dispersion phase u_D are given by the following relationship (Dix 1971):

$$C_d = X_d \left[1 + \left(\frac{1}{X_d} - 1 \right) \left(\frac{\rho_d}{\rho_c} \right)^{0.1} \right] \quad (21)$$

$$u_D = 2.9 \left(\frac{g \tau_{O-W} |\Delta \rho|}{\rho_c^2} \right)^{0.25} \quad (22)$$

Firoozabadi and Ramey's (1988) correlation is used in calculating the surface tension (τ_{O-W}) between oil and water phases. X_d is the apparent volume fraction of the dispersion phase determined by the flowrate of the two-phase flow:

$$X_d = \frac{q_d}{q_d + q_c}, \quad (23)$$

where subscript c denotes a continuous phase.

In our application of oil–water drift-flux model, we assume the flow pattern is W/O to have continuous pressure

and temperature profiles. Oil and liquid holdup can be calculated as follows:

$$y_w = \frac{u_{sw}}{u_w} \quad (24)$$

$$y_o = \frac{u_{so}}{u_o} = 1 - y_w \quad (25)$$

The pressure (momentum) equation for each phase i in a multi-phase environment can be written as (Ayala and Adewumi 2003; Ayala and Alp 2008):

$$\frac{d(y_i \rho_i u_i^2)}{dx} + y_i \frac{dp}{dx} = y_i F_{wi} + y_i F_{gi}, \quad (26)$$

where

$$F_{wi} = \frac{f_{Mi} \rho_i u_i^2}{2d_e g_c} \quad (27)$$

and d_e is phase wetted equivalent diameter (Ayala and Adewumi 2003; Ayala and Alp 2008). By adding the pressure equations for each of the phases, we obtain

$$\frac{d(\sum y_i \rho_i u_i^2)}{dx} + \frac{dp}{dx} = \sum y_i F_{wi} + \sum y_i F_{gi} \quad (28)$$

The temperature equation at steady state for each phase i becomes

$$y_i \rho_i u_i C_{pi} \frac{dT_i}{dx} = \Gamma_i \left[(h_i^* - h_i) + \frac{(u_i^2 + u_i^{*2})}{2} \right] + \frac{2(1-\gamma)}{r} Q_i - y_i u_i F_{wi} + y_i \rho_i u_i C_{pi} \eta_{si} \frac{dp_i}{dx} \quad (29)$$

By adding all thermal equations for each individual phase, and assuming the same local pressure and temperature for each phase, the final form of the temperature equation for the drift-flux model becomes:

$$\begin{aligned} \frac{dT}{dx} = & \frac{\sum_i \Gamma_i}{\sum_i y_i \rho_i u_i C_{pi}} \left[(h_i^* - h_i) + \frac{(u_i^2 + u_i^{*2})}{2} \right] \\ & + \frac{2(1-\gamma)}{r \sum_i y_i \rho_i u_i C_{pi}} U(T^* - T) \\ & - \frac{\sum_i y_i u_i}{\sum_i y_i \rho_i u_i C_{pi}} F_{wi} + \frac{\sum_i y_i \rho_i u_i C_{pi} \eta_{si}}{\sum_i y_i \rho_i u_i C_{pi}} \frac{dp}{dx} \end{aligned} \quad (30)$$

The solution procedure of two-phase flow system is given in Appendix B.

Case study

A sensitivity study of the proposed two-phase flow model in several scenarios is conducted. Both homogeneous and drift-flux models have been applied in both oil–water and oil–gas flows. In the oil–water flow cases, a thermal effect of water entry on wellbore is discussed; while in the oil–

Table 1 Openhole wellbore descriptions

Inner diameter (in)	2.5
Wellbore length (ft)	4000
Inclination (degree)	0
Relative roughness	0.027

Table 2 Fluid compositions

Component	mol %
Oil	
C1	0.6
C3	0.1
C6	0.1
C10	0.1
C15	0.05
C20	0.05
Gas	
C1	0.886
C2	0.049
C3	0.025
nC4	0.01
nC5	0.01
N2	0.02

gas flow cases, another thermal effect of an oil–gas mixture production at different gas flow rates is analyzed. An openhole wellbore condition is initially applied in the case study. A perforated wellbore type is also applied in oil–gas flow case to compare the sensitivity of thermal response between two wellbore types. Tables 1, 2 give the openhole wellbore description and fluids compositions. For consistency purposes, reservoir pressure and temperature for all the cases were taken with the values of 3900 psia and 190 °F, respectively.

Oil–water flow problem

In the oil–water flow system, we specify oil and water productions along the wellbore. Water entered the wellbore at different locations including toe, middle and heel of wellbores. The inflow fluid in this case is either oil or water. The oil and water flowrate specifications under study are shown in Figs. 1, 2, 3.

Homogeneous model results

The homogeneous oil–water flow model is initially applied in the openhole wellbore condition. Figure 4 gives water holdup profiles in three cases. When water enters at the toe of

Fig. 1 Oil and water production along wellbore-water entered at toe

Fig. 4 Water holdup-homogeneous model

Fig. 2 Oil and water production along wellbore-water entered at middle

Fig. 3 Oil and water production along wellbore-water entered at heel

Fig. 5 Pressure profiles at different water entry locations-homogeneous model

the wellbore, its holdup is first maintained at 1 because there is no oil production during that time. As oil production begins, water's cumulative production does not change and its holdup begins to decrease. When water enters wellbore at the middle and heel of wellbore, its holdup is zero until it begins to produce. When fluid reaches the heel of the wellbore, the water holdup is the same in all three cases, i.e., about 0.24. Figure 5 shows pressure responses in all three cases with comparison of a single-phase oil case. Compared to single-phase oil flow pressure response, the pressure profiles of two-phase cases are continuous and have no obvious differences. Therefore, one could not recognize the entry of water using pressure response profiles alone.

Figures 6, 7, 8 present temperature responses in all three cases with comparisons of the single-phase cases. When water enters the wellbore at different locations, different temperature responses are observed from Figs. 6, 7, 8. The

Fig. 6 Temperature profile comparison for water entered at toe-homogeneous model

Fig. 7 Temperature response comparison for water entered at middle-homogeneous model

Fig. 8 Temperature response comparison for water entered at heel-homogeneous model

Fig. 9 Comparison of overall temperature contribution between single-phase oil and oil–water flow cases

temperature profile begins to deviate from the single-phase fluid case at the location where water enters the wellbore. In order to show the detailed thermal behavior of single-phase oil and oil–water case, Fig. 9 is plotted to give the overall temperature contribution of the two cases (single-phase oil and oil–water flow with water enters at toe). Compared to single-phase oil case, oil–water flow is less heated by the friction effect and less cooled by the isentropic coefficient. The combination of three factors result in a smaller temperature increment compared to single-phase oil case, in which the water entry could be detected. Therefore, temperature response profiles in wellbore flow may be utilized to interpret the water entry phenomenon during production.

Drift-flux model results

The main difference between the drift-flux model and the homogeneous model is that the drift-flux model considers slip in evaluating phase holdup and velocity. In oil–water flow, to make each thermal profile in continuous format, we assume that the flow pattern in the flow system is W/O, which means that the water phase is dispersed in the continuous oil phase. Water holdup profiles are given in Fig. 10. Compared to the water holdup calculated by the homogeneous model in Figs. 4, 10 has a similar trend of water holdup. Figure 11 is generated to show the difference between these two results.

Let us consider the entry of water at the mid-section depicted in Fig. 11. It is shown that the homogeneous model over-predicts the water holdup along the wellbore, compared to the drift-flux model. That is because the two models utilize different algorithms in calculating the phase holdup. In the homogeneous model, the phase holdup is calculated directly by the cumulative production of each phase, whereas in the drift-flux model, the phase holdup is

Fig. 10 Water holdup-oil-water drift-flux model

Fig. 11 Comparison of water holdup in two models

calculated by drift-flux correlations. The result of the velocity profile for the middle location case is in Fig. 12. V_o and V_w in the figure give the velocity profiles of oil and water phase, respectively. V_m is the mixture velocity of the two-phase flow calculated in the homogeneous model.

Figure 12 shows that the dispersed water phase travels at a higher velocity than the oil phase along the wellbore, while the mixture velocity has a value between the oil and water velocities. The velocity profiles obey one basic assumption of drift-flux model—one phase is transported at a higher speed than another phase. Pressure profiles in each case are generated by the drift-flux model as shown in Fig. 13. Similar to the pressure profiles in the homogeneous model, pressure profiles are almost overlapped with each other. So, one could not detect the water entry effect and water location from pressure profiles. Figures 14, 15, 16 give the temperature profiles of each case relative to the single-phase oil case. There are similar phenomena in

Fig. 12 Velocity profile for water in toe location-oil-water

Fig. 13 Pressure profiles at different water entry locations-oil-water drift-flux model

temperature profiles generated by the drift-flux model compared to the homogeneous model. Temperature of two-phase flow deviates from that of single-phase flow predictions at the location where water begins to enter.

Figure 17 gives the comparison of temperature profiles of two models with water entering wellbore at the toe. It is shown that the temperature change in the drift-flux model is larger than that of the homogeneous model. Therefore, the homogeneous model tends to under-predict wellbore thermal behavior compared to the drift-flux model.

Oil-gas flow problem

In following oil-gas two-phase flow problem, the thermal effect of gas appearance during oil production is discussed. We assume that gas enters the oil production zone at each segment of the wellbore and is mixed with oil during

Fig. 14 Temperature profile comparison for water entered at toe-drift-flux model

Fig. 17 Temperature comparison of two models with water entering at toe-oil-water drift-flux model

Fig. 15 Temperature profile comparison for water entered at middle-drift-flux model

Fig. 18 Oil and gas phase production

Fig. 16 Temperature profile comparison for water entered at heel-drift-flux model

production. Gas enters the wellbore at different flowrates in three cases. The cumulative productions of oil and gas phase are shown in Fig. 18. With the same oil production, gas flowrates are specified in three types—high, medium and low.

Homogeneous model results

Similarly, we first show the result of the homogeneous model in the openhole wellbore type. Gas holdup is given in Fig. 19. Since gas enters the wellbore simultaneously at each segment, the cumulative production of gas makes its holdup increase from toe to heel. The larger the gas flowrate, the higher the holdup gas phase will be. Figure 20 shows pressure profiles in three cases. With the same oil flowrate, the largest gas flowrate case results in the largest pressure drop, then the medium and low gas flowrates.

Fig. 19 Gas holdup-homogeneous model

Fig. 20 Pressure profiles in different gas flowrate-homogeneous model

Figure 21 shows the temperature profiles in three cases. It is interesting to find that in three cases, from toe to heel, temperature first increases then decreases due to the effect of gas entry. As demonstrated in the single-phase case study, oil is heated while gas is cooled along the wellbore. When the two-phase flows come together, the fluid mixture at the first half of the wellbore experiences heating like the oil phase; then it is cooled like the gas phase. Since a higher gas flowrate leads to a larger pressure drop, the oil–gas mixture in the largest gas production has the largest range of temperature changes. Due to the cooling effect in the gas phase, it is easy to diagnose entry of the gas during oil production from its temperature profile. Figure 22 shows the comparison of overall temperature contributions between two

Fig. 21 Temperature profiles in different gas flowrate-homogeneous model

Fig. 22 Comparison of overall temperature contribution between single-phase oil and oil–gas flow cases

cases (single-phase oil and oil–gas flow with highest gas rate). It is observed that compared to single-phase case, oil–gas flow experiences more frictional heating and isentropic cooling. The energy exchange cools the flow at the same time. The combination of three factors gives a different thermal behavior of oil–gas flow compared to single-phase oil flow.

Drift-flux model results

In the drift-flux model, the gas holdup is evaluated with drift-flux techniques in Fig. 23. Hold-up trends for the drift-flux model are similar to those shown by the homogeneous model in Fig. 19.

Fig. 23 Gas holdup-oil-gas drift-flux model

Fig. 24 Comparison of gas holdup in two model

Figure 24 is given to compare the difference between holdup results in the two models. As illustrated, when slip is considered in the drift-flux model, the gas holdup becomes smaller compared with the homogeneous case. Again, this is caused by different algorithms in evaluating phase holdup, and the homogeneous model tends to over-predict the gas holdup. The velocity profile in the high gas flowrate case is given in Fig. 25. As expected, due to the slip between the two phases, the gas phase has a higher velocity than that of the oil phase, and the mixture velocity in homogeneous model is also between two velocity profiles. Figures 26, 27 present pressure and temperature profiles in the drift-flux model. The pressure and temperature profiles have similar trends compared to those in the homogeneous model shown in Figs. 20, 21.

Fig. 25 Velocity profile in high gas flowrate case-oil-gas drift-flux model

Fig. 26 Velocity profile in high gas flowrate case-oil-gas drift flux model

Figure 28 gives the temperature profile comparison between two models in the high flowrate case. As is shown, the temperature change of the drift-flux model is larger than that of the homogeneous model. Using different protocols to solve thermal response between two models, the homogeneous model tends to under-predict temperature change compared to the drift-flux model.

Well completion effect

The results discussed so far for the two-phase flow are in openhole wellbore type. For thermal model in an openhole wellbore, the pipe's open ratio γ is 1, so that energy

Fig. 27 Temperature profiles in different gas flowrate-oil-gas drift-flux model

Fig. 28 Temperature comparison of two models with high gas flowrate

exchange only appears in the mass exchange part. In this case, we introduce heat conduction into our model for a perforated wellbore type and compare the result with the openhole wellbores. Appendix C shows the development of overall heat transfer coefficient calculations. Fluid and perforated wellbore properties are given in Table 3 (Yoshioka et al. 2005a). A perforated wellbore type has been applied in the same oil–gas flow case. Pressure and temperature results are generated by the drift-flux model.

Figure 29 shows pressure profiles in three oil–gas cases. Similar profiles can be found compared to the openhole case in Fig. 26. However, due to smaller roughness of the wellbore, the pressure drop of perforated wellbore fluid in the figure is relatively smaller than that of the openhole wellbore fluid. Figure 30 gives temperature profiles in this case. Temperature change in a perforated wellbore is not as significant as that in an openhole wellbore. Two reasons

Table 3 Perforated wellbore description and fluid properties (Yoshioka et al. 2005a)

Pipe open ratio	0.02
Oil conductivity (BTU/hr ft F)	0.797
Gas conductivity (BTU/hr ft F)	0.0116
Water conductivity (BTU/hr ft F)	0.3886
Cement conductivity (BTU/hr ft F)	4.021
Casing conductivity (BTU/hr ft F)	6.933
Oil–gas interfacial tension (dyne/cm)	10
Cement diameter (in)	5
Casing diameter (in)	3.5
Relative roughness	0.01

Fig. 29 Pressure profiles in perforated wellbores-oil-gas drift-flux model

Fig. 30 Temperature profiles in perforated wellbores-oil-gas drift-flux model

can be considered for this fact. First, the smaller pressure drop weakens the effect of the isentropic thermal coefficient, leading to the wellbore fluid being less cooled.

Second, the heat conduction from reservoir to wellbore always has an opposite effect in determining the overall trend of temperature change. For example, in this case, the first half of the wellbore from toe to the middle is being heated, while the heat conduction cools the wellbore fluid because the reservoir temperature is lower; the remaining half of the wellbore from middle to heel is being cooled while heat conduction heats the wellbore because the reservoir temperature is higher. Combination of these two factors results in a non-sensitive thermal response of a perforated wellbore fluid.

Concluding remarks

Based on our studies on two-phase flow wellbore fluids systems, it is shown that the η_s -driven model can be implemented to analyze two-phase wellbore flow thermal behaviors during the production of oil, gas and water. The Entry of the undesired phases including water and gas can be detected via the temperature profiles. For perforated wellbores, the thermal response is not as sensitive as in the openhole case. Results show that our models (homogeneous and drift-flux) can be applied to effectively predict and interpret wellbore fluid thermal behaviors at steady state. Further experiments are needed to test the performance of the two types of models. The proposed models can be further developed for transient flow conditions to analyze the early time regime. In addition, to match field data, the reservoir model is necessary to be coupled with the wellbore model to generate more realistic flowrate, reservoir pressure and temperature as inputs before calculating the wellbore temperature profile. Also, flash calculation can be applied in every block of the wellbore in an oil–gas two-phase flow system to have a more accurate evaluation of gas entry effect and a better estimation of oil and gas production on the surface.

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Appendix A: two-phase flow variables

Liquid holdup and gas void fraction

Liquid holdup (y_L) is the fraction of a two-phase flow volume element occupied by the respective liquid phase. Similarly, the gas void fraction (y_G) is the fraction of the volume element that is occupied by the gas phase (Shoham 2006). For oil-gas two-phase flow:

$$y_O + y_G = 1 \quad (31)$$

and for the oil-water two-phase flow:

$$y_O + y_W = 1 \quad (32)$$

We take the oil-gas two-phase flow as an example in our following discussion of two-phase flow variables and models.

Superficial velocity

Superficial velocity describes the volumetric flow rate per unit area, which is the volumetric flux of the phase. Superficial velocity of oil and gas are

$$u_{SO} = \frac{q_O}{A} \quad (33)$$

$$u_{SG} = \frac{q_G}{A}, \quad (34)$$

where A is the cross-sectional area of the wellbore.

Mixture velocity

The mixture velocity refers to the total volumetric flow rate of both phases per unit area. In oil and gas flow, mixture velocity is given as

$$u_m = u_{SO} + u_{SG} \quad (35)$$

Actual velocity

The actual velocity of a specific phase is its volumetric flowrate divided by the actual cross-sectional area occupied by the phase. The actual velocity of the oil and gas phases is

$$u_O = \frac{q_O}{A_O} = \frac{u_{SO}}{y_O} \quad (36)$$

$$u_G = \frac{q_G}{A_G} = \frac{u_{SG}}{y_G} \quad (37)$$

Appendix B: solution procedure

In this section, the solution procedure of the proposed model is discussed in both its single-phase and two-phase flow form at steady-state conditions. Figure 31 shows a simplified schematic of a discretized wellbore.

Fig. 31 Schematics of a discretized wellbore

Homogeneous model

For two-phase flow, we implement two models (homogeneous and drift-flux) in our results. The difference between these two models is the procedure to calculate phase holdup and velocity profiles. In the homogeneous model, take oil-gas flow as an example, oil holdup is obtained as

$$y_{O_j} = \frac{q_{O_j}}{q_{O_j} + q_{G_j}} \quad (38)$$

The mixture velocity is calculated as

$$u_{m_j} = u_{SO_j} + u_{SG_j} = \frac{q_{O_j} + q_{G_j}}{A} \quad (39)$$

To solve pressure in the two-phase flow system, the solution procedure is the same as for the single-phase flow, except that fluid properties are replaced by mixture properties. Finite difference form of pressure equation is

$$\frac{p_{j+1} - p_j}{\Delta x_j} = - \left(\frac{\rho_{m_{j+1}} u_{m_{j+1}}^2 - \rho_{m_j} u_{m_j}^2}{\Delta x_j g_c J_0} \right) + \frac{F_{w_j}}{J_0} + \frac{F_{g_j}}{J_0} \quad (40)$$

Following expression can be used to solve for pressure in each segment:

$$p_j = p_{j+1} + C_j,$$

where

$$C_j = \Delta x_j \left[-\frac{F_{w_j}}{J_0} - \frac{F_{g_j}}{J_0} \right] + \left[\frac{\rho_{m_{j+1}} u_{m_{j+1}}^2 - \rho_{m_j} u_{m_j}^2}{g_c J_0} \right] \quad (41)$$

The finite difference form of the temperature equation in each phase can be written as:

$$\begin{aligned} \frac{T_{j+1} - T_j}{\Delta x_j} = & \frac{\sum_i \Gamma_{ij} \left[\left(h_j^* - h_j \right) + \frac{(u_{m_j}^2 + u_j^{*2})}{2g_c J_1} \right]_i}{u_{m_j} \sum_i (y_j \rho_j C_{p_j})_i} \\ & + \frac{2(1-\gamma)}{r u_{m_j} \sum_i (y_j \rho_j C_{p_j})_i} U_j (T_j^* - T_j) \\ & - \frac{1}{\sum_i (y_j \rho_j C_{p_j})_i} \frac{F_{w_j}}{J_2 J_0} + \frac{\sum_i (y_j \rho_j C_{p_j} \frac{\alpha_j T_j}{\rho_j C_{p_j} J_2})_i p_{j+1} - p_j}{\sum_i (y_j \rho_j C_{p_j})_i \Delta x_j} \end{aligned} \quad (42)$$

Similarly to the single-phase solution procedure, one can solve the equation (42) using the following expression:

$$T_j = \frac{T_{j+1} + \Delta x_j (-D_j - E_j T^* + F_j)}{1 + \Delta x_j (-E_j + G_j)}, \quad (43)$$

where

$$D_j = \frac{\sum_i \Gamma_{ij} \left[\left(h_j^* - h_j \right) + \frac{(u_{m_j}^2 + u_j^{*2})}{2g_c J_1} \right]_i}{u_{m_j} \sum_i (y_j \rho_j C_{p_j})_i} \quad (44)$$

$$E_j = \frac{2(1-\gamma)}{r u_{m_j} \sum_i (y_j \rho_j C_{p_j})_i} U_j \quad (45)$$

$$F_j = \frac{1}{u_{m_j} \sum_i (y_j \rho_j C_{p_j})_i} \frac{F_{w_j}}{J_2 J_0} \quad (46)$$

$$G_j = \frac{\sum_i (y_j \rho_j C_{p_j} \frac{\alpha_j T_j}{\rho_j C_{p_j} J_2})_i p_{j+1} - p_j}{\sum_i (y_j \rho_j C_{p_j})_i \Delta x_j} \quad (47)$$

Drift-flux model

For oil-water flow in drift-flux model, u_G and y_G are calculated simultaneously from

$$u_{G_j} = C_{O_j} u_{m_j} + u_{D_j}, \quad (48)$$

where

$$\begin{aligned} C_{O_j} = & \frac{2}{1 + (Re_{m_j}/1000)^2} \\ & + \frac{1.2 - 0.2 \sqrt{\rho_{G_j}/\rho_{O_j}} (1 - \exp(-18 y_{G_j}))}{1 + (1000/Re_{m_j})^2} \end{aligned} \quad (49)$$

$$u_{D_j} = A \cos \theta + B \left(\frac{g(\tau_{O-G})_j |\Delta \rho_j|}{\rho_{O_j}^2} \right)^{0.25} \sin \theta \quad (50)$$

From these equations, u_G and y_G are calculated iteratively where y_G is initialized as an input to get C_0 . Velocity and holdup of the oil phase are then calculated as

$$y_{O_j} = \frac{u_{SO_j}}{u_{O_j}} = 1 - y_{G_j}$$

For oil-water flow, water velocity is calculated as

$$u_{W_j} = C_{d_j} u_{m_j} + u_{D_j}, \quad (51)$$

where

$$C_{d_j} = X_{d_j} \left[1 + \left(\frac{1}{X_{d_j}} - 1 \right) \left(\frac{\rho_{W_j}}{\rho_{O_j}} \right)^{0.1} \right] \quad (52)$$

$$X_{d_j} = \frac{q_{W_j}}{q_{W_j} + q_{O_j}}$$

$$u_{D_j} = 2.9 \left(\frac{g(\tau_{O-W})_j |\Delta \rho_j|}{\rho_{O_j}^2} \right)^{0.25}$$

Water holdup is given by

$$y_{w_j} = \frac{u_{sw_j}}{u_{w_j}} \quad (53)$$

Velocity and holdup of oil phase are calculated as

$$y_{O_j} = \frac{u_{SO_j}}{u_{O_j}} = 1 - y_{w_j} \quad (54)$$

The discretization of the pressure equation yields

$$\begin{aligned} \frac{p_{j+1} - p_j}{\Delta x_j} = & - \left(\frac{\sum_i (y_{j+1} \rho_{j+1} u_{j+1}^2)_i - \sum_i (y_j \rho_j u_j^2)_i}{\Delta x_j g_c J_0} \right) \\ & + \frac{\sum_i (y_j F_{w_j})_i}{J_0} + \frac{\sum_i (y_j F_{g_j})_i}{J_0} \end{aligned} \quad (55)$$

from where following expression can be used to solve for pressure in each segment:

$$p_j = p_{j+1} + C_j, \quad (56)$$

where

$$\begin{aligned} C_j = \Delta x_j \left[- \frac{\sum_i (y_j F_{w_j})_i}{J_0} - \frac{\sum_i (y_j F_{g_j})_i}{J_0} \right] \\ + \left[\frac{\sum_i (y_{j+1} \rho_{j+1} u_{j+1}^2)_i - \sum_i (y_j \rho_j u_j^2)_i}{g_c J_0} \right] \end{aligned} \quad (57)$$

Finally, finite difference form of temperature equation is expressed as

$$\begin{aligned} \frac{T_{j+1} - T_j}{\Delta x_j} = & \frac{\sum_i \Gamma_{ij} \left[(h_j^* - h_j) + \frac{(u_j^2 + u_j^{*2})}{2g_c J_1} \right]_i}{\sum_i (y_j \rho_j u_j C_{p_j})_i} \\ & + \frac{2(1 - \gamma)}{r \sum_i (y_j \rho_j u_j C_{p_j})_i} U_j (T_j^* - T_j) \\ & - \frac{\sum_i (y_j u_j F_{w_j})_i}{\sum_i (y_j \rho_j u_j C_{p_j})_i J_2 J_0} \\ & + \frac{\sum_i (y_j \rho_j u_j C_{p_j} \frac{\alpha_j T_j}{\rho_j C_{p_j} J_2})_i p_{j+1} - p_j}{\sum_i (y_j \rho_j u_j C_{p_j})_i \Delta x_j} \end{aligned} \quad (58)$$

Similarly to the homogeneous model solution procedure, one can solve the equation (58) using the following expression:

$$\begin{aligned} T_j &= \frac{T_{j+1} + \Delta x_j (-D_j - E_j T^* + F_j)}{1 + \Delta x_j (-E_j + G_j)} \\ D_j &= \frac{\sum_i \Gamma_{ij} \left[(h_j^* - h_j) + \frac{(u_j^2 + u_j^{*2})}{2g_c J_1} \right]_i}{\sum_i (y_j \rho_j u_j C_{p_j})_i} \\ E_j &= \frac{2(1 - \gamma)}{r \sum_i (y_j \rho_j u_j C_{p_j})_i} U_j \\ F_j &= \frac{\sum_i (y_j u_j F_{w_j})_i}{\sum_i (y_j \rho_j u_j C_{p_j})_i J_2 J_0} \\ G_j &= \frac{\sum_i (y_j \rho_j u_j C_{p_j} \frac{\alpha_j T_j}{\rho_j C_{p_j} J_2})_i p_{j+1} - p_j}{\sum_i (y_j \rho_j u_j C_{p_j})_i \Delta x_j} \end{aligned} \quad (59)$$

Appendix C: equations of overall heat transfer coefficient

The relationship between wellbore fluid temperature and reservoir fluid temperature can be written as follows (Yoshioka et al. 2005a, b):

$$T - T^* = \frac{\dot{Q}}{2\pi(1 - \gamma)} \left[\frac{\ln(\frac{r_c}{r_e})}{K_c} + \frac{\ln(\frac{r_{cem}}{r_c})}{K_{cem}} + \frac{1}{r C_h} \right], \quad (60)$$

where C_h is heat transfer coefficient of fluid, $\dot{Q}$ (BTU/ft hr) is heat transfer rate, K is thermal conductivity and subscript c and cem denote casing and cement, respectively. Based on the equation above, the overall heat transfer coefficient can be expressed as

$$U = \frac{\dot{Q}}{(T^* - T) 2\pi r (1 - \gamma)} = \left[\frac{r \ln(\frac{r_c}{r_e})}{K_c} + \frac{r \ln(\frac{r_{cem}}{r_c})}{K_{cem}} + \frac{1}{C_h} \right]^{-1} \quad (61)$$

According to Yoshioka et al. (2005a), for laminar flow, heat transfer coefficient is

$$C_h = 3.656 \frac{K_{fl}}{2r} \quad (62)$$

While for turbulent single-phase or oil–water two-phase flow, the following formula is used

$$C_h = \frac{(\frac{f}{2})(Re - 1000) Pr}{1 + 12.7(\frac{f}{2})^{0.5}(Pr^{2/3} - 1)} \frac{K_{fl}}{2r}, \quad (63)$$

where Pr is fluid Prandtl number, given as

$$Pr = \frac{C_p \mu}{K} \quad (64)$$

Re, Pr and K_f are determined using mixture properties.

For oil–gas two-phase flow, correlation from Kim and Ghajar (2006) is applied. A flow pattern factor (F_P) is introduced to reflect the real shape of oil-gas interface:

$$F_P = (1 - y_G) + y_G F_s^2 \quad (65)$$

F_s refers to shape factor defined in equation (37):

$$F_s = \frac{2}{\pi} \tan^{-1} \left(\sqrt{\frac{\rho_G (u_G - u_O)^2}{g d (\rho_O - \rho_G)}} \right) \quad (66)$$

The heat transfer coefficient of oil-gas two-phase flow is introduced in equation (67):

$$C_h = F_P C_L \left\{ 1 + c \left(\left[\frac{x}{1-x} \right]^k \left[\frac{1-F_P}{F_P} \right]^l \frac{Pr_G^m \mu_G^n}{Pr_O \mu_O} \right) \right\}, \quad (67)$$

where c , m , n , p , q are constant which are determined experimentally. C_L is the liquid heat transfer coefficient that comes from Sieder and Tate (1936), for turbulent flow:

$$C_L = 0.027 Re_L^{4/5} Pr_L^{1/3} \left(\frac{K_L}{d} \right) \left(\frac{\mu_B}{\mu_W} \right)_L^{0.14} \quad (68)$$

x is quality which defined as

$$x = \frac{M_G}{M_G + M_O}, \quad (69)$$

where M (lbm/s) is the mass flow rate of each phase.

Appendix D: sensitivity study at different flow condition

In order to show our η_s -driven model's sensitivity to different flow condition, we apply five different total gas flowrates on openhole wellbore case (refer to Table 1), while keeping other parameters as constant. Figure 32 shows accumulated flowrates for each case.

Figures 33, 34, 35 show velocity, pressure and temperature response, respectively. Larger mass influx in the larger flowrate case leads to higher fluid velocities. The pressure drop and temperature drop tend to increase with the increase of flowrate. The increase of velocity leads to a larger friction force to the system, thus causing a larger pressure drop. In addition, the temperature drop from heel to toe increases from $\Delta T \approx 0.48^\circ\text{F}$ to $\Delta T \approx 0.77^\circ\text{F}$ because the larger pressure drop causes larger isentropic

Fig. 32 Accumulated gas flowrate

Fig. 33 Velocity profiles for flowrate study-gas phase

Fig. 34 Pressure profiles for flowrate study-gas phase

Fig. 35 Temperature profiles for flowrate study-gas phase

coefficient cooling effect. In summary, larger flowrate in the wellbore results in a more significant temperature response.

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